Table of Contents

Vibs Example - 5DOF Lumped Mass Shear Beam

Beam Details and Analysis

jdv 06232015; 07232015; 08162015; 11122015

```
clear all
```

Beam Parameters

```
% section
b = 1;                  % in
h = 12;                 % in
I = b*h^3/12;           % in^4
A = b*h;                % in^2
E = 29e6;               % psi

% geometry
L = 100*12;             % ft -> in; total length
nn = 5;                 % number of inner dof (discretization)
nel = nn+1;             % number of beam elements
mchk = 1;               % 1 = lumped mass, else = continuous

% mass
ro = .29;               % density [lb/in^3]
grav = 386.4;           % in/sec^2;
mbar = A*ro;            % lbf/in - weight
mbar = mbar/grav;       % lbm/in (m = f/a)
```

Build Beam

```
% build bernoulli beam
[K,M] = beam_builder(E,I,L,nel,mbar,mchk);
```

```
% make shear beam
%   -remove rotation dof
ind = 2:2:length(K);
K = removerows(K,'ind',ind);
K = removerows(K','ind',ind);
M = removerows(M,'ind',ind);
M = removerows(M','ind',ind);
```

Eigen Solution

```
[V,D] = eig(K,M);           % solve
[val,ind] = sort(diag(D));  % sort eigenvalues
V = V(:,ind);               % apply sort to eigenvectors
W = sqrt(val);              % [rad/sec]
F = W/2/pi;                 % [hz]

% decouple system matrices
Mr = V'*M*V; % modal mass, Mr
Kr = V'*K*V; % modal stiffness, Kr


% form mass normalized modeshapes
Vn = zeros(size(V));
ne = size(V,2);     % number of effective modes
for ii = 1:ne
    Vn(:,ii) = V(:,ii)/sqrt(Mr(ii,ii));
end

% get scaled modal mass and stiff
Mr = Vn'*M*Vn;
Kr = Vn'*K*Vn;

% add proportional damping
dampr = [.05 .05 .05 .05 .05]';      % damping ratio [% critical damping]
dampf = -dampr.*W;                   % damping factor [rad/sec]
Wn = sqrt(W.^2 + dampf.^2);          % damped natural frequency [rad/sec]
root = dampf + 1j.*Wn;               % form positive poles
```

Form Residues

Get residue [A]r for each mode r from eigenvectors to form partial fraction expansion

```
% form radial frequency vector
ns = 2^9;                   % number of spectral samples
w = linspace(-150,150,ns);  % [rad/sec]
inLoc = 1:5;                % index for input locations (columns of H)
outLoc = 1:5;               % index for output locations (rows of H)
no = length(outLoc);        % number of outputs
ni = length(inLoc);         % number of inputs
ne = 5;                     % number of effective modes

% define modal scaling for unit mass
```

```
%  notes: -true unity mass due to mass normalize eigenvector
%         -5 modes solved for
Qr = 1./(2j.*diag(Mr).*Wn);

% get FRF via residues
AA = zeros(no,ni,ns); HH = zeros(no,ni,ns);
for ii = 1:ne   % loop modes
    % form [A] for mode ii -> [no x ni x ns]
    AA(:,:,ii) = Qr(ii) * V(:,ii) * V(:,ii)';
    for jj = 1:no       % loop outputs
        for kk = 1:ni   % loop inputs
            out = outLoc(jj); % output DOF index
            in  = inLoc(kk);  % input  DOF index
            for ll = 1:ns % loop spectral lines

                % form [H] - add mode ii contribution -> [no x ni x ns]
                %             complex conjugate
                tt = AA(out,in,ii) ./ (1j*w(ll) - root(ii)) + ...
                          conj(AA(out,in,ii))./(1j*w(ll) - conj(root(ii)));

                % add each mode for total response
                HH(jj,kk,ll) = HH(jj,kk,ll) + tt;

            end
        end
    end
end

% convert HH [no x ni x ns] -> H [ns x no*ni] (legacy format)
H = zeros(ns,no*ni);
hInd = 1:no*ni;
hInd = reshape(hInd,no,ni);
for ii = 1:ns
    for jj = 1:no
        for kk = 1:ni
            H(ii,hInd(jj,kk)) = HH(jj,kk,ii);
        end
    end
end
```

Summary

System State Information:

```
fprintf('Mass: \n');
disp(M);

fprintf('Stiffness: \n');
disp(K);

fprintf('Natural Frequencies [Hz]: \n');
disp(F);
```

```
fprintf('Damped natural frequencies [rad/sec]: \n');
disp(Wn);

fprintf('Modal Mass: \n');
disp(Mr);

fprintf('Modal Stiffness: \n');
disp(Kr);

fprintf('Complex Roots:\n')
fprintf('Pole %d\t Damping Factor: %6.3f\t Positive Pole: %6.3f\n',...
    [1:length(root); real(root)'; imag(root)'])
```

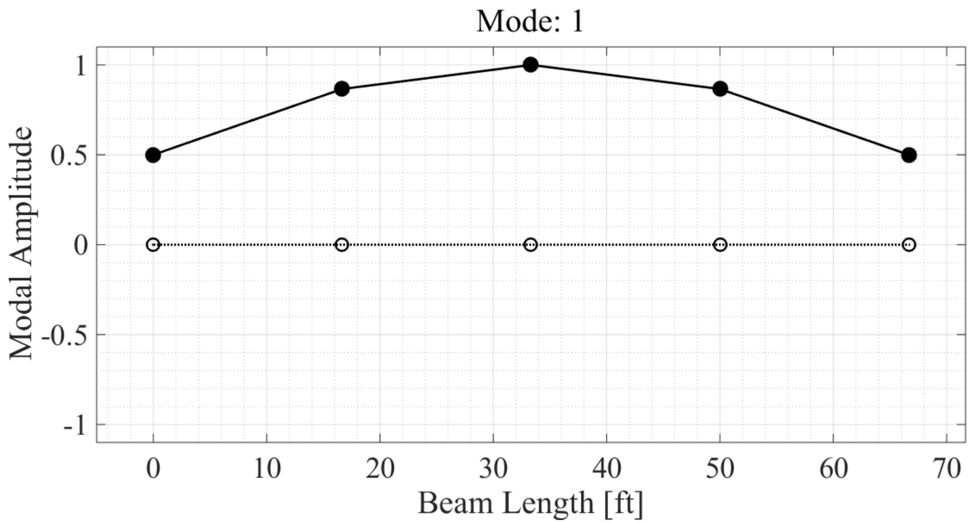
```
Mass:
    1.8012         0         0         0         0
         0    1.8012         0         0         0
         0         0    1.8012         0         0
         0         0         0    1.8012         0
         0         0         0         0    1.8012

Stiffness:
       12528        -6264            0            0            0
       -6264        12528        -6264            0            0
           0        -6264        12528        -6264            0
           0            0        -6264        12528        -6264
           0            0            0        -6264        12528

Natural Frequencies [Hz]:
    4.8583
    9.3856
   13.2732
   16.2563
   18.1315

Damped natural frequencies [rad/sec]:
   30.5639
   59.0448
   83.5020
  102.2687
  114.0659

Modal Mass:
    1.0000   -0.0000    0.0000    0.0000    0.0000
   -0.0000    1.0000   -0.0000         0   -0.0000
    0.0000   -0.0000    1.0000   -0.0000   -0.0000
    0.0000         0   -0.0000    1.0000    0.0000
    0.0000   -0.0000   -0.0000    0.0000    1.0000

Modal Stiffness:
   1.0e+04 *

    0.0932   -0.0000    0.0000    0.0000    0.0000
   -0.0000    0.3478   -0.0000   -0.0000   -0.0000
    0.0000   -0.0000    0.6955   -0.0000   -0.0000
```

```
    0.0000    -0.0000         0    1.0433    0.0000
    0.0000    -0.0000   -0.0000    0.0000    1.2979

Complex Roots:
Pole 1  Damping Factor: -1.526  Positive Pole: 30.564
Pole 2  Damping Factor: -2.949  Positive Pole: 59.045
Pole 3  Damping Factor: -4.170  Positive Pole: 83.502
Pole 4  Damping Factor: -5.107  Positive Pole: 102.269
Pole 5  Damping Factor: -5.696  Positive Pole: 114.066
```

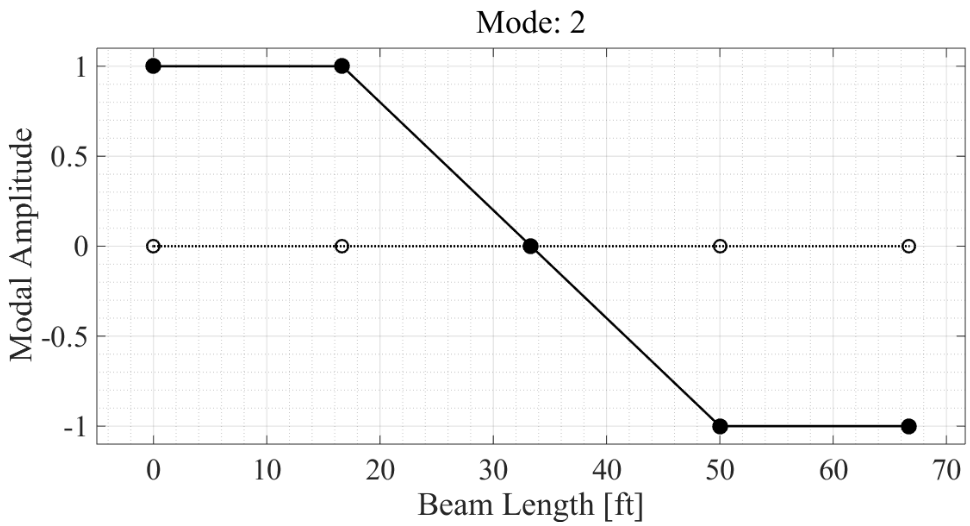
Beam Eigenvectors

```
% create figure
fh = figure;
ah = axes;
fh.PaperPositionMode = 'auto';
fh.Position = [100 100 1300 600];

% loop to plot
for ii = 1:5
    fprintf('Mode: %d',ii);
    beam_plotshape(ah,V(:,ii),1,L,nn);
    title(['Mode: ' num2str(ii)],...
        'fontweight','normal')
    snapnow
end
```

```
Mode: 1
```

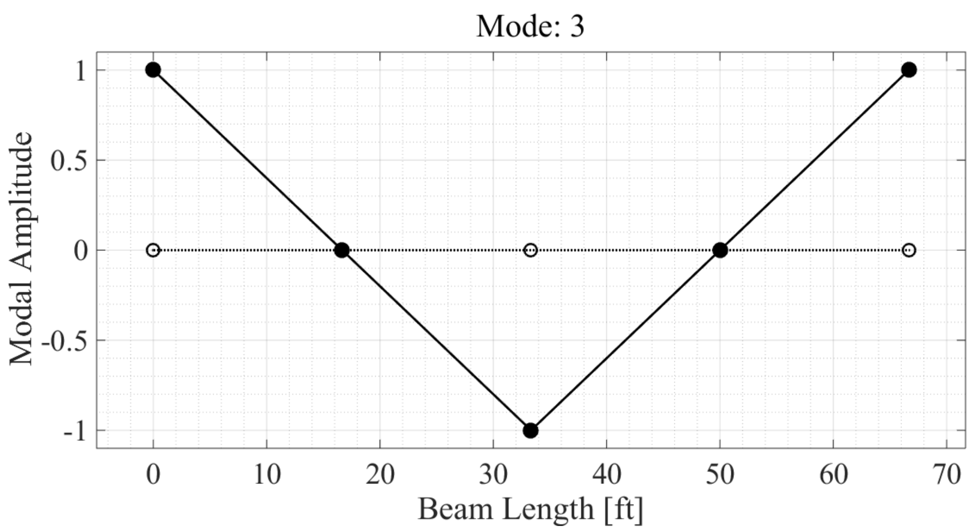



```
Mode: 2
```



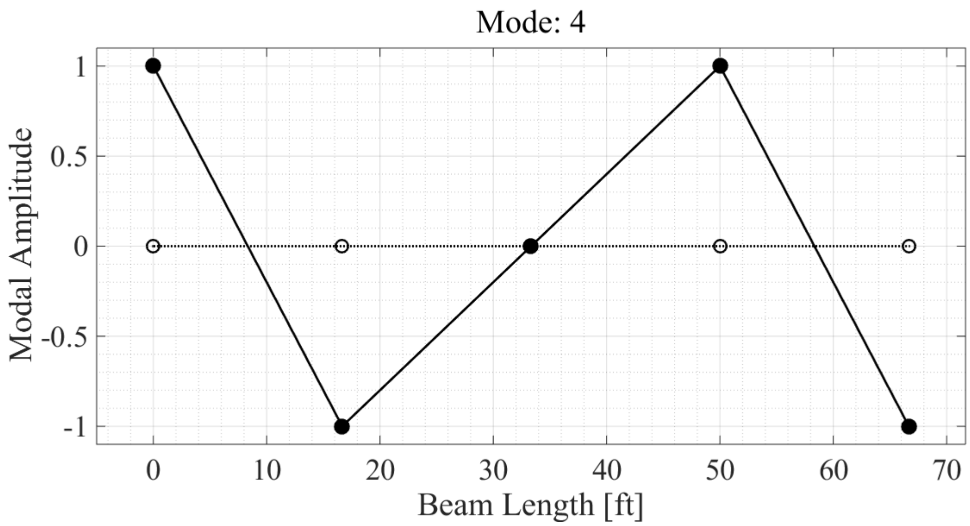
Mode: 2
Modal Amplitude
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
Beam Length [ft]

`Mode: 3`

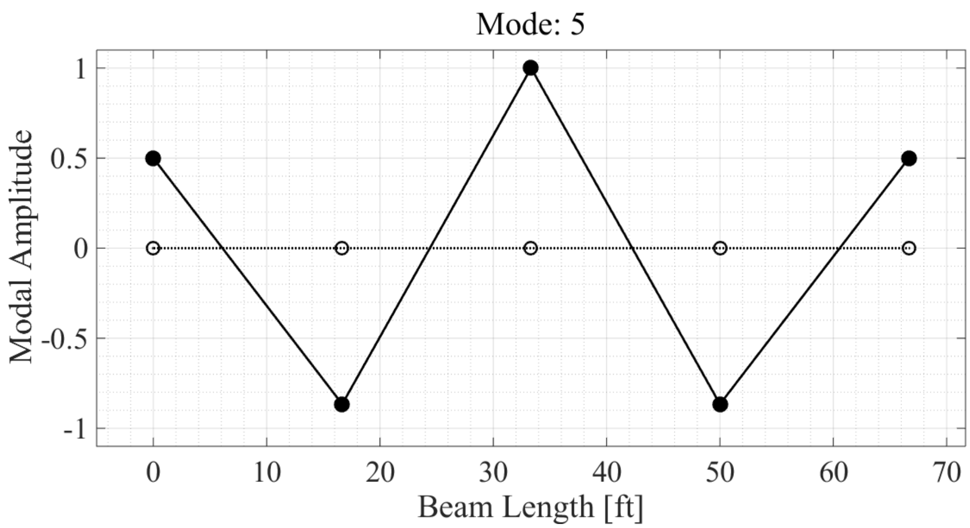


Mode: 3
Modal Amplitude
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
Beam Length [ft]

`Mode: 4`



Mode: 5

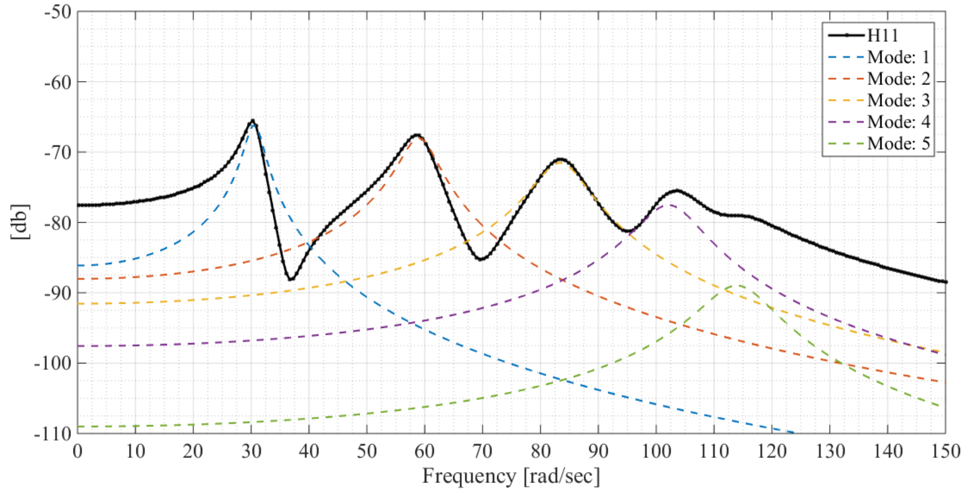


H11 - FRF

Input 1, Output 1 Driving Point

```
% index dof
in  = 1;    % H column index
out = 1;    % H row index

% form frf from residues and poles
[Hs,hh] = vibsFRF(AA,root,in,out,w);

% plot frf
fprintf('H%d%d Magnitude\n',out,in);
vibsFRFplot(Hs,hh,in,out,w);
snapnow
```

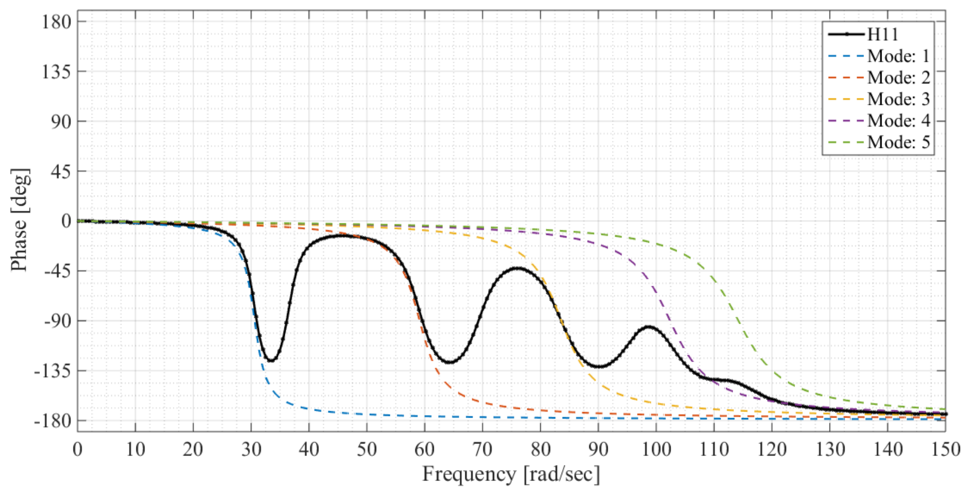
H11 Magnitude



H11 - Phase

```
% plot phase
fprintf('H%d%d Phase\n',out,in);
vibsPhaseplot(Hs,hh,in,out,w);
snapnow
```

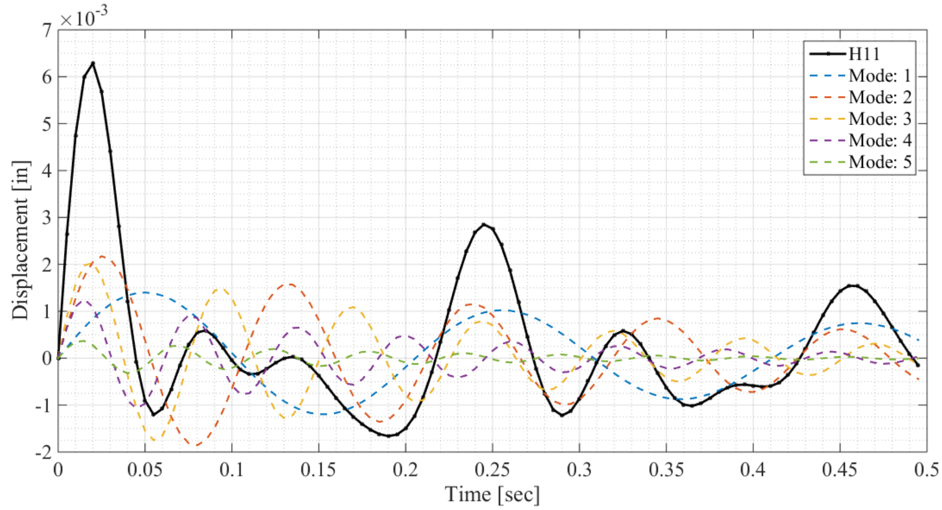
H11 Phase



H11 - IRF

```
% plot irf
fs = 200; % sampling freq
l = .5;     % length [sec]
[hs,h] = vibsIRF(AA,root,in,out,fs,l);
fprintf('H%d%d - Impulse Response Function\n',out,in);
vibsIRFplot(hs,h,in,out,fs,l);
snapnow
```

H11 - Impulse Response Function

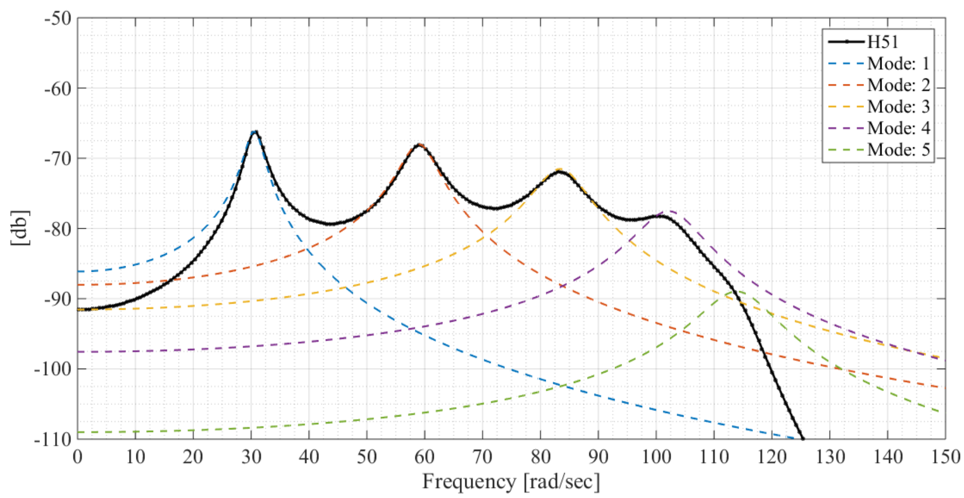


H51 - FRF

Input 1, Output 5 Symmetric DOF

```
% index dof
in  = 1;     % H column index
out = 5;     % H row index

% form frf from residues and poles
[Hs,hh] = vibsFRF(AA,root,in,out,w);

% plot frf
fprintf('H%d%d Magnitude\n',out,in);
vibsFRFplot(Hs,hh,in,out,w);
snapnow
```

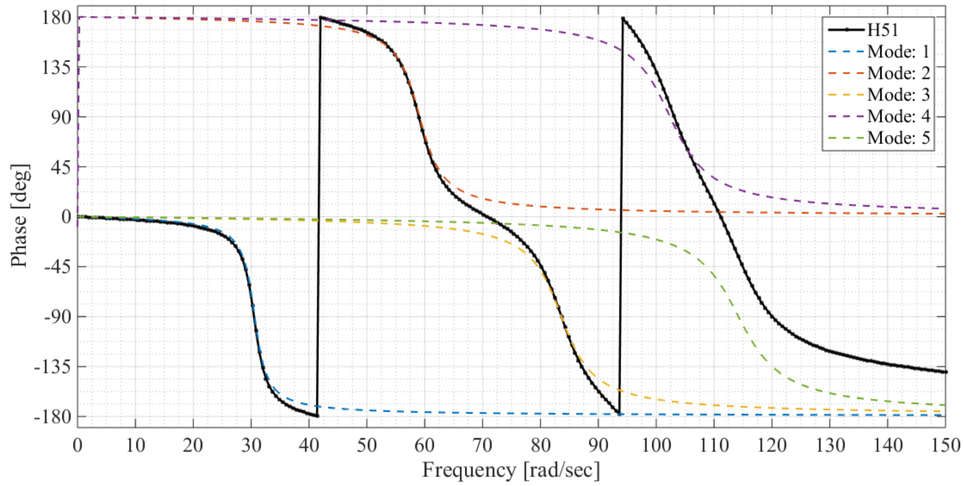
H51 Magnitude



H51 - Phase

```
% plot phase
fprintf('H%d%d Phase\n',out,in);
vibsPhaseplot(Hs,hh,in,out,w);
snapnow
```

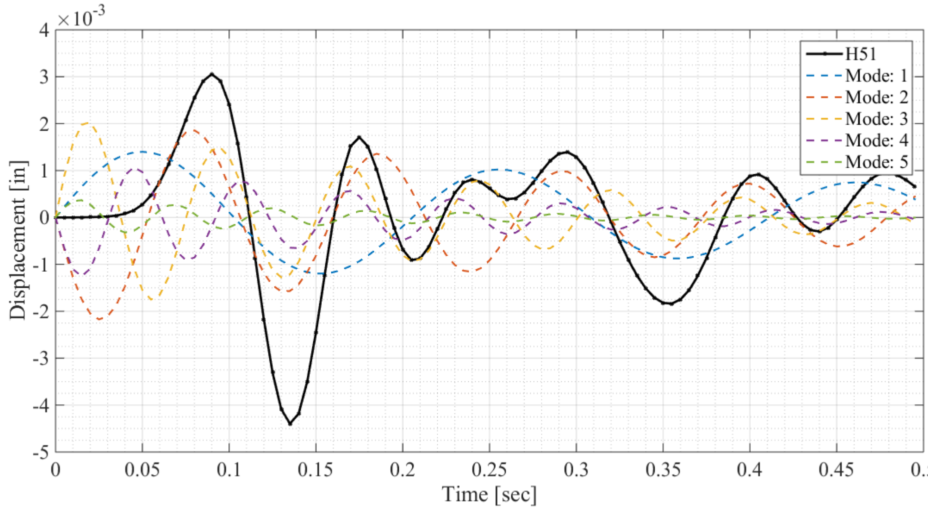
H51 Phase



H51 - IRF

```
% plot irf
fs = 200; % sampling freq
l = .5;    % length [sec]
[hs,h] = vibsIRF(AA,root,in,out,fs,l);
fprintf('H%d%d - Impulse Response Function\n',out,in);
fh = vibsIRFplot(hs,h,in,out,fs,l);
snapnow
```

H51 - Impulse Response Function



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