AMTH 4910: Senior Project

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1 Derive P_x for GH skewed t

Let

$$\hat{R}_n \equiv \tilde{R}_n - R_f$$

where $\tilde{R}_n$ has the GH skewed t distribution. Let

$$\hat{R}_M \equiv \tilde{R}_M - R_f$$

where $\tilde{R}_M$ is normally-distributed. We want a formula for

$$P_x(R) = P(\hat{R}_M + x\hat{R}_n \leq R)$$

By assumption 13, the skewed asset return is independent of the J risky asset returns. So,

$$\begin{aligned} P(\hat{R}_M + x\hat{R}_n \leq R) &= \int_{\mathbb{R}} P(\hat{R}_M \leq R - x\hat{R}_n \mid \hat{R}_n = k) f_{\hat{R}_n}(k)\, dk \\ &= \int_{\mathbb{R}} P(\hat{R}_M \leq R - xk) f_{\hat{R}_n}(k)\, dk \\ &= \int_{\mathbb{R}} \Phi\left(\frac{R - xk - \mu_M}{\sigma_M}\right) \cdot f_{\hat{R}_n}(k)\, dk \end{aligned}$$

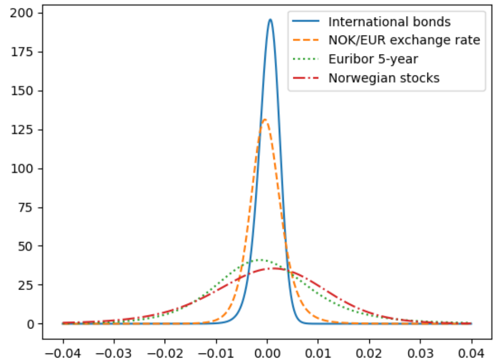
Alternatively, if I use the `ghyp` package in `R`, it's simple to calculate

$$\begin{aligned} P(\hat{R}_M + x\hat{R}_n \leq R) &= \int_{\mathbb{R}} P\left(\hat{R}_n \leq \frac{R-k}{x}\right) \cdot f_{\hat{R}_M}(k)\, dk \\ &= \int_{\mathbb{R}} P\left(\hat{R}_n \leq \frac{R-k}{x}\right) \cdot \phi(k)\, dk \end{aligned}$$

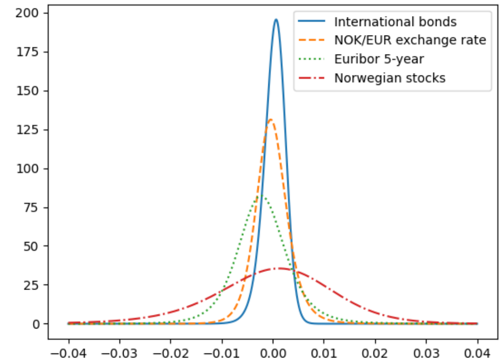
1.1 Questions

- Is there a reason that I want to code the GH skewed t pdf from scratch?
- Is the Aas and Haff parameterization worse for some reason?
- Is it possible that Aas and Haff misreported a parameter? How can I check? (Figure 1)
- How was p_n found in Stocks as Lotteries?[1]
  - I assume it is easy to find μ_M via a root-finding algorithm on $V(\hat{R}_M)$.
  - Getting p_n s.t. $V(\hat{R}_M + x\hat{R}_n) < 0 \ \ \forall 0 < x \neq x^*$ seems harder.

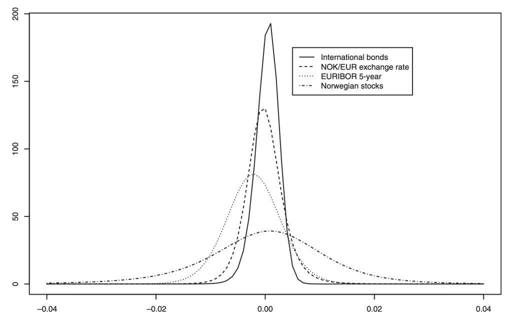
[1] I assume I want p_n and μ_M to be endogenous.



(a) Using reported parameters



(b) Dividing reported δ by 2 for Euribor



(c) Aas & Haff figure

Figure 1: Comparison of δ values for Euribor.

1.2 Answers

- Search over μ until you get double hump shape with two global optima.
- Use the distribution in BJW2021 and recognize that, assuming μ is the search variable and $\nu \equiv 7.5$ (from BJW2021), the equations

$$\text{Var}(()\,R) = \frac{\nu}{\nu-2}S + \frac{2\nu^2}{(\nu-2)^2(\nu-4)}\zeta^2 \tag{1}$$

$$\text{Skew}(()\,R) = \frac{2\zeta\sqrt{\nu(\nu-4)}}{\sqrt{S}(2\nu\zeta^2/S + (\nu-2)(\nu-4))^{\frac{3}{2}}}\left[3(\nu-2) + \frac{8\nu\zeta^2}{S(\nu-6)}\right] \tag{2}$$

  determine the two remaining parameters, ζ and S.
- Can write optimization routine to search over μ or just do it manually to find one with the “double hump” shape.

1.3 Correction to the above

Since we are working with the *excess* return on the skewed asset, we should have

$$\begin{aligned} P(\hat{R}_M + x\hat{R}_n \leq R) &= \int_{\mathbb{R}} P(\hat{R}_M \leq R - x(R_n - R_f) \mid R_n = k) f_{R_n}(k)\, dk \\ &= \int_{\mathbb{R}} P(\hat{R}_M \leq R - x(k - R_f)) f_{R_n}(k)\, dk \\ &= \int_{\mathbb{R}} \Phi\left(\frac{R - x(k - R_f) - \mu_M}{\sigma_M}\right) \cdot f_{R_n}(k)\, dk \end{aligned}$$

2 Manual numerical integration

Let

$$\Phi_k = \Phi\left(\frac{R - x(k - R_f) - \mu_M}{\sigma_M}\right)$$

Want to show that for a vector approximation of my pdf, ξ, such that

$$\vec{1} \cdot \xi \approx 1$$

we have

$$\int_{\mathbb{R}} \Phi_k f_{R_n}(k)\, dk \approx \sum_{\ell \in \text{supp}(\xi)} \Phi_k(\ell)\xi(\ell)$$

This should follow from

$$\int_{\mathbb{R}} |\Phi_k f_{R_n}(k)|\, dk \leq (\|\Phi_k\|_2 \cdot \|f_{R_n}\|_2)^{\frac{1}{2}}$$

FILL IN THE GAPS LATER.

Proposed method:

(a) Get sample grid for f_{R_n} on which

$$\int_{\mathbb{R}} f_{R_n} \approx 1$$

and the function is reasonably well-approximated. Use `adaptive` package. Denote this grid with

$$\mathcal{K} = \{k_0, k_1, \ldots, k_N\}$$

(b) Evaluate Φ_k on $\mathcal{K}$ and approximate the convolution with `np.trapezoid` to find $P_x(R)$. That is

$$\int_{\mathbb{R}} \Phi_k f_{R_n}(k)\, dk \approx \frac{1}{2} \sum_{i=0}^{N-1} \left(\Phi_{k_i} f_{R_n}(k_i) + \Phi_{k_{i+1}} f_{R_n}(k_{i+1})\right)(k_{i+1} - k_i)$$

This has truncation *and* quadrature error.

(c) Evaluate $V_x(R)$ with `adaptive`'s integral approximation scheme. Need to figure out how to bound error here, but seems stable.

Idea: We can bound the error on the term $\Phi_k(\ell)\xi(\ell)$ by noting that $\Phi_k(\ell) \in [0, 1]$ everywhere. So $\Phi_k \notin L^1(\mathbb{R})$ but

$$\begin{aligned} |\Phi_k| \leq 1 &\implies |\Phi_k| \cdot |f_{R_n}| \leq |f_{R_n}| \\ &\implies \int_{\mathbb{R}} |\Phi_k| \cdot |f_{R_n}| \leq \int_{\mathbb{R}} |f_{R_n}| \end{aligned}$$

2.1 Notes

- $V_x(R)$ is unstable in `n_points` for small values, but seems to stabilize for values >4000. How to formalize?
- $P_x(R)$ is too computationally expensive to calculate with built-in integration algorithms like `scipy.integrate.quad`.
- The PDF truncation is done w.r.t. tail mass. So

$$\int_{-\infty}^{L} f \, dk + \int_{U}^{\infty} f \, dk \leq \text{tol}$$

- The PDF grid $\mathcal{K}$ is not currently normalized to integrate to exactly 1 with trapezoid rule. But with large enough sample size `n_points` (order of 4000), it is 1 within 1×10^{-13} or better.
- The numerical values of $\text{Var}(X)$ and $\text{Skew}(X)$ are way off for $X \sim \text{GHt}$. For example, in the demo `ipynb` files, we have

$$\begin{aligned} E(X) &= 1.014772 \neq \widehat{E}(X) \approx 1.014774 \\ \text{Var}(X) &= 0.1915 \neq (X) \approx 0.191492 \\ \text{Skew}(X) &\approx 5.794 \neq (X) \approx 5.6032578 \end{aligned}$$

  are these actual values important? They are affected by the loss of tail mass far from 0 (tested by widening truncation bounds). I would assume this is not an issue since the tail mass is negligible, so the final V_x integral is unaffected.

3 Estimating Skewness

Questions:

- Options prices versus returns data?
  - I think returns data since options prices already reflect risk premia.
- Best dataset?
- The market and riskless asset characteristics are annual in the model, but annual returns data is too sparse. Bootstrap from daily returns?
- What measure should I use
  - The usual $E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$ measure might not exist, right?
  - Issues of time-varying volatility / skewness?

3.1 Answers

(a) Bloomberg database

(b) WRDS dataset on daily/monthly returns

(c) Distribution of daily returns, sample from it 365 times to get annual return, make distribution that way

(d) Don't worry about GARCH

(e) FINANCE: Strong coursework –> interest in research –> Cormac RA, describe & indicate contribution –> own research in thesis (what are you interested in going forward)

4 Derive distribution parameters from $\mathrm{E}(X)$, $\mathrm{Var}(X)$, $\mathrm{Skew}(X)$

We have

$$\begin{aligned}
\mathrm{E}(X) &= \mu + \frac{\nu}{\nu-2}\zeta \\
\mathrm{Var}(X) &= \frac{\nu}{\nu-2}S + \frac{2\nu^2}{(\nu-2)^2(\nu-4)}\zeta^2 \\
\mathrm{Skew}(X) &= \frac{2\zeta\sqrt{\nu(\nu-4)}}{\sqrt{S}\left(2\nu\zeta^2/S + (\nu-2)(\nu-4)\right)^{\frac{3}{2}}}\left[3(\nu-2) + \frac{8\nu\zeta^2}{S(\nu-6)}\right]
\end{aligned}$$

Taking the moments as given, we'd like to solve for the parameters of the distribution. So,

$$S = \frac{\nu-2}{\nu}\left[\mathrm{Var}(X) - \frac{2\nu^2}{(\nu-2)^2(\nu-4)}\zeta^2\right]$$

I do this with a nonlinear solver.

5 Empirical approach

Currently, I am using CRSP data from January 1, 2020 to December 31, 2024 and bootstrapping a return distribution using the period return column (daily, monthly, or annual) provided by CRSP. I treat the bootstrap distribution as the population (not a sample) when performing calculations. Below, I describe each approach and then the questions I have.

5.1 Daily returns bootstrap

Denote the daily return of firm i on day t as $r_{i,t}$. Let $i \in I$ and $t \in T$.

For example, let the firms be the MAG7, then $i \in \{1, 2, 3, 4, 5, 6, 7\}$ for some enumeration of NVDA, MSFT, GOOGL, AAPL, META, TSLA, AMZN. Also, if we consider these firms from January 1, 2020 to December 31, 2024, we have $t \in \{0, 1, \ldots, 1825\}$. Then the r.v. r is distributed uniformly as

$$r \sim \{r_{i,t} : i \in I,\ t \in T\}$$

this is our approximation of the daily returns distribution for a general AI stock. We can approximate an annual return by sampling from this distribution 365 times. Then we have the r.v. R defined by

$$1 + R = \Pi_{k=1}^{365}(1 + r(k))$$

where $r(k)$ is the k^{th} sample from the daily distribution. If we repeat this process many times (10,000 times in practice), we can build an annual return distribution. Then,

$$\text{Skew}(R) = \frac{m_3}{m_2^{3/2}}$$

the Fisher-Pearson coefficient of skewness, where $m_i = \frac{1}{10{,}000}\sum_{n=1}^{10{,}000}(R(n) - \overline{R})^i$ is the biased i^{th} central moment. This gives nearly the same result as the more compact formula

$$\text{Skew}(R) = \text{E}\left(\left(\frac{R - \mu}{\sigma}\right)^3\right)$$

The procedure described above tends to give

$$\begin{aligned} 1.6 \leq \text{Skew}(R) &\leq 2 \\ 0.84 \leq \text{Var}(R) &\leq 0.92 \\ \text{E}(R) &\approx 75\% \end{aligned}$$

For all calculations, I treat the bootstrap distribution as the population, not a sample.

5.1.1 Questions

(a) Is there anything wrong with using the daily returns of the MAG7 for the bootstrap—should I stick to just one firm?

(b) Should I treat the bootstrap distribution as a sample or the population?

(c) Should I try to reduce the range of bootstrapped skewness values?

(d) It's hard to find return data up to the present for free. NASDAQ has free open and close prices, but only for every *trading* day. Compustat has a "daily return factor" variable that I can probably use to back out daily returns, but is that the best way?

(e) I can find an equilibrium for various skewness/variance levels by editing the Prospect Theory parameters. For example, I find a heterogeneous holdings equilibrium (with negative e.v.) for $\mathrm{Skew}(R) = 1.4$ and $\mathrm{Var}(R) = 0.45$ and

$$\alpha = 0.7 \quad \delta = 0.6 \quad \lambda = 1.25$$

is this too cooked up?

(f) Should I worry about the i.i.d. assumption and do a more sophisticated approach (e.g., a block bootstrap)?

5.2 Monthly returns bootstrap

For bootstrapping from monthly returns, I use the same method as for the daily bootstrap with minor variations. First, t is denoted in months, i.e., $t \in \{0, 1, \ldots, 59\}$ and $r_{i,t}$ is the monthly return of stock i. Second, the annual return r.v. R is defined

$$1 + R = \Pi_{k=1}^{12}(1 + r(k))$$

Everything else is the same. This procedure tends to give

$$\begin{aligned} 1.4 \leq \mathrm{Skew}(R) &\leq 1.6 \\ 0.36 \leq \mathrm{Var}(R) &\leq 0.38 \\ \mathrm{E}(R) &\approx 46\% \end{aligned}$$

5.2.1 Questions

(a) Are these numbers similar enough to daily return bootstrap, or do I need to rethink approach?

5.3 Cross-sectional approach

Take 31 AI-associated stocks, and use the Compustat monthly return (`trt1m`) for each one to calculate the trailing 12-month return. Use these 32 values to estimate the skewness and variance, finding

$$\begin{aligned} \mathrm{Skew}(R) &\approx 2.2 \\ \mathrm{Var}(R) &\approx 0.88 \end{aligned}$$

These results are robust to any single value, verified by leave-one-out calculations. For the calculations in this approach, I treat the vector of returns as a sample, not the population.

5.3.1 Questions

(a) Moderate levels of skewness $\implies x^* \approx \sigma_M$?

6 Update November 8, 2025

6.1 Cross-sectional approach

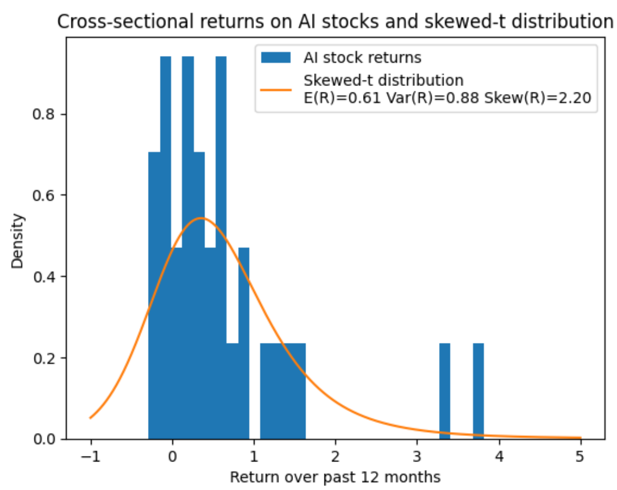
Take 31 AI-associated stocks, and use the Compustat monthly return (`trt1m`) for each one to calculate the trailing 12-month return (to present). Use these 31 values to estimate the skewness and variance, finding

$$\begin{aligned} \text{Skew}(R) &\approx 2.2 \\ \text{Var}(R) &\approx 0.88 \end{aligned}$$

These results are robust to any single value, verified by leave-one-out calculations. For the calculations in this approach, I treat the vector of returns as a sample, not the population.

6.1.1 Questions

(a) Below is a histogram of these returns overlaid with a skewed-t distribution with the empirical expectation, variance, and skew. These don't look so similar to me, and I can improve the visual similarity by fudging the parameters of the skewed-t distribution (e.g., halving the variance). I'm concerned about putting a chart like this in the writeup since the visual similarity isn't great. Is this something I need to address?



(b) The cross-sectional skewness and variance do not seem to support a heterogeneous holdings equilibrium for either your 2021 PT parameters or those of Kahneman and Tversky. However, if I set

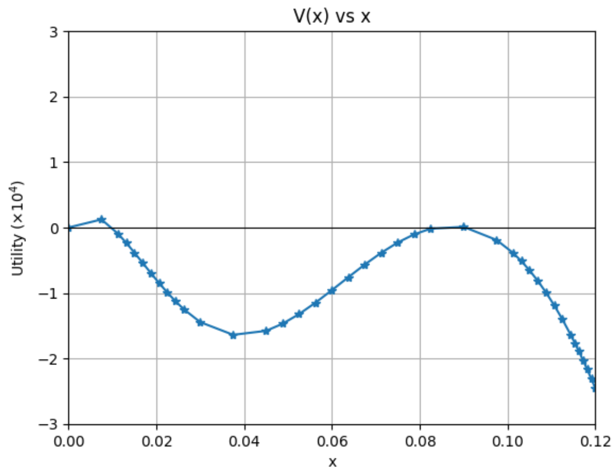
$$\alpha = 0.88 \quad \delta = 0.65 \quad \lambda = 1.5$$

the equilibrium exists. Argh!

- This seems similar to the result in the final paragraph of page 2082 in Stocks as Lotteries, where you find that heterogeneous holdings equilibria only exist for $\alpha \geq 0.845$ (meaning that the parameters in your 2021 paper won't support heterogeneous holdings since $\alpha = 0.7$).

6.2 Miscellaneous

(a) In the graph below, the utility is increasing near 0, then follows the double hump path. I think it's just a manifestation of the usual gains to diversification result, but haven't seen that shape yet so figured I'd share.



(b) This is mentioned in Stocks as Lotteries, but just to confirm: is it a general rule that the expected excess return of the skewed asset must be negative for a heterogeneous holdings equilibrium to exist?

(c) How would you recommend defining the "minimum skewness required to get a double hump"?

- My first thought is to maintain $\text{Var}(R) = 0.88$ and vary the skewness, but I want to make sure the interpretation is acceptable. This seems to suggest that investors correctly perceive the variance but overestimate the skewness. Is that problematic?
- Also, assuming the answer to bullet 6.2(b) is that the excess return of the skewed asset must be negative, my strategy for finding the minimum skewness would be to set the excess return to 0 and vary the skewness until I find a double hump (keeping variance constant). This must be the minimum skewness, since reducing it more would mean investors demand greater excess return, but that excess return wouldn't support heterogeneous holdings since it must be positive.

(d) If my proposal in 6.2(c) is the correct strategy, then the minimum skewness for a heterogeneous holdings equilibrium with your 2021 PT parameters and my cross-sectional skewness and variance is $\text{Skew}(R) = 3.83795$.

(e) I've confirmed that the daily returns have auto-correlation of about -9%, and monthly only -5.6%.

7 Forward-looking skewness

7.1 Bulleted plan

My broad idea is to estimate the skewness $\widehat{\gamma}$ and variance $\widehat{\sigma}^2$ for some single stock, then plug those parameters into the skewed-t distribution—denoted[2] $\text{GHt}(\mu, \gamma, \sigma^2)$—to get a skewed asset distribution given by

$$\widetilde{R} \sim \text{GHt}(\mu, \widehat{\gamma}, \widehat{\sigma}^2)$$

and adjust the expected value μ to find a heterogeneous holdings equilibrium. My approach to estimate the parameters is bulleted below along with some questions:

(a) Choose some single stock.

- I'm currently considering Google since they have DeepMind. Or maybe Meta since they've been investing so heavily? Nvidia seems like the most natural choice at first, but not sure about the interpretation of dominance since they are already basically a monopoly on hardware. Curious what you think.

(b) Use current analyst low, medium, and high price estimates/targets to get a base case return distribution.

(c) Assign some small probability q of "victory" in the AI race.

(d) Construct a bullish return estimate using the most optimistic (yet credible) articles I can find.

- Much bullish sentiment seems to be reported as things like EPS, market share, growth %, etc. So I'll probably need some way to translate that into an actual return number.
  - Is it okay to do this as simply as possible, like with the Gordon Growth Model or something even simpler (like just estimating the increase in market cap of the firm as a result of an event like their cloud infrastructure doubling in value)?
  - Do you know of a better method for this (or an alternative I should at least consider)?
- Should I make multiple dominance estimates so that the return distribution has more observations?

(e) Assume the low, medium, high estimates are equally likely, and the probability of victory is q, so the e.v. of the firm is:

$$\widehat{\mu} = \frac{1}{3}(1-q) \cdot (\text{Low} + \text{Medium} + \text{High}) + q \cdot \text{Victory}.$$

I won't use this $\widehat{\mu}$ for except the skewness calculation since the return must be determined in equilibrium.

(f) Use this 4 observation distribution $\widehat{R}$ to define the skewness via

$$\text{E}\left(\left(\frac{\widehat{R} - \widehat{\mu}}{\widehat{\sigma}}\right)^3\right)$$

- This requires a variance estimate $\widehat{\sigma}^2$. Should I also use the 4 observation distribution for that? It seems like that will be biased toward (possibly unrealistically) high variance due to the small sample size.

[2] μ is the e.v. in this formulation, not the parameter of the distribution. Sorry for the notation.

7.2 Attempt for GOOGL

I will perform a three phase DCF valuation, assuming Google achieves superintelligent AI next year. Based on [3] and [2], AGI could lead to a literal singularity in income, where productivity growth rates increase each year as AI performs R&D, quickly diverging to $+\infty$. Let's be more conservative.

Suppose that revenues double each year for the five years following the advent of superintelligence, then for the next fifteen years, growth is high and finite, say at 25% per annum (approximately twice its historical revenue growth rate). After twenty years, growth returns to a reasonable (yet high) 5% annually.

7.2.1 Phase 1: Rapid Scaling

Over the past 12 months, Google's reported free cash flow was approximately 74B. So with our growth assumptions,

$$\begin{aligned} PV_1 &= \sum_{t=1}^{5} \frac{CF_0(1+g_1)^t}{(1+R)^t} \\ &\approx \sum_{t=1}^{5} \frac{74(1+1)^t}{(1+0.08)^t} \\ &\approx 3.34T \end{aligned}$$

which is, in fact, close to the firm's current market capitalization.

7.2.2 Phase 2: Fast growth

We have

$$\begin{aligned} PV_2 &= \sum_{t=6}^{21} \frac{CF_5(1+g_2)^t}{(1+R)^t} \\ &= \sum_{t=6}^{21} \frac{74(2^5)(1+.25)^t}{(1+0.08)^t} \end{aligned}$$

7.2.3 Phase 3: Long-term growth

7.3 Anchoring to a McKinsey article

In [4], McKinsey partners argue that AI could drive innovation in R&D, even claiming:

> For industries whose products consist of intellectual property (IP) or whose R&D processes are closest to scientific discovery, the rate of innovation could potentially be doubled.

estimating that in software, the potential increase in annual EBIT is around 35%. Based on more bullish sources [3, 2], AGI could lead to a literal singularity in income, where productivity growth rates increase each year as AI performs R&D, quickly diverging to $+\infty$. The CEO of Anthropic has claimed that once AGI is achieved, "AI-enabled biology and medicine will allow us to compress the progress that human biologists would have achieved over the next 50-100 years into 5-10 years" [1]. Given that the returns to such a successful technology would be difficult (and maybe meaningless) to estimate, let's be more conservative and apply McKinsey's insights to Google (really its parent company, Alphabet).

The historical average annual revenue growth for Google is around 12%, its operating margin is around 30%, and it trades at $\frac{P}{E} \approx 30$. Since the market capitalization is around 3.4T, this implies $E \approx 113.33$B and revenue ≈ 377.77B.[3]

The following back-of-the-envelope calculation will estimate the firm's revenue in 2030, assuming it wins the race for "AI superintelligence" (in a very conservative sense), find P_{2030} assuming the $\frac{P}{E}$ ratio remains the same, then discount the price back to today using the firm's cost of capital, $\mathrm{E}(R)$, given its $\beta = 1.08$.

Suppose that as a result of leveraging advanced AI, Google achieves dominance for the next 5 years, increasing its revenue growth rate by 100%, from 12% annually to 24% per annum. This gives

$$\text{Revenue}_{2030} \geq 377.77B \cdot 1.24^5 \approx 1107.48B$$

The McKinsey report models the returns from AI as it exists today, not the returns of AGI. So the assumption of doubled revenue growth is reasonable—a firm that has achieved "superintelligence" should expect more productive R&D and rapid growth in popularity of AI offerings like agents and chatbots, as well as highly effective AI-powered advertising.

Now, OpenAI has estimated that they will themselves have revenue of around 125B in 2029 [5], so if Google releases and licenses a superintelligent model that steals market share from OpenAI (and competitors in the AI space), it can expect a boost to annual revenue of, say, 140B by 2030. So,

$$\text{Revenue}_{2030} \approx 1107.48B + 140B = 1247.48B$$

To get earnings, we can assume that the 35% EBIT increase applies entirely to the firm's margin, meaning

$$E_{2030} = (1 + 0.35) \cdot 0.3 \cdot 1247.48B \approx 561.37B$$

so assuming the $\frac{P}{E}$ ratio remains near 30,

$$P_{2030} = \frac{P}{E} \cdot 561.37B = 16.841T$$

[3] These estimates align well with official data of \$100B and \$349.8B in earnings and revenue, respectively, for the 12 month period ending September 30, 2025.

Assuming an equity risk premium of 5% and riskless rate of 4.5%, we have

$$\mathrm{E}(R) = R_f + \beta \cdot 5\% = 4.5\% + 5.4\% = 9.9\%$$

so discounting to present, we find

$$PV(P_{2030}) = \frac{16.841T}{1.099^5} \approx 10.5T$$

implying a return of

$$R = 10.5/3.4 - 1 = 209\%$$

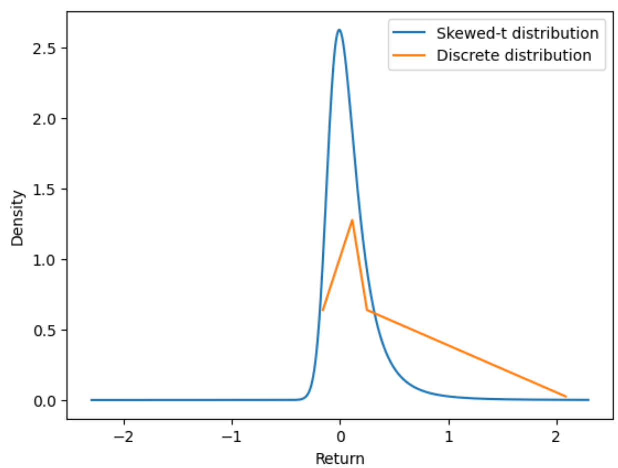
The high analyst share price target is \$350, average is \$312, and low is \$236 (the current price is around \$280). If I model this as the following with $q = 1\%$

$$\begin{aligned} P(\widetilde{R} = \frac{350-280}{280}) = P(\widetilde{R} = \frac{236-280}{280}) &= (1-q)\frac{1}{4} \\ P(\widetilde{R} = \frac{312-280}{280}) &= (1-q)\frac{1}{2} \\ P(\widetilde{R} = 2.09) &= q \end{aligned}$$

I get

$$\mathrm{Skew}\left(\widetilde{R}\right) \approx 4.9, \quad \mathrm{Var}\left(\widetilde{R}\right) \approx 0.06$$

and plotting the distribution of returns alongside a skewed-t distribution with the same moments,



which looks like a good fit!

7.4 Questions

(a) When I calculate P/E, should I use the indexing P_t/E_{t+1} or P_t/E_t? This will make a difference to my discounting equation since my numerator is either P_{2029} or P_{2030}, so my DF is either $\frac{1}{(1+R)^4}$ or $\frac{1}{(1+R)^5}$.

(b) Is it okay to "double-count" McKinsey's estimates? They say that the 35% EBIT is a result of the $2\times$ multiplier on R&D productivity, but they are analyzing existing AI, not AGI. I feel like I count it twice—once when I double revenue growth and once when I widen the profit margin.

(c) Is the revenue addition of 140B from AI products too out-of-nowhere? I based it on OpenAI's revenue projections of 125B/year for 2029, and then multiplied by $1 + \mathrm{E}(R) = 1.099$, Google's discount rate, to get the value of ≈140B in 2030.

(d) Should I make it simpler?

7.5 Comparison to Nvidia dominance trajectory

In 2020, Nvidia had 16.7B in revenue. It had 130.5B revenue in 2024. This is an earnings multiple of

$$130.5/16.7 = 7.8\times$$

as it achieved dominance in the AI chip space. Based on my estimate for Google's earnings assuming it creates AGI and becomes dominant, Google's earnings are multiplied by

$$561.37/113.33 \approx 4.95\times$$

So the scale seems reasonable.

Now to check the constant P/E ratio assumption. In 2020, Nvidia had a P/E ratio between 50 and 80. Now it has a P/E ratio of around 55 (which is near its 10-year average value of 53.5). For comparison, Google has a P/E ratio of 28, near its 10-year average of 27.8.

7.5.1 Questions

(a) Is it reasonable to compare the multipliers on earnings to justify my approach above?

(b) Nvidia's P/E ratio has been fairly volatile. Should I worry about the constant P/E assumption I made above?

7.6 A simpler approach

Let's instead use the Gordon Growth Model (GGM):

$$\frac{P}{D} = \frac{1 + g_D}{r - g_D}$$

suppose a constant payout ratio

$$p = \frac{D_0}{E_0}$$

so that

$$\begin{aligned} P_0 &= \frac{D_1}{r - g_E} \\ &= \frac{pE_0(1 + g_E)}{R - g_E} \\ \implies \frac{P_0}{E_0} &= \frac{p(1 + g_E)}{r - g_E} \end{aligned}$$

Empirically, Google's payout ratio is

$$p = 8\%$$

and since we calculated the firm's discount rate to be $r = 0.099$ and the current P/E ratio is about 28.3, so we can solve

$$28.3 = \frac{0.08(1+g_E)}{0.099 - g_E}$$
$$\implies g_E = \frac{2.7217}{28.38} \approx 0.0959$$

so the market expects Google's long term earnings growth rate to be around 9.59%.

Let's assume that Google discovers "superintelligence" next year, and earnings begin to grow at a rate of 50% per year for the next three years, then 12% for the next twelve years, followed by long-run constant growth at the same $g_E = 0.0959$. We have three phases. In phase 1,

$$PV_1 = \sum_{t=1}^{3} \frac{pE_0(1.5)^t}{1.099^t}$$
$$\approx E_0 \cdot 0.462$$

In phase 2,

$$PV_2 = \sum_{t=4}^{15} \frac{pE_0(1.5^3)(1.12)^{t-5}}{1.099^t}$$
$$\approx E_0 \cdot 2.205$$

Finally in the long run,

$$PV_3 = \frac{1}{1.099^{15}} \frac{pE_0(1.5)^3(1.12)^{12}}{0.099 - 0.0959}$$
$$\approx E_0 \cdot 82.348$$

So in total,

$$P'_{2025} = PV_1 + PV_2 + PV_3 = E_0(0.462 + 2.205 + 82.348) = E_0 \cdot 85.015$$

Comparing this to the original $P_0/E_0 \approx 28.3$, we find that the firm has posted a return of

$$\frac{P'_{2025} - P_0}{P_0} = \frac{E_0 \cdot 85.015 - E_0 \cdot 28.3}{E_0 \cdot 28.3} \approx 200.4\%$$

This is basically the same return as I found in §7.3 (which was 209%), but without any of the storytelling.

7.7 Questions

(a) I can get the same results with plenty of different assumptions. Is it a problem that the value is so loaded on PV_3?

(b) Is the assumption that g_E is unchanged problematic? I assume investors have already priced in some probability of this dominance scenario, so the firm isn't exactly in a steady state currently.

(c) If we say Google has 335B annual revenue today, then the above implies that it will have

$$335B \cdot 1.5^3 \cdot 1.12^{12} \approx 4.4T$$

in revenue 15 years from now. This seems large. Is it okay as something to which investors might assign a 1% probability, or is even that too extreme?

- I could fix this by changing the firm I apply it to—Meta had "just" 164B in revenue for 2024, which would give $2.16T$ in 15 years on the above growth path (assuming margins don't change, so all earnings growth must come from revenue growth).

(d) I'm struggling to justify my growth rate choices for this approach, which is what initially drew me to the original approach I introduced in §7.3. What kind of justification should I offer?

- If I switch firms to Meta, then the current earnings growth rate is around 20%, so I could justify this as something like a short-term doubling in growth, followed by a halving to reflect the fact that so much progress was made in a short time, so there is less growth opportunity "out there."
- As a matter of fact, below I reproduce the analysis for Meta.

7.8 Simple approach for Meta

Empirically, Meta's payout ratio is

$$p = 9\%$$

with a 5-year monthly $\beta = 1.27$, so assuming $R_f = 0.045$ and the equity risk premium is 5%,

$$\mathrm{E}(R) = R_f + \beta \cdot 5\% = 4.5\% + 6.35\% = 10.85\%$$

so the firm's discount rate is $r = 0.1085$ and the current P/E ratio is about 27, so we can solve

$$\begin{aligned} 27 &= \frac{0.09(1+g_E)}{0.1085 - g_E} \\ \implies g_E &\approx 0.104817 \end{aligned}$$

so the market expects Meta's long term earnings growth rate to be around 10.4817%.

Meta's earnings growth tends to be around 20% per year. Let's assume that Meta discovers "superintelligence" next year, and earnings growth triples to a rate of 60% per year for the next three years, then slows down to 12% for the next twelve years (since they will have less "room to grow" after the three years of rapid growth), followed by long-run constant growth at the same $g_E = 0.104817$. We have three phases. In phase 1,

$$\begin{aligned} PV_1 &= \sum_{t=1}^{3} \frac{pE_0(1.6)^t}{1.1085^t} \\ &\approx E_0 \cdot 0.5881 \end{aligned}$$

In phase 2,

$$\begin{aligned} PV_2 &= \sum_{t=4}^{15} \frac{pE_0(1.6^3)(1.12)^{t-5}}{1.1085^t} \\ &\approx E_0 \cdot 2.7705 \end{aligned}$$

Finally in the long run,

$$
\begin{aligned}
PV_3 &= \frac{1}{1.1085^{15}} \frac{pE_0(1.6)^3(1.12)^{12}}{0.1085 - 0.104817} \\
&\approx E_0 \cdot 83.1728
\end{aligned}
$$

So in total,

$$
P'_{2025} = PV_1 + PV_2 + PV_3 = E_0(0.5881 + 2.7705 + 83.1728) = E_0 \cdot 86.5314
$$

Comparing this to the original $P_0/E_0 \approx 27$, we find that the firm has posted a return of

$$
\frac{P'_{2025} - P_0}{P_0} = \frac{E_0 \cdot 86.5314 - E_0 \cdot 27}{E_0 \cdot 27} \approx 220.5\%
$$

This is again basically the same return as I found in §7.3 (which was 209%).

7.8.1 Questions

(a) Thoughts on this analysis?

(b) With these growth rates (assuming no change in profit margins), Meta will be generating revenue of

$$
1.6^3 \cdot 1.12^{12} \cdot 164B \approx 2.617T
$$

in 2040. This seems defensible?

References

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