Week 10 Worksheet

1. Green's Theorem

Find the counterclockwise circulation and outward flux of the field

$$\boldsymbol{F} = (x^2 + 4y)\boldsymbol{i} + (x + y^2)\boldsymbol{j}$$

about the square bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1$.

2. Surface Integrals

2.1 Find the area of the portion of the plane $z = -x$ inside the cylinder $x^2 + y^2 = 4$.

2.2 Integrate $G(x, y, z) = z$ over the cylindrical surface $y^2 + z^2 = 0$, $z \geq 0$, $1 \leq x \leq 4$.

3. Flux Integrals

Find the flux of

$$\mathbf{F}(x, y, z) = -y\boldsymbol{i} + x\boldsymbol{j}$$

over the portion of a sphere of radius a centered at the origin in the first octant in the direction away from the origin.

4. Stokes' Theorem

4.1 Calculate the circulation of

$$\boldsymbol{F} = 2y\boldsymbol{i} + 3x\boldsymbol{j} - z^2\boldsymbol{k}$$

about the circle of radius 3 centered at the origin counterclockwise when viewed from above.

4.2 Calculate the flux of the curl of

$$\boldsymbol{F} = (y - z)\boldsymbol{i} + (z - x)\boldsymbol{j} + (x + z)\boldsymbol{k}$$

in the direction of the outward unit normal of the surface $S : \boldsymbol{r}(r, \theta) = r\cos(\theta)\boldsymbol{i} + r\sin(\theta)\boldsymbol{j} + (9 - r^2)\boldsymbol{k}$, $0 \leq r \leq 3$ and $0 \leq \theta \leq 2\pi$.