MATH 527, HW2 solutions

1) $2x+y-(x+6y)\frac{dy}{dx}=0$

Step ∅: $2x+y+(-x-6y)\frac{dy}{dx}=0$

$M=2x+y \qquad N=-x-6y$

Step 1; Test

$-1\neq 1$

$\frac{\partial M}{\partial y}=1 \qquad \frac{\partial N}{\partial x}=-1$ - $\boxed{\text{„Not exact"}}$ ⊗

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2) $2x-1+(3y+7)\frac{dy}{\partial x}=0$

Step ∅: $M=2x-1 \qquad N=3y+7$

Step1: Test:

$\frac{\partial M}{\partial y}=0 \qquad \frac{\partial N}{\partial x}=0$

„Exact" → so $\exists f(x,y)$ s.t $M=\frac{\partial f}{\partial x}$ and $N=\frac{\partial f}{\partial y}$

Step2: $\frac{\partial f(x,y)}{\partial x}=2x-1 \quad (=M)$

so $f(x,y)=\int(2x-1)\,dx=\underline{x^2-x+g(y)}$ ☆

Step3: $N=\frac{\partial f}{\partial y}$, so $3y+7=\frac{\partial}{\partial y}(x^2-x+g(y))$

$3y+7=g'(y)$

Step4: (find $g(y)$): $\underline{g(y)=\int(3y+7)dy=\frac{3}{2}y^2+7y}$ ☆

Step5: (from ☆s) $f(x,y)=x^2-x+\frac{3}{2}y^2+7y$

Step6: $\boxed{x^2-x+\frac{3}{2}y^2+7y=C}$ ← Sol'n

3) $5t+4y+(4t-8y^3)\frac{dy}{dt}=0$

⓪ $M=5t+4y$ $\quad N=4t-8y^3$

① $\frac{\partial M}{\partial y}=4$ $\quad \frac{\partial N}{\partial t}=4$ $\quad$ ✓ „exact"

so $\exists f(t,y)$ s.t. $M=\frac{\partial f}{\partial t}$ and $N=\frac{\partial f}{\partial y}$

② $\frac{\partial f(t,y)}{\partial t}=M$, so

$$f(t,y)=\int M\,dt=\int(5t+4y)\,dt=\frac{5}{2}t^2+4ty+g(y)$$

③ $N=\frac{\partial f(t,y)}{\partial y}$, so $\quad N=\frac{\partial}{\partial y}\left(\frac{5}{2}t^2+4ty+g(y)\right)$

$$4t-8y^3=4t+g'(y)$$

④ $g'(y)=-8y^3$

$$g(y)=\int -8y^3\,dy=-2y^4$$

⑤ $f(t,y)=\frac{5}{2}t^2+4ty-2y^4$

⑥ $\boxed{\frac{5}{2}t^2+4ty-2y^4=C}$

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4) $x^2-y^2+(x^2-2xy)\frac{dy}{dx}=0$

$M=x^2-y^2$ $\quad N=x^2-2xy$

$\frac{\partial M}{\partial y}=-2y$ $\quad \frac{\partial N}{\partial x}=2x-2y$

$\boxed{\text{Not „exact"}}$

5) $t\dfrac{dy}{dt} = 2te^t - y + 6t^2$

(0) $2te^t - y + 6t^2 + (-t)\dfrac{dy}{dt} = 0$

$M = 2te^t - y + 6t^2$ $\qquad N = -t$

Test: (1) $\dfrac{\partial M}{\partial y} = -1$ $\qquad \dfrac{\partial N}{\partial t} = -1$ $\quad$ ✓ "exact"!

so $\exists\, f(t,y)$ s.t. $M = \dfrac{\partial f}{\partial t}$ and $N = \dfrac{\partial f}{\partial y}$

(2) $N = \dfrac{\partial f}{\partial y}$, so

$f = \int -t\, dy = -ty + g(t)$

(3) $M = \dfrac{\partial f(t,y)}{\partial t}$, so $\quad M = \dfrac{\partial}{\partial t}(-ty + g(t))$

$2te^t - y + 6t^2 = -y + g'(t)$

(4) $g'(t) = 2te^t + 6t^2$

Int. By Parts

$g(t) = \int (2te^t + 6t^2)\,dt = 2te^t - 2e^t + 2t^3$

(5) $f(t,y) = -ty + 2te^t - 2e^t + 2t^3$

(6) $-ty + 2te^t - 2e^t + 2t^3 = C_1$

$$\boxed{y = \dfrac{2te^t - 2e^t + 2t^3 + C}{t}}$$

6) $(x+y)^2+(2xy+x^2-1)\frac{dy}{dx}=0$

$x^2+2xy+y^2+(2xy+x^2-1)\frac{dy}{dx}=0$

$M=x^2+2xy+y^2 \quad N=2xy+x^2-1$

$\frac{\partial M}{\partial y}=2x+2y \quad \frac{\partial N}{\partial x}=2y+2x$ ✓ Exact so $\exists\, f(x,y)$ s.t. $M=\frac{\partial f}{\partial x}$ and $N=\frac{\partial f}{\partial y}$

$f(x,y)=\int M\,dx=\int(x^2+2xy+y^2)dx=\frac{x^3}{3}+x^2y+xy^2+g(y)$

Since $N=\frac{\partial f}{\partial y}$

$2xy+x^2-1=\frac{\partial}{\partial y}\left(\frac{x^3}{3}+x^2y+xy^2+g(y)\right)$

$2xy+x^2-1=x^2+2xy+g'(y)$

$g'(y)=-1$

$g(y)=-y$

$f=\frac{x^3}{3}+x^2y+xy^2-y$

$\boxed{\frac{x^3}{3}+x^2y+xy^2-y=C}$

7) $\sin y-y\sin x+(\cos x+x\cos y-y)\frac{dy}{dx}=0$

$M=\sin y-y\sin x \quad N=\cos x+x\cos y-y$

$\frac{\partial M}{\partial y}=\cos y-\sin x \quad \frac{\partial N}{\partial x}=-\sin x+\cos y$ ✓ exact $\to \exists\, f(x,y)$ s.t. $M=\frac{\partial f}{\partial x}$, $N=\frac{\partial f}{\partial y}$

$f(x,y)=\int M\,dx=\int(\sin y-y\sin x)\,dx=x\cdot\sin y+y\cos x+g(y)$

$N=\frac{\partial f(x,y)}{\partial y}$, so

$\cos x+x\cos y-y=\frac{\partial}{\partial y}(x\sin y+y\cos x+g(y))$

$\cos x+x\cos y-y=x\cos y+\cos x+g'(y)$

$g'(y)=-y$

$g(y)=\int -y\,dy=-\frac{y^2}{2}$

$f(x,y)=x\sin y+y\cos x-\frac{y^2}{2}$

$\boxed{x\sin y+y\cos x-\frac{y^2}{2}=C}$