STAT-S675

Homework 7

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Link to assignment

```
library(ggplot2)
dp <- loadNamespace('dplyr')
import::from(magrittr, `%>%`, `%<>%`)
import::from(ggrepel, geom_text_repel)
import::from(viridis, scale_colour_viridis)
import::from(readr, read_table)

theme_set(ggthemes::theme_base())

source('http://pages.iu.edu/~mtrosset/Courses/675/stress.r')
source('http://pages.iu.edu/~mtrosset/Courses/675/manifold.r')
```

Problem 1

Figure ??(a)

```
# parameters/constants
a <- sqrt(2 - 2 * cos(pi / 3))
b <- sqrt(2 + 2 * cos(pi / 3))

# construct the data
input.df <- dplyr::data_frame(i = seq(6)) %>%
  dp$mutate(theta = (i - 1) * pi / 3) %>%
  dp$mutate(x = cos(theta), y = sin(theta)) %>%
  # attach i+1, i+2, and i+3 for the edge weights
  dp$mutate(x.next = lead(x), y.next = lead(y),
            x.next.2 = lead(x, 2), y.next.2 = lead(y, 2),
            x.next.3 = lead(x, 3), y.next.3 = lead(y, 3))

# plot
figure.a <- ggplot(input.df) +
  coord_fixed() +
  # scale_colour_viridis() +
  scale_colour_distiller(palette = 'Spectral') +
  labs(x = NULL, y = NULL, colour = 'dissimilarity') +
  # edge weights
  geom_segment(aes(x = x, y = y,
                   xend = x.next, yend = y.next,
                   colour = a)) +
  geom_segment(aes(x = x, y = y,
                   xend = x.next.2, yend = y.next.2,
                   colour = b)) +
  geom_segment(aes(x = x, y = y,
```

```
                    xend = x.next.3, yend = y.next.3,
                    colour = 2)) +
  # vertices
  geom_point(aes(x = x, y = y)) +
  geom_text_repel(aes(x = x, y = y, label = i))

figure.a
```

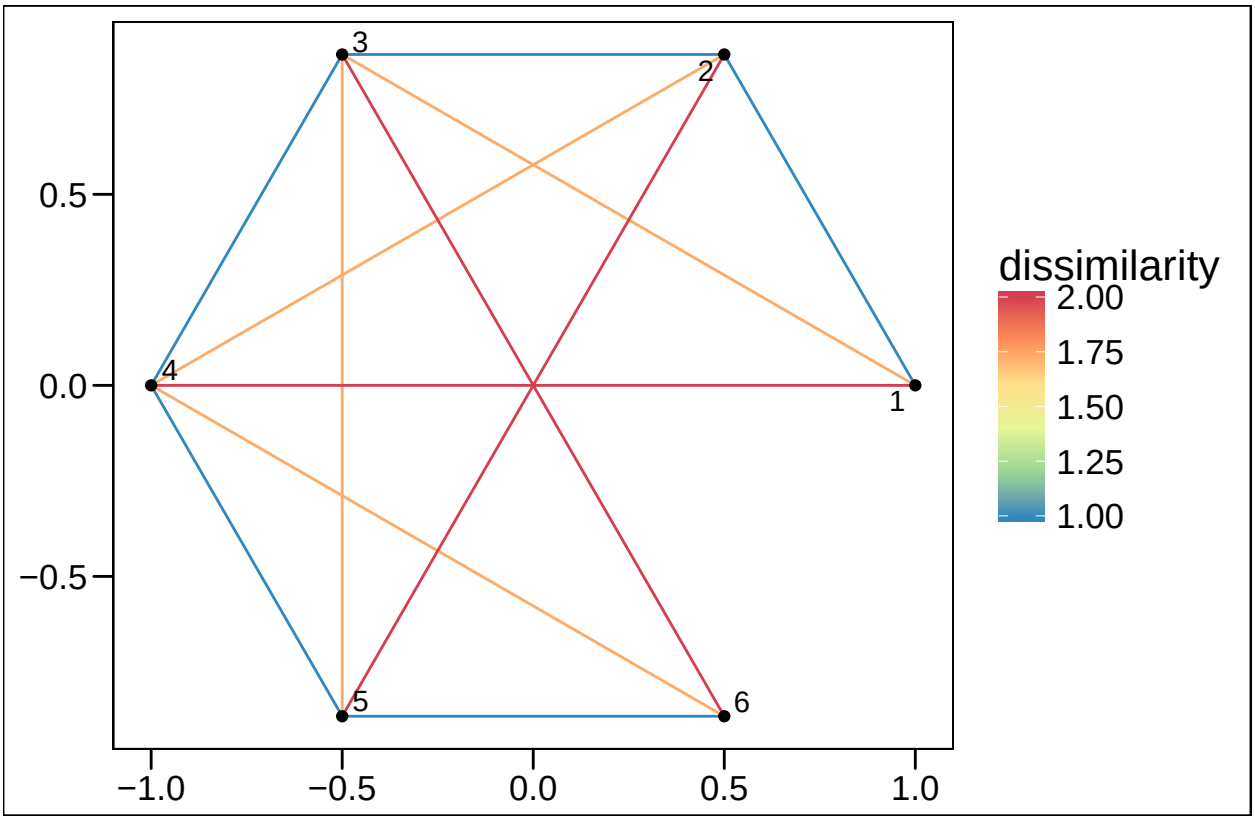
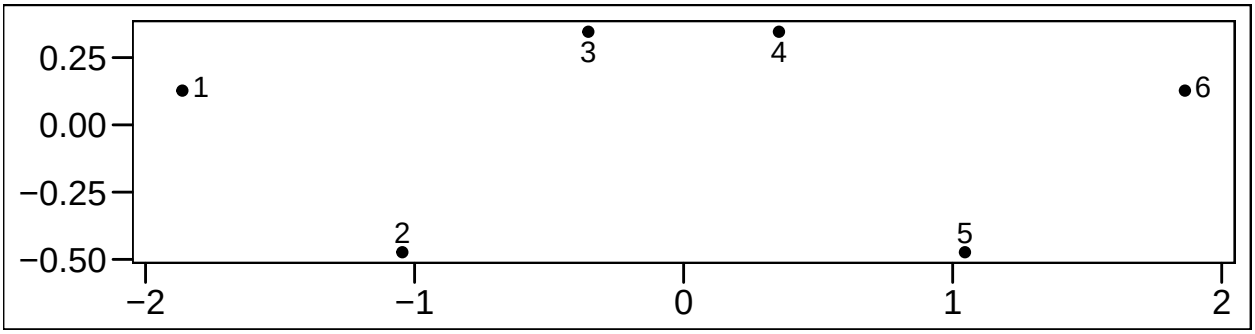


Figure ??(b)

```
# paramters/constants
N.iter <- 20

# specified dissimilarity matrix
Delta <- rbind(c(0, a, b, 2, a + 2, b + 2),
               c(a, 0, a, b, 2, a + 2),
               c(b, a, 0, a, b, 2),
               c(2, b, a, 0, a, b),
               c(a + 2, 2, b, a, 0, a),
               c(b + 2, a + 2, 2, b, a, 0))

# initialize the configuration using CMDS
X <- cmdscale(as.dist(Delta))
stress <- mds.stress.raw.eq(X, Delta)
gma.df <- as.data.frame(X) %>%
  dp$transmute(id = seq(6), x = V1, y = V2, iter = 0, stress)
```

```
# GMA optimization
for (i in seq(N.iter)) {
  # iterate
  X <- mds.guttman.eq(X, Delta)
  # compute stress
  stress <- mds.stress.raw.eq(X, Delta)
  # compile into data frame
  temp.df <- as.data.frame(X) %>%
  dp$transmute(id = seq(6), x = V1, y = V2, iter = i, stress)
  # attach data frame to original
  gma.df %<>% dp$bind_rows(temp.df)
}

# plot final configuration
gma.df %>%
  dp$filter(iter == N.iter) %>%
  ggplot() +
  labs(x = NULL, y = NULL) +
  coord_fixed() +
  geom_point(aes(x = x, y = y)) +
  geom_text_repel(aes(x = x, y = y, label = id))
```

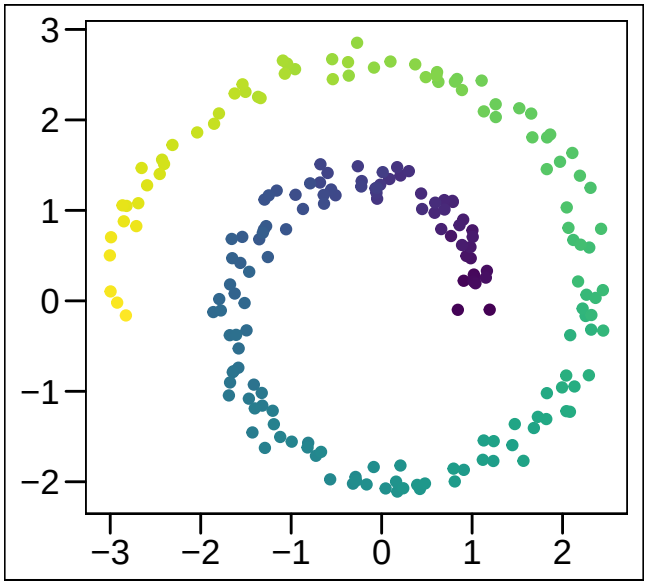


Problem 2

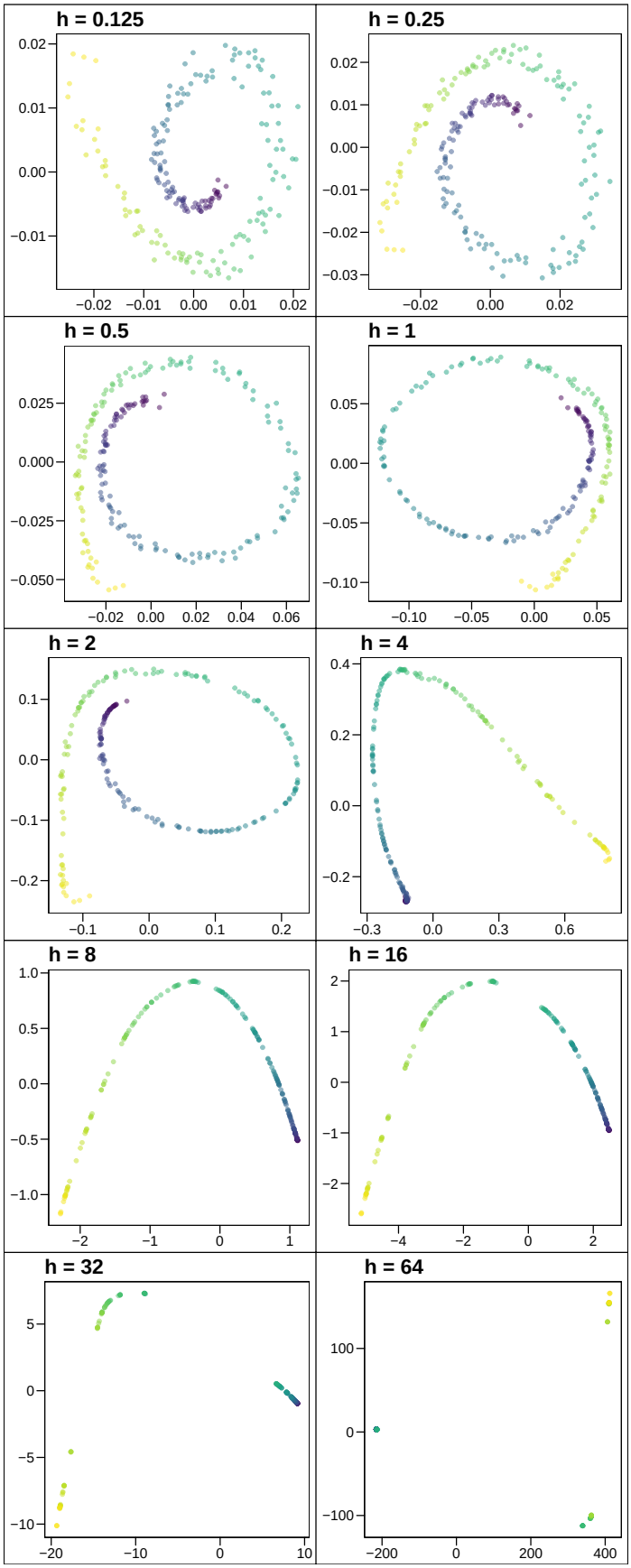
[Exercise 6.8.1 from the text]

```
# load data
spiral.df <- read_table('http://pages.iu.edu/~mtrosset/Courses/675/X.spiral',
                        col_names = FALSE)

# save the number of rows of the data
# for when we want the 2 smallest eigenvalues
n <- nrow(spiral.df)

# plot the data
spiral.df %>%
  dp$mutate(id = as.numeric(rownames(.))) %>%
  ggplot() +
  viridis::scale_colour_viridis() +
  geom_point(aes(x = X1, y = X2, colour = id)) +
  coord_fixed() +
```

```
  theme(legend.position = 'none') +
  labs(x = NULL, y = NULL)
```




```
# values of h to try
h.vector <- 2 ** seq(-3, 6)

eigenmaps <- lapply(h.vector, function(h) {
  # i <- i + 1
  W <- exp(-h * as.matrix(dist(spiral.df)) ** 2)

  L <- graph.laplacian(W)

  L.eigen <- eigen(L)

  eigenmap <- cbind(L.eigen$vectors[, n - 1] / sqrt(L.eigen$values[n - 1]),
                    L.eigen$vectors[, n - 2] / sqrt(L.eigen$values[n - 2])) %>%
    as.data.frame() %>%
    dp$mutate(id = as.numeric(rownames(.))) %>%
    ggplot() +
    geom_point(aes(x = V1, y = V2, colour = id),
               alpha = .5) +
    # coord_fixed() +
    viridis::scale_colour_viridis() +
    labs(x = NULL, y = NULL, title = paste('h =', h)) +
    theme(legend.position = 'none')
  return(eigenmap)
})

.gridarrange <- function(...) gridExtra::grid.arrange(..., ncol = 2)
do.call(.gridarrange, eigenmaps)
```


h = 0.125
h = 0.25
h = 0.5
h = 1
h = 2
h = 4
h = 8
h = 16
h = 32
h = 64

Problem 3