LECTURE 1: Introduction

Dynamic
- → General → "involving bodies in motion"
- → structures → "involving problems where acceleration and velocity cannot be ignore" (e.g. non-static problems)

→ Structural Dynamics

Excitation		Structure		Response
Live load Wind Seismic Waves Blast/Impact Rot. Machinery	→	Bldg. Bridge Dam Pipeline.	→	Rigid body motion + Body* Deformation

*Most str. dyn. is concerned here.

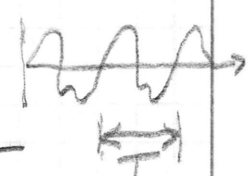
→ Excitation

→ Deterministic – excitation is accurately known.

vs

→ "Random" – only know certain characteristics of the excitation. (μ, δ)

→ Periodic – the excitation repeats in time.



Period = time to repeat, T

vs.

$1/T$ = frequency = f.

→ Non-periodic – the excitation does not repeat in time.

→ Live load – due to people, trucks, trains, etc.

→ "Random" in nature

shocks, roadway, surface, etc.

→ Static load + dyn. allowance.

used for design.

→ typ not analyzed dynamically.

→ Non-periodic (typ.)

→ except is special situations.

→ soldiers marching

→ human-structure interaction

→ synchronized motion
- cheering
- exercise, etc.

→ Wind load – due to drag & lift

→ "Random" in nature.

→ Low-rise structures
→ translate to static force.
→ dependent on velocity

→ Slender structures.
→ wind tunnel test

→ Non-periodic (ty)

→ except for "flutter" & aerodynamic instability.

→ <u>Siesmic</u> → due to release of tetonic forces

→ typically the largest forces civil structures experience

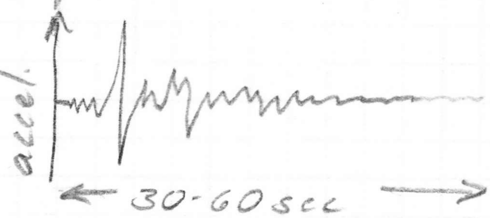
last eq. → Deterministic

↳ measured through seismographs "near-field" vs "far-field"

next eq. → "Random"

↳ synthetic ground motion generation.

→ Non-periodic.



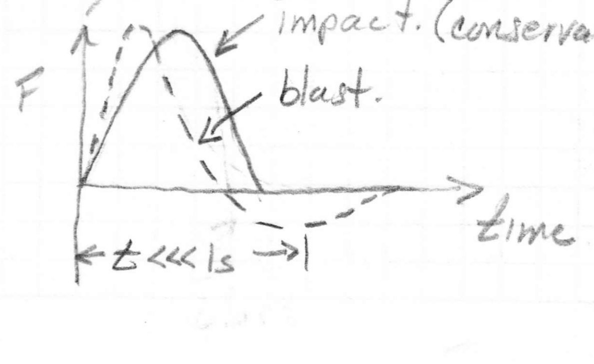
→ <u>Blast</u>/<u>Impact</u> → due to explosion of collision with a mass traveling @ a velocity.

→ Deterministic

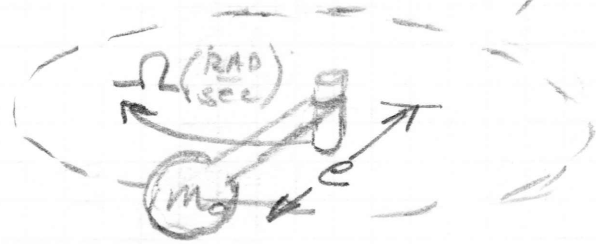
- if the size, type & location of explosion is known.
- if the mass, velocity, direction, stiffness, etc. of the impact mass is known.

Aside: Impacts are also used to test structures.
determine actual properties

→ Non-periodic.



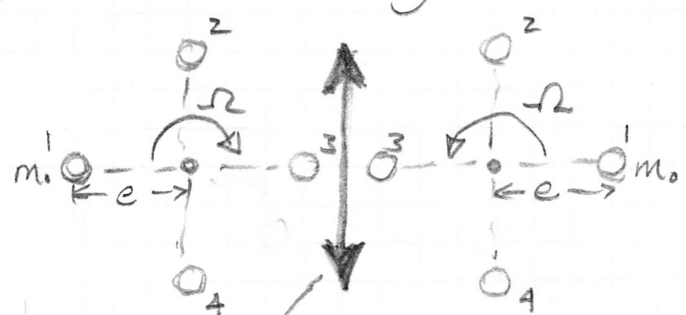
→ Rotating Machinery – due to mass rotating eccentrically or oscillating.



Aside: Rotating or linear shakers are also used for struct testing.

$P_0 = e m_0 \Omega^2$ → the direction of the force is always outward.

→ Balance rotating masses



(1) Forces cancel
(2) Forces add
(3) Forces cancel
(4) Forces add.

horiz.

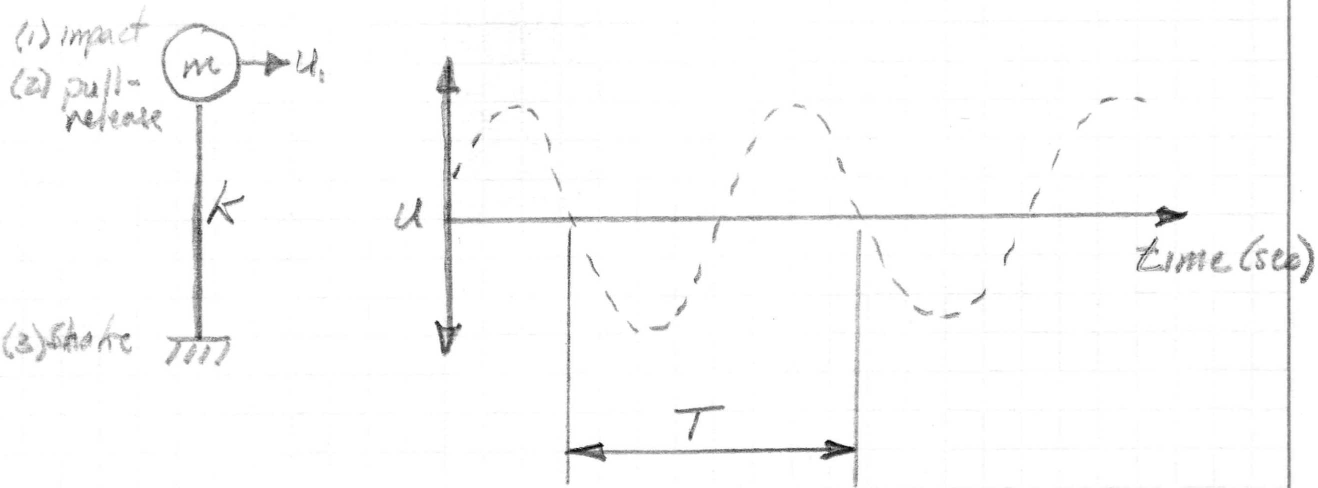
Plan view

resultant is a unidirectional force

$$P(t) = 2 e m_0 \Omega^2 \cos(\Omega t)$$

→ Deterministic
→ Periodic.

→ In Class Experiment



T = period (sec) ⇒ time required to go through a complete cycle.

f = frequency (1/sec or Hz) ⇒ number of complete cycles per second.

$$\boxed{f = \frac{1}{T}}$$

ω = circular frequency (Rad/sec)

⇒ Note: every 2π RAD a sine or cosine will go through 1 complete cycle.

$\boxed{\omega = 2\pi f}$ ⇒ motion can be modeled as

$$u \approx \underbrace{A \sin(\omega t)}_{\text{will have a frequency of } f \text{ (Hz)}}$$

→ Using (1) to (3) above (for excitation) determine if the frequency of vibration of the physical model changes.

→ Estimate the period, frequency & circular frequency of the physical models in the following states.

(1) Current configuration.
(2) Mass at mid-hieght
(3) 2 Masses @ mid-hieght
(4) 2 Masses @ the top.

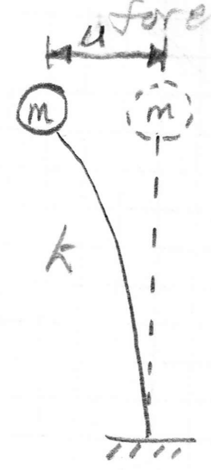
→ Comment on the trends.

→ <u>What</u> <u>Causes</u> <u>Vibration</u>???

⇒ Transfer of energy from potential to kinetic and back

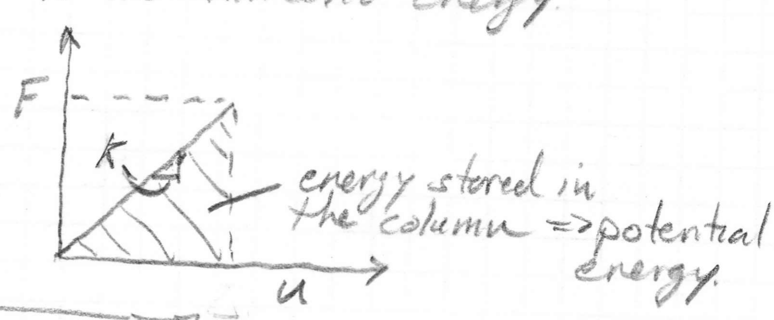
⇒ Without energy dissipation (damping) the resulting oscillations would continue forever.

(1)



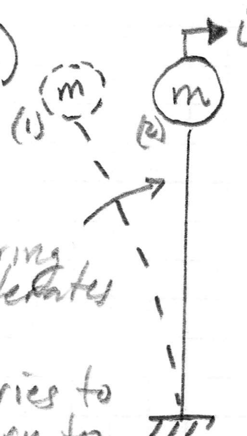
→ Mass displaced by Δ and held stationary.

→ Velocity is zero so there is no kinetic energy.



$\boxed{PE = \frac{1}{2} k u^2}$ (Note $F = ku$)

(2)



Spring accelerates mass
↳ tries to return to zero displacement. ("restoring" force)

→ Spring returns mass to zero displacement.

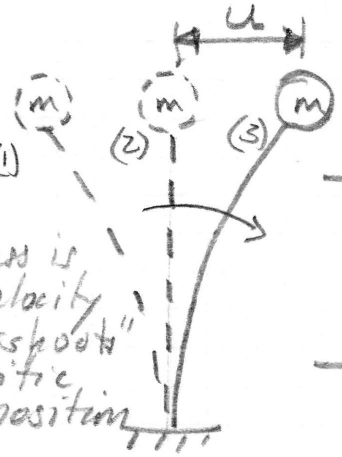
→ Where did the energy go?

→ Displacement is zero so there is no potential energy.

→ w/o damping, all the energy is transferred into kinetic energy ⇒ moving mass.

$$\boxed{KE = \frac{1}{2} m \dot{u}^2}$$

(3)



@(2) mass is at max velocity ⇒ "overshoots" the static equil. position

→ Mass overshoots static equilibrium and deforms the spring in the opposite direction.

→ To stop the mass, all kinetic energy must be transferred by to potential energy
→ assuming no damping.

$$\boxed{PE = \frac{1}{2} k \Delta^2}$$

↓

→ the spring the tries to return to zero disp. and thus accelerates the mass in the other direction.

→ ... this energy transfer process continues...

Total Energy = $PE(t) + KE(t)$

→ @ position (1) & (3)

$PE(t) = PE_{MAX}$ & $KE(t) = 0$

$PE_{MAX} = \frac{1}{2} K(u_{MAX})^2$ = Total Energy (1)

→ @ position (2)

$KE(t) = KE_{MAX}$ & $PE(t) = 0$

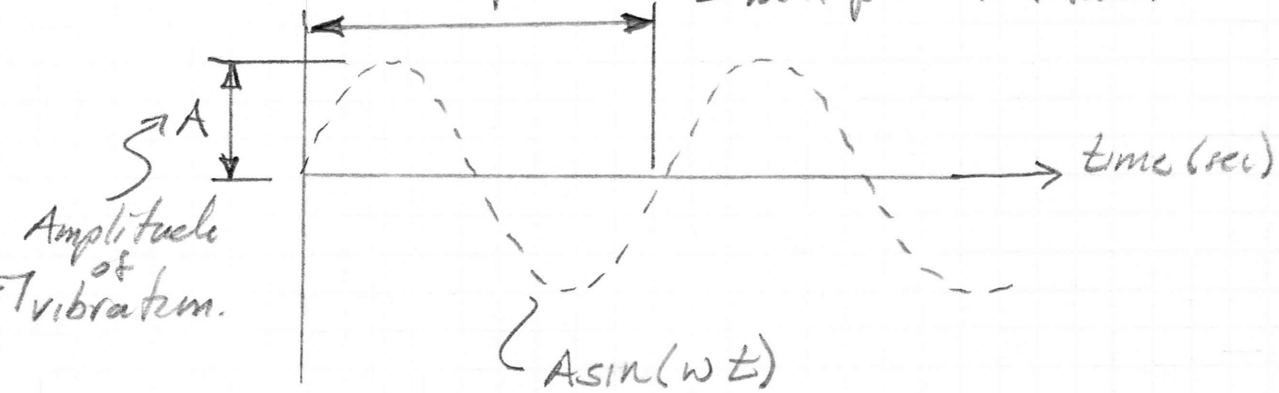
$KE_{MAX} = \frac{1}{2} m(\dot{u}_{MAX})^2$ = Total Energy (2)

→ (1) = (2)

$$\frac{1}{2} k(u_{MAX})^2 = \frac{1}{2} m(\dot{u}_{MAX})^2 \qquad (3)$$

⇒ Assume harmonic motion

↑ we'll prove this later.



↑ circular frequency = $2\pi f = \frac{2\pi}{T}$

$u = A\sin(\omega t)$

$\dot{u} = \frac{du}{dt} = A\omega\cos(\omega t)$

since sin & cos vary between -1 & 1

$$\left.\begin{aligned} u_{MAX} &= A \\ \dot{u}_{max} &= A\omega \end{aligned}\right\} \quad (4)$$

→ Subst. (4) ⇒ (3)

$$\frac{1}{2}k(A)^2 = \frac{1}{2}m(A\omega)^2$$

$$kA^2 = mA^2\omega^2$$

$$\boxed{\omega = \sqrt{\frac{k}{m}} \quad (\text{RAD/sec})} \; *$$

→ Observations

→ As stiffness increase, ω increases.

→ As mass increases, ω decreases.

→ Does this make sense???

→ Back eq (4), for $A = 1$.

$u_{MAX} = 1$

$\dot{u}_{MAX} = \omega$

velocity is dependent on A, but for a constant A, it is proportional to ω.

→ When mass increases, the velocity has to drop

Total Energy = Constant.

→ Since velocity drops, so does ω

$$\dot{u}_{MAX} \sim \omega$$

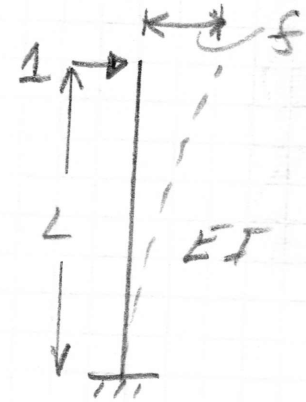
→ When stiffness increases, the velocity has to increase.

$$\text{Total Energy} = \frac{1}{2}k(u_{MAX})^2$$

$k\uparrow \Rightarrow$ Total Energy $\uparrow$

$$\therefore \dot{u}_{MAX}\uparrow \Rightarrow \omega\uparrow$$

→ Calculate ω



$$f = \int_0^L x \frac{x}{EI}\,dx = \left.\frac{x^3}{3EI}\right|_0^L$$

$$f = \frac{L^3}{3EI} \Rightarrow \underline{\underline{k = \frac{3EI}{L^3}}}$$

Single mass $= \dfrac{2.25\,lb}{g}$ $\quad (g = 386\ ^{in}/_{sec^2})$

$$I = \frac{\pi r^4}{4} = \frac{\pi (1/8)^4}{4} = 1.9175 \times 10^{-4}\,in^4$$

$$EI = (29{,}000{,}000\text{ PSI})(1.9175 \times 10^{-4}\,in^4) = 5{,}561\ lb \cdot in^2$$

→ Single mass @ top. ($L = 34''$, $m = 2.25/g$)

$$\omega = \sqrt{\frac{\frac{3(5561)}{(34)^3}}{\frac{2.25}{386}}} = \underline{8.53\ \text{RAD/SEC}}$$

→ Double mass @ top ($L = 32.5''$; $m = 4.5/g$)

$$\omega = \sqrt{\frac{\frac{3(5561)}{(32.5)^3}}{\frac{4.5}{386}}} = \underline{6.45\ \text{RAD/SEC}}$$