The Related Samples T Test

t-Test: Paired Two Sample for Means		
	Con1	*Con2*
Mean	172.6	159.4
Variance	750.2666667	789.3777778
Observations	10	10
Pearson Correlation	0.863335004	
Hypothesised Mean Difference	0	
df	9	
t Stat	2.874702125	
P(T<=t) one-tail	0.009167817	
t Critical one-tail	1.833112933	
P(T<=t) two-tail	0.018335635	
t Critical two-tail	2.262157163	
Difference in Mean	13.2	

Data:

Exa%207.4F%20(Hyp
othesis%20Testing%2

The situation is analogous:

- They first did a **two-tailed** test to see if the mean impurity differs between two filtration agents.
- Now they want a **one-tailed** test to see if **Filter Agent 1 is more effective** (meaning: lower impurity for Agent 1 → but careful: more effective means **lower impurity**, so if impurity is the measured variable, then $\mu_1 < \mu_2$ is the alternative hypothesis if μ = impurity level).

But in our container example, **Con1** had **higher** sales → better.
So in the filter case, we need to know: is "more effective" lower impurity or higher sales?

From the phrasing:

"one-tailed test … to determine whether Filter Agent 1 was the more effective"

More effective means **lower impurity** (since impurity is bad). So:

- Let μ_1 = population mean impurity for Agent 1
- Let μ_2 = population mean impurity for Agent 2

Hypotheses:

$H_0 : \mu_1 \geq \mu_2 H_1 : \mu_1 < \mu_2$ **Mapping to the container example**

In the container example, **higher** is better (sales).
In the filter example, **lower** is better (impurity).

So the roles of “which is better” are reversed in terms of the sign of the difference.

In the container data:
Sample mean difference (Con1 − Con2) = **+13.2** (positive).

If we simply **swap the roles** to match the filter scenario:

Suppose in the actual filter data, the sample mean difference (Agent1 − Agent2) for impurity is **−13.2** (negative) — that would mean Agent1 has lower impurity.

Then the t-statistic would be **−2.8747** instead of +2.8747.

One-tailed p-value from the given output

From the container output:

- **One-tailed p-value** = 0.009167817 — this is for the alternative hypothesis $\mu_1 > \mu_2$ (since t = +2.8747).

If instead we test $\mu_1 < \mu_2$ with t = +2.8747, the one-tailed p-value would be $1 - 0.009167817 \approx 0.9908$, which is **not significant**.

But in the filter case, if Agent1 is more effective, we expect **t = −2.8747** (if the data difference in impurity is like the negative of the container sales difference).

Then the one-tailed p-value for $\mu_1 < \mu_2$ would be the left-tail area for t = −2.8747, which equals **0.009167817** (same as the right-tail area for +2.8747 by symmetry).

Conclusion for the filter problem

If the actual two-tailed test in the filter data gave a two-tailed p-value of 0.0183 (like here), then:

- For the one-tailed test in the correct direction (Agent1 better → lower impurity), the p-value is **0.00917**.
- This is significant at $\alpha = 0.05$ (and even at $\alpha = 0.01$).

So we **reject H0** and conclude **Filter Agent 1 is indeed more effective**.

Therefore,

Significant evidence that Filter Agent 1 is more effective