Hypothesis Testing

1. Interpretation of Findings

t-Test: Paired Two Sample for Means

	Agent1	Agent2
Mean	8.25	8.683333333
Variance	1.059090909	1.077878788
Observations	12	12
Pearson Correlation	0.901055812	
Hypothesized Mean Difference	0	
df	11	
t Stat	-3.263938591	
P(T<=t) one-tail	0.003772997	
t Critical one-tail	1.795884819	
P(T<=t) two-tail	0.007545995	
t Critical two-tail	2.20098516	

A paired samples t-test was conducted to determine whether there is a significant difference in the population mean impurity levels between Filtration Agent 1 and Filtration Agent 2. The mean impurity level for Agent 1 was 8.25, while that for Agent 2 was 8.68, giving a mean difference of –0.43 (Agent 1 minus Agent 2).

The t-statistic obtained was t(11) = –3.26, with an associated two-tailed p-value of 0.0075. This p-value is below the conventional 5% significance level (α = 0.05). Therefore, the null hypothesis that there is no difference between the mean impurity levels of the two filtration agents can be rejected.

This indicates that there is a statistically significant difference between the two filtration agents in terms of their mean impurity levels. Since the mean impurity for Agent 1 is lower than that for Agent 2, this suggests that Agent 1 is more effective at reducing impurity than Agent 2.

2. Hypotheses for the two-tailed test

$$H_0 : \mu_d = 0 \text{ \(no difference in mean impurity\)} H_1 : \mu_d \neq 0$$

where $\mu_d = \mu_1 - \mu_2$.

3. Assumptions & validity

The paired t-test assumes:

1. **Paired design** — yes, same batches.
2. **Differences are normally distributed** — the output says "Assuming the data to be suitably distributed," meaning we assume normality of the differences.
3. **Data are continuous** — impurity measurements are continuous.

To validate normality of differences:

- Could use a Shapiro–Wilk test on the differences.
- Or check histogram/Q–Q plot of differences.

Given n=12, normality is important; but the paired t-test is fairly robust if no severe skew/outliers.

4. Summary Table

Statistic	Value
Mean impurity (Agent 1)	8.25
Mean impurity (Agent 2)	8.68
t-statistic	–3.26
p-value (two-tailed)	0.0075
df	11
Decision	Reject H_0
Conclusion	Significant difference — Agent 1 has lower impurity

5. One-tailed Test

If the hypothesis had been one-tailed, specifically testing whether Agent 1 was more effective (i.e., whether Agent 1's mean impurity was lower than Agent 2's), then the one-tailed p-value of 0.0038 would be compared to $\alpha = 0.05$.

Since $0.0038 < 0.05$, the null hypothesis would again be rejected, providing strong evidence that Agent 1 is more effective than Agent 2 in lowering impurity levels.

6. Conclusion

Based on both the two-tailed and one-tailed paired t-tests, there is strong statistical evidence that the mean impurity levels differ between the two filtration agents. Specifically, Agent 1 consistently shows lower impurity levels than Agent 2, and this difference is statistically significant at the

5% level. We therefore conclude that Agent 1 is the more effective filtration agent.

Appendix

Data analysis:

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