

# Manifold Clustering in the Setting of Generalized Random Dot Product Graphs

SDSS Lightning Presentation

May 2023



John Koo,  
Postdoctoral Fellow,  
Indiana University

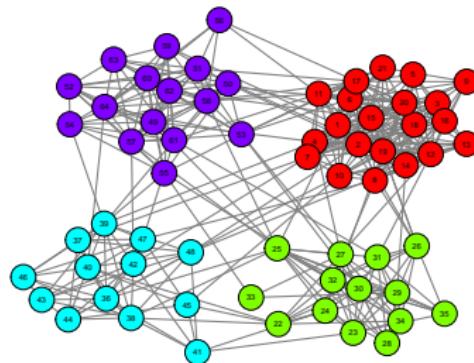


Minh Tang,  
Assistant Professor,  
NC State University



Michael W. Trosset,  
Professor of Statistics,  
Indiana University

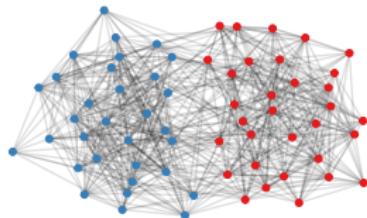
# Community Detection for Networks



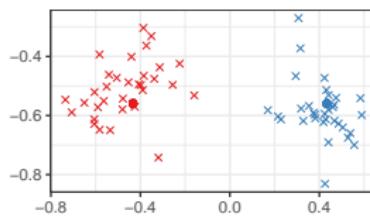
How might we cluster the nodes of a network?

# Connecting Block Models to the GRDPG

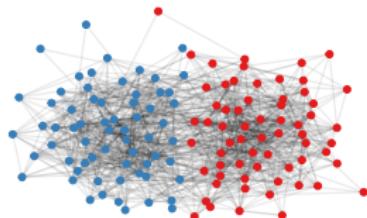
SBM



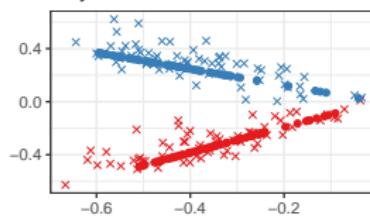
Point Masses



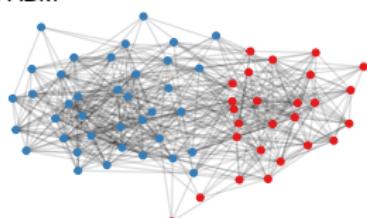
DCBM



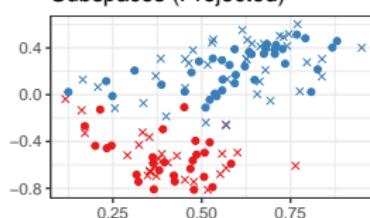
Rays



PABM

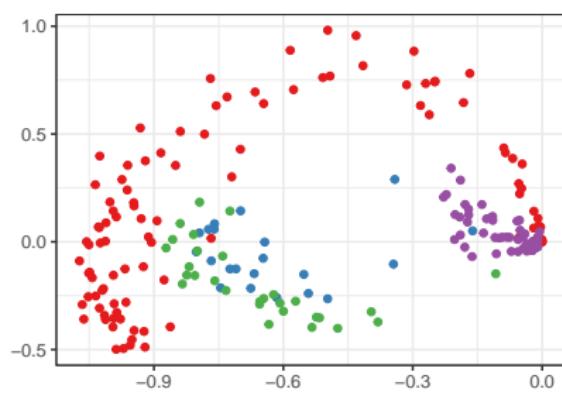
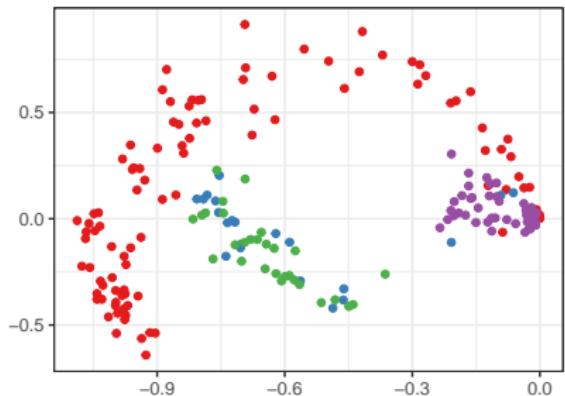
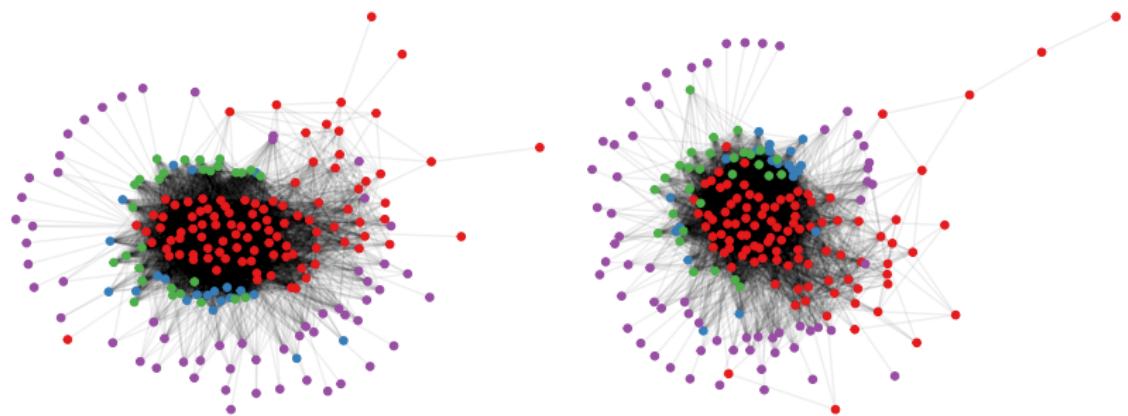


Subspaces (Projected)



- K-means clustering
- Gaussian mixture models
- K-means with cosine similarity
- GMM on angles
- Orthogonal Spectral Clustering
- Sparse Subspace Clustering

# Nonlinear Community Structure



## Manifold Block Model

Let  $p, q \geq 0$ ,  $d = p + q \geq 1$ ,  $1 \leq r < d$ ,  $K \geq 2$ , and  $n > K$  be integers. Define manifolds  $\mathcal{M}_1, \dots, \mathcal{M}_K \in \mathcal{X}$  for

$\mathcal{X} = \{x, y \in \mathbb{R}^d : x^\top I_{p,q} y \in [0, 1]\}$  each by continuous function  $g_k : [0, 1]^r \rightarrow \mathcal{X}$ . Define probability distribution  $F$  with support  $[0, 1]^r$ . Then the following mixture model is a *manifold block model*:

1. Draw labels  $z_1, \dots, z_n \stackrel{\text{iid}}{\sim} \text{Categorical}(\alpha_1, \dots, \alpha_K)$ .
2. Draw latent vectors by first taking  $t_1, \dots, t_n \stackrel{\text{iid}}{\sim} F$  and then computing each  $x_i = g_{z_i}(t_i)$ .
3. Compile the latent vectors into data matrix  $X = [x_1 \mid \dots \mid x_n]^\top$  and define the adjacency matrix as  $A \sim \text{GRDPG}_{p,q}(X)$ .

# Manifold Block Model

1.  $z_1, \dots, z_n \stackrel{\text{iid}}{\sim} \text{Categorical}(1/2, 1/2)$
2.  $t_1, \dots, t_n \stackrel{\text{iid}}{\sim} \text{Uniform}(0, 1)$
3.  $x_i = g_{z_i}(t_i)$ 
  - $g_1(t) = [t^2, 2t(1-t)]^\top$
  - $g_2(t) = [2t(1-t), (1-t)^2]^\top$
4.  $A \sim \text{GRDPG}_{2,0}(X)$

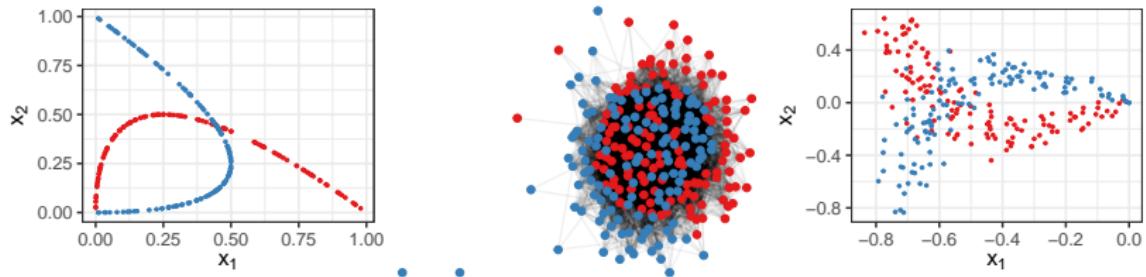


Figure 1: Latent vectors on intersecting curves (left), along with an RDGP drawn from this configuration (center) and its ASE (right).

# *K*-Curves Clustering

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## Algorithm 1: *K*-curves clustering.

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**Data:** Adjacency matrix  $A$ , number of communities  $K$ , embedding dimensions  $p$ ,  $q$ , stopping criterion  $\epsilon$

**Result:** Community assignments  $1, \dots, K$ , curves  $g_1, \dots, g_K$

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1 Compute  $X$ , the ASE of  $A$  using the  $p$  most positive and  $q$  most negative
   eigenvalues and their corresponding eigenvectors.
2 Initialize community labels  $z_1, \dots, z_n$ .
3 repeat
4   for  $k = 1, \dots, K$  do
5     Define  $X_k$  as the rows of  $X$  for which  $z_i = k$ .
6     Fit curve  $g_k$  and positions  $t_{k_i}$  to  $X_k$  by minimizing
          $\sum_{k_i} \|x_{k_i} - g_k(t_{k_i})\|^2$ .
7   end
8   for  $k = 1, \dots, K$  do
9     Assign  $z_i \leftarrow \arg \min_\ell \|x_i - g_\ell(t_i)\|^2$ .
10  end
11 until the change in  $\sum_k \sum_{i \in C_k} \|x_i - g_k(t_i)\|^2$  is less than  $\epsilon$ 
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# Example

