

S626

HW1

John Koo

Problem 2

Part a

Using $P(A) = \sum_i P(A \cap B_i)$

occupation	P(occupation)
farm	0.11
operatives	0.279
craftsmen	0.277
sales	0.099
professional	0.235

Part b

Similar to part (a)

occupation	P(occupation)
farm	0.023
operatives	0.26
craftsmen	0.24
sales	0.125
professional	0.352

Part c

Using $P(A|B) = \frac{P(A \cap B)}{P(B)}$

occupation	P(son's occupation father is farmer)
farm	0.164
operatives	0.318
craftsmen	0.282
sales	0.073
professional	0.164

Part d

Similar to part (c)

occupation	P(father's occupation son is farmer)
farm	0.783
operatives	0.087
craftsmen	0.043
sales	0.043
professional	0.043

Problem 3

Part a

We can drop all the parts that do not depend on x

$$p(x|y, z) = \frac{p(x, y, z)}{p(y, z)} \propto p(x, y, z) \propto f(x, z)g(y, z)h(z) \propto f(x, z)$$

Part b

Similar to part (a), we can say:

$$p(y|x, z) \propto p(x, y, z) \propto g(y, z)$$

Part c

$$p(x, y|z) = \frac{p(x, y, z)}{p(z)} \propto p(x, y, z) \propto f(x, z)g(y, z)h(z) \propto f(x, z)g(y, z) \propto p(x, z)p(y, z) = p(x|z)p(z)p(y|z)p(z) \propto p(x|z)p(y|z)$$

Suppose $p(x, y|z) \neq p(x|z)p(y|z)$, i.e., $p(x, y|z) = kp(x|z)p(y|z)$. Since $\int p(x, y|z)dx dy = 1$, $k = (\int p(x|z)p(y|z)dx dy)^{-1}$. But $\int p(x|z)p(y|z)dx dy = (\int p(x|z)dx)(\int p(y|z)dx) = (1)(1) = 1 \implies k = 1$. Therefore, $p(x, y|z) = p(x|z)p(y|z)$.

Problem 4

Part a

$$P_A(E) = \begin{cases} 0 & \text{6 is not the observed outcome} \\ 1 & \text{6 is the observed outcome} \end{cases} \text{ since person } A \text{ already observed the outcome.}$$

On the other hand, let's say person B assumes that the die is fair. Then $P_B(E) = 1/6$.

Part b

If person A is ignorant of soccer, then they may assign a uniform distribution across all teams. Of course, A may not know how many teams compete, so a guess might be 200, which is approximately the number of countries in the world. So perhaps $P_A(E) = 1/200$.

On the other hand, if person B is knowledgeable about soccer, then they may know that there are 32 teams that qualify for the World Cup and that Germany is almost guaranteed to qualify, so perhaps $P_B(E) = 1/32$. Person B probably also knows that Germany won 2 out of 10 most recent World Cups, so perhaps $P_B(E) = 1/5$.