STAT-S620

Assignment 2

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1.7.7

Since each box can hold any number of balls, there are 20^{12} possible arrangements. On the other hand, if we restrict each box to at most one ball, then there are only $\frac{20!}{(20-12)!}$ possible arrangements. Therefore, the probability is $\boxed{\dfrac{20!/8!}{20^{12}} \approx 0.015}$.

1.8.6

If A and B sit net to each other, then there are $n-1$ slots. On the other hand, all possible seating arrangements is equal to $\binom{n}{2}$. Then the probability that A and B sit next to each other is $\frac{n-1}{\binom{n}{2}} = \boxed{\dfrac{2}{n}}$.

1.8.8

Number of ways to seat people: $\binom{n}{k}$
Number of ways for k people to sit all together doesn't depend on k (as long as $k < n$ since it's cyclic. Then we can see that there are n ways everyone can all sit together.

Then the probability is $\boxed{\dfrac{n}{\binom{n}{k}}}$.

1.8.10

$$\boxed{\frac{\binom{22}{8}\binom{2}{2}}{\binom{24}{10}} \approx 0.163}$$

1.8.12

$P(\text{same team}) = P(\text{both on team of 10}) + P(\text{both on team of 25})$
$= \boxed{\dfrac{\binom{33}{8}\binom{2}{2}}{\binom{35}{10}} + \dfrac{\binom{33}{23}\binom{2}{2}}{\binom{35}{10}} \approx 0.58}$

1.8.17

By symmetry, this is just $4 \times P(\text{first player gets 4 Aces})$

$= \boxed{4\frac{\binom{4}{4}\binom{48}{9}}{\binom{52}{13}} \approx 0.011}$

1.9.8

Total ways to distribute cards: $\binom{52}{13,13,13,13}$
Number of ways to distribute such that each player gets 3 face cards: $\binom{12}{3,3,3,3}$
Number of ways to distribute the rest: $\binom{40}{10,10,10,10}$

Then the probability is: $\boxed{\frac{\binom{12}{3,3,3,3}\binom{40}{10,10,10,10}}{\binom{52}{13,13,13,13}} \approx 0.032}$

Not from text

There are $\binom{7+4-1}{4-1}$ ways to distribute the 7 points to the 4 students.

On the other hand, let's say each student already has one point. Then there are three points left over and there are still four students. Then there are $\binom{3+4-1}{4-1}$ ways to distribute those three remaining points among the four students. Therefore, the probability is:

$$\boxed{\frac{\binom{6}{3}}{\binom{10}{3}} = \frac{1}{6}}$$