

# **Station Location: An Extensible Model for Charging Station Placement**

Control #88455

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## 1 Introduction

Fuel-based vehicles are responsible for a significant amount of greenhouse gas emissions globally every year. If electric vehicles are used as an alternative form of transportation, they reduce such greenhouse gas emissions and overall global environmental footprint.<sup>1</sup> With the rapid increase of electric vehicle purchase and use within the last 5 to 10 years, the need for guidance on implementation methods and transition timelines has expanded as well.

In this paper, we establish an efficient model for determining the number and placement of charging stations subject to an assortment of high impact geographically dependent factors. Our goal is to create an extensible model that can be implemented trans-nationally with simple readjustments of "modifier" inputs such as a map of population per square kilometer, climate effects, or wealth distributions.

## 2 Statement of Problem

Our aim is to create a highly extendible model that will prioritize the placement of charging stations in response to easily implemented "modifiers"<sup>1</sup> derived from a simple distribution of your variable of choice. Among the variables we sample are the increased need for charging stations in highly populated areas, climate effects on battery mileage, and the different capabilities of destination charging stations versus supercharging stations.

Additionally, the model will direct the implementation plans of a nation transitioning to all electric vehicles in response to the strength of the aforementioned variables in their country. In this way, our method seeks to be effective for a variety of countries and conditions and also cost-efficient for the budget they have for the initiative.

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<sup>1</sup>refers to geographically dependent functions that measure a count of some factor at particular latitude and longitude.

## 2.1 Assumptions

Due to the huge amount of factors involved, creating a relatively simple model for generating a charging station plan faces several hurdles. Below are the assumptions we make about the countries we apply our model to:

### 2.1.1 Budgetary Constraints

Although the model will prioritize charging station locations according to the variables assigned, we will not adjust the model directly using the country's ability to build. Instead, we will infer the building capabilities relative to other comparable countries' building rates.

### 2.1.2 Road Placement

The model's charging station location suggestions will not be dependent on the transportation infrastructure of our country of choice. We will assume that the model, being dependent on population, will place charging stations reasonably close to important roadways given high resolution data. Frequently traveled roadways often drive the construction of housing districts. Indeed, looking ahead to Scheme 4, one can see the roadways of Ireland traced out by a population map. However, our model could be tweaked in ways that does not allow the placement of charging stations in underpopulated areas, as discussed in subsection 3.1.1 (Derivation of  $d$ ).

### 2.1.3 Amount of Chargers at Charging Stations

We assume a uniform amount of chargers at every charging station. While there exists data on the charger capacity of every charging station in the United states which could be easily implemented in our model to adjust for the local density of future charging stations, we will not naively assume a capacity for the charger stations that are added by the model. Additionally, other countries of interest do not keep such data on hand or publicly available.

## 2.2 Theoretical Charging Station Placement

The average electric vehicle can travel as much as 322 kilometers on a fully charged battery, or around 161 kilometers at minimum. An instinctual approach to capture this constraint would be to simply uniformly distribute the charging stations across the entirety of the nation in an 161 kilometer-spaced grid. However, this method is neither cost-effective nor considerate of any variables that may change the need for a station. Regardless, this radius will provide a basis to our model – inevitably, charging stations should be separated by a maximum of 161 kilometers to allow transportation to every part of the country. In order to make this placement dependent on user-defined modifiers, we will utilize an iterative approach to adding new charging stations.

Assume that a charging station has been placed in a major city. The placement of a new potential charging station,  $p$ , must neither be too far away from the initial point (outside of  $r_{\max}$ ) nor too close (within  $r_{\min}$ ). User defined modifiers will change either  $r_{\max}$  or  $r_{\min}$  for every location. For our model's purposes, modifiers must be continuous surfaces generated from interpolating distribution information about a factor of choice at a particular geographical point (e.g. population). Since  $r_{\min}$  determines how *close together* points can be at a particular location, it will be determined by *density* modifiers. In other words,  $r_{\min}$  will be adjusted due to some perceived increase in demand. Since  $r_{\max}$  determines how *far* a car can travel at a particular location, it will be determined by *physical* modifiers, such as the effect of average temperature on an electric car's range. The area between  $r_{\min}$  and  $r_{\max}$  will be the *permissive region* in which the potential charging station  $p$  will be admitted.

To determine how far points are from each other, we will utilize the Haversine Formula,<sup>2</sup>

$$\text{hav}(\theta) = \sin^2\left(\frac{\theta}{2}\right)$$

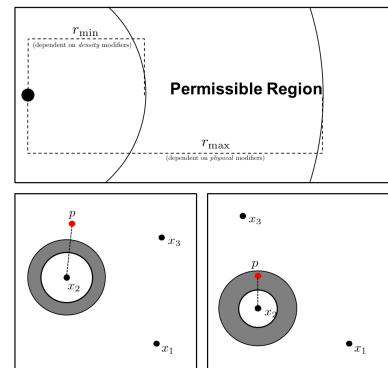


Figure 1: A potential point  $p$  must be within the permissible region of the closest charging station to be admitted as a valid charging station.

which uses the latitude and longitude of two points in order to calculate their circular distance from one another on the surface of the sphere<sup>2</sup> on which they reside.<sup>3</sup> In the case where there are multiple points of comparison, a potential point  $p$  will be compared to the closest point. Therefore the minimum distance from all points,

$$v = \min \left\| \sum_{i=1}^k c_i - p \right\|_{\text{hav}},$$

must be within the permissible region. Here,  $p$  and  $x_i$  represent the charging location of the potential charging station and the surrounding established charging stations, respectively.

Using this distance,  $r_{\min}$  can be expressed as follows:

$$r_{\min} = r_{\max} - \sum_{i=0}^{k_1} d_i D_{\text{mod}_i} \quad (1)$$

Here,  $D_{\text{mod}_i}$  refers to all  $k_1$  density modifiers and  $d_i$  is a matching appropriate scaling factor derived from the impact of the modifier on desired charging station density. Similarly,  $r_{\max}$  can be expressed as

$$r_{\max} = \text{range} - \sum_{i=0}^{k_2} p_i P_{\text{mod}_i}, \quad (2)$$

where  $P_{\text{mod}_i}$  enumerates the  $k_2$  physical modifiers with their corresponding scaling factors  $p_i$ . Note that both modifiers are dependent on the location of the point that point  $p$  is being compared against.<sup>3</sup>

We will randomly generate a large pool of points onto the nation in question. Then, these points will be iteratively confirmed or denied based on the choice function

$$F_{\text{keep}}(v) = \begin{cases} \text{keep}, & r_{\min}(x) \leq v \leq r_{\max}(x) \\ \text{omit}, & \text{otherwise} \end{cases}.$$

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<sup>2</sup>the earth is an oblate spheroid in actuality, but this approximation is widely used.

<sup>3</sup>A linear formulation is used to accommodate meager computational resources. More detailed analysis would be performed by allowing the terms to have different degrees of effect on  $r_{\max}$  and  $r_{\min}$

These new admitted points will then serve to check for new potential points.

### 2.3 Generating an Implementation plan

One of the major reasons for designing the random iteration-based point selection was for its ability to automatically generate a feasible implementation plan. That is, the order of accepted points suggests the placement order subject to the chosen modifiers *while* solving the optimal distribution. See section 4 (Growth of Tesla Charging Stations in the U.S.) for a more direct example of this ability.

The implementation plan is sensitive to initial values since it only recommends charging stations close to previously placed charging stations. Therefore the initialization of the model must be chosen with intention. In all cases, we have placed a collection of first charging stations in all top 10% most populous cities as might be expected.

## 3 Relevant Modifiers

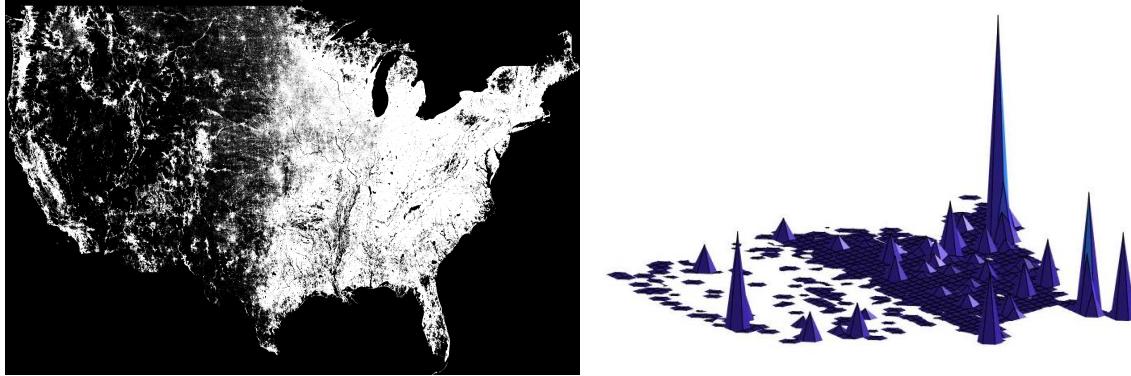
During our research, population and temperature maps were among the highest impact and easily obtainable modifiers. While population demonstrates a need for higher density of charging stations, colder temperatures have been shown to decrease battery life by as much as 33%.

Several other common-sense modifiers, such as elevation, are not significant; any battery life lost while driving uphill often regenerates while rolling downhill. Additionally, population does a good job of representing most other factors involved with the distribution of charging stations, including road structure and geographical terrain.

In the following section, we will use values calculated for the U.S. as an example.

### 3.1 Effects of Population

There should be a higher density of charging stations in more populous areas. Utilizing a raster data set containing population counts with 30 arc-second resolution,<sup>4</sup> our model uses piecewise cubic interpolation to generate a continuous surface of population ( $\text{pop}(x, y)$ ) at every longitude and latitude across the U.S.



Scheme 1: U.S. Census Grids (Summary File 1) contains grids of demographic and socioeconomic data from the year 2010. The grids have a resolution of 30 arc-seconds (0.0083 decimal degrees), or approximately 1 square km. A low-poly rendering of the surface(*right*) is derived from a piecewise cubic interpolation of the population data (*left*) and includes around 22 million data points.

This population count would directly affect  $r_{\min}$  as a density modifier:

$$r_{\min} = r_{\max} - d * \text{pop}(x, y) \quad (3)$$

The population constant  $d$  will scale the density of the population at that point and determine how to adjust the range by lessening the minimum with respect to said population size. Models for differing numbers of electric car owners can be easily accounted for by merely reducing the impact of the population term by a given percentage.  $r_{\max}$  is not affected because population has no effect on how far an electric vehicle can travel.

### 3.1.1 Derivation of $d$

Our model calculates population density in terms of square kilometers. The maximum population of any given square kilometer in the United States is 6,198 excluding New York City<sup>4</sup>. At first glance, a simple way to scale  $r_{\min}$  in terms of population is to subtract the portion of the population out of the maximum 6,198 in terms of  $r_{\max}$  from  $r_{\max}$  itself as follows:

$$r_{\min} = r_{\max} - \frac{r_{\max}}{6,198} \text{pop}(x, y)$$

However, consider the case where the population is zero. This would mean  $r_{\min} = r_{\max}$ , so there could be no charging stations within a non-populous region, despite the possibility of needing to travel through it.

To counteract this possibility, we could multiply the previous formula by a “cushioning” factor to obtain

$$r_{\min} = 0.96 \left( r_{\max} - \frac{r_{\max}}{6,198} \text{pop}(x, y) \right),$$

so that even at full population capacity, there is some room for a point to be placed within low population regions with reduced probability ( $\approx 5\%$ ). As the percentage decreases, the permissible region will increase and allow some base percentage of charging stations in any area, regardless of modifiers. Likewise, the higher that this percentage is, the more reliant the model is on the density modifiers. This cushion must be chosen for the approximate density of road infrastructure in your country of choice.

Now consider the case where population is at its maximum, 6,198. This would result in  $r_{\min} = 0$ , meaning there would be no constriction on how close charging stations could be to one another. Thus, we must find some constant  $c_1 \in [0, 1]$  within

$$r_{\min} = 0.96 \left( r_{\max} - c_1 \frac{r_{\max}}{6,198} \text{pop}(x, y) \right)$$

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<sup>4</sup>New York is an extreme population density outlier.

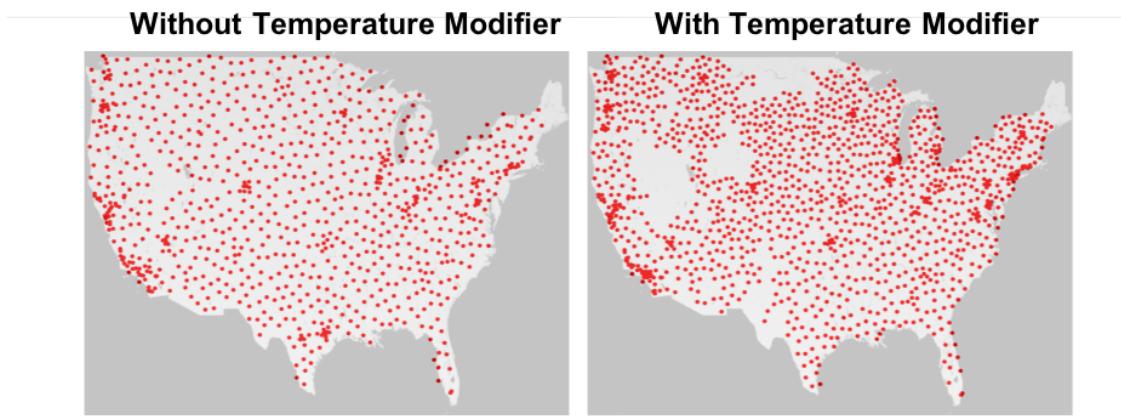
assessing the maximum impact that population can have on the minimum radius for a specified minimal distance between charging stations.

We decide that in such conditions, it would be reasonable to expect a minimum radius of 1 kilometer before encountering another charging station. In this case, we simply need to solve for when

$$r_{\min} = 1 = 0.96 \left( r_{\max} - c_1 \frac{r_{\max}}{6,198} \text{pop}(x, y) \right),$$

yielding  $c_1 = \left( 1 - \frac{1}{0.96r_{\max}} \right)$ . Plugging  $c_1$  back into (3), we find that  $d = c_1 \frac{r_{\max}}{6,198}$ .

### 3.2 Effects of Temperature



Scheme 2: Graphic demonstrating the effect of a temperature modifier on the model generated placement of charging stations. As the temperature decreases further north, battery performance suffers, necessitating a higher density of charging stations.

As temperature drops, so too does the mobility of electrons throughout the vehicle's battery. This, combined with the strain of heating the inside of the car, dramatically alters the efficiency of an electric vehicle's battery. Similarly, temperature increase can lower battery potential as the tires stick to the ground more and cause the battery to exert more energy than usual.<sup>5</sup> In this sense, temperature is a *physical* modifier and can be represented by a continuous piecewise interpolation  $\text{temp}(x, y)$ .

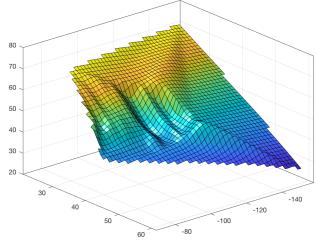
Therefore temperature effects on charger placement can be formalized as

$$r_{\max} = \text{range} - p * \text{temp}(x, y),$$

where range might be taken to be  $\approx 161$  kilometers, a distance nearly all electric vehicles brands and models can travel on a full battery.

### 3.2.1 Attenuating Temperature

Figure 2: Low-poly rendering of the heat surface of the U.S. constructed by piecewise cubic interpolation. The  $z$ -axis is in units of Fahrenheit, while  $x$  and  $y$  correspond to longitude and latitude, respectively.



Now, on average, an electric vehicle battery performs best at a temperature of 70 degrees.<sup>5</sup> Any decrease or increase in temperature will decrease the battery's potential, and thus lessen  $r_{\max}$ . Adjusting linearly<sup>5</sup> for this fact, our formula becomes

$$r_{\max} = \text{range} - c_2 \frac{|70 - \text{temp}(x, y)|}{70},$$

where  $c_2$  is the weight of temperature strength on the overall loss in potential battery range. The general effect on battery by temperature is about a 25% drop in functionality.<sup>5</sup> Therefore, we let  $c_2 = 0.25$  so that the weight of temperature is relative to its maximum potential damage to battery life. This gives the final formula:

$$r_{\max} = \text{range} - 0.25 \frac{|70 - \text{temp}(x, y)|}{70}. \quad (4)$$

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<sup>5</sup>linear approximation has a relatively high  $R^2$ .

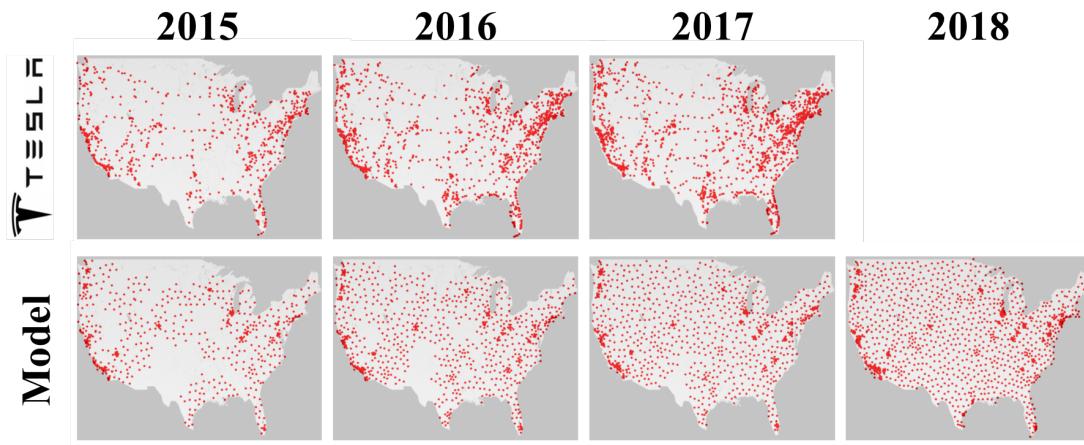
## 4 Growth of Tesla Charging Stations in the U.S.

### 4.1 Tesla Introduction

Though the number of electric vehicle owners in the United States has steadily increased throughout the last decade, there are still, as of 2017, less than 1% of vehicle owners that drive electric cars.<sup>6</sup> Tesla has established a network of chargers with many kinds of businesses across the globe.<sup>7</sup> These chargers are meant to charge electric vehicles over an extended period of time, from a few hours to all throughout the night. Hotels, restaurants, shopping malls and more are partnering with Tesla by purchasing the chargers and then having them installed on their property at their own cost and paying for the accompanying electricity costs. This way, Tesla is able to expand its network of charging stations to a plethora of Tesla owners, while business owners can attract more revenue from customers willing to stop at their location and spend money while their vehicles charge. Since the most likely places to see these chargers are at peoples' homes and frequently visited businesses, it would be reasonable to find them in the more populated regions of the country, designated in our model by higher concentration of charging stations. Any differences between Supercharging stations and Destination stations can be simulated by simply changing range in Equation (4).

### 4.2 Tesla's Track to Convert to All-Electric

Scheme 3 contrasts Tesla's charging station placement over time relative to our model-generated placement plan assuming that an identical number of charging stations are placed each year. In the current state of Tesla's charging station placement, as seen in Scheme 3, Tesla has similar placement as our model in highly populated cities. However, for a full conversion to all-electric vehicles, there would need to be charging station access everywhere, not just in the most populous areas. Otherwise, there would be electric vehicle owners in less populated areas that would have no access at all to charging stations and therefore could



Scheme 3: Graphic contrasting Tesla's placement of charging stations versus the model's prediction over the years 2015 (1,081 charging stations), 2016 (2,160 charging stations), 2017 (3,001 charging stations), and 2018 (3,700 projected charging stations).

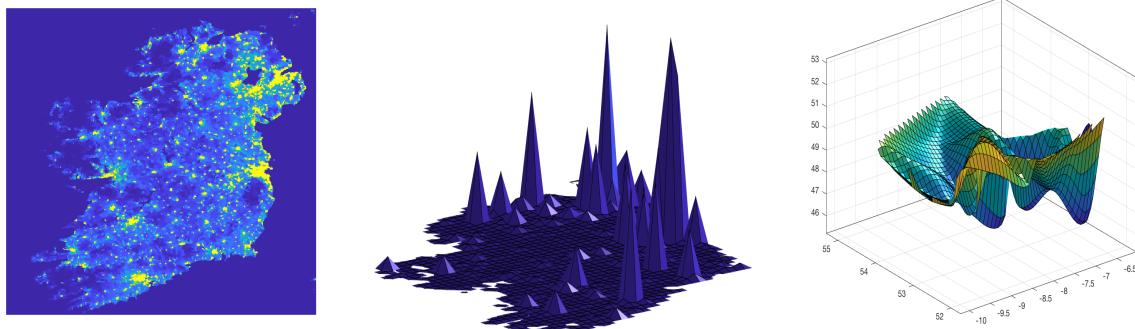
never leave the areas they reside. Our model takes the diminishing returns of placing charging stations in already densely packed areas into account, hence the difference in charging station placement in less populated areas.

Scheme 3 also displays the projected number and placement of charging stations from our model for the year 2018. This amount of charging stations would be idealistic for reaching all necessary regions of the United States. It would be probable that Tesla is on track to accommodate the conversion to all-electric vehicles should it make an effort to spread its access to stations to less populated regions of the nation so that no driver is isolated from the network of chargers. It is likely that Tesla clustered their chargers for advertising and access to business opportunities within large cities. Thus, the concern is not whether or not Tesla has the means to provide enough charging stations for a complete conversion, but rather when they will expand their network to provide access to all parts of the nation.

## 5 Effect of Various Factors on Model Performance

### 5.1 Transition Time: Ireland

#### 5.1.1 Instantaneous Transition



Scheme 4: Low-poly renderings of Population (*middle*) and temperature (*right*) in Ireland derived by piecewise cubic interpolation. The function includes around 22 million points. The  $z$ -axis is in units of population, then Fahrenheit, respectively, while  $x$  and  $y$  correspond to longitude and latitude, respectively. Population density is visible in the heat map (*left*).

Scheme 4 reveals that Ireland has a much more evenly distributed population and less outliers. Therefore, our implementation of charging stations over the entirety of the nation will be less scattered and more uniformly distributed than our plan for the U.S.



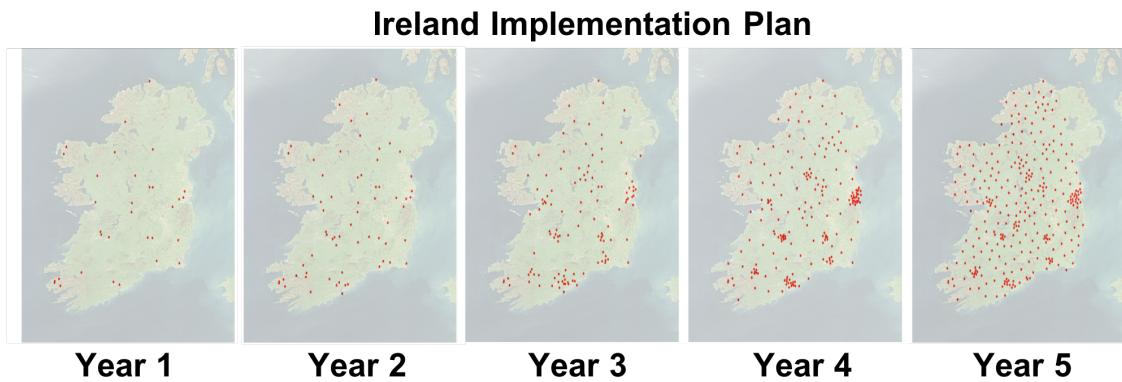
Figure 3: Model-generated implementation of 314 charging stations assuming an instantaneous transition time.

Should Ireland implement our method for charging station placement instantaneously, a general format for station locations is demonstrated in Figure 3. As previously mentioned, our model creates a generally uniform spread of stations throughout the nation, as seen in Figure 3. There are a couple of regions with higher population density that have slightly more concentrated stations as a result, but the rest of the nation has almost identical access.

Temperature data is also taken into account in our implementation of charging station distribution for Ireland. Since

average temperatures in Ireland vary on a very small interval, effect of temperature on our model's output is hardly noticeable. This further reinforces the reason for the relatively equal placement of charging stations across the nation.

### 5.1.2 Time Dependent Implementation



Scheme 5: Model-generated electric charger implementation of Ireland shows complete coverage within 6 years, assuming a relative building rate to Tesla. The model first focuses on highly populated areas, and then proceeds to cover Ireland with the density needed to meet the expected car mileage in each area.

Similar to Tesla's instinct of placing the first round of charging stations, our method locates the majority of stations in the most populous areas to both advertise the option of driving electric vehicles and allow for ease of access for our most likely consumer base. From year 1 to year 5, we steadily increase the number of charging stations and see populous areas become more concentrated while other stations are dispersed further apart in less populated regions and places of frequent travel. In order to promote conversion to electric vehicles, we could offer incentives to purchase, such as free parking spaces with charging access to electric vehicle owners and access to an additional lane of freeway, similar to a bus lane, for less clustered travel. These methods, combined with our placement model, will create a reasonable timeline for all-electric vehicle conversion in Ireland within 6 years.

## 5.2 Population Density: China

The current population of China is approximately 1.4 billion. Of that population, 94% reside in eastern china and 6% in the west.<sup>8</sup> Since large regions of China may have no population for kilometers on end, it would be unreasonable and impractical to have charging stations uniformly distributed over those regions as the through-traffic would be essentially non-existent in some places. To account for such vast stretches of land without a formal roadway, an appropriate “cushion” percentage, as discussed in 2.2 (Theoretical Charging Station Placement), could be used. This way, the model is closer attenuated to paths of frequent road travel rather than focused on providing contiguous access, as in the US model.

## 5.3 Varied Terrain: Andes Mountains

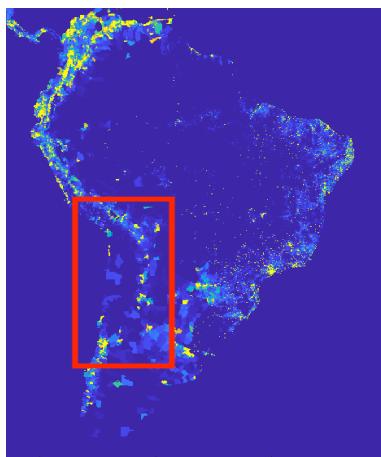


Figure 4: Graphic depicting the population divide caused by the Andes mountains throughout South America.

The Andes Mountains have the highest peaks in the Western Hemisphere. Due to the treacherous terrain the Andes presents,<sup>9</sup> the population in this area is split between the coastal area to the West and the rest of the continent to the East.<sup>9</sup> Our model inherently accounts for terrain through the use of population density. This is just one example of how terrain affects population distribution.

## 5.4 Extreme Climates: Saudi Arabia

Saudi Arabia experiences extremely hot weather in many areas of the country. In winter, the average temperature in parts of the coast is as high as 83°F. In the summer, in some areas, the temperature reaches upwards of 120°F, and thus electric cars in this climate would have lower battery ranges. Our model accounts for the temperature effect in our maximum radius to increase the accuracy of our model. Since our

model is centered on the ideal temperature for electric vehicles and the amount of battery loss that can occur based on variance from this ideal, our model would provide optimized placements for charging stations based on this computed data.

## 5.5 Technological Advancement

Technological advancements have continued to flourish and expand as the 21st century progresses. Though some of these developments may affect the way our model should function, not all of them are significant.

### 5.5.1 Hyperloops

Consider the concept of a hyperloop, where a sleek, aerodynamic bullet train glides through a rail system at speeds exceeding 700 miles per hour.<sup>10</sup> This type of transportation would revolutionize the way humans traveled long distances and cross-country. However, a hyperloop would have essentially no effect on our model for determining electric vehicle charging stations. A hyperloop is only effective when traveling distances of at least 900 miles.<sup>10</sup> Since everyone presumably travels much more frequently at lesser distances and often times just to work and run errands and go back home, this concept will not be sufficient in meeting those needs. As such, our model will run the same whether or not the hyperloop exists.

### 5.5.2 Ride-sharing Services

When it comes to ride-sharing services, the most popular companies are no strangers to electric vehicle use. In fact, Lyft has already begun allowing contracted drivers in certain areas to rent electric vehicles to use for ride-sharing purposes. It would be no surprise to see a full transition to electric vehicles in the ride-sharing industry within the next couple decades.<sup>11</sup> Thus, a shift of this nature would require adjustment to our model. More electric vehicles on the roads of cities and metropolitan areas, not only for personal passenger

vehicles but also for business purposes, would require a greater number of local charging stations. Depending on which cities were most dominated by these ride-sharing services, more charging stations would be needed to accommodate the increase in electric vehicle traffic and concentration.

### 5.5.3 Universal Batteries

At some point in the evolution of electric vehicle engineering, it would be ideal to have a universal battery so as to enable the possibility of rapid battery-swapping stations. If this was accomplished, the risk of getting stranded because of a depleted battery would diminish significantly. With services like AAA that could come to those in distress with a fully charged battery to replace the dead one, drivers would be less anxious about getting stuck without access to charging stations.<sup>12</sup> As such, our model would be able to expand its maximum radius per charging station, resulting in less charging stations nation-wide. With this decrease in charging stations, there could also be an additional variable to determine placement of battery swapping stations to be positioned in places where gaps between charging stations are more expansive.

## 6 Conclusions & Future Directions

As more of the population switches to electric cars in response to environmental stress, an increasing need for national implementation plans is essential. Our model seeks to provide informed recommendations on charging station implementation in response to providing various “modifiers” on demand and vehicle performance. While there exists many possible frameworks for such a model, choosing a random iteration-based model affords a customizable design allowing facile implementation of datasets of the users choice. This is achieved by allowing modifiers, in the form of some continuous surface describing the variable at a given location, to affect either  $r_{\max}$ , the maximum distance a charger can be from another charger

at a particular location, or  $r_{\min}$ , the minimum distance a charger can be from another charger at a particular location.

During our research, we found that population and temperature rank among the most impactful factors on charging station placement, followed closely by wealth distribution. Population modifies how densely the chargers should be placed ( $r_{\min}$ ), while temperature modifies the range of the car on a full battery( $r_{\max}$ ). By implementing these factors, we were able to mimic previous implementation plans from Tesla with high accuracy as well as derive their future growth for 2018 to ensure cross-country travel.

Though our model is generally accurate for placing stations near roadways based on population data, there is no factor to account for the possibility that our model places a charging station in an inaccessible area that is not near a roadway. With more time, this could be remedied in a few ways. If we could create a surface derived from roadway data, this factor would fit easily within the model's parameters as a modifier. Otherwise, one could create an interpolated vector field from position vectors centered at every major city representative of migration patterns to other parts of the U.S. Then, the flux of this vector field at a particular location would represent the average movement of people moving through the point on their shortest travel path. If the flux were high, this would indicate an increased need for charging stations. In addition to possibly deriving roadway structure, this approach has the added benefit of anticipating how frequently the roads are used.

Another source of improvement could be found in our formulation of density modifiers. Although we've made reasonable linear approximations, such as our temperature modifier derivation, modifiers could potentially have an appreciable nonlinear effect on battery performance or demand. These changes are likely to increase algorithmic complexity, which is one of the reasons we chose to approximate this nuance.

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