

# STAT-S620

## Assignment 3

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### 2.1.9

#### Part a

There are 4 cards left, 3 of which are red. Therefore, the probability is just  $\boxed{\frac{3}{4}}$ .

#### Part b

Since all but one card are red, we are guaranteed to get at least one red card. Therefore, this is just the probability that both are red, which is  $\frac{4}{5} \times \frac{3}{4} = \boxed{\frac{3}{5}}$ .

### 2.1.14

$G$  : good working order

$W$  : wearing down

$N$  : needs maintenance

$D$  : defective  $D^c$ : not defective

Then  $P(D) = P(D|G)P(G) + P(D|W)P(W) + P(D|N)P(N) = (.02)(.8) + (.1)(.1) + (.3)(.1) = \boxed{0.056}$

### 2.2.6

$A$ : win first lottery

$B$ : win second lottery

Then  $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$   
 $= .01 + .02 - (.01)(.02) = \boxed{0.0298}$

### 2.2.7

#### Part a

$A$  : student  $A$  is in class

$B$  : student  $B$  is in class

Then  $P(A \cup B) = P(A) + P(B) - P(A)P(B) = .8 + .6 - (.8)(.6) = \boxed{0.92}$

## Part b

$$P(A|(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = .8/.92 \approx \boxed{0.87}$$

### 2.2.13

$$P(X = 1) = \binom{10}{1}.01^1.99^9 \approx \boxed{0.091}$$

### 2.2.14

$$P(X \geq 1) = 1 - P(X = 0) = 1 - .99^{10} \approx \boxed{0.096}$$

### 2.2.15

Find  $n \in \mathbb{N}$  s.t.  $1 - .99^n \geq .8$

$$\text{Then } n = \lceil \frac{\log(.2)}{\log(.99)} \rceil = \boxed{161}$$

### 2.3.4

$$C \text{ denotes the event of having cancer. Then } P(C|+) = \frac{P(+|C)P(C)}{P(+|C)P(C)+P(+|C^c)P(C^c)} \approx \boxed{2 \times 10^{-4}}$$

### 2.3.13

## Part a

$F$  : coin is fair

We also have  $P(F|HH) = \frac{1}{5}$  from an example in the text.

$$\text{Then } P(F|HHH) = \frac{P(F|HH)P(H|FHH)}{P(F|HH)P(H|FHH)+P(F^c|HH)P(H|F^cHH)}$$

If the coin is fair, then the probability of heads is  $1/2$  regardless of the previous flips. If the coin is not fair, then the probability of heads is  $1$  regardless of the previous flips. Then this is  $P(F|HHH) = \frac{1/5 \times 1/2}{1/5 \times 1/2 + 4/5 \times 1}$

$$= \boxed{\frac{1}{9}}.$$

## Part b

If any coin flip lands tails, then it cannot be the biased coin. Therefore,  $P(F|HHHT) = \boxed{1}$ .

### 2.5.20

$$P(B \text{ wins}) = P(B \text{ wins on second turn}) + P(B \text{ wins on fourth turn}) + \dots$$

We can see that  $P(B \text{ wins on } i^{\text{th}} \text{ turn}) = P(i - 1 \text{ failures})P(\text{success})$  where  $i = 1, 3, 5, \dots$

$$P(i - 1 \text{ failures}) = \Pi_j^{i-1} P(\text{failure}) = \Pi_j^{i-1} 5/6 = (5/6)^{i-1}.$$

$$\text{Then } P(B \text{ wins}) = \sum_{i=1} (5/6)^{2i-1} (1/6) = (1/6)(5/6) \sum_{i=0} ((5/6)^2)^i = \frac{5/36}{1-25/36} = \boxed{\frac{5}{11}}.$$

### 2.5.24

$$P(A|L) = \frac{P(L|A)P(A)}{P(L|A)P(A)+P(L|B)P(B)+P(L|C)P(C)} = \boxed{\frac{1}{5}}.$$