STAT-S631

Exam 2

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Statement

On my honor, I have not had any form of communication about this exam with any other individual (including other students, teaching assistants, instructors, etc.).

Signed: *John Koo*

```r
# packages
import::from(magrittr, `%>%`, `%<>%`)
dp <- loadNamespace('dplyr')
library(ggplot2)
import::from(car, Anova, ncvTest)

# plotting stuff
theme_set(theme_bw())

# read data
flight.df <- read.table('~/dev/stats-hw/stat-s631/takehome2.txt')
head(flight.df)
```

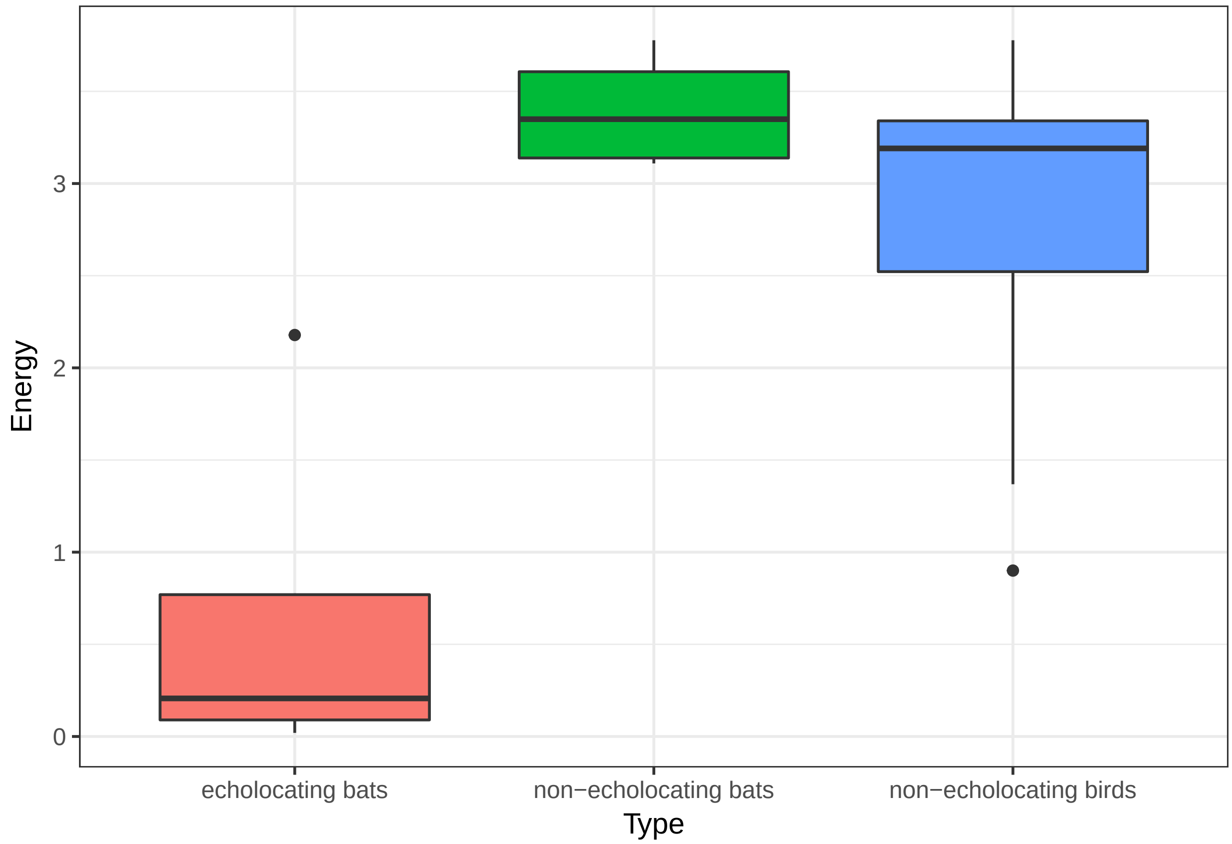
```
   Mass                   Type    Energy
1 779.0  non-echolocating bats 3.7773481
2 628.0  non-echolocating bats 3.5496174
3 258.0  non-echolocating bats 3.1484534
4 315.0  non-echolocating bats 3.1090610
5  24.3 non-echolocating birds 0.9001613
6  35.0 non-echolocating birds 1.3686394
```

```r
summary(flight.df)
```

```
      Mass                            Type        Energy
 Min.   :  6.70   echolocating bats     : 4   Min.   :0.0198
 1st Qu.: 63.35   non-echolocating bats : 4   1st Qu.:1.9758
 Median :266.50   non-echolocating birds:12   Median :3.1179
 Mean   :262.68                               Mean   :2.4822
 3rd Qu.:391.00                               3rd Qu.:3.3399
 Max.   :779.00                               Max.   :3.7773
```

Problem 1

```r
ggplot(flight.df) +
  geom_boxplot(aes(x = Type, y = Energy, group = Type, fill = Type)) +
  scale_colour_brewer(palette = 'Set1') +
  guides(fill = FALSE)
```




```r
mod.1 <- lm(Energy ~ Type, data = flight.df)
summary(mod.1)
```

```
Call:
lm(formula = Energy ~ Type, data = flight.df)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.88718 -0.39944  0.02359  0.49323  1.52531 

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)    
(Intercept)                  0.6528     0.4224   1.546 0.140585    
Typenon-echolocating bats    2.7433     0.5973   4.593 0.000259 ***
Typenon-echolocating birds   2.1345     0.4877   4.377 0.000411 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8447 on 17 degrees of freedom
Multiple R-squared:  0.5953,	Adjusted R-squared:  0.5477 
F-statistic:  12.5 on 2 and 17 DF,  p-value: 0.0004576
```

`Energy` for `Type` = `"non-echolocating bats"` and `"non-echolocating birds"` is significantly different than the baseline, `Type` = `"echolocating bats"`, for usual values of α. However, the plot suggests that there isn't a significant difference between `"non-echolocating bats"` and `"non-echolocating birds"`. We can test this as follows:

```r
L <- c(0, 1, -1)
c <- 0
beta.hat <- mod.1$coefficients
V.hat <- vcov(mod.1)

F.stat <- t(L %*% beta.hat - c) %*%
  solve(L %*% V.hat %*% L) %*%
  (L %*% beta.hat - c)
F.stat
```

```
         [,1]
[1,] 1.558157
```

```r
1 - pf(F.stat, 1, mod.1$df.residual)
```

```
          [,1]
[1,] 0.2288543
```

So we fail to reject the null hypothesis that $\beta_1 = \beta_2$. In other words, we cannot say that the difference in `Energy` is statistically significant between `Types` `"non-echolocating bats"` and `"non-echolocating birds"`. But since they are both significantly different from the baseline, we can say it is reasonable to use `Type` as a regressor. We might want to try this model:

```r
flight.df %<>%
  dp$mutate(type = dp$if_else(Type != 'echolocating bats',
                              'non-echolocating bats/birds',
                              'echolocating bats'))
mod.2 <- lm(Energy ~ type, data = flight.df)
summary(mod.2)
```

```

Call:
lm(formula = Energy ~ type, data = flight.df)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.0394 -0.3994  0.1981  0.4803  1.5253 

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)    
(Intercept)                       0.6528     0.4289   1.522 0.145309    
typenon-echolocating bats/birds   2.2867     0.4795   4.769 0.000153 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8577 on 18 degrees of freedom
Multiple R-squared:  0.5582,	Adjusted R-squared:  0.5337 
F-statistic: 22.74 on 1 and 18 DF,  p-value: 0.0001534
```

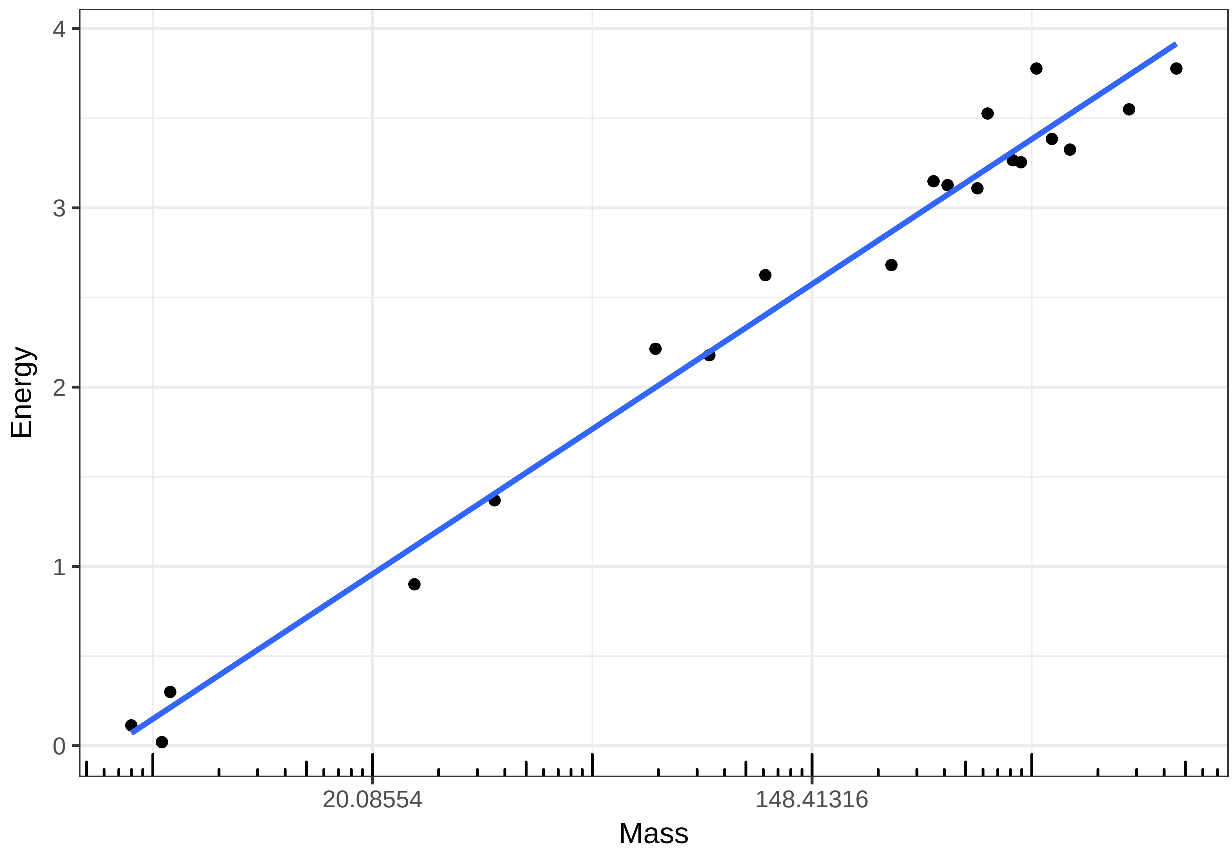
```r
anova(mod.2, mod.1)  # should be the same as before
```

```
Analysis of Variance Table

Model 1: Energy ~ type
Model 2: Energy ~ Type
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     18 13.242
```

```
2      17 12.130  1     1.1118 1.5582 0.2289
```

Problem 2

```
ggplot(flight.df) +
  geom_point(aes(x = Mass, y = Energy)) +
  scale_x_continuous(trans = 'log') +
  annotation_logticks(sides = 'b') +
  stat_smooth(aes(x = Mass, y = Energy), method = 'lm', se = FALSE)
```




```
lin.mod <- lm(Energy ~ log(Mass), data = flight.df)
quad.mod <- lm(Energy ~ log(Mass) + I(log(Mass) ** 2), data = flight.df)
anova(quad.mod)
```

```
Analysis of Variance Table

Response: Energy
               Df  Sum Sq Mean Sq F value    Pr(>F)
log(Mass)       1 29.3919 29.3919 920.597 3.024e-16 ***
I(log(Mass)^2)  1  0.0401  0.0401   1.257    0.2778
Residuals      17  0.5428  0.0319
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the scatterplot, it appears that there is no reason to use higher order terms. The ANOVA test confirms this.

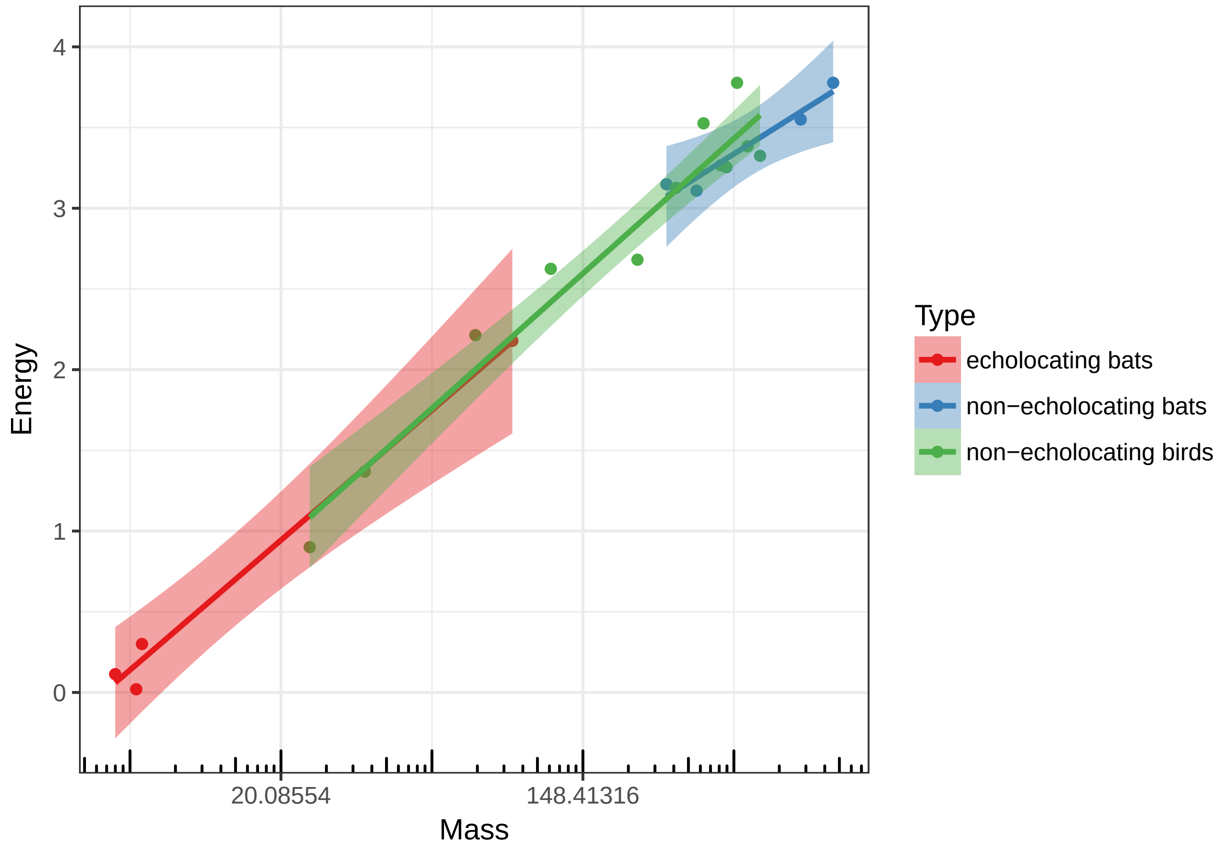
```r
summary(lin.mod)
```

```

Call:
lm(formula = Energy ~ log(Mass), data = flight.df)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.21143 -0.14422 -0.04284  0.09681  0.37695 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.46826    0.13716  -10.71  3.1e-09 ***
log(Mass)    0.80861    0.02684   30.13  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.18 on 18 degrees of freedom
Multiple R-squared:  0.9806,    Adjusted R-squared:  0.9795 
F-statistic: 907.6 on 1 and 18 DF,  p-value: < 2.2e-16
```

A linear model using `Mass` appears to be appropriate in this case.

Problem 3

```r
ggplot(flight.df) +
  geom_point(aes(x = Mass, y = Energy, colour = Type)) +
  scale_colour_brewer(palette = 'Set1') +
  scale_x_continuous(trans = 'log') +
  annotation_logticks(sides = 'b') +
  stat_smooth(aes(x = Mass, y = Energy, fill = Type, colour = Type),
              method = 'lm') +
  scale_fill_brewer(palette = 'Set1')
```



From the scatterplot, it appears that changes in `Energy` explained by `Type` is pretty much all captured by `Mass`. We can test this:

```r
summary(flight.df)
```

```
      Mass                           Type        Energy      
 Min.   :  6.70   echolocating bats     : 4   Min.   :0.0198  
 1st Qu.: 63.35   non-echolocating bats : 4   1st Qu.:1.9758  
 Median :266.50   non-echolocating birds:12   Median :3.1179  
 Mean   :262.68                               Mean   :2.4822  
 3rd Qu.:391.00                               3rd Qu.:3.3399  
 Max.   :779.00                               Max.   :3.7773  
     type          
 Length:20         
 Class :character  
 Mode  :character  
```

```r
full.mod <- lm(Energy ~ log(Mass) * Type, data = flight.df)
Anova(full.mod)
```

```
Anova Table (Type II tests)

Response: Energy
                Sum Sq Df  F value    Pr(>F)    
log(Mass)      11.5770  1 321.0305 4.748e-11 ***
```

```
Type            0.0296  2   0.4100    0.6713
log(Mass):Type  0.0484  2   0.6718    0.5265
Residuals       0.5049 14
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(lin.mod, full.mod)
```

```
Analysis of Variance Table

Model 1: Energy ~ log(Mass)
Model 2: Energy ~ log(Mass) * Type
  Res.Df     RSS Df Sum of Sq      F Pr(>F)
1     18 0.58289
2     14 0.50487  4  0.078023 0.5409 0.7084
```

Type II test (`Anova`):

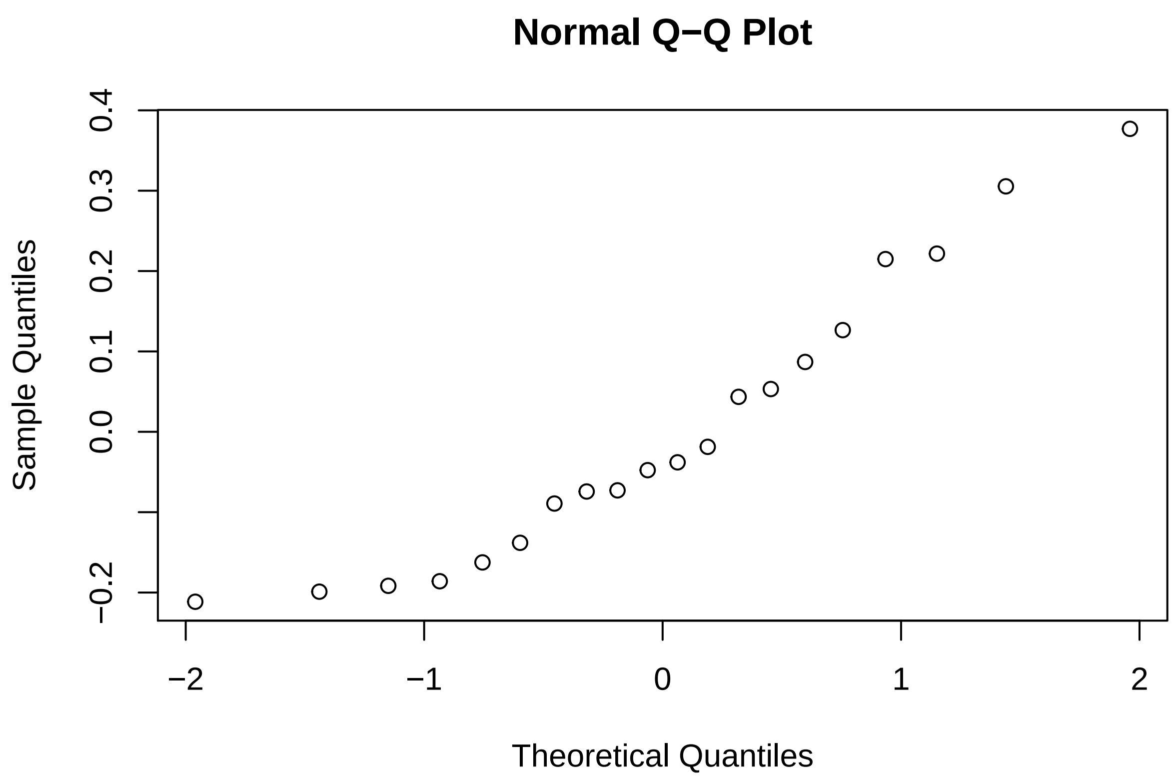
- A model excluding `log(Mass)` (and the interaction term, per the marginality principle), i.e., the model `Energy ~ Type`, is significantly different than the full model.
- A model excluding `Type` (and the interaction term, per the marginality principle), i.e., the model `Energy ~ Mass`, is not significantly different than the full model.
- A model excluding the interaction term, i.e., `Energy ~ Mass + Type`, is not significantly different than the full model.

Type I test (`anova`): The model `Energy ~ log(Mass)` is not significantly different from the full model. This is consistent with the scatterplot. So we can conclude that the most appropriate model is `Energy ~ log(Mass)`.

Problem 4

Part a

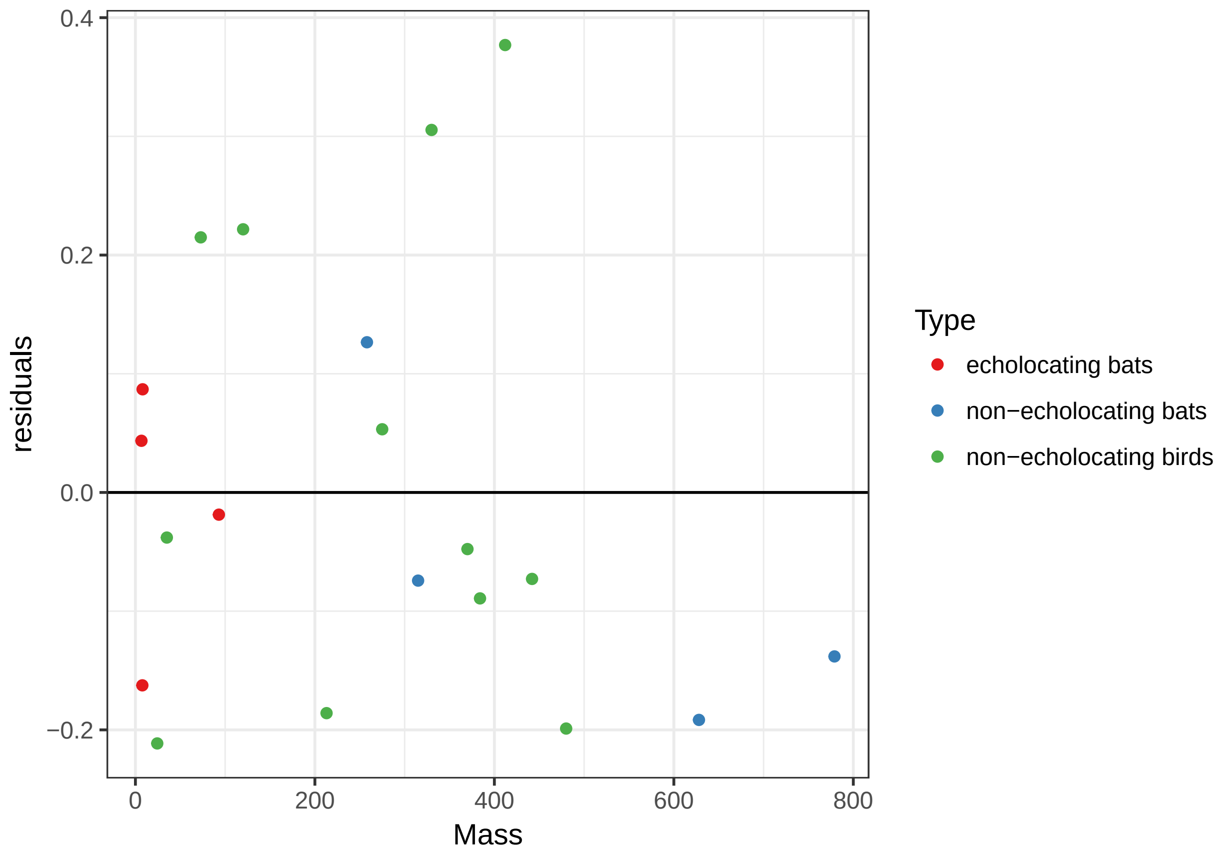
```
# quick tests for normality
qqnorm(lin.mod$residuals)
```




```
shapiro.test(lin.mod$residuals)
```

```
	Shapiro-Wilk normality test

data:  lin.mod$residuals
W = 0.92436, p-value = 0.1202
```

```
flight.df %<>%
  dp$mutate(energy.pred = predict(lin.mod, flight.df),
            resid = Energy - energy.pred)

ggplot(flight.df) +
  geom_point(aes(x = Mass, y = resid, colour = Type)) +
  geom_abline(slope = 0) +
  labs(y = 'residuals') +
  scale_colour_brewer(palette = 'Set1')
```




```
ncvTest(lin.mod, ~ log(Mass), data = flight.df)
```

```
Non-constant Variance Score Test
Variance formula: ~ log(Mass)
Chisquare = 0.6779911    Df = 1     p = 0.4102793
```

```
ncvTest(lin.mod, data = flight.df)
```

```
Non-constant Variance Score Test
Variance formula: ~ fitted.values
Chisquare = 0.6779911    Df = 1     p = 0.4102793
```

```
ncvTest(lin.mod, ~ Type, data = flight.df)
```

```
Non-constant Variance Score Test
Variance formula: ~ Type
Chisquare = 1.879569    Df = 2     p = 0.3907121
```

Visually, there appears to be no reason to believe that that variance isn't consistent. The non-constant variance tests confirm this.

Part b

Since we have no reason to believe that the residuals are heteroskedastic, we will just use the OLS estimators.

```
alpha <- .98

mass.t <- unname(lin.mod$coefficients['log(Mass)'])
```

```r
mass.se <- summary(lin.mod)$coefficients['log(Mass)', 'Std. Error']
t.98 <- qt(.5 + alpha / 2, lin.mod$df.residual)
c('lower' = mass.t - t.98 * mass.se,
  'estimate' = mass.t,
  'upper' = mass.t + t.98 * mass.se)
```

```
    lower  estimate     upper
0.7401039 0.8086098 0.8771156
```

```r
# same thing
confint(lin.mod, 'log(Mass)', .98)
```

```
                1 %      99 %
log(Mass) 0.7401039 0.8771156
```