Week 8 Worksheet

1. Cylindrical Coordinates

Evaluate the following integral:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$$

2. Spherical Coordinates

2.1 Evaluate the following integral:

$$\int_{0}^{2\pi}\int_{0}^{\pi/4}\int_{0}^{2}(\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

2.2 Find the average value of the function $f(r,\theta,z)=r$ over the solid ball bounded by the sphere $r^2+z^2=1$ (that is, the ball bound by $x^2+y^2+z^2=1$).

3. Generalized Coordinate Transforms

Evaluate the integral:

$$\int_{0}^{2/3}\int_{y}^{2-2y}(x+2y)e^{y-x}\,dx\,dy$$

4. Line Integrals

4.1 Evaluate the following integral along the given curve:

$$\int_C \sqrt{x^2+y^2}\,ds,\quad \boldsymbol{r}(t)=(4\cos t)\boldsymbol{i}+(4\sin t)\boldsymbol{j}+3t\boldsymbol{k},\ -2\pi\le t\le 2\pi$$

4.2 Find the line integral of

$$f(x,y,z)=\frac{\sqrt{3}}{x^2+y^2+z^2}$$

over the curve $\boldsymbol{r}(t)=t\boldsymbol{i}+t\boldsymbol{j}+t\boldsymbol{k}$ and interval $1\le t\le\infty$.