HW 10 ungraded
Math 527, spring 2018, University of New Hampshire

These are practice problems meant to give you some practice solving 3-d linear systems and in linearizing nonlinear systems.

Problems 1-3. Find the general solution of the linear system. Boldface indicates a vector. E.g. $\mathbf{x}$ is a vector with components x, y, z (or x_1, x_2, x_3 if you prefer).

1. $\mathbf{x}' = \begin{pmatrix} 1 & 4 & 2 \\ 4 & -1 & -2 \\ 0 & 0 & 6 \end{pmatrix} \mathbf{x}$

2. $\mathbf{x}' = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \mathbf{x}$

3. $\mathbf{x}' = \begin{pmatrix} 2 & 5 & 1 \\ -5 & -6 & -4 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x}$

Problem 4. The Van der Pol oscillator is the second-order nonlinear ordinary equation

$$x'' - \mu(1 - x^2)x' + x = 0$$

where μ is a nonnegative parameter and primes indicate differentiation in time, e.g. $x' = dx/dt$.

(a) Convert the 2nd-order nonlinear ODE into a nonlinear system $\mathbf{x}' = f(\mathbf{x})$ where $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ using the substitution $y = x'$.

(b) Show that the origin $(x, y) = (0, 0)$ is the only equilibrium of $\mathbf{x}' = f(\mathbf{x})$ for all μ.

(c) Linearize the dynamics $\mathbf{x}' = f(\mathbf{x})$ for small perturbations about the origin. Your answer should be in the form

$$\mathbf{x}' = Df\,\mathbf{x}$$

where Df is the matrix of partial derivatives of f evaluated at the origin and $\mathbf{x}$ is assumed small.

(d) Find the general solution of the linear system from (c) for $\mu = 1$. Is the equilibrium at the origin stable or unstable?

(e) The eigenvalues of Df and hence the nature of the solutions of (c) change form at a couple specific values of $\mu \geq 0$. Determine what these values are, and then specify what kind of solution of (c) occurs over what region of μ.

For example, your asnwer might be "for $0 \leq \mu < 11$, the solutions to the linearized dynamics are stable oscillations, due to complex eigenvalues with negative real part, and for $11 \leq \mu$, the solutions are stable real-valued exponentials, because both eigenvalues are real and negative."

This problem is easier than it looks!