

The INDEPENDENT Samples T Test (Exercise 7.2)

1. Understanding the test performed

F-Test Two-Sample
for Variances

	Male	Female
	<i>Income</i>	<i>33.1</i>
		44.422
Mean	52.91333333	03
		191.28
Variance	233.1289718	14
Observations	60	59
df	59	58
F	1.218774896	
P(F<=f) one-tail	0.226087561	
F Critical one-tail	1.5435835	

From the output:

- An **F-test for variances** was done first:
 - $F = 1.2188$,
 - $p\text{-value (one-tail)} = 0.2261$,
 - $F_{\text{critical}} = 1.5436$.

Since $p > 0.05$, we **do not reject** H_0 that the variances are equal.

But the subsequent **t-test** used “**Unequal Variances**” (Welch’s t-test) anyway.

t-Test: Two-Sample Assuming Unequal
Variances

	<i>Male</i>	<i>Female</i>
	52.913333	44.233
Mean	33	33
	233.12897	190.17
Variance	18	58
Observations	60	60
Hypothesised Mean Difference	0	
df	117	
	3.2679000	
t Stat	01	
	0.0007112	
P(T<=t) one-tail	86	

	1.6579816
t Critical one-tail	59
P(T<=t) two-tail	0.0014225
	72
	1.9804475
t Critical two-tail	99

2. Hypotheses and conclusion

The problem says:

test of whether the population mean income for males exceeds that of females

That's a **one-tailed test**:

$$H_0: \mu_M \leq \mu_F \quad H_1: \mu_M > \mu_F$$

From the output:

One-tailed p-value = 0.000711

Since $p < 0.05$ (and even $p < 0.01$), we **reject** H_0 .

Conclusion: There is strong evidence that the population mean income for males is greater than that for females.

3. Assumptions for the t-test (unequal variances)

1. **Independence:** The male and female samples are independent of each other (not paired).
2. **Normality:** The distribution of income in each group should be approximately normal — especially important because sample sizes are moderate ($n_1=60, n_2=60$), but the t-test is fairly robust to non-normality with such sample sizes.
3. **Random sampling:** The data should come from a random sample from the population.

4. How to validate assumptions

- **Independence:** Known from study design (different individuals, no pairing).
- **Normality:** Check using:
 - Histograms / Q-Q plots of incomes for each group.

- Shapiro-Wilk or Kolmogorov-Smirnov tests (though with $n > 30$, CLT helps).
- **Equal variance not assumed:** The Welch test doesn't assume equal variances, so the F-test's non-significance is just extra info; we already used the safer Welch df.

5. Interpretation of practical significance

The mean difference is about 8.68 (in thousands? the units are not given, but likely same as data).

We can also compute Cohen's d for effect size:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

But since we used unequal variances, maybe use:

$$d = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2 + s_2^2}{2}}}$$

Roughly:

$$d \approx \frac{8.68}{\sqrt{\frac{233.13 + 190.18}{2}}} = \frac{8.68}{\sqrt{211.655}} \approx \frac{8.68}{14.55} \approx 0.60$$

That's a **medium to large** effect size.

Final summary:

The analysis provides strong statistical evidence ($p = 0.00071$) that males have a higher mean income than females in the population. The Welch's t-test was appropriate here given possible unequal variances, though the F-test suggested no significant difference in variances. Assumptions of normality and independence should be checked for rigor. The effect size is substantial.

Appendix A

Data Analysis:



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