PROBLEM SET 3

2022

1. Suppose there are three coffee houses along Main Street. The street is one mile. One hundred residents are uniformly distributed along this stretch. Each resident is willing to buy one cup of coffee per day. A cup of coffee differs only in its location and price and not in any other way. Each customer derives a utility of v = \$3.00 from the cup of coffee. A consumer's (round-trip) cost of travel is quadratic in the (one-way) distance from home to any of the coffee houses.

Starbucks Coffee House is located at either end of the one mile stretch and Esquire Coffee House is located halfway between the two end points of the street. The prices of coffee at Starbucks' two locations are p_0 and p_1 respectively. Esquire's price of coffee is denoted by q. Marginal costs of a cup of coffee are zero.

(a) Determine the location of the two marginal consumers – the one who is indifferent between purchasing from Esquires and Starbucks located at the left end point and the one who is indifferent between purchasing from Esquires and the Starbucks located the right end point.
(b) Derive the best-reply functions to the pricing game in which the coffee houses choose prices simultaneously. Assume that Starbucks can set different prices at its two locations.
(c) Determine the equilibrium prices and market shares.
(d) Suppose Starbucks and Esquires swap houses so that the Starbucks houses are located at one of the endpoints and the halfway point and Esquires is located at the other endpoint. Derive the equilibrium prices and market shares and explain why it differs (if at all) from (c).
(e) Suppose instead of swapping houses, Starbucks sells one of its coffee houses to Seattle Best Coffee. Derive the equilibrium prices and market shares. Explain why it differs (if at all) from (c).

2. A county consists, for all intents and purposes, of two towns, Right and Left. They are connected by a straight road of length one mile. The population is uniformly distributed on this one-mile stretch. Everyone is an imbiber. Jim Beam and Jack Daniels own the only two liquor licenses. Each drinker will go regularly to whichever bar is closest to him. Unfortunately for Jim and Jack, Mr. Nag has convinced the powers that be that if these two gentlemen were unregulated, they would conspire to raise prices. Hence the price of a drink in each bar is equal and fixed at \$p by the county Pickled Brain Commission. Thus, the only variable that Jack and Jim have control over is their location. Each drinker's cost of travel is quadratic in the distance from home to the bar. Assume that the utility obtained from a drink is sufficiently high that market is covered for every pair of locations.

(a) Formulate this location game by defining strategies and payoffs. Let a denote the location of Jim Beam and 1-b denote the location of Jack Daniels. Be sure to derive demands for locations (a, 1-b).
(b) Determine the Nash equilibrium locations of the two establishments and explain the intuition behind your answer.
(c) Are the Nash equilibrium locations socially optimal (i.e., do they minimize total travel costs)? Why or why not? If not, set up the social planner's problem and derive the optimal locations.

3. Consider a model of product differentiation along a line segment of length 1. Consumers are uniformly distributed along the line. Consumers have unit demand with valuation $1 - x$ for a good sold at a distance x from their location. Production costs are zero.

Assume that firm 1 has two products: one (L) located at the left endpoint of the line segment and another (R) at the right endpoint. Firm 1 may charge distinct prices p_{1R} and p_{1L} for its two products. Firm 2 has only a single product, which is also located at the right endpoint, which firm 2 sells at price p_2.

Suppose that the firms compete by simultaneously choosing prices in a one-shot game. Assume that if any consumers are indifferent between a product offered by firm 1 and the product offered by firm 2, they will buy from firm 2.

(a) Find the equilibrium prices and profits of the two firms in equilibrium.
(b) Is firm 1 better or worse off with product R? That is, should firm 1keep or drop product R (assuming no exit costs)? Explain your reasoning.

4. Suppose two firms, L and H, are each capable of producing a good at a constant marginal cost of zero. Assume that the goods the two firms produce differ in quality, with exogenously fixed quality levels s_L and s_H satisfying $s_H > s_L$.

Assume that a unit mass of consumers is considering purchasing one unit of the good. The utility that a consumer type θ gets from purchasing good i at price p_i is

$$U(p_i,\theta) = \theta s_i - p_i.$$

Assume that the taste parameter θ is uniformly distributed on the interval $[\theta_1, \theta_2]$ and that $\theta_2 > 2\theta_1$. Each consumer's utility is zero if she buys neither good.

Consider the game in which two firms simultaneously choose prices p_L and p_H and then each consumer decided between purchasing one unit from firm L, one unit from firm H, or buying neither good.

(a) For what prices will every consumer buy one product, with each firm receiving a positive market share?
(b) Assuming prices are always in the range described above, find the firms' best-reply functions and solve for the Nash equilibrium. Show that the following bound is needed:

$$(1/3)(\theta_2 - 2\theta_1)(s_H - s_L) \leq \theta_1 s_L.$$

5. Due to advances in information technology, firms are increasingly able to learn about a consumer's willingness-to-pay and set a personalized price, i.e., offer different consumers different prices based on their individual willingness-to-pay. What is the welfare consequences of personalized prices?

To study this question, suppose there is a unit mass of consumers uniformly distributed on the unit interval [0, 1] and two firms, with firm 1 located at the left end point and firm 2 at the right end point. A consumer at location x values firm 1's product at $v_1 = V - x$ and firm 2's product at $v_2 = V - (1 - x)$ where V is large enough that the market is covered in equilibrium. Unit costs are normalized to zero. The two firms set prices simultaneously and then each consumer chooses whether to buy from firm 1 or firm 2.

(a) Compute the Nash equilibrium under uniform pricing.
(b) What is consumer surplus, profits, and welfare?
(c) Compute the Nash equilibrium under personalized pricing.
(d) What is the consumer surplus, profits, and welfare? Who benefits from personalized pricing?
(e) Do you think this result would continue to hold in a model where the market is not covered? I want informed intuition, not calculations please!

FYI: Problems 2,3, and 5 are based on articles.