UPPAAL-SMC: Statistical Model Checking for Priced Timed Automata *

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This paper offers a survey of UPPAAL-SMC, a major extension of the real-time verification tool UPPAAL. UPPAAL-SMC allows for the efficient analysis of performance properties of networks of priced timed automata under a natural stochastic semantics. In particular, UPPAAL-SMC relies on a series of extensions of the statistical model checking approach generalized to handle real-time systems and estimate undecidable problems. UPPAAL-SMC comes together with a friendly user interface that allows a user to specify complex problems in an efficient manner as well as to get feedback in the form of probability distributions and compare probabilities to analyze performance aspects of systems. The focus of the survey is on the evolution of the tool – including modeling and specification formalisms as well as techniques applied – together with applications of the tool to case studies.

1 Introduction

Quantitative properties of stochastic systems are usually specified in logics that allow one to compare the measure of executions satisfying certain temporal properties with thresholds. The model checking problem for stochastic systems with respect to such logics is typically solved by a numerical approach [3, 14] that iteratively computes (or approximates) the exact measure of paths satisfying relevant sub-formulas; the algorithms themselves depend on the class of systems being analyzed as well as the logic used for specifying the properties.

Another approach to solve the model checking problem is to *simulate* the system for finitely many runs, and use *hypothesis testing* to infer whether the samples provide a *statistical* evidence for the satisfaction or violation of the specification [40]. The crux of this approach is that since sample runs of a stochastic system are drawn according to the distribution defined by the system, they can be used to get estimates of the probability measure on executions. Those techniques, also called *Statistical Model Checking techniques* (SMC) [26, 36, 40, 35], can be seen as a trade-off between testing and formal verification. In fact, SMC is very similar to Monte Carlo used in industry, but it relies on a formal model of the system. The core idea of SMC is to monitor a number of simulations of a system whose behaviors depend on a stochastic semantic. Then, one uses the results of statistics (e.g. sequential hypothesis testing or Monte Carlo) together with the simulations to get an overall estimate of the probability that the system

*The paper is supported by VKR Centre of Excellence – MT-LAB and the IDEA4CPS center established on a grant from Danish National Research Foundation

M. Massink and H. Wiklicky (Eds.):Workshop on Quantitative Aspects of Programming Languages and Systems 2012 (QAPL 2012)
EPTCS 85, 2012, pp. 1–16, doi:10.4204/EPTCS.85.1

will behave in some manner. While the idea resembles the one of classical Monte Carlo simulation, it is based on a formal semantic of systems that allows us to reason on very complex behavioral properties of systems (hence the terminology). This includes classical reachability properties such as “can I reach such a state?”, but also non trivial properties such as “can I reach this state x times in less than y units of time?”. Of course, in contrast with an exhaustive approach, such a simulation-based solution does not guarantee a result with 100% confidence. However, it is possible to bound the probability of making an error. Simulation-based methods are known to be far less memory and time intensive than exhaustive ones, and are sometimes the only option [41, 27].

Statistical model checking is now widely accepted in various research areas such as software engineering, in particular for industrial applications [5, 33, 18], or even for solving problems originating from systems biology [17, 29]. There are several reasons for this success. First, SMC is very simple to understand, implement, and use. Second, it does not require extra modeling or specification effort, but simply an operational model of the system, that can be simulated and checked against state-based properties. Third, it allows us to verify properties [15, 16, 5] that cannot be expressed in classical temporal logics. Finally, SMC allows to approximate undecidable problems. This latter observation is crucial. Indeed most of emerging problems such as energy consumption are undecidable [24, 9] and can hence only be estimated. SMC has been applied to a wide range of problems that goes from embedded systems[15] and systems biology [15, 16] to more industrial applications [5].

In a series of recent works [22, 13, 21], we have investigated the problem of Statistical Model Checking for networks of Priced Timed Automata (PTA). PTAs are timed automata, whose clocks can evolve with different rates, while[1] being used with no restrictions in guards and invariants. In [21], we have proposed a natural stochastic semantic for such automata, which allows to perform statistical model checking. Our work has later been implemented in UPPAAL-SMC, that is a stochastic and statistical model checking extension of UPPAAL. UPPAAL-SMC relies on a series of extensions of the statistical model checking approach generalized to handle real-time systems and estimate undecidable problems. UPPAAL-SMC comes together with a friendly user interface that allows a user to specify complex problems in an efficient manner as well as to get feedback in the form of probability distributions and compare probabilities to analyze performance aspects of systems.

The objective of this paper is to offer a survey of UPPAAL-SMC. This includes modeling and specification formalism as well as techniques applied – together with applications of the tool to case studies.

Structure of the paper In Section 2, we introduce the formalism of networks of Priced timed automata. Section 3 provides an overview of some existing statistical model checking algorithms, while Sections 4 and 5 introduce the GUI and give some details on the engine of UPPAAL-SMC. Finally, Section 6 presents a series of applications for the tool-set and Section 7 concludes the paper.

2 Modeling Formalism

The new engine of UPPAAL-SMC [22] supports the analysis of Priced Timed Automata (PTAs) that are timed automata whose clocks can evolve with different rates in different locations. In fact, the expressive power (up to timed bisimilarity) of NPTA equals that of general linear hybrid automata (LHA) [1], rendering most problems – including that of reachability – undecidable. We also assume PTAs are input-enabled, deterministic (with a probability measure defined on the sets of successors), and non-zeno.

[1]in contrast to the usual restriction of priced timed automata [7, 2]

PTAs communicate via broadcast channels and shared variables to generate Networks of Price Timed Automata (NPTA).

Fig. 1 provides an NPTA with three components A, B, and T as specified using the UPPAAL GUI. One can easily see that the composite system $(A|B|T)$ has the transition sequence:

$$\big((A_0,B_o,T_0),[x=0,y=0,C=0]\big) \xrightarrow{1} \xrightarrow{a!}$$
$$\big((A_1,B_0,T_1),[x=1,y=1,C=4]\big) \xrightarrow{1} \xrightarrow{b!}$$
$$\big((A_1,B_1,T_2),[x=2,y=2,C=6]\big),$$

demonstrating that the final location T_3 of T is reachable. In fact, location T_3 is reachable within cost 0 to 6 and within total time 0 and 2 in $(A|B|T)$ depending on when (and in which order) A and B choose to perform the output actions $a!$ and $b!$. Assuming that the choice of these time-delays is governed by probability distributions, a measure on sets of runs of NPTAs is induced, according to which quantitative properties such as *"the probability of T_3 being reached within a total cost-bound of 4.3"* become well-defined.

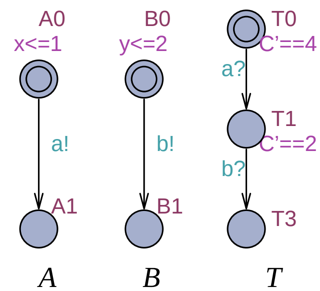


Figure 1: An NPTA, $(A|B|T)$.

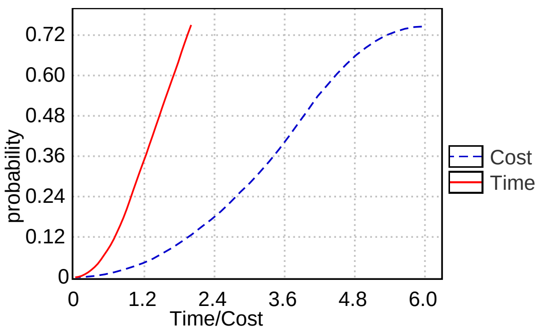


Figure 2: Cumulative probabilities for `time` and `Cost`-bounded reachability of T_3.

In our early works [21], we provide a natural stochastic semantics, where PTA components associate probability distributions to both the time-delays spent in a given state as well as to the transition between states. In UPPAAL-SMC uniform distributions are applied for bounded delays and exponential distributions for the case where a component can remain indefinitely in a state. In a network of PTAs the components repeatedly race against each other, i.e. they independently and stochastically decide on their own how much to delay before outputting, with the "winner" being the component that chooses the minimum delay. For instance, in the NPTA of Fig. 1, A wins the initial race over B with probability 0.75.

As observed in [21], though the stochastic semantic of each individual PTA in UPPAAL-SMC is rather simple (but quite realistic), arbitrarily complex stochastic behavior can be obtained by their composition when mixing individual distributions through message passing. The beauty of our model is that these distributions are naturally and automatically defined by the network of PTAs.

The Hammer Game To illustrate the stochastic semantics further consider the network of two priced timed automata in Fig. 3 modeling a competition between the two players Axel and Alex both having to hammer three nails down. As can be seen by the representing `Work`-locations the time (-interval) and rate of energy-consumption required for hammering a nail depends on the player and the nail-number. As expected Axel is initially quite fast and uses a lot of energy but becomes slow towards the last nail, somewhat in contrast to Alex. To make it an interesting competition, there is only *one* hammer illustrated by repeated competitions between the two players in the `Ready`-locations, where the slowest player has to wait in the `Idle`-location until the faster player has finished hammering the next nail. Interestingly, despite the somewhat different strategy applied, the best- and worst-case completion times are identical for Axel and Alex: 59 seconds and 150 seconds. So, there is no difference between the two players and their strategy, or is there?

a) Axel

b) Alex

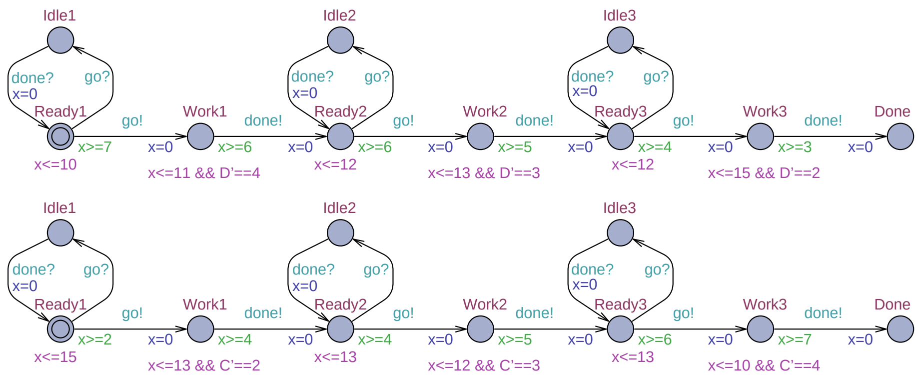
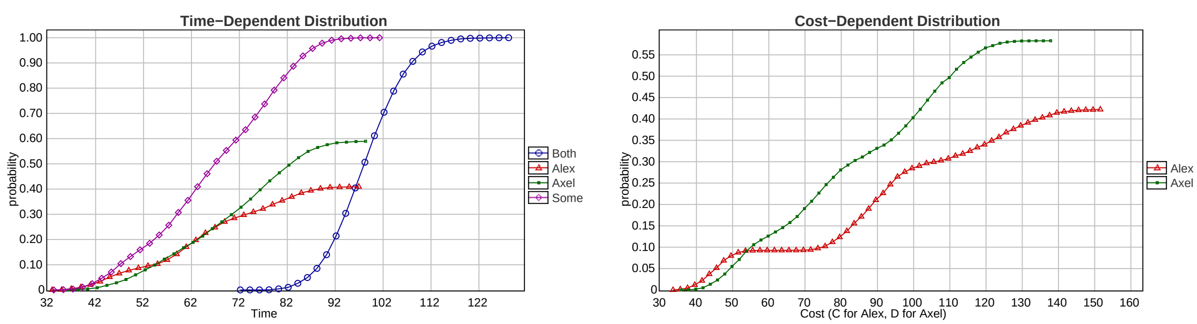


Figure 3: 3-Nail Hammer Game between Axel and Alex.

Assume now that a third person wants to bet on who is the more likely winner – Axel or Alex – given a refined semantics, where the time-delay before performing an output is chosen stochastically (e.g. by drawing from a uniform distribution) and independently by each player (component).

Under such a refined semantics there is a significant difference between the two players (Axel and Alex) in the Hammer Game. In Fig. 4a) the probability distributions for either of the two players winning before a certain time is given. Though it is clear that Axel has a higher probability of winning than Alex (59% versus 41%) given unbounded time, declaring the competition a draw if it has not finished before 50 seconds actually makes Alex the more likely winner. Similarly, Fig. 4b) illustrates the probability of either of the two players winning given an upper bound on energy. With an unlimited amount of energy, clearly Axel is the most likely winner, whereas limiting the consumption of energy to maximum 52 “energy-units” gives Alex an advantage.



a) b)

Figure 4: Time- and Cost-dependent Probability of winning the Hammer Game

Extended Input Language UPPAAL-SMC takes as input NPTAs as described above. Additionally, there is support for other features of the UPPAAL model checker’s input language such as integer variables, data structures and user-defined functions, which greatly ease modeling. UPPAAL-SMC allows the user to specify an arbitrary (integer) rate for the clocks on any location. In addition, the automata support branching edges where weights can be added to give a distribution on discrete transitions. It is important to note that rates and weights may be general expressions that depend on the states and not just simple constants.

To illustrate the extended input language, we consider a train-gate example. This example is available in the distributed version of UPPAAL-SMC. A number of trains are approaching a bridge on which there is only one track. To avoid collisions, a controller stops the trains. It restarts them when possible to make sure that trains will eventually cross the bridge. There are timing constraints for stopping the trains modeling the fact that it is not possible to stop trains instantly. The interesting point w.r.t. SMC is to define the arrival rates of these trains. Figure 5(a) shows the template for a train. The location `Safe` has no invariant and defines the rate of the exponential distribution for delays. Trains delay according to this distribution and then approach and synchronize with `appr[i]!` with the gate controller. Here we define the rational $\frac{1+id}{N^2}$ where *id* is the identifier of the train and N the number of trains. Rates are given by expressions that can depend on the current states. Trains with higher *id* arrive faster. Taking transitions from locations with invariants is given by a uniform distribution. This happens in `Appr`, `Cross`, and `Start`, e.g., it takes some time picked uniformly between 3 and 5 time units to cross the bridge. Figure 5(b) shows the gate controller that keeps track of the trains with an internal queue data-structure (not shown here). It uses functions to queue trains (when a train is approaching while the bridge is occupied in `Occ`) or dequeue them when possible (when the bridge is free and some train is queued).

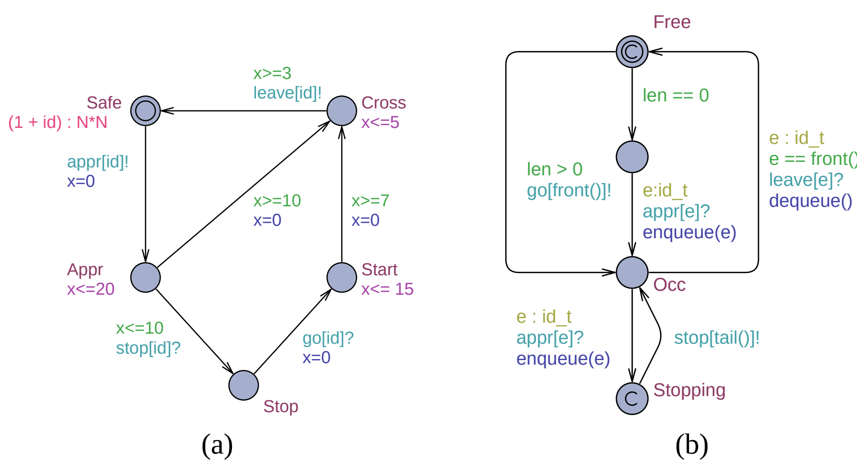


Figure 5: Template of a Train (a) and the Gate Controller (b).

Floating Point Arithmetic For modeling certain systems, e.g., biological systems, integer arithmetic shows its precision limits very quickly. The current engine implements simple arithmetic operations on clocks as floating point variables. This allows various tricks, in particular the tool can compute nontrivial functions using small step integration. For example, Figure 6(a) shows a timed automaton with floating point arithmetic. The clocks `sin_t` and `cos_t` are used to compute $sin(t)$ and $cos(t)$ using simple facts

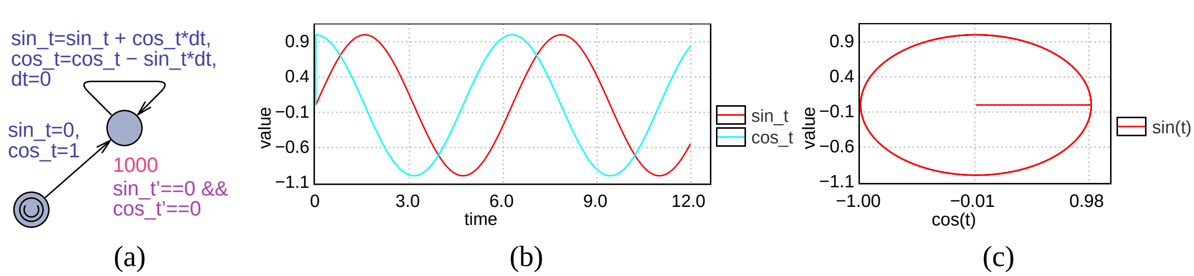


Figure 6: How to use clock arithmetic to integrate complex functions.

as $sin(t+dt) \approx sin(t) + sin'(t)dt$ for small steps of $dt \to 0$, whereas $sin'(t) = cos(t)$ and $sin(0) = 0$, and similarly for $cos(t)$. The interesting trick on the model is the high exponential rate (1000) that tells the engine to take small (random) time steps and record the duration in clock `dt`. The other clocks are stopped and updated on transition. The value evolution of variables `sin_t` and `cos_t` in terms of time are plotted in Figure 6(b). Figure 6(c) shows `sin_t` values with corresponding `cos_t` which form almost perfect circle. These plots are rendered using value monitoring features described in Section 4.

3 Properties and Queries

For specifying properties of NPTAs, we use weighted temporal properties over runs expressed in the logic WMTL$_{\leq}$[10] (*Weighted Metric Temporal Logic*), defined by the grammar $\varphi ::= ap \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathsf{O}\varphi \mid \varphi_1 \mathsf{U}^x_{\leq d} \varphi_2$, where ap is an atomic proposition, d is a natural number and x is a clock. Here, the logical operators are interpreted as usual, and O is a next state operator. An WMTL$_{\leq}$-formula $\varphi_1 \mathsf{U}^x_{\leq d} \varphi_2$ is satisfied by a run if φ_1 is satisfied on the run until φ_2 is satisfied, and this will happen before the value of the clock x increases with more than d. For an NPTA M we define $\mathbb{P}_M(\psi)$ to be the probability that a random run of M satisfies ψ.

The problem of checking $\mathbb{P}_M(\psi) \geq p$ ($p \in [0,1]$) is unfortunately undecidable in general [2]. For the sub-logic of cost-bounded reachability problems $\mathbb{P}_M(\Diamond_{x \leq C}\phi) \geq p$, where ϕ is a state-predicate, x is a clock and C is bound, we approximate the answer using simulation-based algorithms known under the name of statistical model checking algorithms. We briefly recap statistical algorithms permitting to answer the following three types of questions:

1. *Hypothesis Testing:* Is the probability $\mathbb{P}_M(\Diamond_{x \leq C}\phi)$ for a given NPTA M greater or equal to a certain threshold $p \in [0,1]$?
2. *Probability evaluation:* What is the probability $\mathbb{P}_M(\Diamond_{x \leq C}\phi)$ for a given NPTA M?
3. *Probability comparison:* Is the probability $\mathbb{P}_M(\Diamond_{x \leq C}\phi_2)$ greater than the probability $\mathbb{P}_M(\Diamond_{y \leq D}\phi_2]$?

From a conceptual point of view solving the above questions using SMC is simple. First, each run of the system is encoded as a Bernoulli random variable that is true if the run satisfies the property and false otherwise. Then a statistical algorithm groups the observations to answer the three questions. For the qualitative questions (1 and 3), we shall use sequential hypothesis testing, while for the quantitative question (2) we will use an estimation algorithm that resemble the classical Monte Carlo simulation. The two solutions are detailed hereafter.

Hypothesis Testing This approach reduces the qualitative question to testing the hypothesis $H : p = \mathbb{P}_M(\Diamond_{x \leq C}\phi) \geq \theta$ against $K : p < \theta$. To bound the probability of making errors, we use strength parameters α and β and we test the hypothesis $H_0 : p \geq p_0$ and $H_1 : p \leq p_1$ with $p_0 = \theta + \delta_0$ and $p_1 = \theta - \delta_1$. The interval $p_0 - p_1$ defines an indifference region, and p_0 and p_1 are used as thresholds in the algorithm. The parameter α is the probability of accepting H_0 when H_1 holds (false positives) and the parameter β is the probability of accepting H_1 when H_0 holds (false negatives). The above test can be solved by using Wald's *sequential hypothesis testing* [39]. This test computes a proportion r among those runs that satisfy the property. With probability 1, the value of the proportion will eventually cross $\log(\beta/(1-\alpha)$ or $\log((1-\beta)/\alpha)$ and one of the two hypothesis will be selected. In UPPAAL-SMC we use the following query: `Pr[`*bound*`](`ϕ`)>=`p_0, where *bound* defines how to bound the runs. The three ways to bound them

[2] Exceptions being PTA with 0 or 1 clocks.

are 1) implicitly by time by specifying `<=`M (where M is a positive integer), 2) explicitly by cost with x`<=`M where x is a specific clock, or 3) by number of discrete steps with `#<=`M. In the case of hypothesis testing p_0 is the probability to test for. The formula ϕ is either `<>` q or `[]` q where q is a state predicate.

Probability Estimation This algorithm [26] computes the number of runs needed in order to produce an approximation interval $[p-\varepsilon, p+\varepsilon]$ for $p = Pr(\psi)$ with a confidence $1-\alpha$. The values of ε and α are chosen by the user and the number of runs relies on the Chernoff-Hoeffding bound. In UPPAAL-SMC we use the following query: `Pr[`*bound*`](`ϕ`)`

Probability Comparison This algorithm, which is detailed in [21], exploits an extended Wald testing. In UPPAAL-SMC, we use the following query: `Pr[`$bound_1$`](`ϕ_1`)>=Pr[`$bound_2$`](`ϕ_2`)`.

In addition to those three classical tests, UPPAAL-SMC also supports the evaluation of expected values of min or max of an expression that evaluates to a clock or an integer value. The syntax is as follows: `E[`*bound*;*N*`](min:`*expr*`)` or `E[`*bound*;*N*`](max:`*expr*`)`, where *bound* is as explained in this section, *N* gives the number of runs explicitly, and *expr* is the expression to evaluate. For this property, no confidence is given (yet).

Full WMTL$_\leq$ Regarding implementation, the reader shall observe that both of the above statistical algorithms are trivially implementable. To support the full logic of WMTL$_\leq$is slightly more complex as our simulation engine needs to rely on monitors for such logic. In [10], we proposed an extension of UPPAAL-SMC that can handle arbitrary formulas of WMTL$_\leq$. Given a property φ, our implementation first constructs deterministic under- and over-approximation monitoring PTAs for φ. Then it puts these monitors in parallel with a given model M, and applies SMC-based algorithms to bound the probability that φ is satisfied on M.

4 Graphical User Interface

Besides short 'yes' or 'no' answers and probability estimates, UPPAAL-SMC verifier also provides a few statistical measures in terms of time (or cost), including frequency histogram, average time (or cost), probability density distribution, cumulative probability distribution (the last two with confidence intervals, e.g. using the Clopper-Pearson method [19]).

These statistical data can also be superposed onto a single plot for comparison purposes using the plot composer tool. Figure 7 shows the superposed probability distributions of trains 0, 3 and 5 crossing from our train-gate example. On the left side of the plot composer window the user can select a particular data to be added to the plot and on the right side the user can see the superposed plot and can also change some details such as labels, shapes and colors.

Monitoring Expressions UPPAAL-SMC now allows the user to visualize the values of expressions (evaluating to integers or clocks) along runs. This gives insight to the user on the behavior of the system so that more interesting properties can be asked to the model-checker. To demonstrate this on our previous train-gate example, we can monitor when `Train(0)` and `Train(5)` are crossing as well as the length of the queue. The query is `simulate 1 [<=300]{Train(0).Cross,Train(5).Cross,Gate.len}`. This gives us the plot of Figure 8. Interestingly `Train(5)` crosses more often (since it has a higher arrival rate). Secondly, it seems unlikely that the gate length drops below 3 after some time (say 20), which is

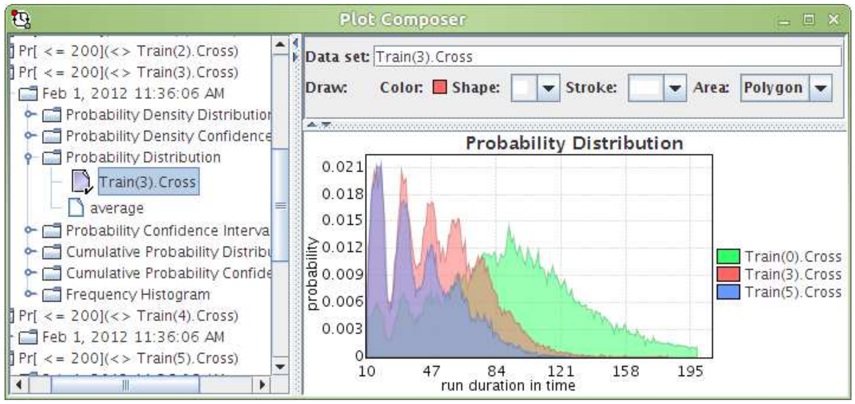


Figure 7: Snapshot of the plot composer displaying three probability distributions.

not an obvious property from the model. We can confirm this by asking `Pr[<=300](<> Gate.len < 3 and t > 20)` and adding a clock `t`. The probability is in $[0.102, 0.123]$.

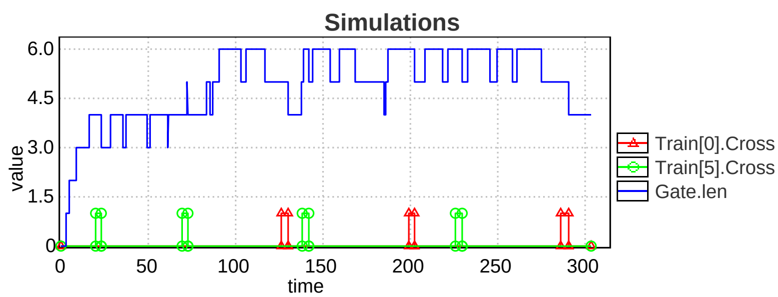


Figure 8: Visualizing the gate length and when `Train(0)` and `Train(5)` cross on one random run.

As a second example to illustrate this feature, we consider the modeling of chemical reactions. Figure 9(a) and 9(b) show two symmetric timed automata that model the concentrations of reactants `a` and `b` (here as integers). The exponential rate for taking the transition is given by the concentration of `a` and `b`. Figure 9(c) shows the evolution of the system when it is started with `a=99` and `b=1`: `a` is consumed to produce `b` and vice-versa, and the concentrations oscillate.

The simulations are obtained by querying `simulate 1 [<=10]{a,b}`. Figure 9(c) is showing one evolution of `a` and `b` over time. The tool can also plot clouds of trajectories, which is useful to identify patterns in the behavior, as shown in figure 9(d).

It is important to notice that generating such curves is not as trivial as it seems. In fact, on such models, if the exponential rates are higher, then the time steps are much smaller, which generates a lot of points, up to consuming several GB of memory. Drawing such plots is not practical. The tool would not work due to out-of-memory problems or in the best case will take around 30s to transfer the data and several seconds for every redraw. To solve this the engine applies an on-the-fly filtering of the points based on the principle that if two points are too close to each other to be distinguished on the screen, then they are considered to be the same. A resolution parameter is used to define the maximal resolution of the plot and eliminates the memory and speed problems completely (down to almost not measurable).

This plot in Figure 6(b) is obtained by asking `simulate 1 [<=12]{sin_t,cos_t}` to the model-checker. Interestingly, UPPAAL-SMC can generate a run bounded by any clock so we can also plot

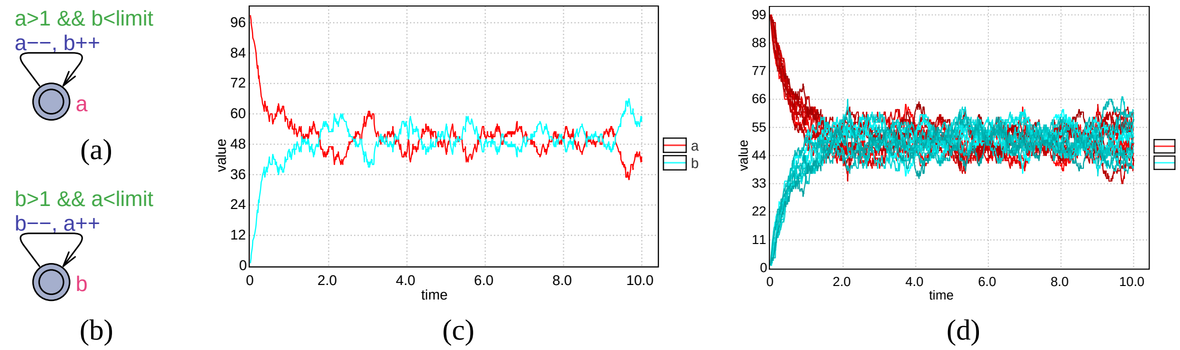


Figure 9: Evolution of the concentrations of two reactants `a` and `b`.

`simulate 1 [cos_t<=1]{sin_t}` and obtain a circle as shown in Figure 6(c).

5 Engine

The actual techniques to achieve the current performance of the tool were never exposed before. In this section, we present a few key optimizations to implement the algorithms presented and new features that were not available in earlier versions of UPPAAL-SMC.

Distributed SMC The problem in distributing the implementation of the sequential SMC algorithm is that a *bias* may be introduced. The reason is that sequential testing relies on collecting outcomes of the generated runs on-the-fly. If some computation cores generate some accepting runs faster, which is possible if rejecting runs happen to be longer or simply more expensive to compute, then the result will be biased. The solution of this problem is to force all the cores to generate the same amount of simulations. The paper [40] proposes a method to ensure this by splitting the simulations into batches of the same size, and this method has been generalized and implemented in UPPAAL-SMC [13]. The distributed implementation gives a linear speed-up in the number of cores used.

Detection of States When choosing the delays, the engine does not know if it will *skip* the state that should be observed by the query or not. This problem is present when picking delays to take transitions as well. For example, the query could be `<> A.critical and x >= 2 and x <= 3` where `x` is a clock. The engine should not delay 4 time units from a state where `x=0` because the first possible transition is enabled at this point. Special care is taken to make sure that the formula is part of the next *interesting* points that are computed when choosing the delays. Now comes the question of how to detect those interesting points in both the formula and the guards.

The technique we use follows the decorator pattern where we evaluate guards (for detecting which transitions will be enabled in the future) and formulas in the query to keep track of the lower bounds. We wrap a state inside a decorator state that keeps track of the constraints on-the-fly, only remembering the bounds that we need. The point of the technique here is to avoid *symbolic* states that would require zones typically implemented with different bound matrices.

Early Termination The engine checks for query on-the-fly on every generated run. If a query is satisfied then the computation of the run is stopped before it reaches the specified bound. In addition, in order to give the user a way to stop runs earlier, the engine supports an *until* property: `p U q` can be queried instead of `<> q` and cut the runs as soon as `p` stops to hold.

Dependencies and Reuse of Choice When a process takes an action, it may not affect other processes, which means that from a stochastic point-of-view, picking a new delay from scratch or *reusing* the old (random) choice is equivalent. The engine exploits this independence: it remembers the previous delays chosen by the processes and invalidates them when dependent transitions are taken. A process has its delay invalidated if there is a dependency with another transition being taken, which happens in case of synchronization or a dependency through a clock rate, invariant, guard, or update. A static analysis is made at the granularity of *how transitions affect processes*[3].

The result is that whenever a process *needs* to pick a delay, it does so. Whenever a process takes a transition, the processes that may be affected by it must pick a new delay at the next step. Otherwise, processes *reuse* their choices from the previous step in the simulation[4].

Checking the query `Pr[<=300](<> Train(0).Cross and (forall (i:id_t) i!=0 imply Train(i).Stop))` to evaluate the probability of `Train(0)` crossing while all the others are stopped gives the results in Table 1 for different numbers of trains. The results are obtained with the parameter $\varepsilon = 0.005$ and the probability results agree with or without reuse within ε. The experiments are made on a core i7 at 2.66GHz. This optimization is designed to improve on systems with large number of components, which is shown by the increasing improvement relative to verifications without reuse.

Trains	5	10	20	40
Proba.	0.985-0.995	0.286-0.297	0-0.008	0-0.005
Time$^-$	3.9s	17.3s	41.1s	98.1s
Time$^+$	3.5s	14.8s	33.2s	74.8s
Gain	10.2%	14.4%	19.2%	23.8%

Table 1: Probability and time results without (-) and with (+) reuse.

6 Case-Studies

In this section we evaluate the applicability of the developed techniques on practical case studies.

Robot Control In paper [10] we considered a case – explored in [4] – of a robot moving on a two-dimensional grid. Each field of the grid is either `normal`, on `fire`, cold as `ice` or it is a wall which cannot be passed. Also, there is a `goal` field that the robot must reach. The robot is moving in a random fashion i.e. it stays in a field for some time, and then moves to a neighboring field at random (if it is not a wall).

We are interested in the probability that the robot reaches its goal location without staying on consecutive fire fields for more than one time unit and on consecutive ice fields for more than two time units. This property is captured by the WMTL$_{\leq}$ formula $\varphi \equiv (\varphi_1 \wedge \varphi_2) \mathsf{U}^{\tau}_{\leq 10} \mathtt{goal}$, where τ is a special clock that grows with rate 1 and is never reset, and:

$$\varphi_1 \equiv \mathtt{ice} \implies \Diamond^{\tau}_{\leq 2}(\mathtt{fire} \vee \mathtt{normal} \vee \mathtt{goal})$$
$$\varphi_2 \equiv \mathtt{fire} \implies \Diamond^{\tau}_{\leq 1}(\mathtt{ice} \vee \mathtt{normal} \vee \mathtt{goal})$$

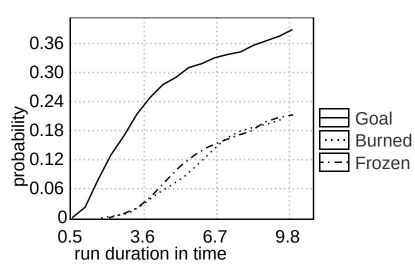


Figure 10: Cumulative Probability

We applied UPPAAL-SMC to compute the probability of the robot reaching the goal φ, staying too long in the fire or too long on the ice. Figure 10 shows the cumulative distribution for these probabilities.

[3] We judge that keeping track of the dependencies down to the locations may have a too large overhead.

[4] If time elapses then of course the delays chosen are updated.

Firewire. IEEE 1394 High Performance Serial Bus or Firewire for short is used to transport multimedia signals among a network of consumer devices. The protocol has been extensively studied (see [37] for comparison) and in particular [31] uses probabilistic timed automata in PRISM [30]. In paper [22] we adopt the model from [31] and demonstrate how UPPAAL-SMC can be used to evaluate fairness of a node becoming a root (leader) with respect to the mode of operation. UPPAAL-SMC provides two methods for comparing probabilities: estimating the probabilities and then comparing them, or using indirect probability comparison from [39], which is more efficient. Figure 11 contains a resulting plot of estimated probabilities (red and blue lines) and a comparison (yellow area). The red and blue probability estimates appear very close to each other in entire range, while the yellow area shows that at the beginning the probabilities are indistinguishable (yellow area is at 0.5 level), then the *fast* node has higher probability to become a *root* (at 1.0 level), and later the probabilities become too close to be distinguishable again (at 0.5 level).

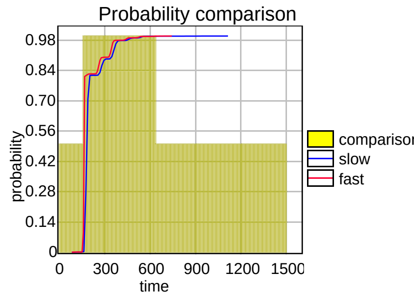


Figure 11: Probability Comparison

Bluetooth [34] is a wireless telecommunication protocol using frequency-hopping to cope with interference between the devices in the wireless network. In paper [22] we adopted the model from [23], annotated the model to record the power utilization and evaluated the probability distributions of likely response times and energy consumption. Figure 12 shows that after 70s the cost of a device operation is at least 2440 energy units and the mean is about 2853 energy units.

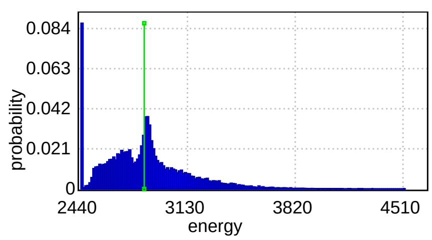


Figure 12: Energy consumption.

Lightweight Medium Access Protocol (LMAC) [38] is a communication scheduling protocol based on time slot distribution for nodes sharing the same medium. The protocol is designed having wireless sensor networks in mind: it is simple enough to fit on a modest hardware and at the same time robust against topology reconfiguration, minimizing collisions and power consumption. Paper [25] studies LMAC protocol using classical UPPAAL verification techniques by systematically exploring networks of up to five nodes but the state space explosion prevents formal verification of larger networks. In paper [21] we adopt the model by removing verification optimizations and parameterizing with probabilistic weights, and show how collisions can be analyzed and power consumption estimated using statistical model checking techniques. The study showed that there are still perpetual collisions in a ring topology but the probability that the network will not recover is very low (0.35%). The likely energy consumption of different network topologies is compared in UPPAAL plot (Figure 13), which shows that on average the likely energy consumption after 1000 time units in a ring is higher than in a chain by 10%, possibly due to more collisions in a ring. In [13] distributed techniques are applied in exploring over 10000 larger networks of up to 10 nodes, the worst (star-like) and the best (chain-like) topologies in terms of col-

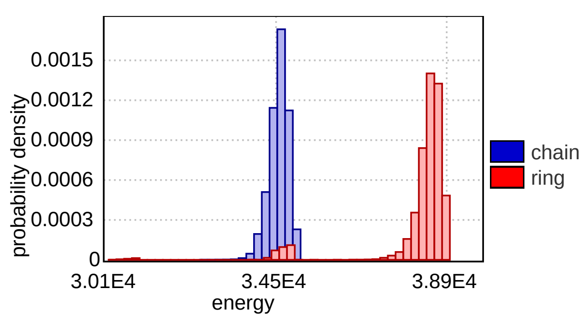


Figure 13: Likely energy consumption.

lisions are identified and evaluated.

Computing Nash Equilibrium in Wireless Ad Hoc Networks One of the important aspects in designing wireless ad-hoc networks is to make sure that a network is robust to the selfish behavior of its participants, i.e. that its configuration satisfies Nash equilibrium (NE).

In paper [11] we proposed an SMC-based algorithm for computing NE for the case when network nodes are modeled by SPTA and an utility function of a single node is equal to a probability that the node will reach its goal. Our algorithm consists of two phases. First, we use UPPAAL-SMC to find a strategy that most likely (heuristic) satisfies NE. In the second phase we apply statistics to test the hypothesis that this strategy actually satisfies NE.

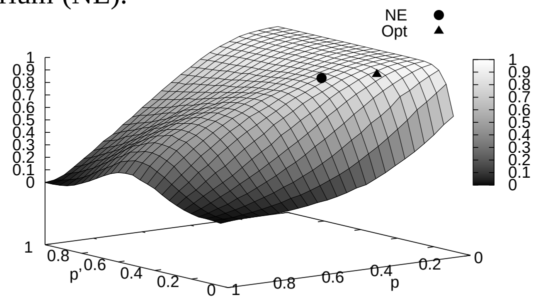


Figure 14: Nash Equilibrium for Aloha CSMA/CD

We applied this algorithm to compute NE for Aloha CSMA/CD and IEEE 802.15.4 CSMA/CA protocols. Figure 14 depicts the utility function plot for the Aloha CSMA/CD protocol with two nodes. Here the p and p' axis correspond to the strategies of the honest and cheater nodes (a strategy defines how persistent these nodes are in sending their data). We see, that NE strategy is slightly less efficient than the symmetric optimal strategy (Opt), but it still results in a high value of the utility function.

Duration Probabilistic Automata In [20] we compared UPPAAL-SMC to Prism [30] in the context of Duration Probabilistic Automata (DPA) [32]. A Duration Probabilistic Automaton (DPA) is a composition of Simple Duration Probabilistic Automata (SDPA). An SDPA is a linear sequence of tasks that must be performed in a sequential order.

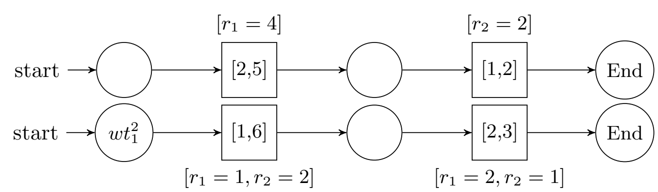


Figure 15: Rectangles are busy states and circles are for waiting when resources are not available. There are $r_1 = 5$ and $r_2 = 3$ resources available.

Each task is associated with a duration interval which gives the possible durations of the task. The actual duration of the tasks is given by a uniform choice from this interval. To model races between the SDPAs we introduce resources to the model such that an SDPA might have to wait for resources before processing a task. When two SDPAs are in waiting position for the same resource, a scheduler decides which SDPA is given the resource in a deterministic manner.

Param.			Estim.				Hyp. Testing			
n	k	m	Prism	Up_p	Up_d	Up_c	Prism	Up_p	Up_d	Up_c
4	4	3	2.7	0.3	0.2	0.2	2.0	0.1	0.1	0.1
6	6	3	7.7	0.6	0.5	0.4	3.9	0.2	0.2	0.3
8	8	3	26.5	1.2	0.9	0.7	16.4	0.5	0.4	0.3
20	40	20	>300				>300	35.5	26.2	20.7
30	40	20	>300				>300	61.2	41.8	33.2
40	40	20	>300				>300	92.2	56.9	59.5
40	20	20	>300				>300	41.1	31.2	26.5
40	30	20	>300				>300	68.8	46.7	46.1
40	55	40	>300				>300			219.5

Table 2: Performance of SMC (sec). The n column is the number of SDPAs, the k column is the number of tasks per SDPA and the m column is the number of resource types in the model. Up_p is the UPPAAL model that matches Prism, Up_d the discrete encoding and Up_c the continuous time encoding.

The comparison with Prism was made by randomly generating models with a specific number of SDPAs and a

specific number of tasks per SDPA and translate these into Prism and UPPAAL models. The Prism model uses a discrete time semantics whereas three models were made for UPPAAL- one with continuous time semantics, one that matches the Prism model as close as possible and one with discrete semantics that makes full use of our formalism.
The queries to the models were *What is the probability of all SDPAs ending within t time units* (Estimation)and *Is the probability that all SDPAs end within t time units greater than* 40% (Hypothesis testing). The value of t is different for each model as it was computed by simulating the system 369 times and represent the value for which at least 60% of the runs finished all their tasks.

The result of the experiments are shown in Table 2 and indicates that UPPAAL is notably faster than Prism, even with a encoding that closely matches that of Prism.

Checking of Distributed Statistical Model Checking As we wrote in Section 5, a naive (and incorrect) distributed implementation of the sequential SMC algorithms might introduce a bias towards the results that are generated by shorter simulations.

The interesting question is how much this bias affects the SMC results. In the paper [12] we answered this question by modeling the naive distributed SMC algorithm in UPPAAL-SMC itself. The comparison was made on the basis of the SPTA model that ends up in the `OK` location after 100 time units with probability 0.58, otherwise it ends up in the `NOK` location after 1 time unit (thus producing `NOK` requires 100 times less time than producing `OK`).

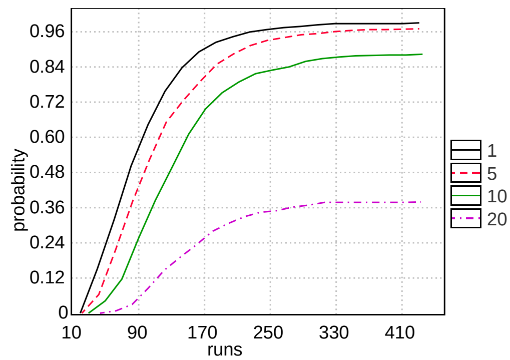


Figure 16: Probability distributions obtained with 1, 5, 10, and 20 cores.

We used UPPAAL-SMC to compute the probability that the naive distributed SMC algorithm will accept the hypothesis `Pr[<=100](`$\Diamond$ `OK)`$\geq$ `0.5`. The results for the different numbers of computational cores are given in the plot at Figure 16. The x axis denotes the total number of runs of the SPTA model on all the cores, and the y axis depicts the probability that an SMC algorithm accepts the hypothesis not later than after this number of runs. It can be observed that the probability of accepting the hypothesis tends (incorrectly) to 0 as the number of computational cores increases.

7 Conclusions

This paper gives an overview of the features of UPPAAL-SMC, our new efficient extension of UPPAAL for Statistical Model Checking. Contrary to other existing SMC-based tool-sets, UPPAAL-SMC allows to handle systems with real-time features. The tool has been applied to a series of case studies that are beyond the scope of classical model checkers. As has been outlined in this paper, UPPAAL-SMC has a large potential for future work and applications.

Among others, the following extensions of UPPAAL-SMC are contemplated.

Floating Point So far the support of floating point is done via misusing and extending clock operations. A better and more general support is needed since the tool has now departed from traditional timed automata and model-checking.

Since the tool now supports floating point arithmetic and we can integrate complex functions, it is a natural extension to add differential equations as well to support hybrid systems in a more general way. To fit with the stochastic semantics (in particular how to pick delays), only simple equations whose analytical solutions are known are planned.

New Applications With the extended expressivity of our hybrid modeling language, our tool can be applied to different domains, in particular for biological systems. UPPAAL-SMC now offers powerful visualization capabilities needed by biologists and a logic to do statistical model-checking.

Another application is to analyze performance of controllers generated by UPPAAL-TIGA [6], in particular their stability or energy consumption. SMC can also be used in the domain of refinement checking, which is in the end just another type of game.

Rare Events Statistical model checking avoids the exponential growth of states associated with probabilistic model checking by estimating properties from multiple executions of a system and by giving results within confidence bounds. Rare properties are often very important but pose a particular challenge for simulation-based approaches, hence a key objective under these circumstances is to reduce the number and length of simulations necessary to produce a given level of confidence. Importance sampling is a well-established technique that achieves this, however to maintain the advantages of statistical model checking it is necessary to find good importance sampling distributions without considering the entire state space. Such problem has been recently investigated for the case of discrete stochastic systems. As an example, in [28] we presented a simple algorithm that uses the notion of cross-entropy to find the optimal parameters for an importance sampling distribution. Our Objective is to extend our results to PTAs by exploiting pure timed model checking to improve the search for efficient distribution.

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