Parameter Estimation for the PABM

First, we note that the $\lambda^{(kl)}$ are only identifiable up to multiplicative constant. Instead, we aim to identify each block $P^{(kl)} = \lambda^{(kl)} (\lambda^{(lk)})^\top$.

Let $M^{(kl)} = \begin{bmatrix} 0 & P^{(kl)} \\ P^{(lk)} & 0 \end{bmatrix}$. We can consider $M^{(kl)}$ as the edge probability matrix of a GRDPG with latent configuration $X = \frac{1}{2} \begin{bmatrix} \lambda^{(kl)} & \lambda^{(kl)} \\ \lambda^{(lk)} & -\lambda^{(lk)} \end{bmatrix}$ and signature $(1, 1)$.

Treating $M^{(kl)}$ as an edge probability matrix, draw $\hat{M}^{(kl)} \sim \mathit{BernoulliGraph}(M^{(kl)})$, and let $\hat{X}$ be its ASE. Then $\hat{M}^{(kl)} = \begin{bmatrix} 0 & A^{(kl)} \\ A^{(lk)} & 0 \end{bmatrix}$ and $\|\hat{X} Q - X\|_{2 \to \infty} = O_P(\frac{(\log n_{kl})^c}{\sqrt{n_{kl}}})$ where $n_{kl} = n_{lk} = n_k + n_l$.

Then we get $\|\hat{M}^{(kl)} - M^{(kl)}\| = \|(\hat{X}Q - X) I_{1,1} X^\top + X I_{1,1} (\hat{X} Q - X)^\top + (\hat{X} Q - X) I_{1,1} (\hat{X} Q - X)^\top\|$
$\leq 2 \|\hat{X} Q - X\| \|I_{1,1}\| \|X\| + \|I_{1,1}\| \|\hat{X} Q - X\|^2$
$= O(\frac{(\log n_{kl})^c}{\sqrt{n_{kl}}})$