John Higgins

Econ 899

21 September, 2022

1   I. Huggett (1993) with enforceable insurance

As in Hugget (1993), suppose there is a continuum of agents with total mass equal to one. Each agent has no initial asset holdings. Each period, agents receive an endowment of a consumption good. The endowment e takes values in $E = \{e_h, e_l\}$, with $e_h > e_l$.

The endowment process for each agent follows a Markov process with $\pi(e' \mid e) = P(e_{t+1} = e' \mid e_t = e) > 0$ for each $e, e' \in E$. We assume each realization of the endowment process is iid and thus independent of all other agents' endowment realizations.

Agents have preferences given by

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{c^{1-\sigma}}{1-\sigma}\right] \quad \sigma > 1$$

where E is the expectation operator with respect to the stochastic endowment process.

Agents are able to purchase state-contingent insurance which pays out based on their realization of their endowment in the next period. That is, agents can buy and sell Arrow securities a_h^{t+1} and a_l^{t+1}, where a_h^{t+1} pays out one unit of the consumption good in period $t+1$ when the agent receives endowment e_h, and a_l^{t+1} pays out one unit when the agent receives e_l. These time t contracts have prices q_h^t and q_l^t, respectively. We assume these contracts are enforceable and thus all agents can credibly commit to fulfilling their obligations.

Agents in time t with current endowment e_h and current holdings a_h^t and a_l^t facing prices q_h^t and q_l^t obey the following constraint:

$$c_t + q_l^t a_l^{t+1} + q_h^t a_h^{t+1} \leq e_h + a_h^t$$

Similarly, agents with endowment e_l have the following constraint:

$$c_t + q_l^t a_l^{t+1} + q_h^t a_h^{t+1} \leq e_l + a_l^t$$

Now that we have set up the problem, we wish to find the competitive equilibrium of such an

John Higgins
Econ 899
21 September, 2022

economy. We start by finding the solution to the planner's problem and then decentralizing it.

Let Π_h^t and Π_l^t indicate the proportion of agents in time t with e_h and e_l, respectively. Assuming the planner is utilitarian and weighs all agents equally, the planner solves

$$\max_{\{c_h^t, c_l^t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \left[\Pi_h^t \frac{(c_h^t)^{1-\sigma}}{1-\sigma} + \Pi_l^t \frac{(c_l^t)^{1-\sigma}}{1-\sigma} \right]$$

$$\text{s.t. } c_h^t, c_h^t \geq 0, \quad \forall t$$

$$\Pi_h^t c_h^t + \Pi_l^t c_l^t \leq \Pi_h^t e_h + \Pi_l^t e_l, \quad \forall t$$

Assuming the non-negativity constraint on consumption doesn't bind, the planner's problem has the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^\infty \beta^t \left[\Pi_h^t \frac{(c_h^t)^{1-\sigma}}{1-\sigma} + \Pi_l^t \frac{(c_l^t)^{1-\sigma}}{1-\sigma} \right] + \lambda_t [\Pi_h^t e_h + \Pi_l^t e_l - \Pi_h^t c_h^t - \Pi_l^t c_l^t]$$

This has the following first order conditions:

$$\beta^t \Pi_h^t (c_h^t)^{-\sigma} \leq \Pi_h^t \lambda_t$$

$$\beta^t \Pi_l^t (c_l^t)^{-\sigma} \leq \Pi_l^t \lambda_t$$

with $\lambda_t \geq 0 \ \forall t$, as well as the complementary slackness condition:

$$\lambda_t [\Pi_h^t e_h + \Pi_l^t e_l - \Pi_h^t c_h^t - \Pi_l^t c_l^t] = 0$$

If we let the inequality constraints bind, we note that we have

$$(c_h^t)^{-\sigma} = (c_l^t)^{-\sigma} = \lambda_t > 0$$

since $c_h^t, c_l^t > 0$ by our initial assumption. This means that the planner will optimally set

$c_h^t = c_l^t = c^t \; \forall t$. It now remains to find c^t. Note that since $\lambda_t > 0 \; \forall t$, it follows that the planner's resource constraint will bind. Hence,

$$\Pi_h^t c^t + \Pi_l^t c^t = \Pi_h^t e_h + \Pi_l^t e_l \iff c^t = \frac{\Pi_h^t e_h + \Pi_l^t e_l}{\Pi_h^t + \Pi_l^t} = \Pi_h^t e_h + \Pi_l^t e_l$$

since $\Pi_h^t + \Pi_l^t = 1$.

Furthermore, if the Markov process is assumed to have a stationary distribution, we note that $\Pi_h^t = \Pi_h$ and $\Pi_l^t = \Pi_l$ for all t, for some Π_h and Π_l such that $\Pi_h + \Pi_l = 1$ (since there is a continuum of agents). This means that the allocation is constant across time (since the aggregate endowment is constant). Therefore, we have that

$$\forall t, c^t = \bar{c} = \Pi_h e_h + \Pi_l e_l$$

In summary, the planner sets consumption for all agents to be the same, and the allocation depends only on the aggregate endowment which is constant across time.

For concreteness, we use the parameterization specified in part II which has $e_h = 1$, $e_l = 0.5$, $\Pi_l = 0.0566$, and $\Pi_h = 0.9434$. Hence,

$$\bar{c} = 0.9434 \cdot 1 + 0.0566 \cdot 0.5 = 0.9717$$

We now decentralize this solution using the Arrow-security setup specified previously. A competitive equilibrium involves agents maximizing their utility as described above and markets clearing. We note that, in order to implement the planner's solution, it must be the case that agents sell a_h^{t+1} and buy a_l^{t+1} to insure themselves against adverse shocks (since $e_l < \bar{c} < e_h$).

Let $c(e_t)$ indicate the consumption of an agent with endowment e_t, and $a(e_t)$ indicate their insurance purchase. The asset market clearing condition is that $\Pi_h a(e_h) = -\Pi_l a(e_l)$.

The agent's problem is thus

$$\max_{c(e_t),a(e_{t+1})} E\left[\sum_{t=0}^{\infty} \frac{c(e_t)^{1-\sigma}}{1-\sigma}\right]$$

subject to the sequential budget constraint

$$c(e_t) + qa(e_{t+1}) \le e_t + a(e_t)$$

Their Lagrangian is the following:

$$\mathcal{L} = E\left[\sum_{t=0}^{\infty} \beta^t \frac{c(e_t)^{1-\sigma}}{1-\sigma}\right] + \lambda_t[e_t + a(e_{t+1}) - c(e_t) - qa(e_t)]$$

which has FOCs

$$\beta^t c(e_t)^{-\sigma} = \lambda_t \quad [c(e_t)]$$

$$\lambda_t = q\lambda_{t+1} \quad [a(e_{t+1})]$$

Combining these, we obtain

$$\beta^t c(e_t)^{-\sigma} = q\beta^{t+1} c(e_{t+1})^{-\sigma}$$

Imposing the condition from the planner's allocation that $c(e_t) = c(e_{t+1}) = \bar{c}\ \forall t$, it follows that

$$\beta^t = q\beta^{t+1} \implies q = \beta$$

Hence, the price to a claim on consumption in the next period is given by $q = \beta$. The allocation of consumption is the same as the planner's problem. It remains to find the assets purchased by each type. We note that

$$a(e_h) = \bar{c} - e_h \quad \text{and} \quad a(e_l) = \bar{c} - e_l$$

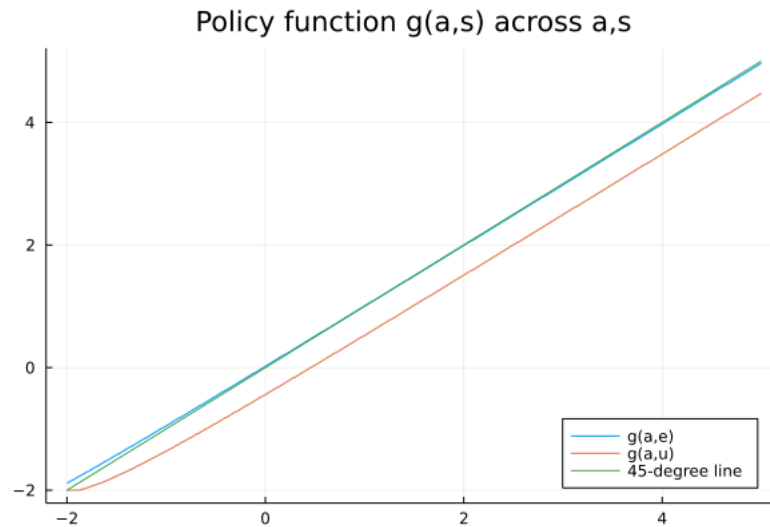Again using the parameterization from part II, this implies that

$$a(e_h) = -0.0283 \quad \text{and} \quad a(e_l) = 0.4717$$

We verify that asset market clearing is satisfied:

$$\Pi_h a(e_h) = (0.9434)(-0.0283) = -0.0267 = -(0.0566)(0.4717) = -\Pi_l a(e_l)$$

2   II. Computing Huggett with incomplete markets

a) The plot of the policy function $g(a,s)$ for each s is included below, alongside a plot of the 45-degree line.

To verify that $\exists \hat{a}$ such that $g(\hat{a}, s) < \hat{a}$, we plot $g(a, s) - a$ for each (a, s) combination in the following figure:

Difference between g(a, s) - a across a,s
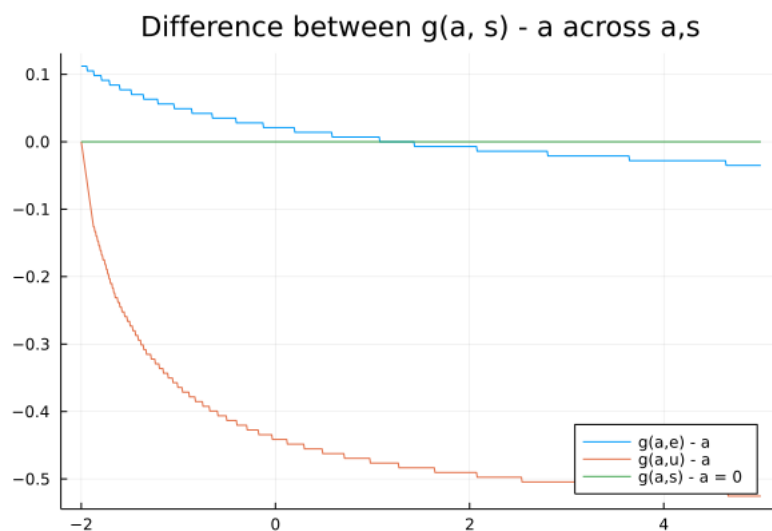
It appears that $\hat{a} \approx 1.3$ or so for employed agents and $\hat{a} = -2$ for unemployed agents.

b) The market-clearing bond price is approximately 0.99427. The stationary cross-sectional distribution associated with this price is plotted below:

Wealth distribution by a,s