Week 9 Worksheet

1. Work

1.1 Find the work done by the force $\boldsymbol{F} = xy\boldsymbol{i} + (y-x)\boldsymbol{j}$ over the straight line from $(1,1)$ to $(2,3)$.

1.2 Find the work done by the gradient of

$$f(x,y) = (x+y)^2$$

counterclockwise around the circle $x^2 + y^2 = 4$ from $(2,0)$ to itself.

2. Flux & Circulation

Find the flux of the fields:

$$\boldsymbol{F}_1 = 2x\boldsymbol{i} - 3y\boldsymbol{j}, \qquad \boldsymbol{F}_2 = 2x\boldsymbol{i} + (x-y)\boldsymbol{j}$$

about the path:

$$r(t) = a\cos(t)\boldsymbol{i} + a\sin(t)\boldsymbol{j},\ 0 \le t \le 2\pi$$

3. Conservative Fields & Potential Functions

3.1 Determine which of the following fields are conservative fields.

(a) $\boldsymbol{F} = y\sin(z)\boldsymbol{i} + x\sin(z)\boldsymbol{j} + xy\cos(z)\boldsymbol{k}$

(b) $\boldsymbol{F} = -y\boldsymbol{i} + x\boldsymbol{j}$

(c) $\boldsymbol{F} = e^x\cos(y)\boldsymbol{i} - e^x\sin(y)\boldsymbol{j} + z\boldsymbol{k}$

3.2 Evaluate the line integral:

$$\int_{(-1,-1,-1)}^{(2,2,2)} \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}$$

3.3 Evaluate the line integral:

$$\int_{(0,2,1)}^{(1,\pi/2,2)} 2\cos(y)dx + \left(\frac{1}{y} - 2x\sin(y)\right)dy + \frac{1}{z}dz$$