M6 3D RECOVERY OF URBAN SCENES - SESSION 3

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GOAL

The goal of this session is to learn and compute the fundamental matrix between two different views of the same picture.

1 INTRODUCTION

First of all, in order to compute the fundamental matrix F, it is necessary to solve the system shown in figure 1. In this system we will need to find 8 different equations as the matrix F has 8 different unknowns.

In order to compute the matrix W, which is the restriction matrix, we need to normalize the points p1 and p2 and compute it as shown in figure 1.

Finally, as a restriction of the system of equations, we cannot set the system equal to 0 as it would not give an optimum solution. In order to avoid it, the system is set as the minimization problem shown in figure 1.

$$\mathbf{W} \cdot \boldsymbol{F} = 0$$

$$\mathbf{W} = (x_{2i} \cdot x_{1i}, x_{2i} \cdot y_{1i}, x_{2i}, y_{2i} \cdot x_{1i}, y_{2i} \cdot y_{1i}, y_{2i}, x_{1i}, y_{1i}, 1)$$

$$min_{\boldsymbol{F}} \ \|\mathbf{W}\boldsymbol{F}\|_2 \ with \ \| \boldsymbol{F} \|_2 = 1$$

Figure 1: System of the Fundamental Matrix

In order to solve this minimization problem, it is necessary to use the SVD decomposition of the matrix W as can be seen in figure 2.

$$\mathbf{W} = UDV^T$$

Figure 2: SVD of the matrix W

As we know from theory that the last column of the matrix V from the SVD is equal to the vector f, this theorem gave us the matrix F. Once we have F (Named Fq in the figure 3) , it is necessary to denormalize it as can be seen in figure 3.

$$F = H'^T F_q H$$

Figure 3: Denormalized F

The principal problem is that as F would be rank 3, the epipolar lines would not coincide in the epipole. In order to solve this problem, it is necessary to pass F to a rank 2 using the SVD as can be seen in figure 4.

$$F_{RANK\_3} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

$$F_{RANK\_2} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$

Figure 4: Differences between Rank 3 and Rank 2 of matrix F

The consequences of using or not the SVD in F in order to convert it in rank 2 are shown in figure 5.

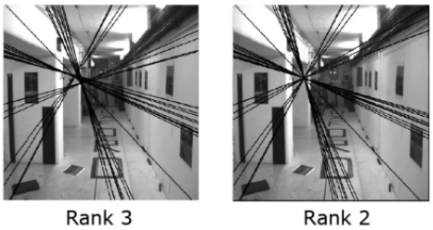

Figure 5: Differences between Rank 3 and Rank 2 of matrix F

In our case, after applying the steps mentioned before, the fundamental matrix F that we obtained was:

$$\begin{bmatrix} -1.9127 & -7.1383 & 3.6951 \\ 7.1383 & -1.9127 & -11.0852 \\ -0.7001 & 11.6639 & 0.0000 \end{bmatrix}$$

Figure 6: Resulting Matrix F

This matrix can be compared with the real fundamental Matrix that is obtained as shown in figure 7. In our case both K1 and K2 are equal to the Identity Matrix I because P1 and P2 are a Rotation and translation. The resulting real fundamental matrix that we obtained is shown in figure 8.

$$F=K'^{-T}[T'_X]RK^{-1}$$

Figure 7: Real Fundamental Matrix F

$$\begin{bmatrix} -0.0518 & -0.1932 & 0.1000 \\ 0.1932 & -0.0518 & -0.3000 \\ -0.0189 & 0.3157 & 0 \end{bmatrix}$$

Figure 8: Resulting Real Fundamental matrix F

Finally, after normalizing both matrices, they gave us the same result shown in figure 9.

$$\begin{bmatrix} -0.1383 & -0.5163 & 0.2673 \\ 0.5163 & -0.1383 & -0.8018 \\ -0.0506 & 0.8436 & 0 \end{bmatrix}$$

Figure 9: Resulting F

2 RESULTS

In our case, we will work with the images shown in figure 10 which correspond to the sequence castle.

Figure 10: Images of the sequence Castle

First of all, it is necessary to compute the correspondences of the Keypoints between the two images and set the optimum direction with RANSAC. These results can be seen in figure 11.

After applying RANSAC, we can compute the 8th point algorithm in order to compute the epipolar lines and the Fundamental Matrix. The Fundamental Matrix estimated that we obtained is shown in figure 6. Then, we chose three random points in the image in order to compute the epipolar lines (these points will be part of the three epipolar lines that are shown in the figures). In figure 12 it can be seen how we can see a point in one image and its correspondence epipolar line in the other one.

(a) Keypoints with SIFT

(b) Keypoints with RANSAC

Figure 11: Keypoints before and after RANSAC

Figure 12: Resulting F

Finally, in figures 13 and 14 we can see the epipolar lines of the different random points chosen. We can now which point is related to each line because of the color of the points.

Figure 13: Resulting F

Figure 14: Resulting F

3 PHOTO SEQUENCING

Photo sequencing paper presents an interesting problem. When a group of people taking pictures of a dynamic event with their mobile phones we can obtain a lot of information about the event. Thus, the set of still images obtained this way is rich in dynamic content but lacks accurate temporal information. They proposed a method for photo-sequencing-temporally ordering a set of still images taken asynchronously by a set of uncalibrated cameras.
As an optional part we were asked to compute a simplified version of this algorithm in order to extract the correct temporal ordering of images in figure 15.

Since we do not have two images taken from the same viewpoint at two

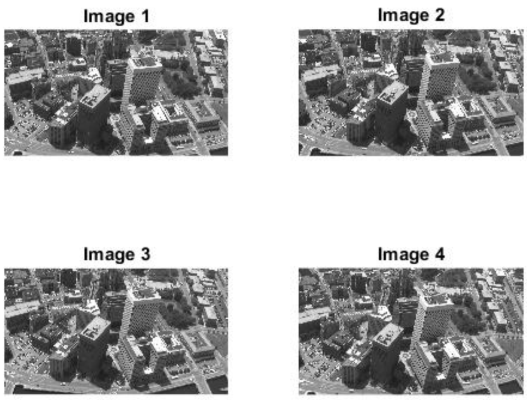

Figure 15: Set of images before ordering

different time instants, we used a manually picked point corresponding to a point in a van and the projection of its 3D trajectory in the reference image.

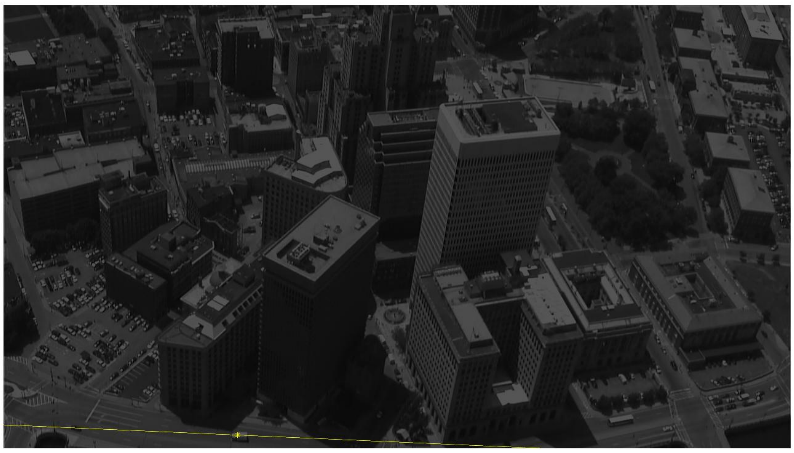

Figure 16: Line between the van point and a point in the 3D trajectory

The line represented in figure 16 intersects with the epipolar lines corresponding to the same dynamic point in the others images. Given as a result the figure 17. This figure has different lines intersecting with the trajectory line (yellow line). The points from intersections will gives us the order of the sequence of images. The final order is, image 1, image 2, image 3 and image 4.

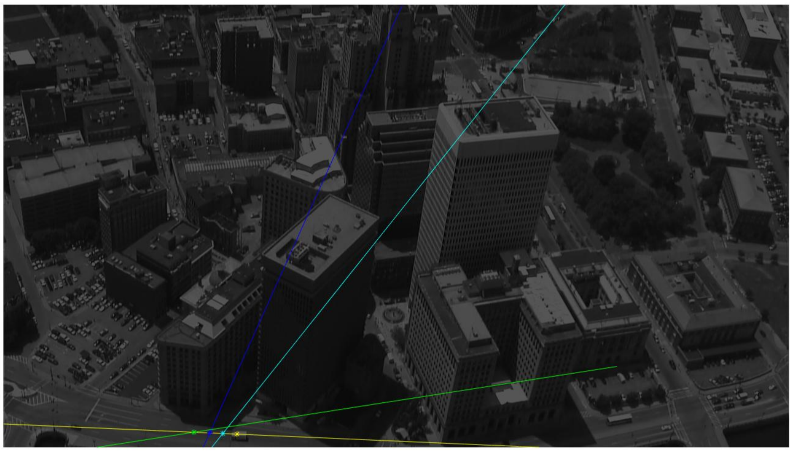

Figure 17: Trajectory line (yellow) intersects epipolar lines from image 2 (cyan), image 3 (blue) and image 4 (green)

REFERENCES

[1] R.I. Hartley and A. Zisserman, Multiple View Geometry in Computer Vision, Cambridge University Press, 2004.