

$$A = \{X \text{ / . } \{x \rightarrow 0, y \rightarrow 0\}, X \text{ / . } \{x \rightarrow a, y \rightarrow 0\}, X \text{ / . } \{x \rightarrow a, y \rightarrow b\}, X \text{ / . } \{x \rightarrow 0, y \rightarrow b\}\};$$
$$\text{Out}[3]=\left\{1-\frac{x}{a}-\frac{y}{b}+\frac{xy}{ab}, \frac{x}{a}-\frac{xy}{ab}, \frac{xy}{ab}, \frac{y}{b}-\frac{xy}{ab}\right\}$$

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c = {{k, 0, 0}, {0, k, 0}, {0, 0, 0}};
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$$\text{Out}[9]=\left\{\left\{-\frac{2 a k}{3 b}+a\left(\frac{k}{b}+\frac{b k}{3 a^2}\right), \frac{a k}{6 b}-\frac{b k}{3 a},-\frac{a k}{6 b}-\frac{b k}{6 a}, \frac{2 a k}{3 b}+a\left(-\frac{k}{b}+\frac{b k}{6 a^2}\right)\right\},\right. \\ \left.\left\{\frac{a k}{6 b}-\frac{b k}{3 a}, \frac{a k}{3 b}+\frac{b k}{3 a},-\frac{a k}{3 b}+\frac{b k}{6 a},-\frac{a k}{6 b}-\frac{b k}{6 a}\right\},\left\{-\frac{a k}{6 b}-\frac{b k}{6 a},-\frac{a k}{3 b}+\frac{b k}{6 a}, \frac{a k}{3 b}+\frac{b k}{3 a}, \frac{a k}{6 b}-\frac{b k}{3 a}\right\},\right. \\ \left.\left\{\frac{2 a k}{3 b}+a\left(-\frac{k}{b}+\frac{b k}{6 a^2}\right),-\frac{a k}{6 b}-\frac{b k}{6 a}, \frac{a k}{6 b}-\frac{b k}{3 a}, \frac{a k}{3 b}+\frac{b k}{3 a}\right\}\right\}$$

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Out[10]= {{1, 2, 6, 5}, {2, 3, 7, 6}, {3, 4, 8, 7}, {5, 6, 10, 9}, {6, 7, 11, 10}, {7, 8, 12, 11}}
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Do[
  K[[connect[[i]], connect[[i]]]] += ke,
  {i, 1, Length[connect]}
];
MatrixForm[K]
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$-\frac{2ak}{3b} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right)$	$\frac{ak}{6b} - \frac{bk}{3a}$	$-\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right)$	$\frac{ak}{6b} - \frac{bk}{3a}$	0	$\frac{2ak}{3b} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right)$
$\frac{ak}{6b} - \frac{bk}{3a}$	$-\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right)$	$\frac{ak}{6b} - \frac{bk}{3a}$	0	$-\frac{ak}{6b} - \frac{bk}{6a}$	$\frac{ak}{3b} +$
0	$\frac{ak}{6b} - \frac{bk}{3a}$	$-\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right)$	$\frac{ak}{6b} - \frac{bk}{3a}$	0	
0	0	$\frac{ak}{6b} - \frac{bk}{3a}$	$\frac{ak}{3b} + \frac{bk}{3a}$	0	
$\frac{2ak}{3b} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right)$	$-\frac{ak}{6b} - \frac{bk}{6a}$	0	0	$-\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right)$	
$-\frac{ak}{6b} - \frac{bk}{6a}$	$\frac{ak}{3b} + \frac{bk}{6a} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right)$	$-\frac{ak}{6b} - \frac{bk}{6a}$	0	$\frac{ak}{3b} - \frac{2bk}{3a}$	$\frac{ak}{3b} +$
0	$-\frac{ak}{6b} - \frac{bk}{6a}$	$\frac{ak}{3b} + \frac{bk}{6a} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right)$	$-\frac{ak}{6b} - \frac{bk}{6a}$	0	
0	0	$-\frac{ak}{6b} - \frac{bk}{6a}$	$-\frac{ak}{3b} + \frac{bk}{6a}$	0	
0	0	0	0	$\frac{2ak}{3b} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right)$	
0	0	0	0	$-\frac{ak}{6b} - \frac{bk}{6a}$	$\frac{ak}{3b} +$
0	0	0	0	0	
0	0	0	0	0	
0	0	0	0	0	
0	0	0	0	0	

In[25]:= **F = ConstantArray[0, {12}];**

$$F[[9]] = u0 \cos\left[\frac{\pi x}{6a}\right] /. \{x \rightarrow 0\};$$

$$F[[10]] = u0 \cos\left[\frac{\pi x}{6a}\right] /. \{x \rightarrow a\};$$

$$F[[11]] = u0 \cos\left[\frac{\pi x}{6a}\right] /. \{x \rightarrow 2a\};$$

$$F[[12]] = u0 \cos\left[\frac{\pi x}{6a}\right] /. \{x \rightarrow 3a\};$$

In[30]:= **Do[**

K[[i]] = Normal@SparseArray[i → 1, {12}]
, {i, {9, 10, 11, 4, 8, 12}}]

In[31]:= **MatrixForm[K]**

Out[31]//MatrixForm=

$$\begin{pmatrix} -\frac{2ak}{3b} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right) & \frac{ak}{6b} - \frac{bk}{3a} & 0 & 0 & \frac{2ak}{3b} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right) \\ \frac{ak}{6b} - \frac{bk}{3a} & -\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right) & \frac{ak}{6b} - \frac{bk}{3a} & 0 & -\frac{ak}{6b} - \frac{bk}{6a} & \frac{ak}{3b} + \\ 0 & \frac{ak}{6b} - \frac{bk}{3a} & -\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right) & \frac{ak}{6b} - \frac{bk}{3a} & 0 & \\ 0 & 0 & 0 & 1 & 0 & \\ \frac{2ak}{3b} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right) & -\frac{ak}{6b} - \frac{bk}{6a} & 0 & 0 & -\frac{ak}{3b} + \frac{bk}{3a} + a\left(\frac{k}{b} + \frac{bk}{3a^2}\right) & \\ -\frac{ak}{6b} - \frac{bk}{6a} & \frac{ak}{3b} + \frac{bk}{6a} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right) & -\frac{ak}{6b} - \frac{bk}{6a} & 0 & \frac{ak}{3b} - \frac{2bk}{3a} & \frac{ak}{3b} + \\ 0 & -\frac{ak}{6b} - \frac{bk}{6a} & \frac{ak}{3b} + \frac{bk}{6a} + a\left(-\frac{k}{b} + \frac{bk}{6a^2}\right) & -\frac{ak}{6b} - \frac{bk}{6a} & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[35]:= **uh = LinearSolve[K, F] /. {a → 1, b → 1, u0 → 1, k → 1} // N**

Out[35]= {0.612842, 0.530737, 0.306421, 0., 0.703, 0.608816, 0.3515, 0., 1., 0.866025, 0.5, 0.}

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In[38]:= ListContourPlot[
  {0, 0, uh[[1]]},
  {1, 0, uh[[2]]},
  {2, 0, uh[[3]]},
  {3, 0, uh[[4]]},
  {0, 1, uh[[5]]},
  {1, 1, uh[[6]]},
  {2, 1, uh[[7]]},
  {3, 1, uh[[8]]},
  {0, 2, uh[[9]]},
  {1, 2, uh[[10]]},
  {2, 2, uh[[11]]},
  {3, 2, uh[[12]]}
], PlotTheme -> "Detailed", AspectRatio -> 2 / 3
]

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