PREAMBLE: This is a sample exam for UNH Math 527 exam 1, spring 2018. You should take this as a *rough* example of what the exam might be like in terms style, problem difficulty, and coverage, *but not a direct indication of the kinds of problems that will appear on the exam.* This exam is perhaps a bit too long for 50 minutes. For a real 50-minute exam I would simplify problem 4, cutting out the derivation of the reduced equation.

Exam 1 sample
Math 527 UNH

instructions	1	2	3	4	total

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Section: 0

INSTRUCTIONS (5 pts)
1. Write your name and section number legibly **in pen** on each page.
2. Show your work and put a box or circle around your final answers.
3. Simplify your answers and find explicit solutions where possible.
4. Your work should be clear and organized.
5. Always write equations.

Problem 1. (25 pts) Identify the type of equation. Find all solutions and specify their intervals.

equation type
separable

$$\frac{dy}{dx} - (x-3)(y+1)^{2/3} = 0$$

$$(y+1)^{-2/3}\frac{dy}{dx} = (x-3) \qquad \text{for } y+1 \neq 0 \text{ : possible singular soln!}$$

$$\int (y+1)^{-2/3}\frac{dy}{dx}\,dx = \int x-3\,dx$$

$$3(y+1)^{1/3} = \frac{1}{2}x^2 - 3x + c$$

$$(y+1)^{1/3} = \frac{1}{6}x^2 - x + c \qquad (\text{aborb } 1/3 \text{ into } c)$$

$$\boxed{y(x) = \left(\frac{1}{6}x^2 - x + c\right)^3 - 1 \qquad -\infty < x < \infty}$$

plus singular solution

$$\boxed{y(x) = -1 \qquad -\infty < x < \infty}$$

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Problem 2. (25 pts) Identify the type of the equation. ~~Find all solutions and specify their intervals~~. Solve the initial value problem + specify its interval

equation type
1^{st} order linear

$$\frac{dy}{dt} + 4y - e^{-t} = 0, \quad y(0) = 4/3$$

$$\frac{dy}{dt} + \underbrace{4}_{p(t)}y = e^{-t}$$

$$\mu(t) = e^{\int 4\,dt} = e^{4t \cancel{+c}}$$

$$e^{4t}\frac{dy}{dt} + 4e^{4t}y = e^{-t+4t}$$

$$\frac{d}{dt}\left[e^{4t}y(t)\right] = e^{3t}$$

$$e^{4t}y(t) = \int e^{3t}\,dt = \frac{1}{3}e^{3t} + c$$

$$y(t) = e^{-4t}\left[\frac{1}{3}e^{3t} + c\right]$$

$$y(t) = \frac{1}{3}e^{-t} + ce^{-4t}$$

$$y(0) = \frac{1}{3} + c = \frac{4}{3} \Rightarrow c = 1$$

$$\boxed{y(t) = \frac{1}{3}e^{-t} + e^{-4t} \qquad -\infty < t < \infty}$$

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Problem 3. (25 pts) Identify the type of the equation and find all solutions.

equation type
exact eqn

$$(3y - xy^{-2})\frac{dy}{dx} + y^{-1} = 0$$

$$\underbrace{y^{-1}}_{M} + \underbrace{(3y - xy^{-2})}_{N}\frac{dy}{dx} = 0$$

Is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$? $\quad \frac{\partial M}{\partial y} = -y^{-2} \quad \frac{\partial M}{\partial x} = -y^{-2}$ Yes! Eqn is exact.

$\Rightarrow \exists\ f(x,y)$ s.t. $M = \frac{\partial f}{\partial x}$ and $N = \frac{\partial f}{\partial y}$.

$$M = \frac{\partial f}{\partial x} \Rightarrow \frac{\partial f}{\partial x} = y^{-1} \Rightarrow f = xy^{-1} + g(y)$$

$$N = \frac{\partial f}{\partial y} \Rightarrow 3y - \cancel{xy^{-2}} = -\cancel{xy^{-2}} + g'(y)$$

$$g'(y) = 3y$$

$$g(y) = \frac{3}{2}y^2 \cancel{+c}^{\,\to 0}$$

so $f(x,y) = xy^{-1} + \frac{3}{2}y^2$ and solns of ODE are level curves of f.

$$\boxed{xy^{-1} + \frac{3}{2}y^2 = C}$$

note that the problem did not ask for a solution interval!

It can be tricky to determine these for implicit solutions.

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Problem 4. (20 pts) Identify the type of the equation and the substitution that reduces it a simpler problem. Make the substitution and derive the reduced equation. **Do not solve the reduced equation!**

equation type	substitution	reduced equation type
Bernoulli	$u = y^{-1}$	1st order linear

$$\frac{dy}{dx} + \frac{y}{x} - x^3 y^2 = 0$$

$$\frac{dy}{dx} + \frac{1}{x} y = x^3 y^2$$

This is Bernoulli with $n = 2$.
Make subs $u = y^{1-n} = y^{-1}$.

(Substitution $u = y^{-1}$ requires $y \neq 0$, which generates singular solution $y(x) = 0$ for $-\infty < x < 0$ or $0 < x < \infty$
- note $\frac{1}{x}$ in eqn ruling out $x=0$ ... but I didn't ask for singular solns.)

$u = y^{-1} \Rightarrow y = u^{-1}$ ($u = y^{-1}$ can't be zero for finite y)

$$\frac{dy}{dx} = -u^{-2} \frac{du}{dx}$$

substituting into ODE...

$$-u^{-2} \frac{du}{dx} + \frac{1}{x} u^{-1} = x^3 u^{-2}$$

mult by $-u^2$

$$\boxed{\frac{du}{dx} - \frac{1}{x} u = -x^3}$$

reduced eqn is 1st order linear