

## Week 6 Worksheet

### 1. Constrained Optimization

1.1 Extrema on a circle Find the extrema of  $f(x, y) = xy$  subject to  $x^2 + y^2 = 10$ .

1.2 Constrained minimum Find the points on the curve  $xy^2 = 2$  nearest the origin.

1.3 Maximizing a product Find the largest product the positive numbers  $x$ ,  $y$ , and  $z$  can have if  $x^2 + y^2 + z^2 = 16$ .

### 2. Constrained Variables

If  $f = x^2 + y - z + \sin t$  and  $x + y = t$ , find:

(a)  $\left(\frac{\partial f}{\partial y}\right)_{x,z}$

(b)  $\left(\frac{\partial f}{\partial y}\right)_{z,t}$

(c)  $\left(\frac{\partial f}{\partial z}\right)_{x,y}$

### 3. Multi-dimensional Integrals

3.1 Compute the following iterated integrals.

(a)  $\int_0^1 \int_1^2 xy e^x dy dx$

(b)  $\int_{-\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y) dx dy$

(c)  $\int_0^4 \int_1^2 \frac{\sqrt{x}}{y^2} dy dx$

3.2 Find the volume of the region bounded above by the elliptical paraboloid  $z = 16 - x^2 - y^2$  and below by the square  $R : 0 \leq x \leq 2, 0 \leq y \leq 2$ .

3.3 Evaluate by reversing the order of integration:  $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x dy dx$

3.4 Find the average height of the paraboloid  $z = x^2 + y^2$  over the square  $0 \leq x \leq 2, 0 \leq y \leq 2$ .