Week 5 Worksheet

1. Limits of Functions of Several Variables

Compute the following limits:

(a) $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$

(b) $\lim_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x}$

2. Partial Derivatives

2.1 Compute $\partial f/\partial x$ and $\partial f/\partial y$.

(a) $f(x,y)=\sqrt{x^2+y^2}$

(b) $f(x,y)=\frac{x}{x^2+y^2}$

2.2 Chain rule Find dy/dx of the following function:

$$x^3-2y^2+xy=0$$

3. Gradients

3.1 Compute the gradient of the following functions:

(a) $f(x,y)=\tan^{-1}\frac{\sqrt{x}}{y}$

(b) $f(x,y)=\frac{x^2}{2}-\frac{y^2}{2}$

(c) $f(x,y)=\ln(x^2+y^2)$

3.2 Find the directional derivative of $h(x,y)=\tan^{-1}(y/x)+\sqrt{3}\sin^{-1}(xy/2)$ at $P_0(1,1)$ in the direction $\boldsymbol{u}=3\boldsymbol{i}-2\boldsymbol{j}$.

4. Unconstrained Optimization

4.1 Find the extrema and saddle points of the following functions over the region (if given) and state if it is a minimum, maximum, or saddle.

(a) $f(x,y)=5xy-7x^2+3x-6y+2$

(b) $f(x,y)=x^4+y^4+4xy$

4.2 Find a and b that maximizes the integral $\int_a^b (24-2x-x^2)dx$