

Fortran routines  
for testing unconstrained optimization software  
with derivatives up to third-order\*

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## Contents

### 1 Introduction

This document gives details about the implementation and usage of a Fortran package that implements the computation of objective function and its first-, second-, and third-order derivatives for the well-known 35 problems proposed by Moré, Garbow and Hillstrom [?, ?].

Originally, Moré, Garbow, and Hillstrom proposed 35 test problems for unconstrained optimization and code for computing the objective function and its first-derivative. The problems were divided into three categories: (a) 14 *systems of nonlinear equations*, cases where  $m = n$  and one searches for  $x^*$  such that  $f_i(x^*) = 0$ ,  $i = 1, 2, \dots, m$ ; (b) 18 *nonlinear least-squares* problems, cases where  $m \geq n$  and one is interested in solving the problem

$$\underset{x \in \mathbb{R}^n}{\text{Minimize}} f(x) = \sum_{i=1}^m f_i^2(x) \quad (1)$$

by exploring its particular structure; and (c) 18 *unconstrained minimization* problems, where one is interested in solving (??) just by applying a general unconstrained optimization solver.

In 1994, Averbukh, Figueira, and Schlick [?] added code to compute the second-order derivative for the 18 unconstrained minimization problems.

We now propose a package that considers (??) for all the 35 test problems and implements its first-, second-, and third-order derivatives.

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## 2 Getting Started

When unzipping the code, the user must get the following directories and files:

```
$(MGH) ..... root directory.  
|_ Makefile  
|_ mgh_doc.pdf ..... documentation PDF file.  
|_ driver1.f08 ..... driver with new routines.  
|_ driver2.f08 ..... driver with alg 566 routines.  
|_ mgh.f08 ..... all the new routines.  
|_ mgh_wrapper.f08 ..... wrapper with alg 566 routines.  
|_ set_precision.f08 ..... precision definitions file.
```

After compiling the code, the user must get the binaries `driver1` and `driver2` and the object files inside `$(MGH)`.

## 3 Compiling the code

To compile the main code,

1. The user must have a Fortran compiler installed and must configure in `$(MGH)/Makefile` the variables `FC` with the Fortran compiler command-line and `FFLAGS` with the desired flags for the chosen Fortran compiler. We tested `gfortran` and `nagfor` compilers. Other Fortran compilers were not tested, but they may work as well.
2. Using the terminal, type `make` in the root directory.

To clean the compiled code, use `make clean`.

## 4 Using the module and compiling code

To use the module, the user must make the following modifications in the code.

1. Choose the precision you want to use in module `$(MGH)/set_precision.f08`. For this, set the parameter `rk` as `kind_s` for single-precision, `kind_d` for double-precision, and `kind_q` for quad-precision.
2. Implement the desired routines (see Section ??).
3. Compile your code

```

$(FC) -I $(MGH) -o your_bin your_code.f08
                                $(MGH)/mgh.o
                                $(MGH)/set_precision.o

```

replacing `$(FC)` by the Fortran compiler you are using. You may need to adjust command-line option `-I`, that stands for the directory where `.mod` files are.

If the user prefer, it is possible to use the classic Algorithm 566 routines, to which we added a new one to compute third-order derivatives. In this case,

1. Choose the precision you want to use in module `$(MGH)/set_precision.f08`. For this, set the parameter `rk` as `kind_s` for single-precision, `kind_d` for double-precision, and `kind_q` for quad-precision.
2. Implement the desired routines in the code (see Section ??).
3. Compile your code

```

$(FC) -I $(MGH) -o your_bin you_code.f08
                                $(MGH)/mgh.o
                                $(MGH)/mgh_wrapper.o
                                $(MGH)/set_precision.o

```

replacing `$(FC)` by the Fortran compiler you are using. You may need to adjust command-line option `-I`, that stands for the directory where `.mod` files are.

## 5 Using the drivers

Two drivers are available to test and validate the code:

1. `$(MGH)/driver1` implements the new routines module. It runs all the 35 problems from the test set. The output is given in `driver1.out` file.
2. `$(MGH)/driver2` implements the algorithm 566 routines. It runs all the 18 unconstrained minimization problems (see [?]). The output is given in `driver2.out` file.

## 6 Routines description

### 6.1 New routines

In order to use the new routines, first of all the user must

1. include the module in the code using `use mgh`.

2. set the number of problem to work with, between 1 and 35, using `mgh_set_problem`,
3. customize `m` and `n` using `mgh_set_dims` or retrieve default values using `mgh_get_dims`,

After that, the user is able to retrieve the initial point using `mgh_get_x0` and compute the objective function and its first-, second-, and third-order derivatives using `mgh_evalf`, `mgh_evalg`, `mgh_evalh`, and `mgh_evalt`, respectively. A detailed description of each routine follows.

#### **subroutine mgh\_set\_problem( user\_problem, flag )**

Sets the problem number. When the user set the problem number, default dimensions (`n` and `m`) for it are automatically set. The subroutines arguments are

`user_problem` is an input integer argument that should contain the problem number between 1 and 35.

`flag` is an output integer argument that contains 0 if the problem number was successfully set or -1 if the `user_problem` is out of the range.

#### **subroutine mgh\_set\_dims( n, m, flag )**

Sets the dimensions for the problem.

`n` is an input optional integer argument, sets the number of variables for the problem set.

`m` is an input optional integer argument, sets the number of equations for the problem set.

`flag` is an output optional integer, in the return contains 0 if the dimensions were set successfully, -1 if `n` is not valid, -2 if `m` is not valid, or -3 if both are not valid.

#### **subroutine mgh\_get\_dims( n, m )**

Gets the dimension for the problem.

`n` is an output optional integer argument with the number of variables for the problem.

`m` is an output optional integer argument with the number of equations for the problem.

#### **subroutine mgh\_get\_x0( x0, factor )**

Gets the initial point for the problem.

`x0` is an output array of length `n` that contains the initial point.

`factor` is an optional real scalar that scales the initial point returned at `x0`.

**subroutine mgh\_evalf( x, f, flag )**

Computes the objective function evaluated at **x**.

- x** is an input real array of length **n**, contains the point in which the objective function must be evaluated.
- f** is an output real that contains the objective function evaluated at **x**.
- flag** is an output integer that contains 0 if the computation was made successfully, -1 if problem is not between 1 and 35, or -3 if a division by zero was made.

**subroutine mgh\_evalg( x, g, flag )**

Computes the gradient of the objective function evaluated at **x**.

- x** is an input real array of length **n**, contains the point in which the gradient must be evaluated.
- g** is an output real array of length **n** that contains the gradient evaluated at **x**.
- flag** is an output integer that contains 0 if the computation was made successfully, -1 if problem is not between 1 and 35, or -3 if a division by zero was made.

**subroutine mgh\_evalh( x, h, flag )**

Computes the Hessian of the objective function evaluated at **x**.

- x** is an input real array of length **n**, contains the point in which the Hessian must be evaluated.
- h** is an output real array of length **n × n** that contains the upper triangle of the Hessian evaluated at **x**.
- flag** is an output integer that contains 0 if the computation was made successfully, -1 if problem is not between 1 and 35, or -3 if a division by zero was made.

**subroutine mgh\_evalt( x, t, flag )**

Computes the third-order derivative tensor of the objective function evaluated at **x**.

- x** is an input real array of length **n**, contains the point in which the third-order derivative must be evaluated.
- t** is an output real array of length **n × n × n** that contains the upper part of the third-derivative evaluated at **x**.
- flag** is an output integer that contains 0 if the computation was made successfully, -1 if problem is not between 1 and 35, or -3 if a division by zero was made.

**subroutine mgh\_get\_name( name )**

Returns the problem name

**name** is a `character(len=60)` output parameter that contains the problem name.

## 6.2 Algorithm 566 Routines + Third derivative computation

**subroutine initpt( n, x, nprob, factor )**

Returns the initial point for a given problem.

**n** is an integer input argument, should contain the dimension of the problem.

**x** is a real output array of length **n**, contains the initial point.

**nprob** is an integer input, contains the number of the problem between 1 and 18.

**factor** is a real input, contains the factor by which the initial point will be scaled.

**subroutine objfcn( n, x, f, nprob )**

Compute the objective function value for a given problem at **x**.

**n** is an integer input argument, should contain the dimension of the problem.

**x** is a real input array of length **n**, contains the point in which the objective function will be evaluated.

**f** is a real output argument that contains the objective function value.

**nprob** is an integer input, contains the number of the problem between 1 and 18.

**subroutine grdfcn( n, x, g, nprob )**

Compute the gradient of the objective function, for a given problem, evaluated at **x**.

**n** is an integer input argument, should contain the dimension of the problem.

**x** is a real input array of length **n**, contains the point in which the objective function will be evaluated.

**g** is a real output array of length **n**, contains the gradient of the objective function value evaluated at **x**.

**nprob** is an integer input, contains the number of the problem between 1 and 18.

**subroutine hesfcn( n, x, hesd, hesu, nprob )**

Compute the Hessian of the objective function, for a given problem, evaluated at **x**.

**n** is an integer input argument, should contain the dimension of the problem.

**x** is a real input array of length **n**, contains the point in which the objective function will be evaluated.

**hesd** is a real output array of length **n**, contains the diagonal of the Hessian.

**hesu** is a real output array of length

$$\frac{n(n - 1)}{2},$$

contains the strict upper triangle of the Hessian stored by columns. The  $i, j$  term of the Hessian,  $i < j$ , is located at the position

$$\frac{(j - 1)(j - 2)}{2} + i$$

at **hesu**.

**nprob** is an integer input, contains the number of the problem between 1 and 18.

**subroutine trdfcn( n, x, td, tu, nprob )**

Compute the third-order derivative tensor of the objective function, for a given problem, evaluated at **x**.

- n** is an integer input argument, should contain the dimension of the problem.
- x** is a real input array of length **n**, contains the point in which the objective function will be evaluated.
- td** is a real output array of length **n**, contains the diagonal of the tensor.
- tu** is a real output array of length

$$\frac{n - 1}{6}((n - 2)(n - 3) + 9(n - 2) + 12),$$

contains the strict upper part of the tensor stored by columns. The  $i, j, k$  term of the tensor,  $i \leq j \leq k$  but not  $i = j = k$ , is located at the position

$$\frac{k - 2}{6}((k - 3)(k - 4) + 9(k - 3) + 12) + \frac{j(j - 1)}{2} + i$$

at **tu**.

**nprob** is an integer input, contains the number of the problem between 1 and 18.

## References

- [1] V. Z. Averbukh, S. Figueroa, and T. Schlick, Remark on Algorithm 566, *ACM Transactions on Mathematical Software*, 20(3):282–285, 1994. DOI 10.1145/192115.192128.
- [2] J. J. Moré, B. S. Garbow, and K. E. Hillstrom, Algorithm 566: FORTRAN Subroutines for Testing Unconstrained Optimization Software, *ACM Transactions on Mathematical Software*, 7(1):136–140, 1981. DOI 10.1145/355934.355943.

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