Popularity Adjusted Block Models are Generalized Random Dot Product Graphs

JSM Speed Presentation

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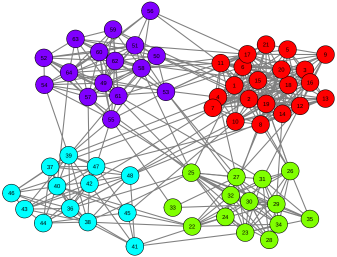
Community Detection for Networks

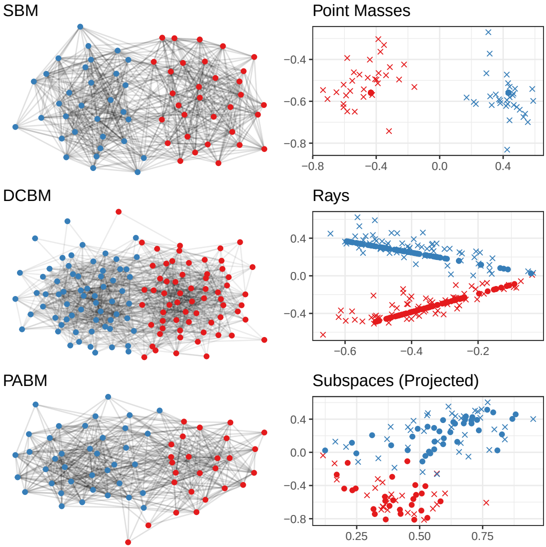


Def Popularity Adjusted Block Model (Sengupta and Chen, 2017):

Let each vertex $i \in [n]$ have K popularity parameters $\lambda_{i1}, ..., \lambda_{iK} \in [0, 1]$.

Then $A \sim$ BernoulliGraph(P) is a PABM if each $P_{ij} = \lambda_{iz_j} \lambda_{jz_i}$.

Connecting Block Models to the GRDPG



- K-means clustering
- Gaussian mixture models

- K-means with cosine similarity
- GMM on angles

- ???

Orthogonal Spectral Clustering

Theorem (KTT): If $P = V \Lambda V^\top$, $B = nVV^\top$, and $z_i \neq z_j$, then $B_{ij} = 0$.

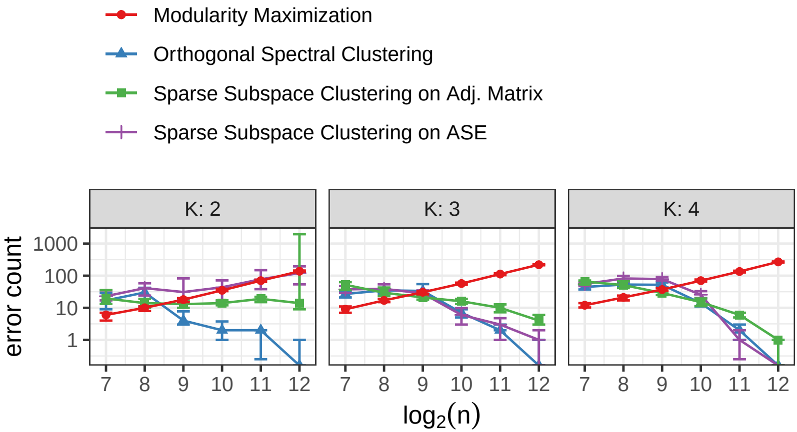
Algorithm: Orthogonal Spectral Clustering:

1. Let V be the eigenvectors of A corresponding to the $K(K+1)/2$ most positive and $K(K-1)/2$ most negative eigenvalues.
2. Compute $B = |nVV^\top|$, applying $|\cdot|$ entry-wise.
3. Construct graph G using B as its similarity matrix.
4. Partition G into K disconnected subgraphs.

Theorem (KTT): For all pairs (i, j) belonging to different communities, $\max\limits_{i,j} B_{ij} = O_P\left(\frac{(\log n)^c}{\sqrt{n \rho_n}}\right)$.

Corollary: OSC results in zero clustering error almost surely as $n \to \infty$.

Simulation Study



Thank you

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arXiv preprint: https://arxiv.org/abs/2109.04010

GitHub repository: https://github.com/johneverettkoo/pabm-grdpg

R package (WIP): https://github.com/johneverettkoo/osc