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Problem 1.

Solution:

a) Given that $P(Q) = a_0 - a_1 Q + \nu$, we compute that the elasticity of demand is given by

$$\varepsilon(P) = -\frac{P(Q)}{P'(Q)Q} = -\frac{a_0 - a_1 Q + \nu}{-a_1 Q} = \frac{a_0 - a_1 Q + \nu}{a_1 Q} = \frac{a_0 + \nu}{a_1 Q} - 1$$

We observe that this is increasing in a_0 and ν and decreasing in a_1 and Q.

b) Using our result from the previous homework, we have that the first order condition for each firm is the following:

$$q_i = \frac{a_0 - a_1(q_i + q_{-i}) + \nu) - (b_0 - \eta)}{a_1}$$

By symmetry, we have that $q_i = q_j = q^* \ \forall i \neq j$ in equilibrium. Hence, we can impose this to solve for q^*:

$$\begin{aligned} q^* &= \frac{(a_0 - a_1 N q^* + \nu) - (b_0 - \eta)}{a_1} \\ \iff (1+N)q^* &= \frac{a_0 + \nu - (b_0 - \eta)}{a_1} \\ \iff q^* &= \frac{a_0 + \nu - (b_0 - \eta)}{(N+1)a_1} \end{aligned}$$

It follows that total industry output is given by

$$Q^* = Nq^* = \frac{N}{N+1} \frac{a_0 + \nu - (b_0 - \eta)}{a_1}$$

Hence, the price is

$$\begin{aligned} P^* &= a_0 - a_1\left(\frac{N}{N+1}\frac{a_0+\nu-(b_0-\eta)}{a_1}\right)+\nu \\ &= a_0 - \frac{N}{N+1}(a_0+\nu-(b_0-\eta))+\nu \\ &= \frac{a_0+\nu+N(b_0-\eta)}{N+1} \end{aligned}$$

We now use these to compute firm profits in equilibrium for fixed N and F:

$$\begin{aligned} \Pi^* &= (P^*-(b_0-\eta))q^* - F \\ &= \left(\frac{a_0+\nu+N(b_0-\eta)}{N+1}-(b_0-\eta)\right)\frac{a_0+\nu-(b_0-\eta)}{(N+1)a_1} - F \\ &= \left(\frac{a_0+\nu-(b_0-\eta)}{N+1}\right)\frac{a_0+\nu-(b_0-\eta)}{(N+1)a_1} - F \\ &= \frac{1}{a_1}\left(\frac{(a_0+\nu-(b_0-\eta))}{N+1}\right)^2 - F \end{aligned}$$

c) The Lerner index L_i is given by

$$L_i = \frac{P^*-C'(q^*)}{P^*} = \frac{\frac{a_0+\nu+N(b_0-\eta)}{N+1}-(b_0-\eta)}{\frac{a_0+\nu+N(b_0-\eta)}{N+1}} = \frac{\frac{a_0+\nu-(b_0-\eta)}{N+1}}{\frac{a_0+\nu+N(b_0-\eta)}{N+1}} = \frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+N(b_0-\eta)}$$

The industry Lerner index is thus

$$L_I = \sum_{i=1}^{N} s_i L_i = \sum_{i=1}^{N}\frac{1}{N}\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+N(b_0-\eta)} = \frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+N(b_0-\eta)}$$

Similarly, we compute that

$$\varepsilon(P^*) = \frac{a_0-a_1Q^*+\nu}{a_1Q^*} = \frac{\frac{a_0+\nu+N(b_0-\eta)}{N+1}}{\frac{N}{N+1}(a_0+\nu-(b_0-\eta)} = \frac{a_0+\nu+N(b_0-\eta)}{N(a_0+\nu-(b_0-\eta))}$$

Finally, since firms have the same output, the HHI is simply $\frac{1}{N}$.

d) To see how many firms enter before it is no longer profitable to do so, we must find the N^* such that firm profits are zero. That is, we need to solve for N^* such that

$$\begin{aligned}
&\frac{1}{a_1}\left(\frac{(a_0+\nu-(b_0-\eta))}{N^*+1}\right)^2 = F \\
\iff &(N^*+1)^2 \frac{1}{a_1 F}\left((a_0+\nu-(b_0-\eta))\right)^2 \\
\iff &N^*+1 = \frac{1}{\sqrt{a_1 F}}(a_0+\nu-(b_0-\eta)) \\
\iff &N^* = \frac{1}{\sqrt{a_1 F}}(a_0+\nu-(b_0-\eta)) - 1
\end{aligned}$$

e) Under perfect collusion, firms agree to jointly produce the monopoly quantity and split the profits evenly. To determine the monopoly quantity, we solve

$$\max_Q P(Q)Q - C(Q)$$

This has first order condition

$$\begin{aligned}
&P'(Q)Q + P(Q) - (b_0-\eta) = 0 \\
\iff & -a_1 Q + a_0 - a_1 Q + \nu - (b_0-\eta) = 0 \\
\iff & 2a_1 Q = a_0 + \nu - (b_0-\eta) \\
\iff & Q = \frac{a_0+\nu-(b_0-\eta)}{2a_1}
\end{aligned}$$

Hence, $Q^c = \frac{a_0+\nu-(b_0-\eta)}{2a_1}$ and thus each firm produces $q^c = \frac{a_0+\nu-(b_0-\eta)}{2Na_1}$. The profit to each firm under collusion is given by

$$\begin{aligned}
\Pi^c &= \frac{1}{N}\left(P(Q^c)-(b_0-\eta)\right)Q^c - F \\
&= \frac{1}{N}\left(\frac{a_0+\nu-(b_0-\eta)}{2}\right)\frac{a_0+\nu-(b_0-\eta)}{2a_1} - F \\
&= \frac{1}{N}\left(\frac{(a_0+\nu-(b_0-\eta))^2}{4a_1}\right) - F
\end{aligned}$$

This implies that firms will enter until the number of firms is equal to N^c, where N^c is defined so that profits under collusion are zero:

$$\frac{1}{N^c}\left(\frac{(a_0+\nu-(b_0-\eta))^2}{4a_1}\right)-F=0 \iff N^c=\frac{1}{F}\left(\frac{(a_0+\nu-(b_0-\eta))^2}{4a_1}\right)$$

We then compute the elasticity of demand at the collusive price as follows:

$$\varepsilon(P^c)=\frac{a_0+\nu}{\frac{a_0+\nu-(b_0-\eta)}{2}}-1=\frac{a_0+\nu+(b_0-\eta)}{a_0+\nu-(b_0-\eta)}$$

The Lerner index for each firm is thus

$$L_i=\frac{s_i}{\varepsilon(P^c)}=\frac{1}{N^c}\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+(b_0-\eta)}$$

Hence, the industry Lerner index is

$$L_I=\sum_{i=1}^{N^c}\frac{1}{N^c}\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+(b_0-\eta)}=\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+(b_0-\eta)}$$

Finally, the HHI is given by $\frac{1}{N^c}$. By inspection, we observe that the industry Lerner index under collusion is higher than the Lerner index under Cournot competition. Indeed,

$$L_I^c=\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+(b_0-\eta)}\leq\frac{a_0+\nu-(b_0-\eta)}{a_0+\nu+N(b_0-\eta)}$$

Furthermore, we argue that $\varepsilon(P^*)>\varepsilon(P^c)$, since

$$\varepsilon(P^*)=\frac{a_0+\nu+N(b_0-\eta)}{N(a_0+\nu-(b_0-\eta))}=\frac{\frac{1}{N}(a_0+\nu)+(b_0-\eta)}{a_0+\nu-(b_0-\eta)}<\frac{a_0+\nu+(b_0-\eta)}{a_0+\nu-(b_0-\eta)}=\varepsilon(P^c)$$

Consequently, since $L_I = \frac{HHI}{\varepsilon(p)}$, it follows that

$$\begin{aligned} L_I^* \leq L_I^c &\iff \frac{HHI^*}{\varepsilon(P^*)} \leq \frac{HHI^c}{\varepsilon(P^c)} \\ &\iff HHI^* \, \varepsilon(P^c) \leq HHI^c \, \varepsilon(P^*) \\ &\iff HHI^* \leq HHI^c \end{aligned}$$

Thus, $HHI^* \leq HHI^c$. Consequently, $N^* \geq N^c$.

In summary, the industry Lerner index, the HHI, and the elasticity of demand are all greater under collusion than under Cournot competition. Furthermore, the number of firms is lower.

□

Problem 2.

Solution: We estimate the below SCP regression for each group of cities:

$$\ln(ObservedLerner_t) = \beta_0 + \beta_1 \ln(ObservedHHI_t) + \epsilon_t$$

Our results are included below: These results indicate that the natural log of the observed

	Estimates			
City group	$\hat{\beta}_0$	se($\hat{\beta}_0$)	$\hat{\beta}_1$	se($\hat{\beta}_1$)
Full sample	-0.786	0.006	0.591	0.004
Active antitrust	-0.586	0.072	0.527	0.044
Inactive antitrust	-0.982	0.013	0.656	0.008

Table 1: SCP regression estimates by sample group, exogenous structure

Lerner index is positively correlated with the natural log of the observed HHI. In the full sample, a one percent increase in the observed HHI is associated with a roughly 0.591 percent increase in the observed Lerner index. In the sample containing only cities with an active antitrust authority, each percent change in observed HHI is associated with an increase of 0.527 percent in the observed Lerner index. In the sample containing only cities without active antitrust enforcement, each percent change in observed HHI is associated with an increase of 0.656 percent in the observed Lerner index.

This indicates that the HHI is generally correlated with the observed Lerner index. Furthermore, the correlation is stronger in cities where it is possible for firms to collude than in cities where collusion is not possible.

The reason for the difference in the estimates between cities with antitrust enforcement and those without is the fact that the equilibrium price and quantity under collusion do not depend on industry size in the city. This means that prices and quantities (and consequently, the Lerner index) will be constant across all cities with collusion. However, the observed HHI in each of those cities still depends on the number of firms in the city. Thus, in the data it will appear as if HHI has less of an impact on observed markups than it does in cities with Cournot competition.

$\beta_1 = 1$ would correspond to the case where market concentration has no impact on markups. To see this, we use the fact that L_I is proportional to HHI. We know that $L_I = \frac{HHI}{\varepsilon(p_t)}$, meaning that we can rewrite the SCP regression in the following manner:

$$\begin{aligned} \ln(ObservedLerner_t) &= \beta_0 + \beta_1 \ln(ObservedHHI_t) + \epsilon_t \\ \iff \ln(HHI_t) - \ln(\varepsilon(p_t)) &= \beta_0 + \beta_1 \ln(HHI_t) + \epsilon_t \\ \iff \ln(\varepsilon(p_t)) &= -\beta_0 - (1 - \beta_1)\ln(HHI_t) + \epsilon_t \end{aligned}$$

Thus, $\beta_1 = 1$ would indicate that industry concentration (as quantified by HHI) would have no impact on the elasticity of demand and thus no impact on observed markups. This is essentially a null result for the SCP regression. Would we expect $\beta_1 = 1$? No, since we know that the HHI and elasticity of demand are decreasing in the number of firms N for both the Cournot and collusive outcomes. As such, we would expect these to be correlated in the data. This is backed up by our regression results, which indicate that $\beta_1 \neq 1$ and therefore that the elasticity of demand (and thus markups) is positively associated with industry concentration.

Another way to think about this is the following: β_1 is essentially the elasticity of L_t with respect to HHI_t. As such, we have that

$$\beta_1 = \frac{d\ln(L_t)}{d\ln(HHI_t)} = \frac{HHI_t}{L_t} \cdot \frac{dL_t}{dHHI_t}$$

If $\beta_1 = 1$, this would imply that a one percent change in HHI_t is associated with a one percent change in L_t. □

Problem 3.

Solution: We estimate the SCP regression across the 1000 cities assuming all firms compete in Cournot competition with $a_0 = 5, a_1 = 1, F = 1$, and $b_0 = 1$. Furthermore, in each city we either have $\nu \sim U([-1,1])$ and $\eta = 0$ or $\nu = 0$ and $\eta \sim U([-1,1])$. Additionally, we assume that firms enter until profits are zero, meaning that the number of firms in each city varies is endogeneously determined by the realized ν and η parameters. The following table summarizes the results of these regressions.

	Estimates			
Parameter values	$\hat{\beta}_0$	se($\hat{\beta}_0$)	$\hat{\beta}_1$	se($\hat{\beta}_1$)
$\nu \sim U([-1,1]), \eta = 0$	-0.692	0.186	0.0008	0.172
$\nu = 0, \eta \sim U([-1,1])$	-2.301	0.074	-1.532	0.067

Table 2: SCP regression estimates by parameter configurations, endogenous structure, Cournot competition

a) When $\nu \sim U([-1,1])$ and $\eta = 0$, we see that $\hat{\beta}_1$ is very small and statistically insignificant.

b) When $\nu = 0$ and $\eta \sim U([-1,1])$, we see that $\hat{\beta}_1$ is negative and statistically significant.

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c) Why do we observe such a stark difference in estimates by varying η instead of ν? The answer lies in how each variable affects both the HHI and the Lerner index. The key difference from problem 2 is that we impose that market structure is endogenous. Namely, firms enter each city until it is no longer profitable to do so. From our answers to problem 1, we know that the number of firms entering is given by the following:

$$N^* = \frac{1}{\sqrt{a_1 F}}(a_0 + \nu - (b_0 - \eta)) - 1$$

Plugging in the parameter values for this problem, we obtain

$$N^* = 5 + \nu - 1 + \eta - 1 = 3 + \nu + \eta$$

We now want to determine the Lerner index for the industry as a function of η. Taking $\eta = 0$ and substituting the other parameter values into our result from the previous section, we have that

$$L_I = \frac{5 + \nu - 1}{5 + \nu + (3 + \nu)} = \frac{4 + \nu}{8 + 2\nu}$$

We also compute that

$$HHI = \frac{1}{3 + \nu}$$

As before, we have that

$$\beta_1 = \frac{HHI}{L_I} \frac{dL_I/d\nu}{dHHI/d\nu}$$

We can compute that

$$\frac{dL_I}{d\nu} = \frac{(8 + 2\nu) - 2(4 + \nu)}{(8 + 2\nu)^2} = 0$$

Hence, we have that $\beta_1 = 0$ based on the model parameters. Of course, is different in the regression since the observations are noisy, but this is why we get a result which is very close to zero in our estimate for part a.

A similar analysis holds for when $\eta \sim U([-1,1])$ and $\nu = 0$. We observe that

$$N^* = 3 + \eta$$

Consequently,

$$HHI = \frac{1}{3+\eta}$$

Furthermore,

$$L_I = \frac{4+\eta}{5+(3+\eta)(1-\eta)} = \frac{4+\eta}{8-2\eta-\eta^2} = \frac{4+\eta}{(4+\eta)(2-\eta)} = \frac{1}{2-\eta}$$

Hence,

$$\beta_1 = \frac{HHI}{L_I}\frac{dL_I/d\nu}{dHHI/d\nu} = \frac{2-\eta}{3+\eta}\frac{-(3+\eta)^2}{(2-\eta)^2} = -\frac{3+\eta}{2-\eta}$$

Taking the expectation over ν which is uniformly distributed over $[-1,1]$, we obtain

$$E[\hat{\beta}_1] = \int_{-1}^{1} \frac{-(3+\eta)}{2(2-\eta)}\, d\eta \approx -1.746$$

which is reasonably close to our estimated value. Of course, the measurement error in the data-generating process can account for this discrepancy. But this is the theoretical reason why the coefficient for β_1 is so much lower when we allow η to vary.

□