STAT-S620

Assignment 6

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```
import::from(magrittr, `%>%`)
```

4.1.3

```
dplyr::data_frame(age = c(18, 19, 20, 21, 25),
                  count = c(20, 22, 4, 3, 1)) %>%
                  {sum(.$age * .$count) / sum(.$count)}
```

```
[1] 18.92
```

Then $E[X] = \boxed{18.92}$

4.1.4

```
'the girl put on her beautiful red hat' %>%
  strsplit(' ') %>%
  unlist() %>%
  sapply(nchar) %>%
  mean()
```

```
[1] 3.75
```

The average number of words is $\boxed{3.75}$.

4.1.8

$E[XY] = \int_{x=0}^1 dx \int_{y=0}^x xy \times 12y^2 dy$
$= \int_0^1 x dx \int_0^x 12y^3 dy$
$= \int_0^1 x(3y^4|_0^x) dx$
$= \int_0^1 3x^5 dx$
$= 3/6 = \boxed{1/2}$

4.2.3

Since $X_i \sim Unif(0, 1)$, $E[X_i] = 1/2$ and $E[X_i^2] = \int_0^1 x^2 dx =$ \$ 1/3\$.

$E[(X_1 - 2X_2 + X_3)^2] = E[X_1^2 - 4X_1X_2 + 2X_1X_3 + 4X_2^2 - 4X_2X_3 + X_3^2] = E[X_1^2] + 4E[X_2^2] + E[X_3^2] - 4E[X_1X_2] + 2E[X_1X_3] - 4E[X_2X_3] = 1/3 + 4/3 + 1/3 - 4E[X_1X_2] + 2E[X_1X_3] - 4E[X_2X_3]$ $= \boxed{2 - 4E[X_1X_2] + 2E[X_1X_3] - 4E[X_2X_3]}$

If we assume that each X_i are independent, then $E[X_iX_j] = E[X_i]E[X_j]$ for $i \neq j$. Then the above becomes $2 - 4(1/2)^2 + 2(1/2)^2 - 4(1/2)^2 = \boxed{1/2}$.

4.2.4

We want to find $E[XY]$. Since X and Y are independent, this is equivalent to $E[X]E[Y]$. From the previous problem, we can say that $E[X] = 1/2$. And $E[Y] = (9+5)/2 = 7$. Therefore, $E[X]E[Y] = \boxed{7/2}$.

4.3.2

From 4.1.4, we saw that $E[X] = 3.75$, so $E[X]^2 = 14.0625$. Then we need to find $E[X^2]$:

```r
'the girl put on her beautiful red hat' %>%
  strsplit(' ') %>%
  unlist() %>%
  sapply(nchar) %>%
  sapply(function(x) x ** 2) %>%
  mean()
```

```
[1] 18.25
```

So $Var(X) = 18.25 - 14.0625 = \boxed{4.1875}$.

4.4.3

From a formula from class, we have $E[(X-\mu)^3] = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3 = \boxed{1}$

4.9.7

$E[Y-X] = \int_0^6 dy \int_0^y (y-x)\frac{x}{36}dx = \frac{1}{36}\int_0^6 dy \int_0^y xy - x^2 dx = \frac{1}{36}\int_0^6 (\frac{x^2y}{2} - \frac{x^3}{3}|_0^y)dy = \int_0^6 \frac{y^3}{216}dy = \frac{y^4}{864}|_0^6 = \boxed{\frac{3}{2}}$

5.2.6

Using $E[X] = np$ if $X \sim Binom(n,p)$:

$E[A+B+C] = E[A] + E[B] + E[C] = 3/8 + 5/4 + 2/2 = \boxed{21/8}$