ECE551 - Homework 6

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1 DTFT of Auto-correlation and Cross-correlation

$$\begin{aligned}
C_{x,y}(\omega) &= \sum_n c_{x,y}[n] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[x[n]y[n]\right] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[x[n](x[n]+w[n])\right] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[x[n]x[n]\right] e^{-jn\omega} + \sum_n \mathbb{E}\left[x[n]w[n]\right] e^{-jn\omega} \\
&= \sum_n a_x[n] e^{-jn\omega} \qquad (\because x[n], w[n] \text{ are uncorrelated}) \\
&= A_x(\omega)
\end{aligned}$$

$$\begin{aligned}
A_y(\omega) &= \sum_n a_y[n] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[y[n]y[n]\right] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[(x[n]+w[n])(x[n]+w[n])\right] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[x[n]x[n]\right] e^{-jn\omega} + \sum_n \mathbb{E}\left[w[n]w[n]\right] e^{-jn\omega} + \sum_n 2\mathbb{E}\left[x[n]w[n]\right] e^{-jn\omega} \\
&= \sum_n \mathbb{E}\left[x[n]x[n]\right] e^{-jn\omega} + \sum_n \mathbb{E}\left[w[n]w[n]\right] e^{-jn\omega} \\
&= A_x(\omega) + A_w(\omega)
\end{aligned}$$

2 Higly Correlated Random Processes

(a)

$$x_1[n] = \begin{cases} A & \text{even } n \\ B & \text{odd } n \end{cases}$$

$$x_2[n] = \begin{cases} A & n \geq 0 \\ B & n < 0 \end{cases}$$

$$\begin{cases} x_3[n+1] = \frac{1}{2}x_3[n] + A \\ x_3[0] = A \end{cases}$$

(b)

3 Adaptive Filter and LMS

4 Regularized Wiener Filter and Leaky LMS

5 Python Problem - Wiener's LMS

6 Python Problem - AR System Identification