Homework #3 **Math 527, UNH spring 2018**
Due Thursday, Feb. 22 in recitation.

Follow the usual instructions on homework submission: Be clear, legible, and organized. Write on loose-leaf paper. Staple together in the upper-left-hand corner, write your name, section #, Math 527 HW2, and date in the upper-right-hand corner.

Problems 1-3. Find the general solution. The "prime" notation indicates differentiation with respect to x: $y' = dy/dx$, etc.

1. $6y'' - 7y' + y = 0$

2. $y'' + 2y' + 3y = 0$

3. $y'' - 6y' + 9y = 0$

Problems 4-6. Solve the initial-value problem. As in problems 1-3, $y' = dy/dx$, etc.

4. $2y'' + y' - 10y = 0, \quad y(1) = 5,\ y'(1) = 2$

5. $4y'' - 4y' + y = 0, \quad y(0) = 0,\ y'(0) = 3$

6. $y'' + y' + 2y = 0, \quad y(0) = 1,\ y'(0) = -2$

Applications of 1st-order equations

Problem 7. A thermometer is removed from a room where the temperature is 70° F and is placed outside where the air temperature is 10° F. Thirty seconds later, the thermometer reads 50° F. What is the reading on the thermometer one minute after being taken outside? How long will it take for the thermometer to reach 15° F?

Approach: use Newton's law of cooling

$$\frac{dT}{dt} = k(T - T_m)$$

where $T(t)$ is the temperature of the thermometer, T_m is the ambient temperature (here $T_m = 10$), and k is a constant of proportionality. Solve the differential equation for $T(t)$ and then use the initial condition $T(0) = 70$ and thirty-second reading $T(30) = 50$ to determine the values of the constants.

Once you have the explicit solution $T(t)$, go back and answer the questions in the problem statement.

Problem 8. A tank contains 200 liters of water in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is pumped into the tank at a rate of 4 L/min. The well-mixed solution is pumped out at the same rate. Derive a differential equation for $A(t)$, the number of grams of salt in the tank at time t. Then solve the differential equation to get an explicit function for $A(t)$. How many grams of salt will be in the tank as $t \to \infty$?

Approach: $dA/dt = R_{\text{in}} - R_{\text{out}}$, where R_{in} is the rate of salt flowing in, and R_{out} is the rate of salt flowing out, both in grams per min. A little thinking yields

$$R_{\text{in}} = 1 \text{ g/L} \cdot 4 \text{ L/min} = 4 \text{ g/min}$$

and

$$R_{\text{out}} = \frac{A(t) \text{ g}}{200 \text{ L}} \cdot 4 \text{ L/min} = \frac{A}{50} \text{ g/min}$$

Drop the g/min units to get $R_{\text{in}} = 4$ and $R_{\text{out}} = A(t)/50$. Plug these back into the equation for dA/dt. Then solve the differential equation and initial value problem.

Once you have the explicit solution $A(t)$, go back and answer the questions in the problem statement.

Problem 9. Two chemicals A and B combine in reaction to form a chemical C. The rate of production of C is proportional to the amounts of chemicals A and B present at any given instant. Two grams of A combine with one gram of B to form three grams of C. Initially, there are 100 grams of A, 50 grams of B, and 0 grams of C. Ten minutes later, 10 grams of C have been formed. Derive a function $C(t)$ for the number of grams of C at time t. How many grams of C are formed in the limit of infinite time? How long does it take for half that amount of C to be produced?

The reaction is governed by the relation

$$\frac{dC}{dt} = kA(t)B(t)$$

One gram of C is composed of 2/3 grams A and 1/3 gram B. Given the initial concentrations of A,B,C, a little thinking produces these equations: $A(t) = 100 - 2/3\ C(t)$ and $B(t) = 50 - 1/3\ C(t)$. Plug those into the equation for dC/dt and solve the resulting initial value problem.

Once you have the explicit solution $C(t)$, go back and answer the questions in the problem statement.

Problems 7 and 8 are taken from Zill's textbook, exercises 3.1 #13 and #20.