$$\boxed{\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}} \rightarrow \text{Generalized Hooke's Law}$$

$$C_{ijkl} = C_{jikl}$$
$$C_{ijkl} = C_{ijlk}$$
$$C_{ijkl} = C_{klij}$$
$$\downarrow$$
21 components

$$C = \frac{\partial w}{\partial \varepsilon_{ij}\,\partial \varepsilon_{kl}} = \frac{\partial}{\partial \varepsilon_{ij}}\left(\frac{\partial w}{\partial \varepsilon_{kl}}\right) = \frac{\partial}{\partial \varepsilon_{kl}}\left(\frac{\partial w}{\partial \varepsilon_{ij}}\right)$$

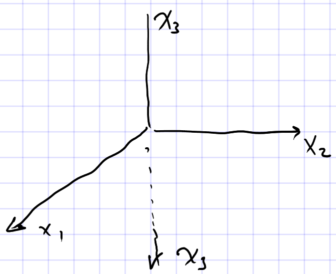
Voight Notation

$$\vec{\sigma} = \bar{\bar{C}}\,\vec{\varepsilon}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1131} & C_{1112} \\ & C_{2222} & C_{2233} & C_{2223} & C_{2231} & C_{2212} \\ & & C_{3333} & C_{3323} & C_{3331} & C_{3312} \\ & \text{symm.} & & C_{2323} & C_{2331} & C_{2312} \\ & & & & C_{3131} & C_{3112} \\ & & & & & C_{1212} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix}$$

"triclinic"

Consider a plane of symm.



$x_1 - x_2$ plane is a plane of symm.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\sigma' = R\sigma R^T = \begin{bmatrix} \sigma_{11} & \sigma_{12} & -\sigma_{31} \\ & \sigma_{22} & -\sigma_{23} \\ & & \sigma_{33} \end{bmatrix}$$

similarly for

$$\varepsilon' = R\varepsilon R^T \qquad \boxed{\varepsilon'_{31} = -\varepsilon_{31}} + \boxed{\varepsilon'_{23} = -\varepsilon_{23}}$$

$$\sigma'_{11} = C_{1111}\varepsilon'_{11} + C_{1122}\varepsilon'_{22} + C_{1133}\varepsilon'_{33} + 2C_{1123}\varepsilon'_{23} + 2C_{1131}\varepsilon'_{31} + 2C_{1121}\varepsilon'_{12}$$

$$\sigma'_{11} = C_{1111}\varepsilon_{11} + C_{1122}\varepsilon_{22} + C_{1133}\varepsilon_{33} - 2C_{1123}\varepsilon_{23} - 2C_{1131}\varepsilon_{31} + 2C_{1121}\varepsilon_{12}$$

$$\sigma_{11} = C_{1111}\varepsilon_{11} + C_{1122}\varepsilon_{22} + C_{1133}\varepsilon_{33} + 2C_{1123}\varepsilon_{23} + 2C_{1131}\varepsilon_{33} + 2C_{1121}\varepsilon_{12}$$

$$\sigma'_{11} - \sigma_{11} = 0$$

$$0 = -4C_{1123}\varepsilon_{23} - 4C_{1131}\varepsilon_{31}$$

$$C_{1123} = C_{1131} = 0$$

Using similar arguments

$$C_{2223} = C_{2231} = C_{3323} = C_{3331} = 0$$

$$\bar{\bar{C}} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & 0 & 0 & C_{1112} \\ & C_{2222} & C_{2233} & 0 & 0 & C_{2212} \\ & & C_{3333} & 0 & 0 & C_{3312} \\ & \text{symm.} & & C_{2323} & C_{2331} & 0 \\ & & & & C_{3131} & 0 \\ & & & & & C_{1212} \end{bmatrix}$$

monoclinic material → 13 independent constants

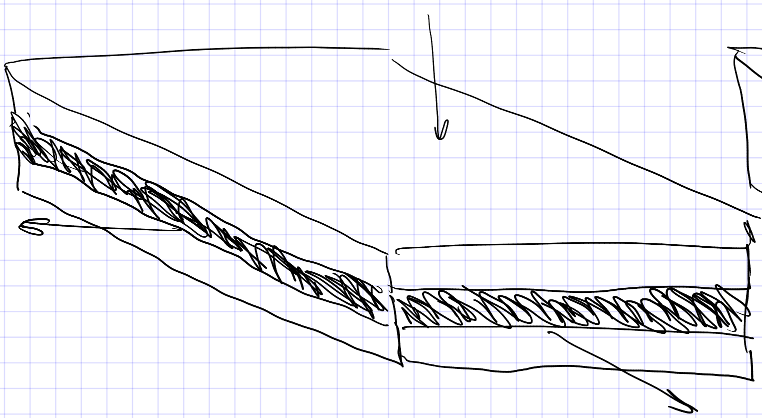
If 3 orthogonal planes of symm.

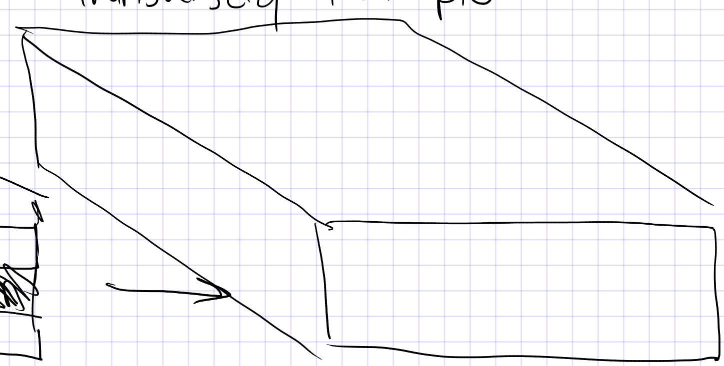
$$C_{1122} = C_{2223} = C_{2231} = C_{2212} = 0$$

9 independent constants → orthotropic material

If there exists an axes about which a material has identical properties, then $\underline{\underline{5}}$ independent constants

transversely isotropic



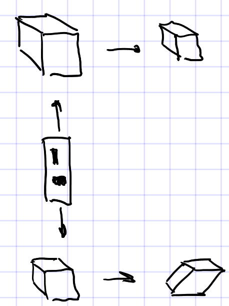


For a material in which every plane is a plane of symm,

Isotropic Material

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\end{Bmatrix} = \begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ & \lambda+2\mu & \lambda & 0 & 0 & 0\\ & & \lambda+2\mu & 0 & 0 & 0\\ & & & \mu & 0 & 0\\ & \text{symm.} & & & \mu & 0\\ & & & & & \mu\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\\ 2\varepsilon_{12}\end{Bmatrix}$$

$$\lambda = \frac{2\mu\nu}{1-2\nu} = \frac{\mu(E-2\mu)}{3\mu-E} = K - \frac{2}{3}\mu$$



λ → Lame's constant

K → Bulk Modulus

E → Young's Modulus "elastic"

$\mu(G)$ → shear modulus

ν → Poisson's ratio

$K(\nu, \cdot)$,P $\nu = 0.5$ $K \to \infty$

For isotropic materials

$$C_{ijkl} = \lambda\,\delta_{ij}\delta_{kl} + \mu(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk})$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} = [\lambda\,\delta_{ij}\delta_{kl} + \mu\,\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk}]\,\varepsilon_{kl}$$

$\delta_{il}\,\delta_{jk}\,\varepsilon_{kl}$

$\delta_{il}\,\varepsilon_{jl}$

ε_{ji}

$$\boxed{\sigma_{ij} = \lambda\,\delta_{ij}\,\varepsilon_{kk} + 2\mu\,\varepsilon_{ij}}$$

Let $j = i$

$$\sigma_{ii} = \lambda\,\overset{3}{\cancel{\delta_{ii}}}\,\varepsilon_{kk} + 2\mu\,\varepsilon_{kk}$$

$$\sigma_{kk} = (3\lambda + 2\mu)\,\varepsilon_{kk} \Rightarrow \boxed{\varepsilon_{kk} = \frac{\sigma_{kk}}{(3\lambda+2\mu)}}$$

$$\boxed{\varepsilon_{ij} = \frac{-\nu}{E}\,\sigma_{kk}\,\delta_{ij} + \frac{1+\nu}{E}\,\sigma_{ij}}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$