Midterm 1 Review Questions

1. The Algebra of Vectors

2. Dot Products

3. Cross Products

4. Lines and Planes

5. Vector-Valued Functions

6. Velocity and Acceleration Vectors

1. Given

$$\vec{r}(t) = (4\cos t)\vec{i} + (4\sin t)\vec{j} + 3t\vec{k}$$

find the arc length from $(4,0,0)$ to $(0,4,\frac{3}{2}\pi)$ (Problem 11 from Section 13.3 of textbook)

2. Given the parameterized trajectory

$$\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + (1-\cos t)\vec{k}$$

determine whether the acceleration of the trajectory is always parallel to the plane described by $x+z=1$ (similar Problem 17.c from Section 13.3 of textbook)

7. $\vec{T}$, $\vec{N}$, and $\vec{B}$ Vectors, Curvature, and Torsion

1. Find $\vec{T}$, $\vec{N}$, and κ (curvature) for the space curve

$$\vec{r}(t) = (e^t\cos t)\vec{i} + (e^t\sin t)\vec{j} + 2\vec{k}$$

(Problem 11 from Section 13.4 of textbook)

2. For the same trajectory, find $\vec{B}$, and τ (torsion)
(Problem 11 from Section 13.4 of textbook)
Hint: thinking of torsion intuitively might save you a bunch of time on this one...

8. Matrix Operations

1. Assuming that $\boldsymbol{A}$ and $\boldsymbol{A}^T$ is invertible, prove that

$$(\boldsymbol{A}^T)^{-1} = (\boldsymbol{A}^{-1})^T$$

2. Compute the inverse of the following matrix:

$$A = \begin{bmatrix} 3 & 10 \\ 3 & 3 \end{bmatrix}$$

3. Compute the determinant of the following matrix:

$$A = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

9. MATLAB

1. The `A = diag(v)` function in MATLAB takes a vector `v` and outputs a matrix `A` such that `v` is on the diagonal. In MATLAB, we often want to *vectorize* our computations such that we avoid loops and write our code as matrix multiplications, so a function such as `diag()` is extremely useful.

   Suppose we have a column vector v, and we want to build a matrix B with n rows such that v^T is every row of the matrix. Write a piece of MATLAB code that does this using

   (a) Two loops

   (b) One loop

   (c) No loops

   Assume that `v` and `n` have already been declared and are stored in MATLAB's memory.

2. Recall the interpolation problem discussed in the week 4 ACE worksheet. In this problem, we are given data points (x_1, y_1), (x_2, y_2), ... , (x_n, y_n), and want to fit an n^{th} degree polynomial such that:

$$c_0 + c_1 x_i + c_2 x_i^2 + \cdots + c_n x_i^n = y_i$$

   for all $i = 1, ..., n$.

   (a) Write this as a matrix equation. (This was the same question as in the linear algebra review sheet.)

   (b) For a given system $\boldsymbol{Ax} = \boldsymbol{b}$, the *backslash operator* will solve a system as `x = A\b`.
   Given `x` and `y` vectors (assume these are already the MATLAB virtual machine's memory), write code to generate the `A` matrix for the interpolation problem stated above.