STAT-S620

Assignment 3

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2.1.9

Part a

There are 4 cards left, 3 of which are red. Therefore, the probability is just $\boxed{\frac{3}{4}}$.

Part b

Since all but one card are red, we are guaranteed to get at least one red card. Therefore, this is just the probability that both are red, which is $\frac{4}{5} \times \frac{3}{4} = \boxed{\frac{3}{5}}$.

2.1.14

G : good working order
W : wearing down
N : needs maintenance

D : defective D^c: not defective

Then $P(D) = P(D|G)P(G) + P(D|W)P(W) + P(D|N)P(N) = (.02)(.8) + (.1)(.1) + (.3)(.1) = \boxed{0.056}$

2.2.6

A: win first lottery
B: win second lottery

Then $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$
$= .01 + .02 - (.01)(.02) = \boxed{0.0298}$

2.2.7

Part a

A : student A is in class
B : student B is in class

Then $P(A \cup B) = P(A) + P(B) - P(A)P(B) = .8 + .6 - (.8)(.6) = \boxed{0.92}$

Part b

$P(A|(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = .6/.92 \approx \boxed{0.652}$

2.2.13

$P(X = 1) = \binom{10}{1}.01^1.99^9 \approx \boxed{0.091}$

2.2.14

$P(X \geq 1) = 1 - P(X = 0) = 1 - .99^{10} \approx \boxed{0.096}$

2.2.15

Find $n \in \mathbb{N}$ s.t. $1 - .99^n \geq .8$

Then $n = \lceil \frac{\log(.2)}{\log(.99)} \rceil = \boxed{161}$

2.3.4

C denotes the event of having cancer. Then $P(C|+) = \frac{P(+|C)P(C)}{P(+|C)P(C)+P(+|C^c)P(C^c)} \approx \boxed{2 \times 10^{-4}}$

2.3.13

Part a

F : coin is fair

We also have $P(F|HH) = \frac{1}{5}$ from an example in the text.

Then $P(F|HHH) = \frac{P(F|HH)P(H|FHH)}{P(F|HH)P(H|FHH)+P(F^c|HH)P(H|F^cHH)}$

If the coin is fair, then the probability of heads is 1/2 regardless of the previous flips. If the coin is not fair, then the probability of heads is 1 regardless of the previous flips. Then this is $P(F|HHH) = \frac{1/5 \times 1/2}{1/5 \times 1/2 + 4/5 \times 1}$ $= \boxed{\frac{1}{9}}$.

Part b

If any coin flip lands tails, then it cannot be the biased coin. Therefore, $P(F|HHHT) = \boxed{1}$.

2.5.20

$P(B \text{ wins}) = P(B \text{ wins on second turn}) + P(B \text{ wins on fourth turn}) + ...$

We can see that $P(B \text{ wins on } i^{th} \text{ turn}) = P(i-1 \text{ failures})P(\text{success})$ where $i = 1, 3, 5, ...$

$P(i-1 \text{ failures}) = \Pi_j^{i-1} P(\text{failure}) = \Pi_j^{i-1} 5/6 = (5/6)^{i-1}$.

Then $P(B \text{ wins}) = \sum_{i=1} (5/6)^{2i-1}(1/6) = (1/6)(5/6) \sum_{i=0} ((5/6)^2)^i = \frac{5/36}{1-25/36} = \boxed{\frac{5}{11}}$.

2.5.24

$P(A|L) = \frac{P(L|A)P(A)}{P(L|A)P(A)+P(L|B)P(B)+P(L|C)P(C)} = \boxed{\frac{1}{5}}$.