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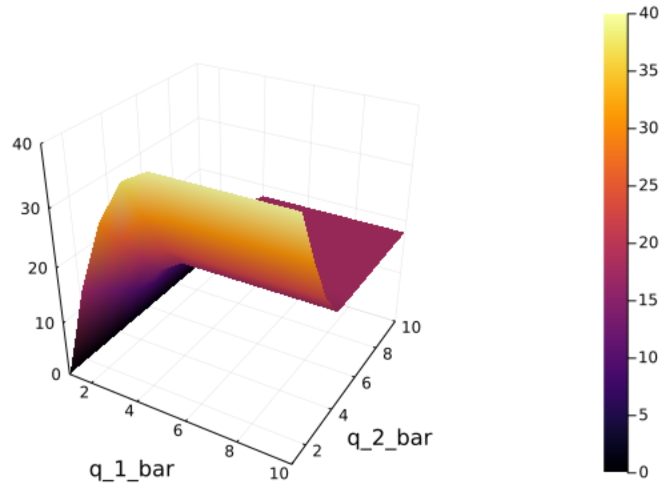
Econ 899
20 December, 2022

Problem 1.

Solution: The state space is the Cartesian product $\{0, 5, \ldots, 45\} \times \{0, 5, \ldots, 45\}$. □

Problem 2.

Solution: We calculated the payoffs in the static game and plotted the payoffs for firm 1 (without loss of generality, since they are symmetric):



□

Problem 3.

Solution: The Markov Perfect Equilibrium is characterized by the following two conditions:

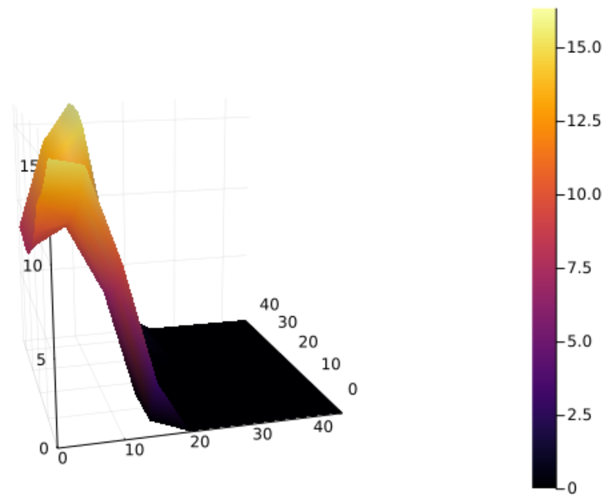
$$x_i(\omega) = R_i(x_j(\omega) \mid \omega) \quad \text{for all } i, j = 1, 2 \tag{1}$$

$$V_i^x(\omega) = \pi_i(\omega) - x_i(\omega) + \beta \sum_{\bar{q}_i'} W_i^x(\bar{q}_i' \mid \omega) P(\bar{q}_i' \mid \bar{q}_i, x_i) \quad \text{for all} i, j = 1, 2 \tag{2}$$

where $R_i(\cdot \mid \omega)$ is i's best response function, $\pi_i(\cdot)$ is the payoffs of the static game, $W_i^x(\bar{q}_i) = \sum_{\bar{q}_2'} V_i^x(\bar{q}_i, \bar{q}_j') P(\bar{q}_j' \mid \bar{q}_j, x_j)$ represents the continuation value. □

Problem 4.

Solution: We have plotted the strategy surface below:

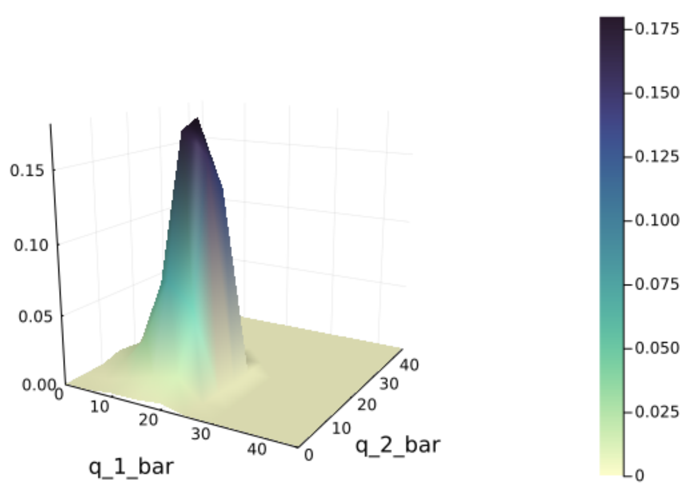


We also used the investment policy function to compute the transition matrix $Q(\omega' \mid \omega)$.

□

Problem 5.

Solution: Starting at $\omega = (0,0)$ and simulating the evolution of the industry over 25 years 1,000 times, we obtain the following distribution over states $(\bar{q}_{1,25}, \bar{q}_{2,25})$:



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