Week 2 Worksheet

1. Unit Vectors

1.1 Describe conceptually what a unit vector is.

1.2 Compute the unit vectors for the following vectors.

(a) $\vec{v} = \langle 1, 1, 1 \rangle$

(b) $\vec{w} = \langle 0, -1, -1 \rangle$

(c) $\vec{u} = \langle 10, 8, -7 \rangle$

1.3 The unit vector from point $A = (0, 2, 3)$ to $B = (1, 6, -2)$.

1.4 If $\vec{v}$ and $\vec{w}$ are orthogonal, will their unit vectors also be orthogonal? Why?

2. Vector Operations

Compute the following.

$$\vec{v} = \langle 1, 2, 5 \rangle, \quad \vec{w} = \langle 3, -4, 2 \rangle$$

(a) $2\vec{v} - \vec{w}$

(b) $(2\vec{v}) \cdot \vec{w}$

(c) The unit vector of $\vec{v} \times \vec{w}$

$$\vec{v} = \langle 1, 0, 0 \rangle, \quad \vec{w} = \langle \sqrt{3}, \sqrt{3}, \sqrt{3} \rangle$$

(d) $\vec{v} \cdot \left(\frac{1}{\sqrt{3}} \vec{w} \right)$

(e) $\vec{w} \cdot \vec{w}$

(f) The angle between $\vec{v}$ and $\vec{w}$.

3. Projections

Compute the following.

$$\vec{v} = \langle 1,2,5 \rangle, \quad \vec{w} = \langle 3,-4,2 \rangle$$

(a) $\text{proj}_{\vec{v}}\, \vec{w}$

(b) $\text{proj}_{\vec{w}}\, \vec{v}$

$$\vec{v} = \langle 1,0,0 \rangle, \quad \vec{w} = \langle \sqrt{3}, \sqrt{3}, \sqrt{3} \rangle$$

(c) $\text{proj}_{\vec{v}}\, \vec{w}$

(d) $\text{proj}_{\vec{w}}\, \vec{v}$

4. Lines and Planes

4.1 Compute the area enclosed by the parallelogram defined by:

$$A(0,0) \quad B(7,3) \quad C(9,8) \quad D(2,5)$$

4.2 Compute the area enclosed by the triangle defined by:

$$A(0,0) \quad B(-2,3) \quad C(3,1)$$

4.3 Find the equation for the line through $(1,2,1)$ in the direction of $\vec{v} = \langle 0,1,0 \rangle$

4.4 Find the equation for the plane through $(1,2,1)$ with normal $\vec{n} = \langle -1,0,1 \rangle$

5. Vector-valued functions

Compute the velocity and acceleration vectors of the following. In (c), also compute the tangent vector at the given point.

(a) $\vec{r}(t) = (1+t)\vec{i} + \frac{t^2}{\sqrt{2}}\vec{j} + \frac{t^3}{3}\vec{k}$

(b) $\vec{r}(t) = \sec(t)\vec{i} + \tan(t)\vec{j} + t\,\vec{k}$

(c) $\vec{r}(t) = \ln(t)\vec{i} + \frac{t-1}{t+2}\vec{j} + t\ln(t)\vec{k}$, and $t_0 = 1$