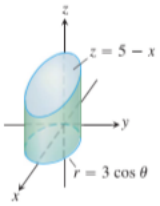
Final Review Problems

1. Triple Integrals

1. (TC 15.6 #14) Find the center of mass and moment of inertia about the x-axis of a thin plate bounded by the curves $x = y^2$ and $x = 2y - y^2$ with density $\delta(x, y) = y + 1$.
2. (TC 15.7 #16) Find the volume of the shape in the following figure.



3. (TC 15.7 #33) Find the volume of the solid between the sphere $\rho = \cos\phi$ and hemisphere $\rho = 2, z \geq 0$.

2. Generalized Coordinate Transform

1. (TC 15,8 #2) Find the value of the Jacobian $\partial(x, y)/\partial(u, v)$ for the system:

$$u = x + 2y$$
$$v = x - y$$

2. (TC 15.8 #14) Evaluate the following integral:

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x - y)e^{(2x-y)^2}\, dx\, dy$$

3. Line Integrals

1. (TC 16.1 #12) Evaluate $\int_C \sqrt{x^2 + y^2}\, ds$ along the curve $\boldsymbol{r}(t) = 4\cos t\,\boldsymbol{i} + 4\sin t\,\boldsymbol{j} + 3t\,\boldsymbol{k}$ for $-2\pi \leq t \leq 2\pi$.
2. (TC 16.1 #28) Evaluate $f(x, y) = \frac{x+y^2}{\sqrt{1+x^2}}$ over the curve $C : y = x^2/2$ from $(1, 1/2)$ to $(0, 0)$.

4. Work

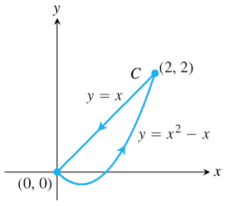
1. (TC 16.2 #20) $\boldsymbol{F} = 2y\,\boldsymbol{i} + 3x\,\boldsymbol{j} + (x + y)\boldsymbol{k}$ over $\boldsymbol{r}(t) = \cos t\,\boldsymbol{i} + \sin t\,\boldsymbol{j} + (t/6)\boldsymbol{k}$, $0 \leq t \leq 2\pi$.
2. (TC 16.3 #8) Find the work done by the field $\boldsymbol{F} = (y + z)\boldsymbol{i} + (x + z)\boldsymbol{j} + (x + y)\boldsymbol{k}$ when moving on a linear path from $(0, 0, 0)$ to $(2, 4, 5)$.
3. (TC 16.3 #28) Find the potential function for $\boldsymbol{F} = e^x \ln y\,\boldsymbol{i} + \left(\frac{e^x}{y} + \sin z\right)\boldsymbol{j} + y\cos z\,\boldsymbol{k}$.

5. Flux

1. (TC 16.2 #32) Find the flux of the field $\boldsymbol{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j}$ about the closed semicircular path of radius 2 in the upper half plane.

6. Green's Theorem

1. (TC 16.4 #11) Find the circulation and flux of $\boldsymbol{F} = x^3y^2\boldsymbol{i} + \frac{1}{2}x^4y\boldsymbol{j}$ about the path in the following figure:



2. (TC 16.4 #12) Find the circulation and flux of the field $\boldsymbol{F} = \frac{x}{1+y^2}\boldsymbol{i} + \tan^{-1} y\boldsymbol{j}$. about the unit circle centered at the origin.

7. Surface Integrals

1. (TC 16.5 #38) Find the area of the band cut from the paraboloid $x^2 + y^2 - z = 0$ by the plans $z = 2$ and $z = 6$.

2. (TC 16.6 #8) Integrate $H(x, y, z) = yz$ over the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

8. Flux in 3D

1. (TC 16.6 # 22) Find the outward flux of $\boldsymbol{F} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ across the sphere of radius 1.

9. Stokes' Theorem

1. (TC 16.7 #2) Find the circulation of $\boldsymbol{F} = 2y\boldsymbol{i} + 3x\boldsymbol{j} - z^2\boldsymbol{k}$ about the circle of radius 3 in the (x, y) plane centered at the origin in the counterclockwise direction.

2. (TC 16.7 #18) Find the flux of the curl of $\boldsymbol{F} = y^2\boldsymbol{i} + z^2\boldsymbol{j} + x\boldsymbol{k}$ across the surface $\boldsymbol{r}(\phi, \theta) = 2\sin\phi\cos\theta\boldsymbol{i} + 2\sin\phi\sin\theta\boldsymbol{j} + 2\cos\phi\boldsymbol{k}$, $0 \le \phi \le \pi/2$, $0 \le \theta \le 2\pi$.

10. Divergence Theorem

1. (TC 16.8 #6) Find the outward flux of $\boldsymbol{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$ across the boundary of the unit cube in the first octant.

2. (TC 16.8 #15) Find the flux of $\boldsymbol{F} = (5x^3 + 12xy^2)\boldsymbol{i} + (y^3 + e^y\sin z)\boldsymbol{j} + (5z^3 + e^y\cos z)\boldsymbol{k}$ across the boundary of the volume between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 2$.