ECE551 - Homework 3-4

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1 Deterministic Correlation Sequences

(a) It is obvious that σ is a delay operator, i.e. $\sigma = T_1$

$$\begin{aligned} \sigma^k x &= (\sigma(\sigma(...(\sigma x)))) \qquad (k \text{ times of } \sigma) \\ \Rightarrow \sigma^k x &= T_k x \\ \Rightarrow (\sigma^k x)[n] &= x[n-k]. \end{aligned}$$

Similarly, $\sigma^{-1}x = T_{-1}x \Rightarrow (\sigma^{-1}x)[n] = x[n+1]$.

(b) Prove or disprove:

i.

$$\begin{aligned} a_x[-k]^* &= \left\langle x, \sigma^k x \right\rangle^* = \left\langle \sigma^k x, x \right\rangle = \sum_{n\in\mathbb{Z}} x[n]^* x[n-k] \\ &= \sum_{n\in\mathbb{Z}} x[n+k]^* x[n-k+k] = \sum_{n\in\mathbb{Z}} x[n+k]^* x[n] = a_x[k] \end{aligned}$$

Hence, $a_x[k] = a_x[-k]^*$.

ii.

iii.

$$\begin{aligned} c_{y,x}[-n]^* &= \left(\sum_{i\in\mathbb{Z}} y[i]x[i+n]^* \right)^* = \sum_{i\in\mathbb{Z}} x[i+n]y[i]^* \\ &= \sum_{i\in\mathbb{Z}} x[i+n-n]y[i-n]^* = \sum_{i\in\mathbb{Z}} x[i]y[i-n]^* = c_{x,y}[n] \end{aligned}$$

Hence, $c_{x,y}[n] = c_{y,x}[-n]^*$.

iv.

$$c_{x,y}[-n]^* = \left(\sum_{i\in\mathbb{Z}} x[i]y[i+n]^*\right)^* = \sum_{i\in\mathbb{Z}} x[i]^*y[i+n]$$
$$= \sum_{i\in\mathbb{Z}} x[i-n]^*y[i] = c_{y,x}[n] \neq c_{x,y}[n].$$

Hence, $c_{x,y}[n] \neq c_{x,y}[-n]^*$.

v.

(c)

2 Studying yet another Linear System

(a)

(b)

3 DTFT Affairs

(a)

(b)

4 The Z-Transform of Autocorrelation

(a)

(b)

5 Some DFT Properties

(a)

(b)

(c)

(d)

(e)

6 Z-Transform of Downsampled Signals

7 Interchange of Multirate Operations and LTI Filtering

(a)

(b)

(c)