**Problem 1.**

*Solution:* I estimate (i) OLS without brand fixed effects, (ii) OLS with brand fixed effects, (iii) IV using the instrumental variables provided in the data, and (iv) IV with brand fixed effects. My procedure for doing so is contained in the Mergers.jl file as well as ols.jl and IV.jl. I include the estimates for $\alpha$ for each regression in Table 1, in addition to their standard errors.

Table 1: Price sensitivity parameter estimates by regression specification

| Regression | $\hat{\alpha}$ | Standard error |
|---|---|---|
| OLS | -29.4550 | 0.2208 |
| OLS w/ FE | -28.9474 | 0.9854 |
| IV | -29.3336 | 0.2208 |
| IV w/ FE | -18.6012 | 0.7693 |

□

**Problem 2.**

*Solution:* I use these estimates to compute the markups predicted by a multi-product Nash-Bertrand equilibrium. To do so, I substitute the estimate for $\alpha$ into each firm's first order condition and solve for the implied markups based on the equilibrium prices and shares. Specifically, following Nevo, we begin by noting that firm $f$'s profits from brands $\mathcal{F}_f$ within a given market are given by

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j)Ms_j(p) - C_f$$

where $s_j(p)$ is brand $j$'s market share, $M$ is the market size, and $C_f$ is the firm's fixed cost. The first order condition for any firm is thus the following:

$$s_j(p) + \sum_{r \in \mathcal{F}_f} (p_r - mc_r)\frac{\partial s_r(p)}{\partial p_j} = 0$$

As this must hold $\forall j$, the set of all firms' first order conditions can be expressed in the following succinct vector representation:

$$s(p) - \Omega(p - mc) = 0$$

where $\Omega = \Omega^* \odot S$, where $S_{jk} = \frac{\partial s_i}{\partial p_k}$, $\Omega^*$ is the ownership matrix with $\Omega^*_{jk} = 1$ if $j, k \in \mathcal{F}_f$ and 0 otherwise, and $\odot$ is the Hadamard (element-wise) product. We note that under the MNL framework, $\frac{\partial s_i}{\partial p_i} = -\alpha s_i(1 - s_i)$ and $\frac{\partial s_i}{\partial p_j} = \alpha s_i s_j$. This is where we will plug in our estimate $\hat{\alpha}$ obtained from our regressions.

We can then invert the system of first order conditions to find the vector of markups $\kappa = p - mc$ (note: I am interpreting markup to mean $p - mc$ and margin to mean $\frac{p-mc}{p}$ and will be using this throughout):

$$\kappa = p - mc = -\Omega^{-1}s(p)$$

The markup of product $j$ in market $t$ is thus given by

$$\kappa_{jt} = -\Omega^{-1}s_{jt}(p)$$

The margins for product $j$ in market $t$ are found by $m_{jt}$, where

$$m_{jt} = \frac{\kappa_{jt}}{p_{jt}} = \frac{p_{jt} - mc_{jt}}{p_{jt}}$$

Finally, the inferred marginal costs for product $j$ in market $t$ are given by

$$mc_{jt} = \kappa_{jt} + p_{jt}$$

We compute all of these quantities for each regression specification and report their mean, median, and standard deviation in Table 2:

Table 2: Summary statistics of markups, margins, and inferred marginal costs, by regression:

| Statistic | OLS | OLS w/ FE | IV | IV w/ FE |
|---|---|---|---|---|
| $\mathrm{E}[\kappa_{jt}]$ | 0.0402 | 0.0409 | 0.0404 | 0.0637 |
| $\mathrm{med}[\kappa_{jt}]$ | 0.0395 | 0.0402 | 0.0396 | 0.0624 |
| $\mathrm{sd}(\kappa_{jt})$ | 0.0050 | 0.0050 | 0.0050 | 0.0079 |
| $\mathrm{E}[m_{jt}]$ | 0.3400 | 0.3460 | 0.3414 | 0.5385 |
| $\mathrm{med}[m_{jt}]$ | 0.3217 | 0.3274 | 0.3231 | 0.5095 |
| $\mathrm{sd}(m_{jt})$ | 0.1040 | 0.1058 | 0.1044 | 0.1646 |
| $\mathrm{E}[mc_{jt}]$ | 0.0855 | 0.0848 | 0.0854 | 0.0606 |
| $\mathrm{med}[mc_{jt}]$ | 0.0837 | 0.0830 | 0.0836 | 0.0601 |
| $\mathrm{sd}(mc_{jt})$ | 0.0300 | 0.0301 | 0.0300 | 0.0310 |

□

**Problem 3.**

*Solution:* I now use the pre-merger estimates of marginal costs, estimated price elasticities, and the continued assumption of multi-product firm Nash-Bertrand post-merger equilibrium to simulate the post-merger equilibrium. The post-merger price vector $p^*$ is the solution to the following equation:

$$p^* = \Omega_m(p^*)s(p^*)$$

where $\Omega_m(p^*)$ is $\Omega_m \odot S(p^*)$ is the element-wise product of the post merger ownership matrix with the matrix $S(p^*)$ (which is the substitution matrix $S(p)$ evaluated at price vector $p^*$). This is a system of 24 nonlinear equations, and there is not a closed form solution for this. However, our approach will iteratively update prices based on firm optimality starting from the initial pre-merger price vector $p_0$ and shares $s(p_0)$. I explicitly outline the procedure below:

1. Change the ownership matrix $\Omega_m^*$ so that, if firms $f$ and $g$ merge, then $\forall j \in \mathcal{F}_f$ and $\forall k \in \mathcal{F}_g$, $\Omega_{mjk}^* = 1$.

2. For each $t \geq 1$, construct the matrix $\Omega_m^t$ defined as the following:

$$\Omega_m^t = \Omega_m^* \odot S(p_{t-1})$$

3. Using the firms' FOC as well as our inferred marginal costs, we know that the optimal markups are given by

$$\kappa_t = p_t - \hat{mc} = \Omega_m^t s(p_t)$$

We can rearrange this to solve for the optimal price for product $j$ in market $t$:

$$p_t = \hat{mc} + \Omega_m^t s_{jt}(p_{t-1})$$

This gives us an updated price vector $p_t$.

4. Find the next iteration's predicted market shares $s(p_t)$ using our regression model to be used in the next iteration of this procedure.

5. We then check if $\|p_t - p_{t-1}\|_\infty < \varepsilon$ for some tolerance parameter $\varepsilon > 0$. If so, then we stop! If not, then we repeat from steps 2-5 until convergence.

While the above operator is not generally a contraction, we will use it nevertheless and see if we converge to a new equilibrium price. I do this procedure for each of the mergers and each of the regression specifications and report the changes in equilibrium price, markups, margins, and market shares relative to the pre-merger case. I first consider the Post-Nabisco merger and include my results in Table 3.

I now do the same for the General Mills-Quaker merger and include my results in Table 4.

□

**Problem 4.**

Table 3: Post-Nabisco merger changes in equilibrium results by regression type:

| Change in moment | OLS | OLS w/ FE | IV | IV w/ FE |
|---|---|---|---|---|
| $\Delta$ E$[p_{jt}]$ | -0.0003 | -0.0010 | -0.0003 | -0.0022 |
| $\Delta$ med$[p_{jt}]$ | -0.0006 | -0.0011 | -0.0005 | -0.0022 |
| $\Delta$ E$[\kappa_{jt}]$ | -0.0003 | -0.0010 | -0.0003 | -0.0022 |
| $\Delta$ med$[\kappa_{jt}]$ | 0.0005 | -0.0002 | 0.0005 | -0.0006 |
| $\Delta$ E$[m_{jt}]$ | -0.0049 | -0.0049 | -0.0009 | -0.0051 |
| $\Delta$ med$[m_{jt}]$ | -0.0037 | -0.0037 | 0.0002 | -0.0054 |
| $\Delta$ E$[s_{jt}]$ | -0.0026 | -0.0026 | -0.0008 | -0.0037 |
| $\Delta$ med$[s_{jt}]$ | -0.0037 | -0.0026 | 0.0034 | 0.0028 |

Table 4: General Mills-Quaker merger changes in equilibrium results by regression type:

| Change in moment | OLS | OLS w/ FE | IV | IV w/ FE |
|---|---|---|---|---|
| $\Delta$ E$[p_{jt}]$ | 0.0014 | 0.0002 | 0.0014 | -0.0005 |
| $\Delta$ med$[p_{jt}]$ | 0.0010 | -0.0001 | 0.0010 | -0.0007 |
| $\Delta$ E$[\kappa_{jt}]$ | 0.0014 | 0.0002 | 0.0014 | -0.0005 |
| $\Delta$ med$[\kappa_{jt}]$ | 0.0025 | 0.0016 | 0.0025 | 0.0012 |
| $\Delta$ E$[m_{jt}]$ | 0.0079 | 0.0017 | 0.0082 | 0.0013 |
| $\Delta$ med$[m_{jt}]$ | 0.0097 | 0.0038 | 0.0100 | 0.0012 |
| $\Delta$ E$[s_{jt}]$ | -0.0014 | -0.0029 | -0.0013 | -0.0040 |
| $\Delta$ med$[s_{jt}]$ | 0.0029 | 0.0023 | 0.0030 | 0.0024 |

*Solution:* One problem with the above analysis is that the MNL framework implies restrictive substitution patterns between goods which may not be realistic. Furthermore, it assumes that consumers' price sensitivity and preferences for product characteristics are independent as well as orthogonal to the Logit shocks. Also, it assumes that the price sensitivity and preference parameters are common across each market, which is not the most reasonable assumption. For example, a market with a large number of affluent people is likely going to have a different level of price sensitivity than a market where consumers tend to have lower wealth. Furthermore, it is reasonable to assume that one's preferences over one product characteristic may be correlated with their preferences over another product characteristic. All of these factors will potentially lead to shortcomings in the predictions of the above analysis. We aim to resolve some of these problems by allowing for correlated consumer heterogeneity using the random coefficients BLP model estimated in the following problem. □

## Problem 5.

*Solution:* I estimate the random coefficients model in Julia using my code listed in model.jl. I used the provided Matlab code to help write my own version. The file has a number of dependencies, which are also included. The results from my own code are summarized in Table 5:

Table 5: Random Coefficients Estimates - Julia code (standard errors in parentheses)

|          | Mean      | $\sigma$    | Income    | Income$^2$ | Age      | Child      |
|----------|-----------|-------------|-----------|------------|----------|------------|
| Constant | -3.475    | 0.3772      | 3.0888    | 0.0        | 1.1859   | 0.0        |
|          | (0.0790)  | (269.846)   | (1.526)   | -          | (8.889)  | -          |
| Price    | -32.497   | 1.848       | 16.598    | -0.659     | 0.0      | 11.625     |
|          | (1.794)   | (890.125)   | (22.447)  | (3.082)    | -        | (396.291)  |
| Sugar    | 0.088     | -0.004      | -0.193    | 0.0        | 0.030    | 0.0        |
|          | (0.079)   | (1.580)     | (1.976)   | -          | (3.655)  | -          |
| Mushy    | -0.533    | 0.081       | 1.468     | 0.0        | -1.514   | 0.0        |
|          | (0.003)   | (1.104)     | (18.051)  | -          | (9.040)  | -          |

We can see that the coefficients are generally very similar to those reported in the results.txt file. There are a few differences (namely, the mean of the sugar and mushy coefficients are a bit off, and the standard errors do differ), the rest of the estimates are consistent with the given results. I also ran the Matlab code and was able to perfectly replicate the results in the results.txt file, which I have included below in Table 6 for completeness:

□

## Problem 6.

Table 6: Random Coefficients Estimates - provided Matlab code (standard errors in parentheses)

|  | Mean | $\sigma$ | Income | Income$^2$ | Age | Child |
|---|---|---|---|---|---|---|
| Constant | -1.860 | 0.377 | 3.089 | 0.0 | 1.186 | 0.0 |
|  | (0.2586) | (0.1295) | (1.207) | - | (1.018) | - |
| Price | -32.437 | 1.848 | 16.598 | -0.659 | 0.0 | 11.625 |
|  | (7.760) | (1.078) | (172.721) | (8.975) | - | (5.220) |
| Sugar | 0.144 | -0.004 | -0.193 | 0.0 | 0.030 | 0.0 |
|  | (0.259) | (0.0124) | (0.045) | - | (0.0363) | - |
| Mushy | 0.773 | 0.081 | 1.468 | 0.0 | -1.514 | 0.0 |
|  | (0.013) | (0.207) | (0.700) | - | (1.107) | - |

Table 7: Summary statistics of markups, margins, and inferred marginal costs, by regression (including BLP):

| Statistic | OLS | OLS w/ FE | IV | IV w/ FE | BLP |
|---|---|---|---|---|---|
| $E[\kappa_{jt}]$ | 0.0402 | 0.0409 | 0.0404 | 0.0637 | 0.0419 |
| $med[\kappa_{jt}]$ | 0.0395 | 0.0402 | 0.0396 | 0.0624 | 0.0404 |
| $sd(\kappa_{jt})$ | 0.0050 | 0.0050 | 0.0050 | 0.0079 | 0.0079 |
| $E[m_{jt}]$ | 0.3400 | 0.3460 | 0.3414 | 0.5385 | 0.3557 |
| $med[m_{jt}]$ | 0.3217 | 0.3274 | 0.3231 | 0.5095 | 0.3290 |
| $sd(m_{jt})$ | 0.1040 | 0.1058 | 0.1044 | 0.1646 | 0.1293 |
| $E[mc_{jt}]$ | 0.0855 | 0.0848 | 0.0854 | 0.0606 | 0.0839 |
| $med[mc_{jt}]$ | 0.0837 | 0.0830 | 0.0836 | 0.0601 | 0.0829 |
| $sd(mc_{jt})$ | 0.0300 | 0.0301 | 0.0300 | 0.0310 | 0.0314 |

*Solution:* I use my estimates of the random coefficients model to repeat question 2 and include the estimates (alongside the previous estimates for comparison) in Table 7:

We see that the the estimated markups are higher under BLP than all of the specifications other than IV with FE, with higher variance as well. Both mean and median margins are also higher (again, except for IV with FE). Finally, the estimated marginal costs under BLP are lower than other specifications (except for IV with FE). This is generally what we'd expect: if certain markets have lower price sensitivity, then firms will set higher prices in equilibrium in response. When firms are able to account for heterogeneity in consumer preferences and price sensitivity, their first order condition will be different than it was in the standard Logit model. Not surprisingly, BLP yields different estimates of markups and inferred marginal costs. (As to why my results for IV with FE seem to be anomalous, I would attribute this to a likely coding error on my part and the true price sensitivity is likely to be higher, which would lead to lower estimates) □

**Problem 7.**

*Solution:* I now repeat the merger simulations using BLP. The results for the Post-Nabisco merger are included in Table 8, whereas the results for the General Mills-Quaker merger are included in Table 9.

Table 8: Post-Nabisco merger changes in equilibrium results by regression type, including BLP:

| Change in moment | OLS | OLS w/ FE | IV | IV w/ FE | BLP |
|---|---|---|---|---|---|
| $\Delta \, \mathrm{E}[p_{jt}]$ | -0.0003 | -0.0010 | -0.0003 | -0.0022 | 0.0002 |
| $\Delta \, \mathrm{med}[p_{jt}]$ | -0.0006 | -0.0011 | -0.0005 | -0.0022 | 0.0000 |
| $\Delta \, \mathrm{E}[\kappa_{jt}]$ | -0.0003 | -0.0010 | -0.0003 | -0.0022 | 0.0002 |
| $\Delta \, \mathrm{med}[\kappa_{jt}]$ | 0.0005 | -0.0002 | 0.0005 | -0.0006 | 0.0001 |
| $\Delta \, \mathrm{E}[m_{jt}]$ | -0.0049 | -0.0049 | -0.0009 | -0.0051 | 0.0011 |
| $\Delta \, \mathrm{med}[m_{jt}]$ | -0.0037 | -0.0037 | 0.0002 | -0.0054 | 0.0005 |
| $\Delta \, \mathrm{E}[s_{jt}]$ | -0.0026 | -0.0026 | -0.0008 | -0.0037 | 0.0000 |
| $\Delta \, \mathrm{med}[s_{jt}]$ | -0.0037 | -0.0026 | 0.0034 | 0.0028 | 0.0000 |

We see that both mergers increase markups, but the GM-Quaker one leads to bigger changes than the Post-Nabisco one. This makes sense, as the GM-Quaker merger is merging two relatively large firms in terms of market share, whereas Post and Nabisco are the smallest two firms in the dataset. It is not surprising that merging two larger firms leads to an increase in markups and margins, as the merged firms experience a greater increase in market power than two smaller firms would, especially when firms are able to take into account how consumer heterogeneity affects substitution patterns. □

Table 9: General Mills-Quaker merger changes in equilibrium results by regression type, including BLP:

| Change in moment | OLS | OLS w/ FE | IV | IV w/ FE | BLP |
|---|---|---|---|---|---|
| $\Delta\,\mathrm{E}[p_{jt}]$ | 0.0014 | 0.0002 | 0.0014 | -0.0005 | 0.0027 |
| $\Delta\,\mathrm{med}[p_{jt}]$ | 0.0010 | -0.0001 | 0.0010 | -0.0007 | 0.0023 |
| $\Delta\,\mathrm{E}[\kappa_{jt}]$ | 0.0014 | 0.0002 | 0.0014 | -0.0005 | 0.0027 |
| $\Delta\,\mathrm{med}[\kappa_{jt}]$ | 0.0025 | 0.0016 | 0.0025 | 0.0012 | 0.0034 |
| $\Delta\,\mathrm{E}[m_{jt}]$ | 0.0079 | 0.0017 | 0.0082 | 0.0013 | 0.0138 |
| $\Delta\,\mathrm{med}[m_{jt}]$ | 0.0097 | 0.0038 | 0.0100 | 0.0012 | 0.0167 |
| $\Delta\,\mathrm{E}[s_{jt}]$ | -0.0014 | -0.0029 | -0.0013 | -0.0040 | 0.0000 |
| $\Delta\,\mathrm{med}[s_{jt}]$ | 0.0029 | 0.0023 | 0.0030 | 0.0024 | 0.0000 |

## Problem 8.

*Solution:* I now estimate the same model using the pyblp Python package. I use tolerance parameter $1e-5$ for the below estimates. I have included my code in the pyblpcode.py file. The output is included below (sorry it is a bit ugly when pasted directly here):

```
 1       Problem Results Summary:
 2     ================================================================================================================
 3     GMM     Objective    Gradient        Hessian            Hessian       Clipped   Weighting Matrix    Covariance Matrix
 4     Step      Value        Norm      Min Eigenvalue    Max Eigenvalue    Shares    Condition Number    Condition Number
 5     ____    _____    _____    _____    _____    _____   _____    _____
 6      2      +6.1E+00     +9.1E-07       +3.3E-05          +2.3E+04          0          +4.8E+07            +7.4E+08
 7     ================================================================================================================
 8
 9     Cumulative Statistics:
10     ================================================================================
11     Computation   Optimizer    Optimization      Objective     Fixed Point   Contraction
12        Time       Converged     Iterations      Evaluations    Iterations    Evaluations
13     _____   _____    _____    _____   _____   _____
14      00:01:34        Yes            91              105           77589        241680
15     ================================================================================
16
17     Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
18     ===================================================================================================================================
19     Sigma:      1          prices         sugar        mushy    |  Pi:       income     income_squared       age
       child
20      ____    _____   _____   _____   _____  |  ____    _____   _____   _____
21       1      +5.4E-01                                           |   1       +2.3E+00       +0.0E+00        +1.3E+00
       +0.0E+00
22              (+1.6E-01)                                         |          (+1.2E+00)                     (+6.5E-01)
23                                                                 |
24     prices   +0.0E+00    +3.1E+00                               | prices    +5.5E+02       -2.8E+01        +0.0E+00
       +1.1E+01
25                          (+1.2E+00)                             |          (+2.5E+02)     (+1.3E+01)
       (+4.1E+00)
26                                                                 |
27     sugar    +0.0E+00    +0.0E+00      -5.0E-03                 | sugar     -3.7E-01       +0.0E+00        +5.1E-02
       +0.0E+00
28                                        (+1.3E-02)               |          (+1.1E-01)                     (+2.5E-02)
29                                                                 |
30     mushy    +0.0E+00    +0.0E+00      +0.0E+00      +7.9E-02   | mushy     +8.1E-01       +0.0E+00        -1.4E+00
       +0.0E+00
31                                                      (+1.8E-01) |          (+7.6E-01)                     (+6.8E-01)
32     ===================================================================================================================================
33
34     Beta Estimates (Robust SEs in Parentheses):
35     ==========
36        prices
37     _____
38      -6.0E+01
39     (+1.4E+01)
40     ==========
```

We see that the coefficients are different than the ones we found previously. This could be due to a different tolerance set for the optimization procedure, or due to differences

in optimization implementation. Or, it could be the case that my optimization algorithm in the previous part found a local minimum of the objective function which wasn't the true minimum, so we may have found a 'false' solution. I played around with changing the tolerance parameter, and the coefficient estimates (for example, price sensitivity) did change. However, I wasn't able to get them as small as $-32$ or so that we previously estimated.

The summary statistics for markups, margins, and marginal costs are included below:

Table 10: Summary statistics, pyblp (compared with prior estimates):

|  | $\mathrm{E}[\kappa_{jt}]$ | $\mathrm{med}[\kappa_{jt}]$ | $\mathrm{sd}(\kappa_{jt})$ | $\mathrm{E}[m_{jt}]$ | $\mathrm{med}[m_{jt}]$ | $\mathrm{sd}(m_{jt})$ | $\mathrm{E}[mc_{jt}]$ | $\mathrm{med}[mc_{jt}]$ | $\mathrm{sd}(mc_{jt})$ |
|---|---|---|---|---|---|---|---|---|---|
| pyblp | 0.0432 | 0.0425 | 0.0100 | 0.3631 | 0.3354 | 0.1274 | 0.0825 | 0.0814 | 0.0299 |
| Julia blp :) | 0.0419 | 0.0404 | 0.0079 | 0.3557 | 0.3290 | 0.1293 | 0.0839 | 0.0829 | 0.0314 |

It is clear that they differ somewhat, which is understandable since the estimated coefficients are different (which will lead to differences in elasticities, which consequently affects firms' first order conditions). However, they are still pretty similar!

I now repeat the merger simulation for both mergers in include the results below in the same table, separated by merger type and compared with my prior estimates using my own code:

Table 11: Changes in equilibrium results after Post-Nabisco merger (top rows) and GM-Quaker (bottom rows), pyblp (compared with prior estimates):

|  | $\Delta\,\mathrm{E}[p_{jt}]$ | $\Delta\,\mathrm{med}[p_{jt}]$ | $\Delta\,\mathrm{E}[\kappa_{jt}]$ | $\Delta\,\mathrm{med}[\kappa_{jt}]$ | $\Delta\,\mathrm{E}[m_{jt}]$ | $\Delta\,\mathrm{med}[m_{jt}]$ | $\Delta\,\mathrm{E}[s_{jt}]$ | $\Delta\,\mathrm{med}[s_{jt}]$ |
|---|---|---|---|---|---|---|---|---|
| pyblp, Post | 0.0002 | 0.0002 | 0.0002 | 0.0005 | 0.0011 | 0.0012 | 0.0 | 0.0 |
| Julia blp :) | 0.0002 | 0.0000 | 0.0002 | 0.0001 | 0.0011 | 0.0005 | 0.0 | 0.0 |
| pyblp | 0.0028 | 0.0026 | 0.0028 | 0.0033 | 0.0143 | 0.0202 | -0.0005 | -0.0004 |
| Julia blp :) | 0.0027 | 0.0023 | 0.0027 | 0.0034 | 0.0138 | 0.0167 | 0.0 | 0.0 |

These estimates are very similar to my previous ones! The Post-Nabisco merger causes very small changes in equilibrium outcomes, whereas the GM-Quaker merger leads to more significant changes. The direction and magnitude of the changes are very close for my Julia implementation and the pyblp implementation, which is encouraging! $\qquad\square$