Homework Assignment 2

Problem 1

1. Write out the conservation of linear momentum equations without using the summation convention, i.e. all components.

Problem 2

For most of the class, we've used a solid mechanics setting to motivate the physics of interest; however, the principles we've derived are general enough to apply to fluids as well. In an ideal nonviscous fluid there can be no shear stress. Hence, the stress tensor is entire hydrostatic, $\sigma_{ij} = -p\delta_{ij}$. Show that this leads to the following form of the momentum equation, known as Euler's equation of motion for a frictionless fluid:

$$-\frac{1}{\rho}\nabla p + \vec{b} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

Problem 3

Using the different forms of conservation of mass, derive an expression for $\frac{dJ}{dt}$ in terms of $\vec{v}$, where $J = \det(\mathbf{F})$

Problem 4

Assume that the internal-energy density can be given as $u = u(\epsilon, T)$, that the heat flux is governed by Fourier's law $\vec{q} = -k(T)\nabla T$, and that $r = 0$. Defining the specific heat $C = \frac{\partial u}{\partial T}$, write the equation resulting from combining these assumptions with the energy- balance equation.

Problem 5

Show that, in an isotropic linearly elastic solid, the principal stress and principal strain directions coincide.

Problem 6

Write the elastic modulus matrix C_{IJ} for an isotropic linearly elastic solid in terms of the Young's modulus E and the Poisson's ratio ν.

Problem 7

Combine the generalized Hooke's law for an isotropic linearly elastic solid with the equations of motion and the definition of small strain in order to derive the equations of motion for such a solid entirely in terms of displacement, using

1. λ and μ
2. G and ν

Problem 8

1. Show that minimizing the integral

$$I = \int_{t_1}^{t_2} L(t, y(t), \dot{y}(t)) \mathrm{d}t$$

   results in the Euler-Lagrange Equation, i.e.

$$\frac{\partial L}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{y}}\right) = 0$$

2. A mass m suspended from a spring with stiffness k has the following kinetic and potential energies,

$$T = \frac{1}{2} m \dot{y}^2 \qquad U = \frac{1}{2} k y^2$$

   Assume $L = T - U$ and use the Euler-Lagrange equation to derive the equation-of-motion for the spring-mass system.