ME321 Ex8.3 Heat Exchanger Analysis

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1 Example 8.3: Heat Exchanger Exergy Analysis

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1.1 Problem Statement

A non-mixing heat exchanger is used to cool 20 kg/s of hot air from 500 K to 300 K using a supply of liquid water at 10°C. The exit temperature of the water is 5°C hotter than its inlet temperature. The ambient temperature and pressure are $T_0 = 20°\mathrm{C}$ and $p_0 = 1\,\mathrm{bar}$.

Determine: * (a) The mass flow rate of the water, kg/s * (b) The rate of heat transfer, kW * (c) The entropy generation, kW/K * (d) The exergy destruction, kW * (e) The second law efficiency * (f) The second law efficiency if the cooling water entered at 20°C instead of 10°C

1.2 Solution

1.2.1 Python Initialization

We'll start by importing the libraries we will use for our analysis and initializing dictionaries to hold the properties we will be usings.

[14]:
```
from thermoJFM import *

properties_dict = {
    'T':'K',
    'p':'kPa',
    'h':'kJ/kg',
    's':'kJ/kg/K',
    'mdot':'kg/s'
}
states = StatesTable(properties_dict, unit_system='SI_C')
for property in states.properties:
    globals()[property] = states.dict[property]

air = FluidProperties('Air', unit_system='SI_C')
water = FluidProperties('Water', unit_system='SI_C')

def show_calc():
    ShowCalculations(locals(),comments=True)
```

1.2.2 Given Parameters

We now define variables to hold our known values.

[15]:
```
T[0] = Q_(20,'degC')
p[0] = Q_(1,'bar')
T[1] = Q_(500,'K')
T[2] = Q_(300,'K')
T[3] = Q_(10,'degC')
T[4] = Q_(15,'degC')
mdot[1] = Q_(20,'kg/s')
mdot[2] = mdot[1]
p[1] = p[0]
p[2] = p[1]
p[3] = p[0]
p[4] = p[3]

sc=ShowCalculations(locals(),comments=True)
```

[15]:

$$T_0 = (20\ \ {}^\circ\mathrm{C}) = 293.15\ \ \mathrm{K} \tag{1}$$

[15]:

$$p_0 = (1\ \ \mathrm{bar}) = 100.0\ \ \mathrm{kPa} \tag{2}$$

[15]:

$$T_1 = 500\ \ \mathrm{K} \tag{3}$$

[15]:

$$T_2 = 300\ \ \mathrm{K} \tag{4}$$

[15]:

$$T_3 = (10\ \ {}^\circ\mathrm{C}) = 283.15\ \ \mathrm{K} \tag{5}$$

[15]:

$$T_4 = (15\ \ {}^\circ\mathrm{C}) = 288.15\ \ \mathrm{K} \tag{6}$$

[15]:

$$\dot{m}_1 = 20\ \ \frac{\mathrm{kg}}{\mathrm{s}} \tag{7}$$

[15]:

$$\dot{m}_2 = \dot{m}_1 = 20 \ \frac{\mathrm{kg}}{\mathrm{s}} \tag{8}$$

[15]:

$$p_1 = p_0 = 100.0 \ \mathrm{kPa} \tag{9}$$

[15]:

$$p_2 = p_1 = 100.0 \ \mathrm{kPa} \tag{10}$$

[15]:

$$p_3 = p_0 = 100.0 \ \mathrm{kPa} \tag{11}$$

[15]:

$$p_4 = p_3 = 100.0 \ \mathrm{kPa} \tag{12}$$

1.2.3 Assumptions

- Negligible changes in kinetic energy
- Negligible changes in potential energy

(a) mass flow rate of water

[16]:
```
# The properties can be can be evaluated at each state based on the temperatuer
↪and pressure
h[1] = air.h(T=T[1],p=p[1])
h[2] = air.h(T=T[2],p=p[2])
h[3] = water.h(T=T[3],p=p[3])
h[4] = water.h(T=T[4],p=p[4])

# The First Law can be used can be used to determine the mass flow rate
mdot[3] = mdot[1]*(h[2]-h[1])/(h[3]-h[4])
mdot[4] = mdot[1]
mdot_water = mdot[3]

sc=ShowCalculations(locals(),comments=True)
```

[16]:
The properties can be can be evaluated at each state based on the temperatuer and pressure

[16]:

$$h_1 = \mathrm{air.h}\left(\mathrm{T} = T_1, \mathrm{p} = p_1\right) = 629.4 \ \frac{\mathrm{kJ}}{\mathrm{kg}} \tag{13}$$

[16]:

$$h_2 = \mathrm{air.h}\left(\mathrm{T} = T_2, \mathrm{p} = p_2\right) = 426.3\ \frac{\mathrm{kJ}}{\mathrm{kg}} \tag{14}$$

[16]:

$$h_3 = \mathrm{water.h}\left(\mathrm{T} = T_3, \mathrm{p} = p_3\right) = 42.118\ \frac{\mathrm{kJ}}{\mathrm{kg}} \tag{15}$$

[16]:

$$h_4 = \mathrm{water.h}\left(\mathrm{T} = T_4, \mathrm{p} = p_4\right) = 63.076\ \frac{\mathrm{kJ}}{\mathrm{kg}} \tag{16}$$

[16]:
The First Law can be used can be used to determine the mass flow rate

[16]:

$$\begin{aligned} \dot{m}_3 &= \frac{\dot{m}_1\left(-h_1 + h_2\right)}{-h_4 + h_3} && (17)\\ &= \frac{\left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(-\left(629.4\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) + \left(426.3\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\right)}{-\left(63.076\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) + \left(42.118\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)} && (18)\\ &= 193.81\ \frac{\mathrm{kg}}{\mathrm{s}} && (19) \end{aligned}$$

[16]:

$$\dot{m}_4 = \dot{m}_1 = 20\ \frac{\mathrm{kg}}{\mathrm{s}} \tag{20}$$

[16]:

$$\dot{m}_{water} = \dot{m}_3 = 193.81\ \frac{\mathrm{kg}}{\mathrm{s}} \tag{21}$$

(b) rate of heat transfer

[17]:
```
# In order to determine the amount of heat transfered between the streams, we
↪can do a first law analysis for a control volume containing just one of the
↪streams.  Here we do this for the air stream
Qdot_1_2 = mdot[2]*h[2]- mdot[1]*h[1]
Qdot = abs(Qdot_1_2)
Qdot = Qdot.to('kW') # hide

sc=ShowCalculations(locals(),comments=True)
```

[17]:
In order to determine the amount of heat transfered between the streams, we can do a first law analysis for a control volume containing just one of the streams. Here we do this for the air stream

[17]:

$$\dot{Q}_{1,2} = -h_1\dot{m}_1 + h_2\dot{m}_2 \tag{22}$$
$$= -\left(629.4\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right) + \left(426.3\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right) \tag{23}$$
$$= -4061.9\ \frac{\mathrm{kJ}}{\mathrm{s}} \tag{24}$$

[17]:

$$\dot{Q} = \mathrm{abs}\left(\dot{Q}_{1,2}\right) = 4061.9\ \mathrm{kW} \tag{25}$$

(c) entropy generation

[18]:
```
# The specific entropies can also be evaluate using the known temperatures and␣
↪pressures
s[1] = air.s(T=T[1],p=p[1])
s[2] = air.s(T=T[2],p=p[2])
s[3] = water.s(T=T[3],p=p[3])
s[4] = water.s(T=T[4],p=p[4])

# The Second Law can be used to solve for the entropy generation
Sdot_gen = mdot[1]*(s[2]-s[1]) + mdot[3]*(s[4]-s[3])
Sdot_gen = Sdot_gen.to('kW/K') # hide

sc=ShowCalculations(locals(),comments=True)
```

[18]:
```
The specific entropies can also be evaluate using the known temperatures and
pressures
```

[18]:

$$s_1 = \mathrm{air.s}\left(\mathrm{T} = T_1, \mathrm{p} = p_1\right) = 4.4087\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})} \tag{26}$$

[18]:

$$s_2 = \mathrm{air.s}\left(\mathrm{T} = T_2, \mathrm{p} = p_2\right) = 3.8905\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})} \tag{27}$$

[18]:

$$s_3 = \mathrm{water.s}\left(\mathrm{T} = T_3, \mathrm{p} = p_3\right) = 0.15108\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})} \tag{28}$$

[18]:

$$s_4 = \mathrm{water.s}\left(\mathrm{T} = T_4, \mathrm{p} = p_4\right) = 0.22445\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})} \tag{29}$$

[18]:

The Second Law can be used to solve for the entropy generation

[18]:

$$\dot{S}_{gen} = \dot{m}_3\left(s_4 - s_3\right) + \dot{m}_1\left(-s_1 + s_2\right) \tag{30}$$
$$= \left(193.81\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\left(0.22445\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right) - \left(0.15108\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right)\right) + \left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(-\left(4.4087\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right) + \left(3.890 \right.\right. \tag{31}$$
$$= 3.8556\ \frac{\mathrm{kW}}{\mathrm{K}} \tag{32}$$

(d) exergy destruction

[22]:
```
# The exergy destruction is always $T_0\dot{S}_gen$
Xdot_dest = T[0]*Sdot_gen

sc=ShowCalculations(locals(),comments=True)
```

[22]:

The exergy destruction is always $T_0\dot{S}_gen$

[22]:

$$\dot{X}_{dest} = \dot{S}_{gen}T_0 \tag{33}$$
$$= \left(3.8556\ \frac{\mathrm{kW}}{\mathrm{K}}\right)(293.15\ \mathrm{K}) \tag{34}$$
$$= 1130.3\ \mathrm{kW} \tag{35}$$

(e) Second Law Efficiency

[24]:
```
# In this case, both streams are approaching the ambient temperature, so both␣
↪streams are decreasing in exergy.  Therefore, there is no recovered exergy␣
↪for this system
Xdot_Rec = Q_(0,'kW')
eta_II = Q_(0,'')

sc=ShowCalculations(locals(),comments=True)
```

[24]:

In this case, both streams are approaching the ambient temperature, so both streams are decreasing in exergy. Therefore, there is no recovered exergy for this system

[24]:

$$\dot{X}_{Rec} = 0\ \mathrm{kW} \tag{36}$$

[24]:

$$\eta_{II} = 0 \tag{37}$$

(f) Second Law Efficiency: Alternate Inlet Temperature

[27]:
```
# We need to recalculate properties to account for the alternate inlet␣
↪temperature
T[5] = Q_(20,'degC')
p[5] = p[4]
T[6] = T[5] + Q_(5,'delta_degC')
p[6] = p[5]
h[5] = water.h(T=T[5],p=p[5])
h[6] = water.h(T=T[6],p=p[6])
s[5] = water.s(T=T[5],p=p[5])
s[6] = water.s(T=T[6],p=p[6])

# For this case, the water stream in increaseing in exergy while the air stream␣
↪is decreasing in exergy.  So we will need the changes in flow exergy for␣
↪each stream
Delta_psi_5_6 = h[6]-h[5] - T[0]*(s[6]-s[5])
Delta_psi_1_2 = h[2]-h[1] - T[0]*(s[2]-s[1])

# A first law analysis can be used to update the water mass flow rate for the␣
↪new inlet temperature
mdot[5] = mdot[1]*(h[2]-h[1])/(h[5]-h[6])
mdot[6] = mdot[5]

# The second law gives the new entropy generation
Sdot_gen_Alt = mdot[1]*(s[2]-s[1]) + mdot[5]*(s[6]-s[5])

# We also have a new exergy destruction
Xdot_dest_Alt = T[0]*Sdot_gen_Alt

# The recovered exergy for this case in the increase in exergy of the water␣
↪stream because it is moving away from the surrounding temperature
Xdot_Rec = mdot[5]*Delta_psi_5_6

# The expended exergy for this case is the decrease in exergy of the air stream␣
↪becuase it is moving toward the surrounding temperature
Xdot_Exp = -mdot[1]*Delta_psi_1_2

# Second law efficiency
eta_II_Alt = Xdot_Rec/Xdot_Exp

sc=ShowCalculations(locals(),comments=True)
```

[27]: We need to recalculate properties to account for the alternate inlet temperature

[27]:

$$T_5 = (20\ \ {}^\circ\mathrm{C}) = 293.15\ \ \mathrm{K} \tag{38}$$

[27]:

$$p_5 = p_4 = 100.0 \ \text{kPa} \tag{39}$$

[27]:

$$\begin{aligned} T_6 &= (5 \ \Delta°\text{C}) + T_5 && (40)\\ &= (5 \ \Delta°\text{C}) + (293.15 \ \text{K}) && (41)\\ &= 298.15 \ \text{K} && (42)\end{aligned}$$

[27]:

$$p_6 = p_5 = 100.0 \ \text{kPa} \tag{43}$$

[27]:

$$h_5 = \text{water.h}\left(\text{T} = T_5, \text{p} = p_5\right) = 84.006 \ \frac{\text{kJ}}{\text{kg}} \tag{44}$$

[27]:

$$h_6 = \text{water.h}\left(\text{T} = T_6, \text{p} = p_6\right) = 104.92 \ \frac{\text{kJ}}{\text{kg}} \tag{45}$$

[27]:

$$s_5 = \text{water.s}\left(\text{T} = T_5, \text{p} = p_5\right) = 0.29646 \ \frac{\text{kJ}}{(\text{K} \cdot \text{kg})} \tag{46}$$

[27]:

$$s_6 = \text{water.s}\left(\text{T} = T_6, \text{p} = p_6\right) = 0.3672 \ \frac{\text{kJ}}{(\text{K} \cdot \text{kg})} \tag{47}$$

[27]:

For this case, the water stream in increaseing in exergy while the air stream is decreasing in exergy. So we will need the changes in flow exergy for each stream

[27]:

$$\begin{aligned} \Delta\psi_{5,6} &= -T_0\left(-s_5 + s_6\right) - h_5 + h_6 && (48)\\ &= -\left(293.15 \ \text{K}\right)\left(-\left(0.29646 \ \frac{\text{kJ}}{(\text{K} \cdot \text{kg})}\right) + \left(0.3672 \ \frac{\text{kJ}}{(\text{K} \cdot \text{kg})}\right)\right) - \left(84.006 \ \frac{\text{kJ}}{\text{kg}}\right) + \left(104.92 \ \frac{\text{kJ}}{\text{kg}}\right) && (49)\\ &= 0.17632 \ \frac{\text{kJ}}{\text{kg}} && (50)\end{aligned}$$

[27]:

$$
\begin{aligned}
\Delta\psi_{1,2} &= -T_0\left(-s_1 + s_2\right) - h_1 + h_2 && (51)\\
&= -\left(293.15\ \mathrm{K}\right)\left(-\left(4.4087\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right) + \left(3.8905\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right)\right) - \left(629.4\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) + \left(426.3\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) && (52)\\
&= -51.176\ \frac{\mathrm{kJ}}{\mathrm{kg}} && (53)
\end{aligned}
$$

[27]:

```
A first law analysis can be used to update the water mass flow rate for the new
inlet temperature
```

[27]:

$$
\begin{aligned}
\dot{m}_5 &= \frac{\dot{m}_1\left(-h_1 + h_2\right)}{h_5 - h_6} && (54)\\
&= \frac{\left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(-\left(629.4\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) + \left(426.3\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\right)}{\left(84.006\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right) - \left(104.92\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)} && (55)\\
&= 194.23\ \frac{\mathrm{kg}}{\mathrm{s}} && (56)
\end{aligned}
$$

[27]:

$$
\dot{m}_6 = \dot{m}_5 = 194.23\ \frac{\mathrm{kg}}{\mathrm{s}} \qquad (57)
$$

[27]:

```
The second law gives the new entropy generation
```

[27]:

$$
\begin{aligned}
\dot{S}_{gen,Alt} &= \dot{m}_5\left(-s_5 + s_6\right) + \dot{m}_1\left(-s_1 + s_2\right) && (58)\\
&= \left(194.23\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(-\left(0.29646\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right) + \left(0.3672\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right)\right) + \left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(-\left(4.4087\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{kg})}\right) + (3 && (59)\\
&= 3.3746\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{s})} && (60)
\end{aligned}
$$

[27]:

```
We also have a new exergy destruction
```

[27]:

$$
\begin{aligned}
\dot{X}_{dest,Alt} &= \dot{S}_{gen,Alt}T_0 && (61)\\
&= \left(3.3746\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{s})}\right)\left(293.15\ \mathrm{K}\right) && (62)\\
&= 989.27\ \frac{\mathrm{kJ}}{\mathrm{s}} && (63)
\end{aligned}
$$

[27]:

The recovered exergy for this case in the increase in exergy of the water stream because it is moving away from the surrounding temperature

[27]:

$$\dot{X}_{Rec} = \Delta\psi_{5,6}\dot{m}_5 \tag{64}$$
$$= \left(0.17632\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\left(194.23\ \frac{\mathrm{kg}}{\mathrm{s}}\right) \tag{65}$$
$$= 34.247\ \frac{\mathrm{kJ}}{\mathrm{s}} \tag{66}$$

[27]:

The expended exergy for this case is the decrease in exergy of the air stream becuase it is moving toward the surrounding temperature

[27]:

$$\dot{X}_{Exp} = -\Delta\psi_{1,2}\dot{m}_1 \tag{67}$$
$$= -\left(-51.176\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)\left(20\ \frac{\mathrm{kg}}{\mathrm{s}}\right) \tag{68}$$
$$= 1023.5\ \frac{\mathrm{kJ}}{\mathrm{s}} \tag{69}$$

[27]:

Second law efficiency

[27]:

$$\eta_{II,Alt} = \frac{\dot{X}_{Rec}}{\dot{X}_{Exp}} \tag{70}$$
$$= \frac{\left(34.247\ \frac{\mathrm{kJ}}{\mathrm{s}}\right)}{\left(1023.5\ \frac{\mathrm{kJ}}{\mathrm{s}}\right)} \tag{71}$$
$$= 0.033461 \tag{72}$$

[28]:

```
ss=ShowSummary(locals())
```

[28]:

$$\Delta\psi_{1,2} = -51.176\ \frac{\mathrm{kJ}}{\mathrm{kg}} \qquad \Delta\psi_{5,6} = 0.17632\ \frac{\mathrm{kJ}}{\mathrm{kg}} \qquad \dot{Q} = 4061.9\ \mathrm{kW} \qquad \dot{Q}_{1,2} = -4061.9\ \frac{\mathrm{kJ}}{\mathrm{s}} \tag{73}$$

$$\dot{S}_{gen} = 3.8556\ \frac{\mathrm{kW}}{\mathrm{K}} \qquad \dot{S}_{gen,Alt} = 3.3746\ \frac{\mathrm{kJ}}{(\mathrm{K}\cdot\mathrm{s})} \qquad \dot{X}_{Exp} = 1023.5\ \frac{\mathrm{kJ}}{\mathrm{s}} \qquad \dot{X}_{Rec} = 34.247\ \frac{\mathrm{kJ}}{\mathrm{s}} \tag{74}$$

$$\dot{X}_{dest} = 1130.3\ \mathrm{kW} \qquad \dot{X}_{dest,Alt} = 989.27\ \frac{\mathrm{kJ}}{\mathrm{s}} \qquad \eta_{II} = 0 \qquad \eta_{II,Alt} = 0.033461 \tag{75}$$

$$\dot{m}_{water} = 193.81\ \frac{\mathrm{kg}}{\mathrm{s}} \tag{76}$$

[28]: <IPython.core.display.HTML object>

[0]:

```
```