CALCULUS INTEGRALS

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a,b]$ and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^a f(x)dx = 0 \text{ and } \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

COMMON INTEGRALS

$$\int k\ dx = kx + C$$

$$\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C,\ n \neq -1$$

$$\int x^{-1}\ dx = \int \frac{1}{x}\ dx = \ln|x| + C$$

$$\int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \ln(x)\ dx = x\ln(x) - x + C$$

$$\int e^x\ dx = e^x + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2}\ du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ du = \sin^{-1}\left(\frac{u}{a}\right) + C$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x\sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x\sum_{k=1}^{n} f(x_k)$$

Midpoint Rule

$$M_n = \Delta x\sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n))$$

TRIGNOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\, d\theta$	$\sqrt{a^2-a^2\sin^2\theta}$ $= a\cos\theta$	$1-\sin^2\theta$ $=\cos^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\, d\theta$	$\sqrt{a^2\sec^2\theta-a^2}$ $= a\tan\theta$	$\sec^2\theta-1$ $=\tan^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\, d\theta$	$\sqrt{a^2+a^2\tan^2\theta}$ $= a\sec\theta$	$1+\tan^2\theta$ $=\sec^2\theta$

APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\ dv = uv - \int v\ du \text{ where } v = \int dv$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)\ dx$$

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CALCULUS

DERIVATIVES AND LIMITS

DERIVATIVE DEFINITION

$$\frac{d}{dx}f(x)=f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))'=c(f'(x))$$

$$(f(x)\pm g(x))'=f'(x)\pm g'(x)$$

$$\frac{d}{dx}(c)=0$$

MEAN VALUE THEOREM

If f is differentiable on the interval (a,b) and continuous at the end points there exists a c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

PRODUCT RULE

$$(f(x)g(x))'=f(x)'g(x)+f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}(f(g(x)))=f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\frac{d}{dx}(x)=1$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x)=a^x\ln(a)$$

$$\frac{d}{dx}(e^x)=e^x$$

$$\frac{d}{dx}(\ln(x))=\frac{1}{x},\ x>0$$

$$\frac{d}{dx}(\ln|x|)=\frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x))=\frac{1}{x\ln(a)}$$

LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x\to -3}\frac{x^2-x-12}{x^2+3x}=\lim_{x\to -3}\frac{(x+3)(x-4)}{x(x+3)}=\lim_{x\to -3}\frac{(x-4)}{x}=\frac{7}{3}$$

L'HOPITAL'S RULE

$$\text{If }\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}\text{ or }\frac{\pm\infty}{\pm\infty}\text{ then }\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n)=n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}(e^{f(x)})=f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)])=\frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)])=f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)])=-f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)])=f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)])=f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)])=\frac{f'(x)}{1+[f(x)]^2}$$

$$\frac{d}{dx}(f(x)^{g(x)})=f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)}+\ln(f(x))g'(x)\right)$$

PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x\to a}[cf(x)]=c\lim_{x\to a}f(x)$$

$$\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$$

$$\lim_{x\to a}[f(x)g(x)]=\lim_{x\to a}f(x)\lim_{x\to a}g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}\quad\text{if }\lim_{x\to a}g(x)\neq 0$$

$$\lim_{x\to a}[f(x)]^n=[\lim_{x\to a}f(x)]^n$$

LIMIT EVALUATION AT $\pm\infty$

$$\lim_{x\to\infty}e^x=\infty\text{ and }\lim_{x\to-\infty}e^x=0$$

$$\lim_{x\to\infty}\ln(x)=\infty\text{ and }\lim_{x\to 0^+}\ln(x)=-\infty$$

$$\text{If }r>0\text{ then }\lim_{x\to\infty}\frac{c}{x^r}=0$$

$$\text{If }r>0\ \&\ x^r\text{ is real for }x<0\text{ then }\lim_{x\to-\infty}\frac{c}{x^r}=0$$

$$\lim_{x\to\pm\infty}x^r=\infty\text{ for even }r$$

$$\lim_{x\to\infty}x^r=\infty\ \&\ \lim_{x\to-\infty}x^r=-\infty\text{ for odd }r$$

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