Models of radiative neutrino mass and lepton-flavour non-universality

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Abstract

This is a summary of what we did. We did x and y.

Publications

Refs. [1–5] below are the journal publications, and preprints authored or co-authored during my PhD candidature. The authors are listed alphabetically in all of the titles.

Journal papers and preprints

[1] R. Foot and J. Gargalionis, *Explaining the 750 GeV diphoton excess with a colored scalar charged under a new confining gauge interaction*, *Phys. Rev. D* **94** (2016), no. 1 011703, [arXiv:1604.06180].

[2] Y. Cai, J. Gargalionis, M. A. Schmidt, and R. R. Volkas, *Reconsidering the One Leptoquark solution: flavor anomalies and neutrino mass*, *JHEP* **10** (2017) 047, [arXiv:1704.05849].

[3] I. Bigaran, J. Gargalionis, and R. R. Volkas, *A near-minimal leptoquark model for reconciling flavour anomalies and generating radiative neutrino masses*, *JHEP* **10** (2019) 106, [arXiv:1906.01870].

[4] J. Gargalionis, I. Popa-Mateiu, and R. R. Volkas, *Radiative neutrino mass model from a mass dimension-11 $\Delta L = 2$ effective operator*, *JHEP* **03** (2020) 150, [arXiv:1912.12386].

[5] J. Gargalionis and R. R. Volkas, *Exploding operators for Majorana neutrino masses and beyond*, [arXiv:2009.13537].

Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD except where indicated clearly,
2. due acknowledgement has been made in the text to all other material used,
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

John Gargalionis, September 2020

Statement of contribution

I did x and someone else did y.

Preface

Particle physics currently finds itself in a strange or exciting place, depending on who you ask. The discovery of a Higgs-like boson at close to 125 GeV has meant both the completion of the Standard Model (SM), and the end of clear signs of new particles at the electroweak scale. Although the Large Hadron Collider (LHC) will continue to collect data well into the next few decades, the mass reach will not increase significantly. The community waits for a new machine, for which there are many candidates and promises, that will continue to push the energy frontier and test theories addressing the many shortcomings of the SM. Time frames for many of these see data taking beginning at the end of my career. If progress is driven by experiment, where do we go from here?

Thankfully, there are already clear signs of new physics in the neutrino sector. The observation of neutrino oscillations, and therefore neutrino masses, is by far the strongest terrestrial evidence demanding an extension of the SM. It is no surprise that a full understanding of the neutrinos has alluded us so far; they are, with the possible exception of the Higgs boson, the most elusive particles currently under laboratory scrutiny. As we move into an era of precision neutrino measurements, now is the right time to take stock of the phenomenologically viable and economic models that explain the pattern of neutrino masses and mixings observed. Armed with the list of possible mechanisms, we can make progress in probing those that are testable and, given that these models are falsified, build circumstantial evidence in favour of those that are not.

Even on the collider front, it is unclear yet that the LHC has left us with the so-called 'nightmare scenario' of a lonely Higgs. Perhaps unexpectedly, the most interesting signs of new physics from CERN have come from the LHC*b* experiment. The now famous 'flavour anomalies' are a collection of theoretically consistent anomalous measurements indicating a departure from the lepton-flavour universality present in the SM. Are these related to the growing evidence for deviations in leptonic anomalous magnetic moments? Might they be clues to a deeper theory of flavour and mass? The Belle II experiment has only just begun taking data, and we wait eagerly for what it has to say on these matters. LHC*b* too will continue to improve its measurements with more collisions; if the anomalies persist, these will be undeniable evidence of physics beyond the SM accessible to the next generation of hadron colliders.

These measurements are tantalising because of their consistency and breadth, but it would not be the first time that physicists have been lead astray, should they disappear with more statistics. Even so, what is perhaps the central result of my doctoral work will remain unchanged: that deviations from lepton-flavour universality in four-fermion operators may be intimately connected to mass generation in the neutrino sector.

Acknowledgements

I would like to thank x and y.

For x and y.

Contents

List of Figures

List of Tables

1 Introduction

The following is a general introduction to the background topics referred to and assumed in subsequent chapters. This includes a review of popular theories of neutrino mass, the current status of neutrino-oscillation parameters, a general introduction to Effective Field Theory (EFT), the experimental situation relevant to the flavour anomalies, and topics peripheral to all of these.

1.1 The Standard Model and neutrinos

Laboratory experiments to date have firmly established the predictive power of the Standard Model (SM) of particle physics, a combined theory of the electroweak and strong interactions described by the gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. It is a model whose probes and predictions span at least 33 orders of magnitude[1] with varying degrees of precision, and these are consistent with almost all known experiments. Although it displays a number of arbitrary features, the dynamics of the theory are mostly fixed by the fundamental principles of gauge theory and Lorentz invariance. Most of this arbitrariness resides in the matter sector of the theory, whose properties

[1] The interval given is from the distance scales probed at the LHC (roughly 10^{-17} cm) to the size of the solar system (roughly 10^{16} cm).

Field	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$Q^{\alpha a i}$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$(\mathbf{2}, \mathbf{1})$
$L^{\alpha i}$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{2}, \mathbf{1})$
$\bar{u}^{\alpha}_{a}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\mathbf{2}, \mathbf{1})$
$\bar{d}^{\alpha}_{a}$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\mathbf{2}, \mathbf{1})$
$\bar{e}^{\alpha}$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{2}, \mathbf{1})$
$(G_{\alpha\beta})^{a}{}_{b}$	$(\mathbf{8}, \mathbf{1}, 0)$	$(\mathbf{3}, \mathbf{1})$
$(W_{\alpha\beta})^{i}{}_{j}$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{3}, \mathbf{1})$
$B_{\alpha\beta}$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{3}, \mathbf{1})$
H^{i}	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1})$

Table 1.1: The SM fields and their transformation properties under the SM gauge group G_{SM} and the Lorentz group written as $SU(2)_+ \otimes SU(2)_-$. The final unbolded number in the 3-tuples of the G_{SM} column represents the $U(1)_Y$ charge of the field, normalised such that $Q = I_3 + Y$. For the fermions a generational index has been suppressed. See Appendix A for details about the mathematical notation used here and throughout this work.

(masses, coupling constants, quantum numbers, *etc.*) are not predicted, but are instead motivated on phenomenological grounds. We show the fields of the SM and their defining properties in Table 1.1, according to the mathematical conventions of Appendix A.

The SM inherits the experimental success of the $SU(2) \otimes U(1)$ theory of the weak interactions, first proposed by Glashow [10] in 1961 as a possible underlying structure for Fermi's theory of beta decay. Before the end of the same decade, Weinberg [11] and Salam [12] had constructed the modern theory of leptons based on the spontaneous breaking of $SU(2)_L \otimes U(1)_Y$ to the electromagnetic symmetry. Interestingly, it seems that these seminal papers went mostly unnoticed (see Fig. 1.1) until the early 1970s, when 't Hooft proved the renormalisability of spontaneously broken gauge theories [13] as a graduate student working under the supervision of Veltman. By the mid 1970s the framework had been extended to include the quarks [14] and the unbroken chromodynamic group, which was successfully shown to reproduce the Bjorken scaling seen in deep-inelastic-scattering experiments through asymptotic freedom [15].

Despite its successes, the SM cannot be the complete theory of fundamental particles and their interactions. It does not explain phenomena such as the baryon asymmetry

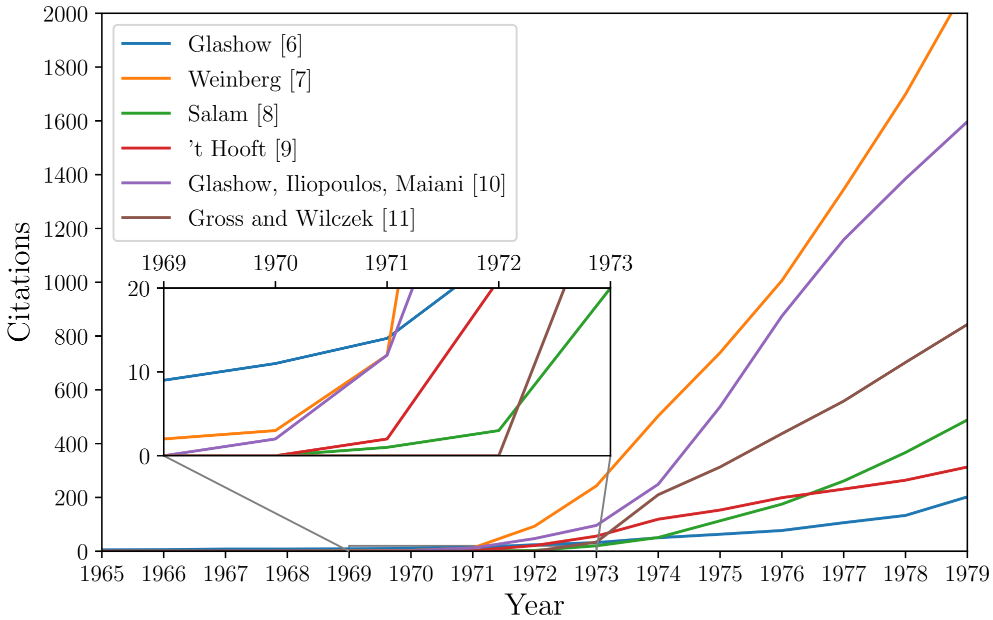

Figure 1.1: The cumulative citation graph for a selection of papers presenting foundational results relevant to the SM. Weinberg's seminal paper 'A model of leptons' (1967) saw an explosion of citations following 't Hooft's work on the renormalisability of gauge theories (1971).

present, and the particle spectrum contains no viable candidate for dark matter. The SM cannot explain why the electric dipole moment of the neutron is so small, why there are three generations of matter or, notably in our case, the origin of neutrino oscillations and the implied small but non-zero neutrino masses.

1.2 Massive neutrinos in experiment and theory

The minimal SM predicts massless neutrinos, a prediction that today sits in contradiction to a wealth of empirical evidence. This evidence could in principle have come from many kinds of experiments, but currently only neutrino oscillations provide strong signs that the masses are non-zero. Below we discuss the phenomenon of neutrino oscillations in the context of the outstandingly successful three-flavour mixing paradigm. We then move on to other probes of neutrino masses, which currently only provide limits on the mass scale. On the theory side, we summarise some popular and motivated extensions of the SM that accommodate massive neutrinos, placing particular emphasis on the direction we have followed in the novel work presented in this thesis. This includes an overview of tree- and loop-level models of Majorana neutrino mass.

1.2.1 Neutrino oscillations

The neutrino flavour eigenstates $\check{\nu}_i = (\nu_e, \nu_\mu, \nu_\tau)$ are defined as the states that couple at charged-current interaction vertices with the corresponding charged lepton. These are the states in which the neutrinos are almost always produced in experiments, and certainly always measured. If neutrinos are massive there is no reason to expect these to coincide with the mass eigenstates $\nu_i = (\nu_1, \nu_2, \nu_3)$. In general, the flavour eigenstates will be an admixture of the propagating fields

$$\check{\nu}_i = U_i{}^j \nu_j \ , \tag{1.1}$$

where the $U_i{}^j$ are elements of the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix [16, 17]. The PMNS matrix is defined such that it diagonalises the neutrino mass matrix:

$$\mathbf{U}^\dagger \mathbf{m}_\nu \mathbf{U}^* = \mathrm{diag}(m_1, m_2, m_3), \tag{1.2}$$

where the m_i are the neutrino masses. Being a 3×3 unitary matrix, $\mathbf{U}$ is in general parametrised by three mixing angles and six phases. Not all of the phases are physical, since the neutrino and charged-lepton fields can be redefined in such a way that five of the phases are removed in the case of Dirac neutrinos. In the presence of a Majorana mass term, only the charged leptons can be rephased. This leaves three physical phases with the two additional ones termed *Majorana phases*. In general

$$\mathbf{U} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{bmatrix} \mathbf{P} \ , \tag{1.3}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and

$$\mathbf{P} = \begin{cases} \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1) & \text{for Majorana neutrinos} \\ \mathbf{1}_{3\times 3} & \text{for Dirac neutrinos} \end{cases} . \tag{1.4}$$

The phase δ_{CP} is often called the *Dirac phase*, while $\alpha_{1,2}$ are the Majorana phases discussed above.

Neutrino oscillation experiments typically involve the production of neutrino flavour states from charged-current processes, *e.g.* leptonic pion decays. Each mass eigenstate evolves in time independently according to the Schrödinger equation: *i.e.* $|\nu_i(t)\rangle = \exp(-iE_i t)|\nu_i(0)\rangle$, for evolution *in vacuo*. This alters the initial superposition away from being a pure flavour eigenstate:

$$|\check{\nu}_i(t)\rangle = \sum_j U_i^{*j} e^{-iE_j t}|\nu_j\rangle \tag{1.5}$$

$$= \sum_{j,k} U_i^{*j} e^{-iE_j t} U_j{}^k |\check{\nu}_k\rangle \, . \tag{1.6}$$

The probability of measuring a specific flavour through the charged-current interaction then oscillates with time:

$$\mathrm{P}(\check{\nu}_m \to \check{\nu}_n) = |\langle\check{\nu}_n|\check{\nu}_m(t)\rangle|^2 = \left|\sum_i U_m^{*i} U_i{}^n e^{-iE_i t}\right|^2 \, . \tag{1.7}$$

The expression can be expanded and the kinematic factors simplified from the fact that the neutrinos are ultra-relativistic. We follow the usual convention and take $E_i = \sqrt{\mathbf{p}^2 + m_i^2} \approx |\mathbf{p}| + m_i^2/(2E)$ with $E = |\mathbf{p}|$. This gives

$$\begin{aligned}\mathrm{P}(\check{\nu}_m \to \check{\nu}_n) = \delta_{mn} &- 4 \sum_{i<j} \mathrm{Re}\left(U_m{}^i U_m^{*j} U_n^{*i} U_n{}^j\right) \sin^2 \frac{\Delta m_{ij}^2 L}{4E} \\ &+ 2 \sum_{i<j} \mathrm{Im}\left(U_m{}^i U_m^{*j} U_n^{*i} U_n{}^j\right) \sin \frac{\Delta m_{ji}^2 L}{2E} \, ,\end{aligned} \tag{1.8}$$

where $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$ are the squared neutrino mass differences and $L = ct$, sometimes called the baseline, is the approximate distance travelled by the particles. To interpret the results of many experiments, it is often sufficient to consider an effective two-flavour oscillation paradigm. In this case, the neutrino-oscillation probabilities are governed by by a single squared mass difference Δm^2 and a single angle θ. Interestingly, the CP-violating phase is completely absent from the two flavour formula:

$$\mathrm{P}(\check{\nu}_m \to \check{\nu}_n)_{n_f=2} = \sin^2(2\theta) \sin^2 \frac{\Delta m^2 L}{4E} \, . \tag{1.9}$$

From the expressions in Eqs. (1.8) and (1.9) a number of properties of the vacuum neutrino oscillations become clear.

1. The neutrino oscillation probabilities depend on the neutrino mass differences, and not on the absolute mass scale. For three flavours, there are only two independent squared mass differences. Typically chosen to be Δm^2_{21} and Δm^2_{31}, although often they are refereed to with the historical names Δm^2_{sol} and Δm^2_{atm}, discussed in detail below.

2. From Eq. (1.8) it is clear that the oscillations only occur if the neutrinos are non-degenerate and the neutrino mixing is non-trivial, *i.e.* if $\Delta m_{ij} \neq 0$ and $\mathbf{U} \neq \mathbf{1}$.

3. The PMNS matrix elements only appear in the combination $U^i_m U^{*j}_m$, to which the Majorana phases contained within the matrix $\mathbf{P}$ do not contribute. This implies that oscillation experiments cannot comment on the Dirac or Majorana nature[2] of the neutrinos. Oscillations can however probe δ_{CP}.

4. In the effective two-flavour mixing paradigm, both θ and Δm^2 appear in such a way that neither the sign of Δm^2 nor the octant of θ can be uniquely determined.

Thus, neutrino oscillations imply that the neutrino masses of at least two of the mass eigenstates are non-degenerate, and therefore only one neutrino could potentially be massless. The largest squared mass difference can therefore be translated into an lower bound on the mass of the heaviest neutrino, which we present later with modern data.

Historically, the effective two-flavour mixing paradigm has provided a good framework for interpreting early indications of neutrino oscillations. Specifically, the solar and atmospheric neutrino puzzles have approximate descriptions in terms of a two-flavour picture. Oscillations $\nu_e \to \nu_{\text{active}}$, where ν_{active} is a coherent superposition of ν_μ and ν_τ, in both matter and vacuum account for the deficit of electron neutrinos measured from the sun, and $\nu_\mu \to \nu_\tau$ oscillations *in vacuo* explain the shortage of muon neutrinos from cosmic-ray-induced production in the upper atmosphere.

The measurement and resolution of these puzzles is an interesting and exciting chapter in the recent history of physics. Experiments as early as the 1960s had noticed a shortage of electron neutrinos coming from the sun relative to the predictions of solar models [18–21], which themselves were subject to much uncertainty [22]. For detection there were three main approaches: Raymond Davis and collaborators [23] pioneered experiments that measured the solar electron-neutrino flux using Chlorine, the Kamiokande and later Super-Kamiokande collaborations [24, 25] used water Cherenkov detectors, and the experiments GALLEX [26] and SAGE [27] had Gallium as the detecting material. All of these experiments showed a deficit of solar electron neutrinos,

[2] Of course, this is already clear from the fact that neutrino oscillations conserve total lepton number, despite breaking the individual familial lepton-number symmetries $L_{e,\mu,\tau}$.

although they were sensitive to neutrinos of different energies. The Sudbury Neutrino Observatory gave the final word on the oscillation solution to the solar neutrino puzzle with accurate confirmation of the electron-neutrino deficit, along with a measurement of the *total* neutrino flux which was found to be in agreement with the solar models [28, 29].

Atmospheric neutrinos were known to come about from helicity-suppressed kaon and pion decays to muons and muon neutrinos. A zenith-angle and energy-dependent suppression in the flux of atmospheric muon neutrinos was measured by the Kamiokande and Irvine–Michigan–Brookhaven experiments [30, 31] in the early 1990s, and after the upgrade to Super-Kamiokande the deficit was confirmed to high precision with results presented at the 'Neutrino 1998' conference [32–35].

The pairs of mixing parameters associated with these two classes of measurements are usually dubbed $\theta_{\text{sol}}, \Delta m^2_{\text{sol}}$ and $\theta_{\text{atm}}, \Delta m^2_{\text{atm}}$. Experimental results find $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}$ and that both θ_{sol} and θ_{atm} are large compared to any angles found in the CKM matrix, the quark mixing matrix. Interpreted in terms of three-flavour mixing, common convention identifies Δm^2_{sol} with the squared mass difference between ν_2 and ν_1, which is known to be positive[3] (*i.e.* $\Delta m^2_{21} > 0$). The solar mixing angle θ_{sol} is then associated with θ_{12}. The atmospheric mixing parameters are identified with $|\Delta m^2_{31}|$ or $|\Delta m^2_{32}|$ and θ_{23}. Of course, three-flavour effects alter the simplistic picture presented here and must be included to interpret measurements of θ_{13} and δ_{CP}, see *e.g.* Ref. [36] and references therein for a more detailed discussion. The picture that emerges from these experiments is then

$$\Delta m^2_{\text{sol}} \approx \Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx |\Delta m^2_{\text{atm}}| \,, \tag{1.10}$$

with which both the *normal mass ordering* $m_1 < m_2 < m_3$ and the *inverted mass ordering* $m_3 < m_1 < m_2$ are consistent.

Neutrino oscillation experiments have continued to probe the squared mass differences and mixing parameters with impressively high precision, see *e.g.* Refs. [6, 37]. Reactor neutrino experiments like KamLAND [38] and long-baseline accelerators, *e.g.* T2K [39] and NOνA [40], are sensitive to all of these parameters, although Δm^2_{atm} and θ_{13} are best measured at short-baseline reactors like Double Chooz [41], RENO [42] and Daya Bay [43]. Today the octant of θ_{12} is certainly known, while θ_{13} is constrained to be close to 0.15. The sign of the atmospheric squared mass difference is still unknown, and therefore so is the mass ordering for the neutrinos. The value of the CP-violating Dirac

[3] We note that the sign of Δm^2_{21} can be known since oscillations in matter are also relevant for the solar squared mass difference, which depart from the simple formula of Eq. (1.9).

phase δ_{CP} is less clear, although there is a preference for a value somewhere between π and $3\pi/2$. The extent of CP-violation in the neutrino sector can be represented in a rephasing-invariant way with the leptonic Jarlskog invariant

$$J_{CP} = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta_{\mathrm{CP}} \;, \tag{1.11}$$

so a value of π would imply no CP-violating effects, while $\delta_{\mathrm{CP}} = 3\pi/2$ would make these maximal. For this work we take the results of the most recent fit to neutrino oscillation data by the NuFit collaboration [6, 7] in the context of the three-flavour paradigm. These results are summarised in Fig. 1.2 separately for the cases of normal and inverted mass ordering. Results including atmospheric neutrino oscillation data from Super-Kamiokande and those not are also distinguished. Two-dimensional projections of the χ^2 fit for the same parameters are shown in Fig. 1.3. These data suggest a leptonic mixing matrix that has a very different form to the CKM matrix, which we call $\mathbf{V}$. We represent this qualitatively by using boxes whose side lengths are scaled to the magnitude of the best-fit values of the parameters in the matrices, the textures are

$$\mathbf{U} \sim \begin{pmatrix} \blacksquare & \scriptstyle\blacksquare & \cdot \\ \cdot & \scriptstyle\blacksquare & \blacksquare \\ \scriptstyle\blacksquare & \scriptstyle\blacksquare & \scriptstyle\blacksquare \end{pmatrix}, \qquad \mathbf{V} \sim \begin{pmatrix} \blacksquare & \cdot & \\ \cdot & \blacksquare & . \\ & . & \blacksquare \end{pmatrix} . \tag{1.12}$$

For the PMNS matrix we take the best fit values for the normal mass ordering including the Super-Kamiokande results, *i.e.* the numbers in the top-left quadrant of Fig. 1.2. The same numbers also imply an upper bound on the mass of the heaviest neutrino at

$$m_{\text{heaviest}} \leq \sqrt{|\Delta m^2_{\text{atm}}|} \approx 0.05\,\text{eV} \;. \tag{1.13}$$

1.2.2 Other experimental probes

Although neutrino oscillations provide a wealth of evidence for non-zero masses for at least two of the neutrino fields, they do not probe the absolute mass scale. There are however kinematic and cosmological probes which bound the neutrino masses, and some of these are mentioned here.

NuFIT 5.0 (2020)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
	$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
	$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{\mathrm{CP}}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
	$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
	$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
	$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
	$\delta_{\mathrm{CP}}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

Figure 1.2: The figure shows a table taken from the latest global fit to neutrino mass and mixing parameters by the NuFit collaboration [6, 7] in the three-flavour picture. The results presented in the upper (lower) panel are obtained by excluding (including) the χ^2 data on atmospheric neutrinos provided by the Super-Kamiokande collaboration (SK). The numbers in the 1st (2nd) column are obtained assuming normal (inverted) neutrino mass ordering. See Ref. [7] for more information.

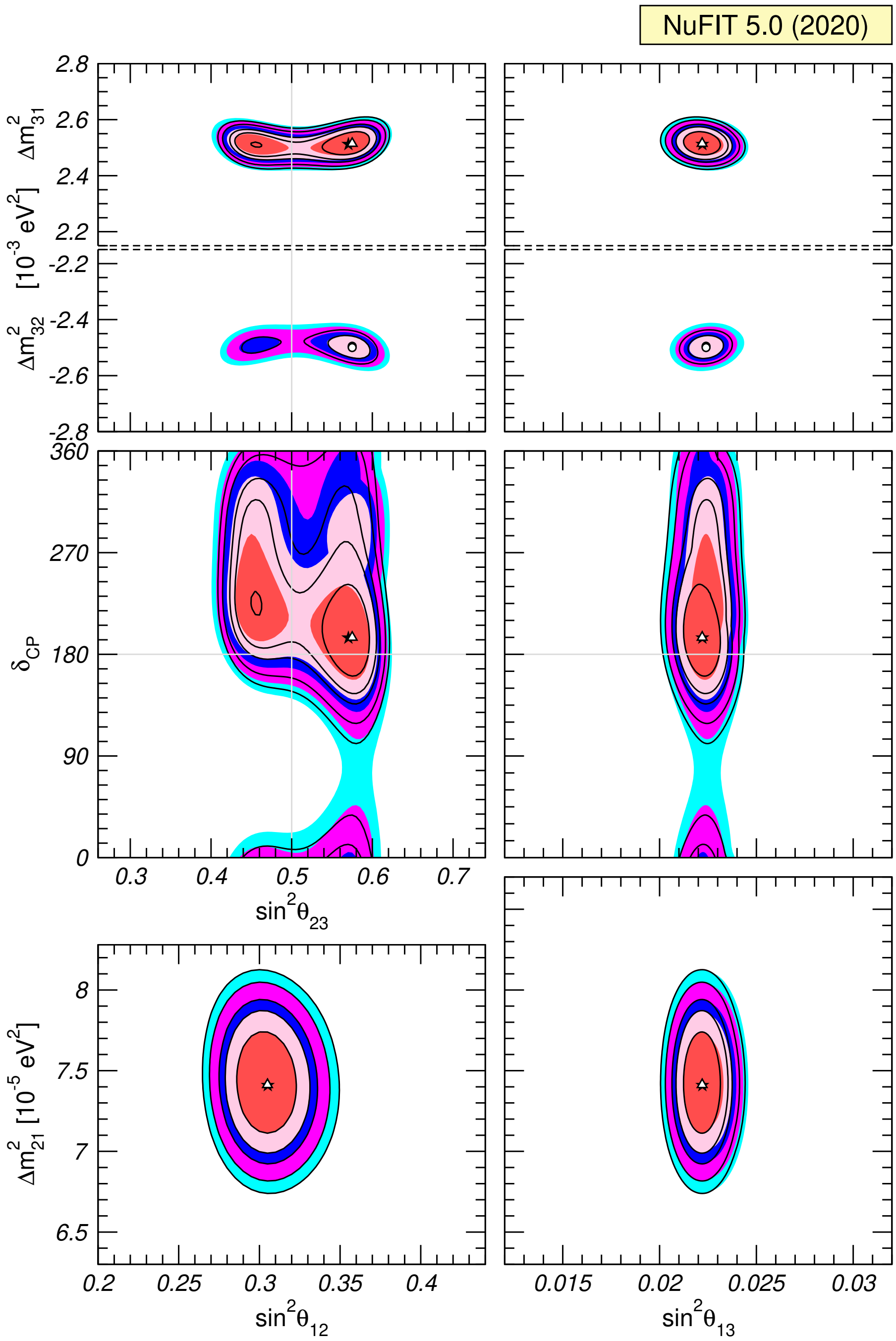

Figure 1.3: The figure shows the two-dimensional allowed regions obtained by the latest fit to the neutrino mass and mixing parameters by the NuFit collaboration [6, 7]. Each plot shows the two-dimensional projection of the allowed region after marginalising with respect to the other parameters. The coloured regions (black contour curves) are obtained by excluding (including) the Super-Kamiomande χ^2 data. The different contours correspond to the two-dimensional allowed regions at 1σ, 90%, 2σ, 99%, 3σ confidence. See Ref. [7] for more information.

Beta decay

A study of the kinematics of beta-decay experiments shows that differences in the energy distribution of the emitted electron are expected for different values of the neutrino mass. Currently these experiments only provide upper bounds on the effective neutrino mass [44]

$$m_\beta \equiv \sqrt{\sum_i |U_1{}^i|^2 m_i^2} \,. \tag{1.14}$$

The best results come from tritium experiments which probe ${}^3\mathrm{H} \to {}^3\mathrm{He} + e + \bar{\nu}_e$. The KATRIN experiment recently presented the most stringent upper bound [45, 46]

$$m_\beta < 1.1\,\mathrm{eV} \,, \tag{1.15}$$

at 90% confidence, with a projected limit of $m_\beta < 0.2\,\mathrm{eV}$ with the full dataset.

Another process with the potential to probe the absolute scale of the neutrino masses, as well as the Majorana phases, is neutrinoless double beta decay ($0\nu\beta\beta$). The process requires the violation of lepton-number by two units, and is therefore intimately tied to the neutrinos' possible Majorana nature. If double-beta decay is seen, the *black box theorem* [47–49] guarantees that the neutrinos pick up a radiative Majorana mass, even if the neutrino masses vanish at tree level. Graphically this is easy to understand: a $0\nu\beta\beta$ operator can always be turned into a neutrino self-energy graph with four loops. The amplitude for the process is proportional to

$$\langle m_\nu \rangle_{\beta\beta} = \sum_i (U_1{}^i)^2 m_i \,, \tag{1.16}$$

which features both the neutrino masses and the Majorana phases. (Of course it may be that the four-loop contribution to the neutrino mass implied by the double-beta-decay operator represents only a small correction to the neutrino masses, which could arise at lower order.) Current limits on $\langle m_\nu \rangle_{\beta\beta}$ are around 0.2 eV, see *e.g.* Ref. [50] for a recent review of the experimental status and prospects. Constraints on $\langle m_\nu \rangle_{\beta\beta}$ are usually presented against the mass of the lightest neutrino mass eigenstate. The behaviour of $\langle m_\nu \rangle_{\beta\beta}$ is very sensitive to the neutrino mass ordering, in such a way that the inverted scenario implies a minimum allowed value of $\langle m_\nu \rangle_{\beta\beta}$, which will begin to be probed by the next generation of experiments. A combined global limit and the projected sensitivity of the DARWIN experiment [9] are shown in Fig. 1.4, which also illustrates the different behaviour of the inverted and normal neutrino mass orderings

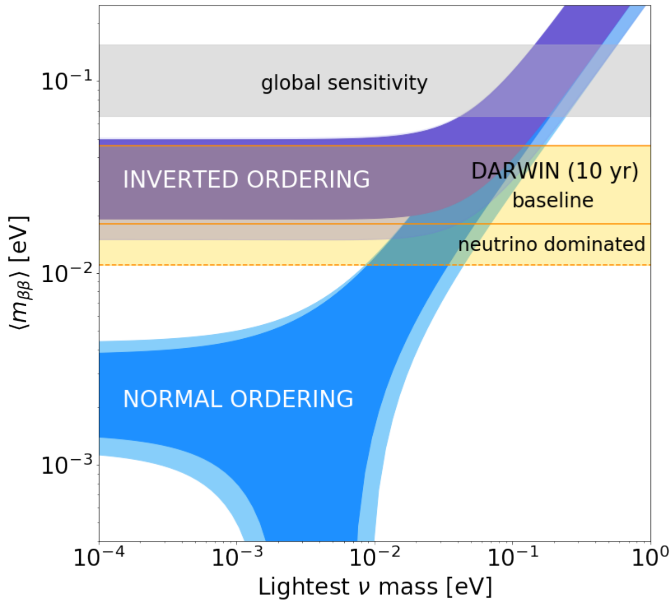

Figure 1.4: The figure shows limits on the effective neutrino mass for different values of m_{lightest} for both normal and inverted mass ordering. The grey region represents the combined sensitivity from a number of leading experiments [8]. The yellow regions are projections for the DARWIN experiment under different background hypotheses. The figure is taken from Ref. [9], and we point the reader there for more information.

in the $\langle m_\nu \rangle_{\beta\beta}$ vs. m_{lightest} plane.

Cosmological limits

The most stringent limits on the sum of the neutrino masses come from cosmology, although they are model-dependent. In the minimal ΛCDM model adjusted for massive neutrinos, the limit implied by the most recent Planck data release [51] is

$$\sum_i m_i < 0.12\,\text{eV}\,, \tag{1.17}$$

at 95% confidence. This is impressively small, and puts pressure on the inverted-ordering scenario, for which $\sum_i m_i \gtrsim 0.1\,\text{eV}$. Excitingly, future cosmological probes will likely make a measurement of the sum of the neutrino masses.

1.2.3 Models of neutrino masses

In the SM the neutrino fields appear only within the lepton doublet L, and one cannot write down—in analogy to the up-type Yukawa—a renormalisable operator that leads to neutrino masses because of the absence of the right-handed fields. A simple model of neutrino mass then involves introducing right-handed neutrino fields $\bar{\nu} \sim (\mathbf{1}, \mathbf{1}, 0)_{(1,2)}$, extending the Yukawa sector of the SM accordingly to

$$-\mathscr{L}_Y \supset y_e \bar{e} L H^\dagger + y_d \bar{d} Q H^\dagger + y_u \bar{u} Q H + y_\nu \bar{\nu} L H \,, \tag{1.18}$$

in a simplified one-flavour picture. This implies Dirac neutrinos with a mass $m_\nu \approx y_\nu v$, and makes mass generation symmetric between the quarks and leptons.

There are a number of problems with this simplistic scenario. First, it is perhaps uncomfortable to suppose that the neutrinos are many orders of magnitude lighter than the charged fermions only because y_ν is a very small number. Indeed, the posited large hierarchy between y_ν and y_u seems to spoil any aesthetic arguments for quark–lepton symmetry in favour of this hypothesis. Although the Yukawa couplings for the other SM fermions span six orders of magnitude, couplings within any one generation are all of similar order. We illustrate this in Fig. 1.5, where the fermion masses are plotted by generation. Whereas one could consider that some underlying theory of flavour may explain, for example, the disparity in scale between the masses of the electron and the top quark, the large mass difference between the electron and the lightest neutrino, although technically natural, seems to cry out for its own explanation.

A second point of criticism with the simple scenario presented above is that it ignores the Majorana mass term for $\bar{\nu}$

$$\mathscr{L} \supset -\tfrac{1}{2}\mu \bar{\nu}\bar{\nu} + \text{h.c.} \tag{1.19}$$

In order to maintain the Dirac neutrino mass with the relation $m_\nu \approx y_\nu v$, the Majorana mass must be forbidden or else chosen to be negligibly small, assumptions adding additional layers of contrivance to the theory. A sensible choice for the scale μ is some high scale Λ associated with the breaking of $\mathrm{U}(1)_{B-L}$. Taking $\mu \gg y_\nu v$, the neutrino-mass matrix takes on the form

$$\mathbf{m}_\nu = \begin{pmatrix} 0 & y_\nu v \\ y_\nu v & \mu \end{pmatrix} \,, \tag{1.20}$$

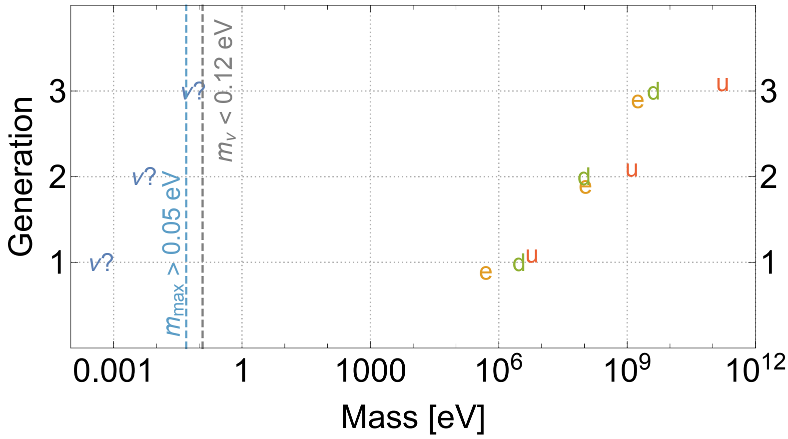

Figure 1.5: The masses of the SM fermions grouped by generation. While the SM Yukawa couplings span a wide range of values, within any specific generation they are all of similar order. The tiny masses of the neutrinos seem to suggest an alternate mass-generation mechanism.

with eigenvalues

$$m_l \approx \frac{y_\nu^2 v^2}{\mu}, \quad m_h \approx \mu \, . \tag{1.21}$$

The assumption $\mu \gg y_\nu v$ implies that $m_l \ll y_\nu v$, and the neutrino is successfully arranged to be much lighter than $y_\nu v$, which can now be taken to be on the order of the charged fermion masses. The theory also leaves us with a neutrino whose mass must be significantly larger than the electroweak scale: $m_h \gtrsim 10^{14}$ GeV, assuming $m_l = 0.05$ eV and $y_\nu = 1$. After transforming into the mass-diagonal basis, the physical fields ν_l and ν_h correspond to Majorana particles. Thus, in the most motivated region of parameter space, the phenomenology of the light neutrinos in even the SM + $\bar{\nu}$ scenario is Majorana. This toy scenario illustrates the mechanism commonly called the *seesaw mechanism*: making the neutrinos very light at the expense of making other fields very heavy. This is discussed more broadly below.

Tree-level models of neutrino mass

The toy seesaw scenario discussed above can be understood more generally by studying the effective theory valid below the scale μ, which does not contain the field $\bar{\nu}$. The leading-order lepton-number-violating (LNV) effects appear at dimension five in the operator

$$\mathscr{L} \supset \frac{\kappa}{\Lambda}(L^i L^j) H^k H^l \cdot \epsilon_{ik}\epsilon_{jl} , \tag{1.22}$$

with κ a dimensionless coefficient. This operator is commonly called the *Weinberg operator*. In the broken phase it gives rise to a Majorana mass for the neutrinos consistent with the seesaw formula:

$$\mathscr{L} \supset \frac{v^2\kappa}{\Lambda}\nu\nu . \tag{1.23}$$

The SM + $\bar{\nu}$ scenario is not the only simplified model that generates the Weinberg operator at tree-level. A simple diagram-topology analysis suggests there are another two seesaw mechanisms in the UV: a model with a scalar isotriplet field $\Xi_1 \sim (\mathbf{1}, \mathbf{3}, 1)_S$, called the type-II mechanism, and another with an isotriplet Majorana fermion $\Sigma \sim (\mathbf{1}, \mathbf{3}, 0)_F$, called type-III. Along with the type-I heavy $\bar{\nu}$ model, these are collectively referred to as the canonical seesaw mechanisms [52–62] and have been studied at length in the literature. They are simple models in that they introduce only very few exotic degrees of freedom and free parameters. However, the high seesaw scale makes these models practically untestable at current and future collider experiments.

Models of Majorana neutrino mass can be made more testable if, instead of the suppression of the neutrino masses coming from a large Λ in Eq. (1.23), the coefficient κ were somehow arranged to be small. Although there are a number of mechanisms to achieve this, we concentrate below on radiative models of Majorana neutrino mass, in which κ is made small through loop and coupling suppression.

Radiative models and their classification

It may be the case that the field content of whatever high-energy theory describes the neutrino masses is such that no neutrino self-energy diagram can be drawn at the tree level. Indeed, this will be the case if there is lepton-number violation by two units ($\Delta L = 2$) from interactions other than those present in the canonical seesaw models. Such models are called radiative, since the neutrino masses arise through loop graphs. The historically important Zee [63] and Zee–Babu [64, 65] models have come to be archetypal radiative scenarios in which the neutrinos gain masses through $\Delta L = 2$ the interactions of exotic scalars at one and two loops respectively. In the Zee model, an additional Higgs doublet $\varphi \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2})$ and a charged scalar $\mathcal{S}_1 \sim (\mathbf{1}, \mathbf{1}, 1)$ are introduced, while the Zee–Babu model contains $\mathcal{S}_1$ and the doubly-charged scalar $\mathcal{S}_2 \sim (\mathbf{1}, \mathbf{1}, 2)$.

The neutrino self-energy diagrams for these models are shown in Fig. 1.6 as examples. The corresponding neutrino-mass matrices are

$$[\mathbf{m}_\nu^{\text{Zee}}]_{rs} = \frac{\mu v^2}{16\pi^2} \sum_t x_{[rt]} I_t [\mathbf{m}_e]_t y_{ts} + (r \leftrightarrow s) \,, \tag{1.24}$$

$$[\mathbf{m}_\nu^{\text{Zee–Babu}}]_{rs} = \frac{\mu' v^2}{(16\pi^2)^2} \sum_{t,u} x_{[rt]} [\mathbf{m}_e]_t z_{\{tu\}} [\mathbf{m}_e]_u x_{[us]} I'_{tu} + (r \leftrightarrow s) \,, \tag{1.25}$$

written in terms of the couplings $x, y, z \in \mathbb{C}$ as shown in Fig. 1.6, and the associated loop functions I and I'. One can see that in these models, the coefficient of the Weinberg operator κ is naturally suppressed with respect to the seesaw formula by SM-fermion masses (charged-lepton masses in this case), exotic couplings, and loop factors.

Such models are economic, since they do not require the imposition of *ad hoc* symmetries, and in many cases make a connection to other unsolved problems of the SM such as the nature of dark matter or the matter–antimatter asymmetry of the Universe. They are also elegant, since the smallness of the neutrino masses emerges as a natural consequence, rather than through the imposed requirement of exceedingly small coupling constants. Radiative models are easier to probe experimentally since the additional loop suppression and products of couplings bring down the allowed scale of the new physics, in some cases to LHC-accessible energy ranges [66]. The Zee–Babu model, for example, is non-trivially constrained by same-sign dilepton searches performed by ATLAS [67–69] and CMS [70–72]. Additionally, the predicted dependence of the neutrino-mass matrix on SM parameters, as is clear from Eqs. (1.24) and (1.25), can make these models very predictive as far as neutrino phenomenology is concerned. For example, the minimal version of the Zee model has the charged-lepton masses arising from couplings to φ, allowing for a simplification of the expression Eq. (1.24) such that the leptonic flavour index t is identified with s. The antisymmetry in the leptonic flavour indices implied by $x_{[rs]}$ must be accounted for by an antisymmetry in the loop integral such that $I \sim I_r - I_s$. The flavour structure then becomes [73]

$$[\mathbf{m}_\nu^{\text{Min. Zee}}]_{rs} \propto x_{[rs]}([\mathbf{m}_e]_r^2 - [\mathbf{m}_e]_s^2) \,, \tag{1.26}$$

which makes clear predictions relevant to neutrino oscillations that in this case rule out the model. For recent reviews of radiative models see Refs. [74, 75].

The Zee and Zee–Babu scenarios are only two of a very large number of radiative models, none of which are *a priori* more likely to be true than any other. In the context of such a large theory-space, it is useful to have an organising principle to aid in the

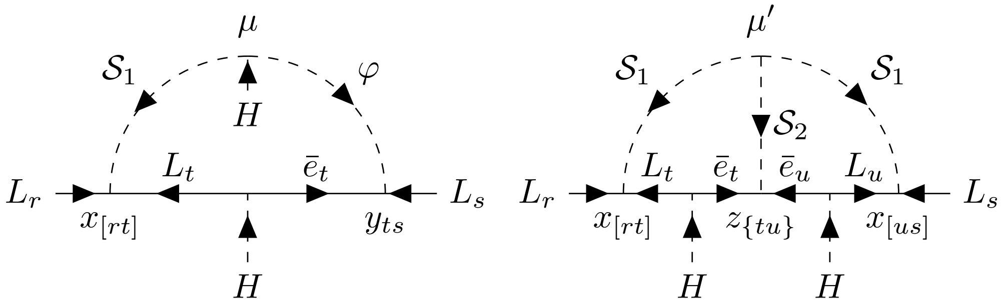

Figure 1.6: The neutrino self-energy diagrams relevant to the Zee (left) and Zee–Babu (right) models.

study and classification of these models, and beginning with $\Delta L = 2$ effective operators has been shown to be an effective strategy.

One approach to this model taxonomy involves studying loop-level completions of the Weinberg operator, and its dimension-$(5 + 2n)$ generalisations

$$\mathcal{O}_1^{\prime\cdots\prime} = (LL)HH(H^\dagger H)^n \, .$$

Here, models can be systematically written down by studying the various topologies able to be accommodated by the operator with increasing number of loops. This is done in such a way that models implying lower-order contributions to the neutrino mass can be discarded [76]. Such an approach has been applied to the Weinberg operator up to three loops [77–79] and to its dimension-seven generalisation at one loop [80]. An alternative and complementary method begins by considering all of the gauge-invariant $\Delta L = 2$ operators in the SM effective field theory (SMEFT), first listed in this context by Babu and Leung (BL) [81] and extended by de Gouvêa and Jenkins (dGJ) [66]. Supposing that the tree-level coefficient of one of these is non-zero at the high scale, neutrino masses will be generated from loop graphs contributing to the mixing of this operator and the Weinberg-like operators $\mathcal{O}_1^{\prime\cdots\prime}$. The process of expanding the operator into a series of UV-complete, renormalisable models that generate the parent operator at tree-level is called *opening up* or *exploding* the operator. The remaining external fields must be looped-off, with additional loops of SM gauge bosons or Higgs fields added as necessary in order to obtain a neutrino self-energy diagram. A model-building formula along these lines has been formulated in Ref. [82], and it has been used to write down all of the minimal, tree-level UV-completions of $\Delta L = 2$ operators at dimension seven [83] corresponding to tree-level and radiative neutrino-mass models. The tree-level completions of the Weinberg-like operators have been written down up to dimension eleven [83–

85]. The operators $\Delta L = 2$ operators in the SM EFT are discussed in more detail in Sec. 1.3.2.

We note that an economic classification scheme, separate from an EFT framework, was presented in Ref. [86] based on the number of exotic degrees of freedom by which the SM is extended. There, the method is applied to the case of radiative models with two exotics[4], and has also been used to study minimal neutrino-mass models compatible with SU(5) unification [87].

1.3 Effective field theories of the SM

In the following we introduce EFTs in general, and then specialise to those built out of SM fields: the Standard Model EFT (SMEFT) and the Weak Effective Theory (WET). We place particular emphasis on the operators appearing at dimension-six, and then at odd mass dimension up to dimension eleven. The odd-dimension operators in the SMEFT are those that organise the building of neutrino-mass models in the tradition of Refs. [66, 81–83]. We begin with prefatory comments on the process of tree-level matching, then move on to discuss the SMEFT and peripheral concepts.

1.3.1 Tree-level matching

Suppose one has a theory with light particle states described by fields π_i and heavy states described by Π_i with a Lagrangian of the form

$$\begin{aligned} \mathscr{L}_{\text{UV}}[\pi, \Pi] &= \mathscr{L}_{\text{kin}}[\pi, \Pi] + \mathscr{L}_{\text{int}}[\pi, \Pi], \text{ with} \\ \mathscr{L}_{\text{int}}[\pi, \Pi] &= \mathscr{L}^{l}[\pi] + \mathscr{L}^{h}[\Pi] + \mathscr{L}^{lh}[\pi, \Pi]. \end{aligned} \tag{1.27}$$

Below the threshold for Π_i production, an effective description of the theory can be used that involves interactions only between the light fields. This effective theory is described by a Lagrangian $\mathscr{L}_{\text{eff}}[\pi]$ involving interactions between the π_i that correspond to diagrams in the full theory containing only heavy internal propagators and light external states. At the classical level, $\mathscr{L}_{\text{eff}}$ can be written down by integrating out the Π_i. Perturbatively this corresponds to expanding the heavy propagators Δ in

[4]Including models with one scalar and one Dirac fermion.

powers of momenta on the heavy mass scale[5] Λ, such that

$$\Delta = \begin{cases} -\dfrac{1}{\Lambda^2}\left(1 + \dfrac{p^2}{\Lambda^2} + \cdots\right) & \text{for} \quad \text{- - - - - - - - - - -} \\ -\dfrac{\delta_\alpha^{\ \beta}}{\Lambda}\left(1 + \dfrac{p^2}{\Lambda^2} + \cdots\right) & \text{for} \quad \beta \longleftarrow\!\!\longrightarrow \alpha \\ -\dfrac{ip \cdot \bar{\sigma}^{\dot{\alpha}\beta}}{\Lambda^2}\left(1 + \dfrac{p^2}{\Lambda^2} + \cdots\right) & \text{for} \quad \beta \longleftarrow \dot{\alpha} \end{cases} . \tag{1.28}$$

In this notation, the arrow-preserving propagator corresponds to the part of the regular four-component fermion propagator proportional to momentum, while the arrow-violating one is the part proportional to the fermion mass. Expressions for the fermion propagators with reversed arrows follow from $\bar{\sigma}^\mu \to \sigma^\mu$ and interchanging dotted and undotted indices (see Ref. [88] Sec. 4.2 for the Lorentz structure).

Equivalently, the integration can be performed using the classical EOM of the Π_i. For some heavy field Π, the linearised solution to its classical EOM can be used to remove it from the Lagrangian completely. This procedure is mildly different for scalars and fermions, and we briefly outline these separately below. In both cases, we begin with a Lagrangian $\mathscr{L}_{\mathrm{UV}}$ for which we imagine that kinetic and mass mixing terms between heavy and light fields have been removed.

There are tree-level contributions to $\mathscr{L}^{\mathrm{eff}}$ as long as there are interaction terms linear in Π. For scalar Π, the UV Lagrangian contains the terms

$$\mathscr{L}_{\mathrm{UV}}[\Pi, \pi] \supset \Pi^\dagger(-D^2 - m_\Pi^2)\Pi + \left(\Pi \frac{\partial \mathscr{L}^{lh}}{\partial \Pi} + \text{h.c.}\right) , \tag{1.29}$$

where $\partial \mathscr{L}^{lh}/\partial \Pi$ is a function only of light fields, and we are neglecting interactions of the form $\Pi^\dagger \Pi f(\pi)$ for the sake of conciseness. The EOM are

$$(-D^2 - m_\Pi^2)\Pi = -\frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger} + \mathcal{O}(\Pi^2) , \tag{1.30}$$

which can be solved for Π^{cl}, the classical field configuration, by inverting the differential

[5]We note that some UV scenarios may have more than one characteristic scale. In this case Λ can be understood as an effective scale which may not necessarily correspond to the mass of a specific particle.

operator on the LHS of Eq. (1.30) and expanding in D^2/m_Π^2:

$$\Pi^{\rm cl} = -\frac{1}{m_\Pi^2}\left(1 - \frac{D^2}{m_\Pi^2} + \cdots\right)\frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger}\,. \tag{1.31}$$

This solution can be substituted back into Eq. (1.29) to give interactions between light fields in the tree-level effective Lagrangian:

$$\mathscr{L}_{\rm eff}[\pi] \supset -\frac{\partial \mathscr{L}^{lh}}{\partial \Pi}\frac{1}{m_\Pi^2}\left(1 - \frac{D^2}{m_\Pi^2} + \cdots\right)\frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger}\,. \tag{1.32}$$

Many concrete examples of this procedure can be found in the literature, see *e.g.* Ref. [89]. The expansion in D^2/m_Π^2 corresponds to the expansion in p^2/Λ^2 in the first case of Eq. (1.28), showing the expansion of the scalar propagator.

Next we sketch out the procedure for a Dirac fermion $\Pi + \bar{\Pi}^\dagger$, where Π and $\bar{\Pi}$ are separate two-component spin-$\frac{1}{2}$ fields transforming oppositely under $G_{\rm SM}$. In this case, the UV theory is described by a Lagrangian like

$$\mathscr{L}_{\rm UV}[\Pi,\pi] \supset i\Pi^\dagger\bar{\sigma}^\mu D_\mu \Pi + i\bar{\Pi}^\dagger\bar{\sigma}^\mu D_\mu\bar{\Pi} + \left(\Pi\frac{\partial \mathscr{L}^{lh}}{\partial \Pi} + \bar{\Pi}\frac{\partial \mathscr{L}^{lh}}{\partial \bar{\Pi}} - m_\Pi\bar{\Pi}\Pi + \text{h.c.}\right) \tag{1.33}$$

Varying the action with respect to the heavy fields gives two coupled EOM:

$$i\bar{\sigma}^\mu D_\mu \Pi - m\bar{\Pi}^\dagger + \frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger} = 0\,, \tag{1.34}$$

$$i\bar{\sigma}^\mu D_\mu \bar{\Pi} - m\Pi^\dagger + \frac{\partial \mathscr{L}^{lh}}{\partial \bar{\Pi}^\dagger} = 0\,. \tag{1.35}$$

Substituting Eq. (1.34) into Eq. (1.35) gives a second-order partial differential equation in Π, analogous to Eq. (1.30). Inverting the differential operator in a similar way gives

$$\Pi^{\rm cl}_\beta = \frac{1}{m_\Pi^2}\left(\epsilon_{\alpha\beta} + \frac{\frac{1}{2}X_{\alpha\beta} - D^2\epsilon_{\alpha\beta}}{m_\Pi^2} + \cdots\right)\left(iD^{\alpha\dot{\alpha}}\frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger_{\dot{\beta}}}\epsilon_{\dot{\alpha}\dot{\beta}} + m_\Pi\frac{\partial \mathscr{L}^{lh}}{\partial \bar{\Pi}_\alpha}\right), \tag{1.36}$$

where the field-strength tensor comes about from a structure like

$$[\sigma^\mu\bar{\sigma}^\nu]_\alpha^{\ \beta} D_\mu D_\nu = \eta^{\mu\nu}D_\mu D_\nu \delta_\alpha^{\ \beta} - 2i[\sigma^{\mu\nu}]_\alpha^{\ \beta} D_\mu D_\nu \tag{1.37}$$

$$= D^2\delta_\alpha^{\ \beta} - \tfrac{1}{2}X_\alpha^{\ \beta}\,. \tag{1.38}$$

Here, and later in this section, the replacement $\bar{\Pi} \to \Pi$ should be understood for Majorana Π. Each contribution corresponds to a particular kind of propagator in the perturbative picture. The first term in the last parenthesis of Eq. (1.36) results from the fermion propagator proportional to momentum: the arrow-preserving fermion propagator shown as the last case of Eq. (1.28). The second term in the same parentheses stems from the fermion propagator proportional to the mass, corresponding to the arrow-violating propagator shown in the middle case of Eq. (1.28). Replacing Π in Eq. (1.33) gives the tree-level effective Lagrangian with the heavy fermion integrated out:

$$
\begin{aligned}
\mathscr{L}_{\text{eff}}[\pi] \supset \frac{\partial \mathscr{L}^{lh}}{\partial \Pi_\beta} \frac{1}{m_\Pi^2} \left(\epsilon_{\alpha\beta} + \frac{\frac{1}{2} X_{\alpha\beta} - D^2 \epsilon_{\alpha\beta}}{m_\Pi^2} + \cdots \right) i D^{\alpha\dot{\alpha}} \frac{\partial \mathscr{L}^{lh}}{\partial \Pi^\dagger_{\dot{\beta}}} \epsilon_{\dot{\alpha}\dot{\beta}} \\
+ \frac{\partial \mathscr{L}^{lh}}{\partial \Pi_\beta} \frac{1}{m_\Pi^2} \left(\epsilon_{\alpha\beta} + \frac{\frac{1}{2} X_{\alpha\beta} - D^2 \epsilon_{\alpha\beta}}{m_\Pi^2} + \cdots \right) \frac{\partial \mathscr{L}^{lh}}{\partial \bar{\Pi}_\alpha} \,.
\end{aligned}
\tag{1.39}
$$

As shown in Eqs. (1.32) and (1.39), expanding in powers of derivatives on heavy masses leads to a tower of local operators of increasing mass dimension d_i organised as a power series in the inverse heavy scale:

$$
\mathscr{L}_{\text{eff}}[\pi] = \mathscr{L}^l[\pi] + \sum_i \frac{C_i}{\Lambda^{d_i - 4}} \mathscr{O}_i[\pi]. \tag{1.40}
$$

The C_i are dimensionless coefficients which are in general calculable if one knows the high-energy theory.

1.3.2 Effective field theories of the SM

Below we discuss EFTs constructed from SM fields and invariant under SM symmetries. The main theory of study is the SMEFT: the gauge- and Lorentz-invariant EFT built from the fields listed in Table 1.1. We also mention the WET, also known as the LEFT (Low-energy Effective Field Theory), for which invariance under $\mathrm{SU}(2)_+ \otimes \mathrm{SU}(2)_- \otimes \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\text{EM}}$ is required. In addition to the difficulties associated with constructing the invariants in an EFT, there are additional intricacies associated with redundancies among operators. These are also discussed below in the context of the SMEFT, although the principles discussed apply more generally. We tie off this section with a discussion of the $\Delta L = 2$ operators in the SMEFT.

The SMEFT at dimension six

Given the broad experimental success of the SM, it is perhaps a sensible assumption that there should be a sizeable mass gap between the electroweak scale and the mass-scale characterising any new physics. In this context, the SMEFT can be a powerful tool for constraining what this new physics might look like in a model-independent way. Indeed, the SMEFT operators at dimension six are already coming to play an increasingly important role in particle phenomenology, and they have become the *de facto* framework for interpreting low-energy constraints on theoretical models, and experimental deviations from SM predictions. For an extensive review, we point the reader to Ref. [90].

It has not been easy to write down a complete basis of operators in the SMEFT [], although this is now a mostly solved problem []. The lowest-dimensional operator appearing in the EFT is also the only dimension-five operator:

$$\mathscr{L}^{(5)} = [C_5]_{\{rs\}}(L_r^i L_s^j)H^k H^l \epsilon_{ik}\epsilon_{jl} + \text{h.c.} \,, \tag{1.41}$$

already discussed briefly in Sec. 1.2.3. The matrix of operator coefficients is necessarily symmetric by Fermi–Dirac statistics. The operator violates lepton-number by two units, and usually gives the dominant contribution to the neutrino mass in Majorana models. Ref. [91] shows that operators in the SMEFT of dimension d satisfy

$$\frac{1}{2}(\Delta B - \Delta L) = d \mod 2 \,, \tag{1.42}$$

and thus odd mass-dimension operators must violate $B-L$ by two units, while operators of even mass-dimension cannot violate $B-L$. So, aside from lepton-number-violating effects, the leading-order deviations from the SM appear at dimension six, where there are many more operators.

The dimension-six operators come in eight general classes: X^3, H^6, H^4D^2, X^2H^2, ψ^2H^3, ψ^2XH, ψ^2H^2D and ψ^4. (Here, X represents a general field-strength tensor, D is a covariant derivative and ψ is a fermion field.) The most common basis found in the literature is the Warsaw basis, which tends to prefer not mixing colour indices in fermion currents.

We do not give a complete listing of these operators here, but instead define them as necessary in the chapters they appear. For the SMEFT we attempt to adhere to the conventions of Ref. [] relevant for the Warsaw basis.

The operators in the notation that we use them throughout the rest of this work are listed in Table 1.2. Each operator is given in four-component notation along with the number of operators for $n_f = 3$ SM-fermion flavours and the operator class.

Operator redundancies and the Hilbert series

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The WET

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Lepton-number-violating SMEFT

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1.4 The flavour anomalies and their explanation

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	Operator	Label	Operator	Label
$(\bar{L}L)(\bar{L}L)$	$(L^\dagger \bar{\sigma}_\mu L)(L^\dagger \bar{\sigma}^\mu L)$	$\mathcal{O}_{ll}$		
	$\left(\bar{q}_L \gamma_\mu q_L\right)\left(\bar{q}_L \gamma^\mu q_L\right)$	$\mathcal{O}_{qq}^{(1)}$	$\left(\bar{q}_L \gamma_\mu \sigma_a q_L\right)\left(\bar{q}_L \gamma^\mu \sigma_a q_L\right)$	$\mathcal{O}_{qq}^{(3)}$
	$\left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{q}_L \gamma^\mu q_L\right)$	$\mathcal{O}_{lq}^{(1)}$	$\left(\bar{l}_L \gamma_\mu \sigma_a l_L\right)\left(\bar{q}_L \gamma^\mu \sigma_a q_L\right)$	$\mathcal{O}_{lq}^{(3)}$
$\left(\bar{R}R\right)\left(\bar{R}R\right)$	$\left(\bar{e}_R \gamma_\mu e_R\right)\left(\bar{e}_R \gamma^\mu e_R\right)$	$\mathcal{O}_{ee}$		
	$\left(\bar{u}_R \gamma_\mu u_R\right)\left(\bar{u}_R \gamma^\mu u_R\right)$	$\mathcal{O}_{uu}$	$\left(\bar{d}_R \gamma_\mu d_R\right)\left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{dd}$
	$\left(\bar{u}_R \gamma_\mu u_R\right)\left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{ud}^{(1)}$	$\left(\bar{u}_R \gamma_\mu T_A u_R\right)\left(\bar{d}_R \gamma^\mu T_A d_R\right)$	$\mathcal{O}_{ud}^{(8)}$
	$\left(\bar{e}_R \gamma_\mu e_R\right)\left(\bar{u}_R \gamma^\mu u_R\right)$	$\mathcal{O}_{eu}$	$\left(\bar{e}_R \gamma_\mu e_R\right)\left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{ed}$
$\left(\bar{L}L\right)\left(\bar{R}R\right)$	$\left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{e}_R \gamma^\mu e_R\right)$	$\mathcal{O}_{le}$	$\left(\bar{q}_L \gamma_\mu q_L\right)\left(\bar{e}_R \gamma^\mu e_R\right)$	$\mathcal{O}_{qe}$
	$\left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{u}_R \gamma^\mu u_R\right)$	$\mathcal{O}_{lu}$	$\left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{ld}$
	$\left(\bar{q}_L \gamma_\mu q_L\right)\left(\bar{u}_R \gamma^\mu u_R\right)$	$\mathcal{O}_{qu}^{(1)}$	$\left(\bar{q}_L \gamma_\mu T_A q_L\right)\left(\bar{u}_R \gamma^\mu T_A u_R\right)$	$\mathcal{O}_{qu}^{(8)}$
	$\left(\bar{q}_L \gamma_\mu q_L\right)\left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{qd}^{(1)}$	$\left(\bar{q}_L \gamma_\mu T_A q_L\right)\left(\bar{d}_R \gamma^\mu T_A d_R\right)$	$\mathcal{O}_{qd}^{(8)}$
$\left(\bar{L}R\right)\left(\bar{R}L\right)$	$\left(\bar{l}_L e_R\right)\left(\bar{d}_R q_L\right)$	$\mathcal{O}_{ledq}$		
$\left(\bar{L}R\right)\left(\bar{L}R\right)$	$\left(\bar{q}_L u_R\right) i\sigma_2 \left(\bar{q}_L d_R\right)^{\mathrm{T}}$	$\mathcal{O}_{quqd}^{(1)}$	$\left(\bar{q}_L T_A u_R\right) i\sigma_2 \left(\bar{q}_L T_A d_R\right)^{\mathrm{T}}$	$\mathcal{O}_{quqd}^{(8)}$
	$\left(\bar{l}_L e_R\right) i\sigma_2 \left(\bar{q}_L u_R\right)^{\mathrm{T}}$	$\mathcal{O}_{lequ}^{(1)}$	$\left(\bar{l}_L \sigma_{\mu\nu} e_R\right) i\sigma_2 \left(\bar{q}_L \sigma^{\mu\nu} u_R\right)^{\mathrm{T}}$	$\mathcal{O}_{lequ}^{(3)}$
B-violating			$\epsilon_{ABC}\left(\bar{d}_R^{c\,A} u_R^B\right)\left(\bar{q}_L^{c\,C} i\sigma_2 l_L\right)$	$\mathcal{O}_{duq}$
			$\epsilon_{ABC}\left(\bar{q}_L^{c\,A} i\sigma_2 q_L^B\right)\left(\bar{u}_R^{c\,C} e_R\right)$	$\mathcal{O}_{qqu}$
			$\epsilon_{ABC}\left(\bar{d}_R^{c\,A} u_R^B\right)\left(\bar{u}_R^{c\,C} e_R\right)$	$\mathcal{O}_{duu}$
			$\epsilon_{ABC}(i\sigma_2)_{\alpha\delta}(i\sigma_2)_{\beta\gamma}\left(\bar{q}_L^{c\,A\alpha} q_L^{B\beta}\right)\left(\bar{q}_L^{c\,C\gamma} l_L^\delta\right)$	$\mathcal{O}_{qqq}$

Table 1.2: Basis of dimension-six operators: four-fermion interactions. Flavor indices are omitted.

non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

1.4.1 Neutral-current anomalies

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1.4.2 Charged-current anomalies

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1.4.3 Anomalous magnetic moment of the muon

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

1.4.4 Scalar and Vector leptoquarks

2

Model building from effective operators

2.1 Introduction

This is a test of *something with an apple* that I would like [11]. The following is $4a + 5 = 13$ some inline math and `this` and we did in in Python.

$$\int_{-\infty}^{\infty} \frac{1}{(2\pi\hbar)^3} \phi(p) dp \, . \tag{2.1}$$

We need some sans serif **words** here too. Then we need to check **what** the bold looks like.

3
Models of radiative neutrino mass

3.1 Introduction

This is a test of *something with an apple* that I would like [11]. The following is $4a + 5 = 13$ some inline math and `this` and we did in in Python.

$$\int_{-\infty}^{\infty} \frac{1}{(2\pi\hbar)^3} \phi(p) dp \,. \tag{3.1}$$

We need some sans serif **words** here too. Then we need to check **what** the bold looks like.

4

The S_1 leptoquark as an explanation of the flavour anomalies

This chapter is based on the publication 'Reconsidering the One Leptoquark scenario: flavour anomalies and neutrino mass,' written in collaboration with Yi Cai, Michael A. Schmidt, and Raymond R. Volkas [2]. We study the potential of the S_1 leptoquark to explain the flavour anomalies and the anomalous magnetic moment of the muon in a new region of parameter space.

4.1 Introduction

A common origin for $R_{D^{(*)}}$ and the anomalous $b \to s$ data is suggested naturally if the former is explained by the effects of the operator $(c^\dagger \bar{\sigma}_\mu b)(\tau^\dagger \bar{\sigma}^\mu \nu)$, related in its general structure by $\mathrm{SU}(2)_L$ invariance to the aforementioned four-fermion effective operator accounting for the $b \to s$ anomalies. A number of models exploring this idea have been suggested in the literature [92–106] (along with many others addressing one or the other anomaly, *e.g.* [107–124]) and among these minimal explanations the Bauer–Neubert (BN) model [93] is one of notable simplicity and explanatory power: a TeV-scale scalar leptoquark protagonist mediating $\bar{B} \to D^{(*)}\tau\bar{\nu}$ at tree-level and the $b \to s$

decays through one-loop box diagrams. The leptoquark transforms under the SM gauge group like a right-handed down-type quark and its pattern of couplings to SM fermions can also reconcile the measured and predicted values of the anomalous magnetic moment of the muon, another enduring tension.

5

Models of neutrino mass and the flavour anomalies

5.1 Introduction

Taken together, the anomalies studied in the previous chapter paint a picture of new physics interacting more strongly with the second and third generations of SM fermions, introducing lepton flavour non-universality and FCNC interactions at energies not significantly higher than the electroweak scale. Interestingly, many of these phenomenological motifs arise naturally in radiative models of neutrino mass, hinting towards the attractive possibility of a common explanation for both phenomena. Previous work has also considered radiative neutrino mass models whose particle content addresses R_K [106, 118, 125–127], $R_{D^{(*)}}$ [100, 106] and $(g-2)_\mu$ [106, 125–128]. In Refs. [100, 118] the flavour anomalies are explained through two light scalar or vector leptoquarks whose couplings to the SM Higgs doublet and fermions prohibit a consistent assignment of lepton number to the leptoquarks such that the symmetry is respected. Thus $\mathrm{U}(1)_L$ is explicitly broken by two units and the neutrinos gain mass at the one-loop level [129], apart from the imposition of any additional symmetries[1]. A general feature of such models is that large amounts of fine-tuning are required to suppress the neutrino mass to the required scale with at least one set of leptoquark–fermion couplings sizeable enough to explain the anomalies.

[1] Mass generation in Ref. [128] occurs at the two-loop level because the Yukawa couplings of one of the leptoquarks to the left-chiral fermions is turned off.

6

The two-photon decay of a scalar-quirk bound state

We use $\digamma$ *to represent the scalar diquark.*

6.1 Introduction

An excess of events containing two photons with invariant mass near 750 GeV has been observed in 13 TeV proton–proton collisions by the ATLAS and CMS collaborations [130, 131]. The cross section $\sigma(pp \to \gamma\gamma)$ is estimated to be

$$\sigma(pp \to \gamma\gamma) = \begin{cases} (10 \pm 3)\ \text{fb} & \text{ATLAS} \\ (6 \pm 3)\ \text{fb} & \text{CMS} \end{cases} \tag{6.1}$$

and there is no evidence of any accompanying excess in the dilepton channel [132]. If we interpret this excess as the two photon decay of a single new particle of mass m then ATLAS data provide a hint of a large width: $\Gamma/m \sim 0.06$, while CMS data prefer a narrow width. Naturally, further data collected at the LHC should provide a clearer picture as to the nature of this excess.

There has been vast interest in the possibility that the diphoton excess results from physics beyond the SM. Most discussion has focused on models where the excess is due to a new scalar particle which subsequently decays into two photons *e.g.* Ref. [133]. The possibility that the new scalar particle is a bound state of exotic charged fermions has also been considered, *e.g.* Refs. [134–138]. Here we consider the case that the 750 GeV state is a non-relativistic bound state constituted by an exotic *scalar* particle χ and its antiparticle, charged under $\mathrm{SU}(3)_c$ as well as a new unbroken non-abelian gauge interaction. Having χ be a scalar rather than a fermion is not merely a matter of taste: In such a framework a fermionic χ would lead to the formation of bound states which (typically) decay to dileptons more often than to photons; a situation which is not favoured by the data.

The bound state, which we denote Π, can be produced through gluon–gluon fusion directly (*i.e.* at threshold $\sqrt{s_{gg}} \simeq M_\Pi$) or indirectly via $gg \to \chi^\dagger\chi \to \Pi +$ *soft quanta* (*i.e.* above Π threshold: $\sqrt{s_{gg}} > M_\Pi$). The indirect production mechanism can dominate the production of the bound state, which is an interesting feature of this kind of theory.

6.2 The model

We take the new confining unbroken gauge interaction to be $\mathrm{SU}(N)$, and assume that, like $\mathrm{SU}(3)_c$, it is asymptotically free and confining at low energies. However, the new $\mathrm{SU}(N)$ dynamics is qualitatively different from QCD as all the matter particles [assumed to be in the fundamental representation of $\mathrm{SU}(N)$] are taken to be much heavier than the confinement scale, Λ_N. In fact we here consider only one such matter particle, χ, so that $M_\chi \gg \Lambda_N$ is assumed. In this circumstance a $\chi^\dagger\chi$ pair produced at the LHC above the threshold $2M_\chi$ but below $4M_\chi$ cannot fragment into two jets. The $\mathrm{SU}(N)$ string which connects them cannot break as there are no light $\mathrm{SU}(N)$-charged states available. This is in contrast to heavy quark production in QCD where light quarks can be produced out of the vacuum enabling the color string to break. The produced $\chi^\dagger\chi$ pair can be viewed as a highly excited bound state, which de-excites by $\mathrm{SU}(N)$-ball and soft glueball/pion emission [139].

With the new unbroken gauge interaction assumed to be $\mathrm{SU}(N)$ the gauge symmetry of the SM is extended to

$$\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(N). \tag{6.2}$$

This kind of theory can arise naturally in models which feature large colour groups [140–

142] and in models with leptonic colour [143–146] but was also considered earlier by Okun [147]. The notation *quirks* for heavy particles charged under an unbroken gauge symmetry (where $M_\chi \gg \Lambda_N$) was introduced in [139] where the relevant phenomenology was examined in some detail in a particular model[1]. For convenience we borrow their nomenclature and call the new quantum number *hue* and the massless gauge bosons *huons* ($\mathscr{H}$).

The phenomenological signatures of the bound states (quirkonia) formed depend on whether the quirk is a fermion or boson. Here we assume that the quirk χ is a Lorentz scalar in light of previous work which indicated that bound states formed from a fermionic χ state would be expected to be observed at the LHC via decays of the spin-1 bound state into opposite-sign lepton pairs ($\ell^+\ell^-$) [139, 146]. In fact, this appears to be a serious difficulty in attempts to interpret the 750 GeV state as a bound state of fermionic quirk particles (such as those of Refs. [134–136]). The detailed consideration of a scalar χ appears to have been largely overlooked[2], perhaps due to the paucity of known elementary scalar particles. With the recent discovery of a Higgs-like scalar at 125 GeV [150, 151] it is perhaps worth examining signatures of scalar quirk particles. In fact, we point out here that the two photon decay is the most important experimental signature of bound states formed from electrically charged scalar quirks. Furthermore this explanation is only weakly constrained by current data and thus appears to be a simple and plausible option for the new physics suggested by the observed diphoton excess.

6.3 Explaining the excess

The scalar χ that we introduce transforms under the extended gauge group (Eq. 6.2) as

$$\chi \sim (\mathbf{3}, \mathbf{1}, Y; \mathbf{N}), \tag{6.3}$$

where we use the normalisation $Q = Y/2$. The possibility that χ also transforms non-trivially under $SU(2)_L$ is interesting, however for the purposes of this letter we focus on the $SU(2)_L$ singlet case for definiteness. Since two-photon decays of non-relativistic quirkonium will be assumed to be responsible for the diphoton excess observed at the LHC, the mass of χ will need to be around 375 GeV.

[1]Some other aspects of such models have been discussed over the years, including the possibility that the $SU(N)$ confining scale is low ($\sim$ keV), a situation which leads to macroscopic strings [148].

[2]The idea has been briefly mentioned in recent literature [137, 149].

We have assumed that χ is charged under $SU(3)_c$ so that it can be produced at tree-level through QCD-driven pair production. We present the production mechanisms in Fig. 6.1. To estimate the production cross section of the bound states, we first consider the indirect production mechanism which we expect to be dominant. Here, a $\chi^\dagger\chi$ pair is produced above threshold and de-excites emitting soft glueballs/pions and hueballs: $gg \to \chi^\dagger\chi \to \Pi + \textit{soft quanta}$. We first consider the case where the confinement scale of the new SU(N) interaction is similar to that of QCD. What happens in this case can be adapted from the discussion in [139], where a fermionic quirk charged under an unbroken SU(2) gauge interaction was considered. As already briefly discussed in the introduction, the $\chi^\dagger\chi$ pairs initially form a highly excited bound state, which subsequently de-excites in two stages. The first stage is the non-perturbative regime where the hue string is longer than Λ_N^{-1}. The second stage is characterised by a string scale significantly less than Λ_N^{-1}: the perturbative Coulomb region. Here the bound state can be characterised by the quantum numbers n and l. De-excitation continues until quirkonium is in a lowly excited state with $l \leq 1$ and n. Imagine first that de-excitation continued until the ground state ($n = 1, l = 0$) is reached. Given we are considering χ to be a scalar, the quirkonium ground state, Π, will have spin 0, and is thus expected to decay into SM gauge bosons and huons. The cross section $\sigma(pp \to \Pi \to \gamma\gamma)$ in this case is then

$$\sigma(pp \to \gamma\gamma) \approx \sigma(pp \to \chi^\dagger\chi) \times \mathrm{Br}(\Pi \to \gamma\gamma). \tag{6.4}$$

Since production is governed by QCD interactions, we can use the values of the pair production cross sections for stops/sbottoms in the limit of decoupled squarks and gluinos [152]. For a χ mass of 375 GeV

$$\sigma(pp \to \chi^\dagger\chi) \approx \begin{cases} 2.6N\ \mathrm{pb} & \text{at 13 TeV} \\ 0.5N\ \mathrm{pb} & \text{at 8 TeV} \end{cases}. \tag{6.5}$$

The branching fraction is to leading order

$$\mathrm{Br}(\Pi \to \gamma\gamma) \simeq \frac{3NQ^4\alpha^2}{\frac{2}{3}N\alpha_s^2 + \frac{3}{2}C_N\alpha_N^2 + 3NQ^4\alpha^2}, \tag{6.6}$$

where $C_N \equiv (N^2-1)/(2N)$, α_N is the new SU(N) interaction strength and we have neglected the small contribution of $\Pi \to Z\gamma/ZZ$ to the total width. Eq. (6.6) also neglects the decay to Higgs particles: $\Pi \to hh$, which arises from the Higgs potential

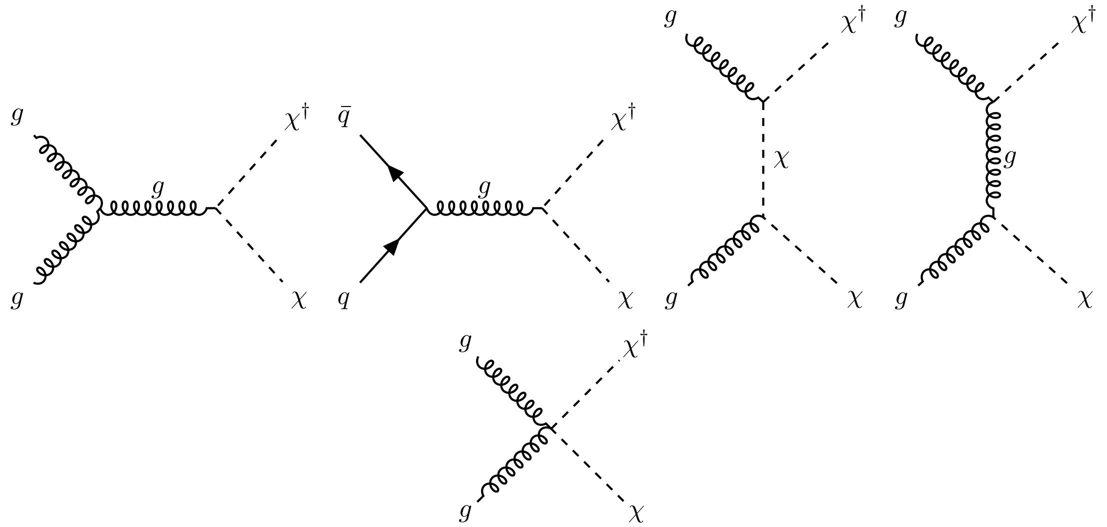

Figure 6.1: Tree-level pair production mechanisms for the scalar quirk χ.

portal term $\lambda_\chi \chi^\dagger \chi \phi^\dagger \phi$. Theoretically this rate is unconstrained given the dependence on the unknown parameter λ_χ, but could potentially be important. However, limits from resonant Higgs boson pair production derived from 13 TeV data: $\sigma(pp \to X \to hh \to bbbb) \lesssim 50\,\text{fb}$ at $M_X \approx 750\,\text{GeV}$ [153, 154] imply that the Higgs decay channel must indeed be subdominant (*c.f.* $\Pi \to gg, \mathscr{H}\mathscr{H}$).

The renormalised gauge coupling constants in Eq. (6.6) are evaluated at the renormalisation scale $\mu \sim M_\Pi/2$. Taking for instance the specific case of $N = 2$, $\alpha_N = \alpha_s \simeq 0.10$ (at $\mu \sim M_\Pi/2$) gives

$$\sigma(pp \to \gamma\gamma) \approx 5 \left(\frac{Q}{1/2}\right)^4 \text{ fb at } 13\,\text{TeV}\,. \tag{6.7}$$

At $\sqrt{s} = 8\,\text{TeV}$ the cross section is around five times smaller. We present the cross section $\sigma(pp \to \Pi \to \gamma\gamma)$ for a range of masses M_Π and different combinations of Q and N in Fig. 6.2. The parameter choice $\alpha_N = \alpha_s$ and $\Lambda_N = \Lambda_{\text{QCD}}$ has been assumed. (The cross section is not highly sensitive to Λ_N, α_N so long as we are in the perturbative regime: $\Lambda_N \lesssim \Lambda_{\text{QCD}}$.) Evidently, for $N = 2$, a χ with electric charge $Q \approx 1/2$ is produced at approximately the right rate to explain the diphoton excess.

In practice de-excitation of the produced quirkonium does not always continue until the ground state is reached. In this case annihilations of excited states can also contribute. However those with $l = 0$ will decay in the same way as the ground state. The only difference is that the excited states will have a slightly larger mass (which we will

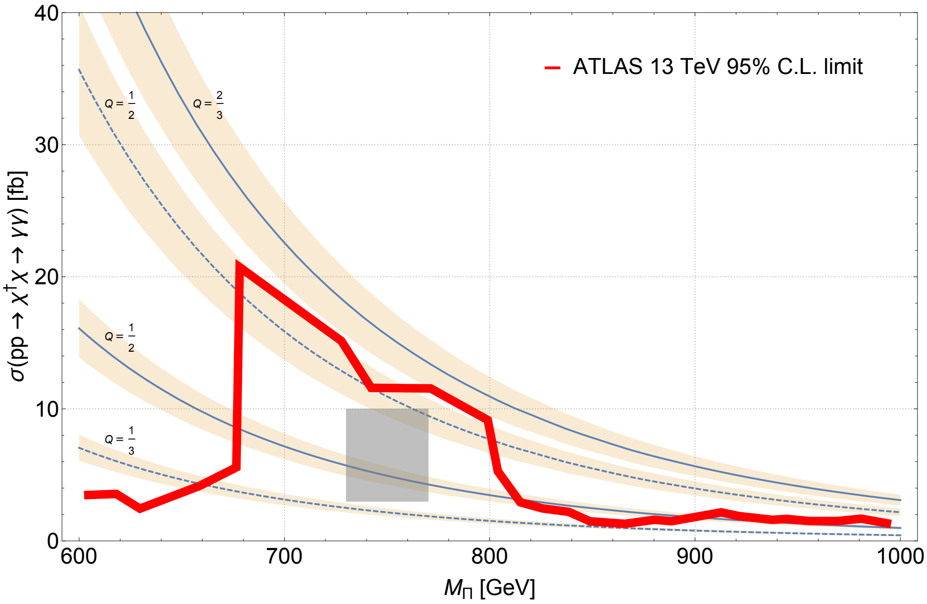

Figure 6.2: The cross section $\sigma(pp \to \Pi \to \gamma\gamma)$ at 13 TeV for a range of quirkonium masses M_Π and charge assignments. Solid lines denote choices of $N = 2$ and dashed lines choices of $N = 5$. The rectangle represents the $\sigma \in [3, 10]$ fb indicative region accommodated by the ATLAS and CMS data. The solid red line is the ATLAS 13 TeV exclusion limit. Uncertainties reflect error associated with the parton distribution functions.

estimate in a moment) due to the change in the binding energy. This detail could be important as it can effectively enlarge the observed width. Annihilation of excited states with non-zero orbital angular momentum could in principle also be important, however these are suppressed as the radial wavefunction vanishes at the origin: $R(0) = 0$ for $l \geq 1$. They are expected to de-excite predominately to $l = 0$ states rather than annihilate [139]. Nevertheless, for sufficiently large α_N the $l = 1$ annihilations: $\Pi \to \mu^+\mu^-$ and $\Pi \to e^+e^-$ could potentially be observable.

The $l = 0$ excited states can be characterized by the quantum number n with binding energies:

$$\frac{E_n}{M_\Pi} = -\frac{1}{8n^2}\left[\frac{4}{3}\bar{\alpha}_s + C_N\bar{\alpha}_N + Q^2\bar{\alpha}\right]^2 . \tag{6.8}$$

The above formula was adapted from known results with quarkonium, *e.g.* [134] (and of course also the hydrogen atom). The coupling constants $\bar{\alpha}_s$, $\bar{\alpha}_N$ and $\bar{\alpha}$ are evaluated at a renomalisation scale corresponding to the mean distance between the particles which is of order the Bohr radius: $a_0 = 4/[(4\bar{\alpha}_s/3 + C_N\bar{\alpha}_N + Q^2\bar{\alpha})M_\Pi]$. The bound state, described by the radial quantum number n has mass given by $M_\Pi(n) = 2M_\chi + E_n$. Considering as an example $N = 2$ and $\bar{\alpha}_N = \bar{\alpha}_s = 0.15$, $\bar{\alpha} = 1/137$ we find the mass difference between the $n = 1$ and $n = 2$ states to be $\Delta M = (E_1 - E_2) \approx 0.01 M_\Pi$. Larger mass splittings will be possible[3] if $\bar{\alpha}_N > \bar{\alpha}_s$, although it has been shown in the context of fermionic quirk models that the phenomenology is substantially altered in this regime [135]. In particular, the hueballs can become so heavy that the decays of the bound state into hueballs is kinematically forbidden.

In the above calculation of the bound state production cross section, we considered only the *indirect* production following pair production of $\chi^\dagger\chi$ above threshold. The bound state can also be produced directly: $gg \to \Pi$, where $\sqrt{s_{gg}} \approx M_\Pi$. The cross section of the ground state direct resonance production is

$$\sigma(pp \to \Pi)_{\mathrm{DR}} \approx \frac{C_{gg}K_{gg}\Gamma(\Pi \to gg)}{sM_\Pi}, \tag{6.9}$$

where C_{gg} is the appropriate parton luminosity coefficient and K_{gg} is the gluon NLO QCD K-factor. For $\sqrt{s} = 13\,\mathrm{TeV}$ we take $C_{gg} \approx 2137$ [133] and $K_{gg} = 1.6$ [155]. The partial width $\Gamma(\Pi \to gg)$ of the $n = 1, l = 0$ ground state is given by

$$\Gamma(\Pi \to gg) = \frac{4}{3} M_\Pi N \alpha_s^2 \frac{|R(0)|^2}{M_\Pi^3}, \tag{6.10}$$

where the radial wavefunction at the origin for the ground state is:

$$\frac{|R(0)|^2}{M_\Pi^3} = \frac{1}{16}\left[\frac{4}{3}\bar{\alpha}_s + C_N\bar{\alpha}_N + Q^2\bar{\alpha}\right]^3. \tag{6.11}$$

Considering again the example of $N = 2$ and $\bar{\alpha}_N = \bar{\alpha}_s = 0.15$, $\bar{\alpha} = 1/137$ we find

$$\sigma(pp \to \Pi)_{DR} \approx 0.40\,\mathrm{pb} \quad \text{at } 13\,\mathrm{TeV}\,. \tag{6.12}$$

Evidently, the direct resonance production cross section is indeed expected to be sub-

[3] Additional possibilities arise if χ transforms non-trivially under $SU(2)_L$, *i.e.* forming a representation N_L. The mass degeneracy of the multiplet will be broken at tree-level by Higgs potential terms along with electroweak radiative corrections. The net effect is that the predicted width of the $pp \to \gamma\gamma$ bump can be effectively larger as there are N_L distinct bound states, Π^i, (of differing masses) which can each contribute to the decay width. Although each state is expected to have a narrow width, when smeared by the detector resolution the effect can potentially be a broad feature.

dominant, around 8% that of the indirect production cross section (Eq. 6.5)[4].

We now comment on the regime where Λ_N is smaller than Λ_{QCD}. In fact, if the SU(N) confining scale is only a little smaller than Λ_{QCD} then a light quark pair can form out of the vacuum, leading to a bound state of two QCD color singlet states: $\chi\bar{q}$ and $\chi^\dagger q$. These color singlet states would themselves be bound together by SU(N) gauge interactions to form the SU(N) singlet bound state. Since only SU(N) interactions bind the two composite states ($\chi\bar{q}$ and $\chi^\dagger q$), it follows that $\frac{4}{3}\bar{\alpha}_s + C_N\bar{\alpha}_N + Q^2\bar{\alpha} \to C_N\bar{\alpha}_N + (Q-Q_q)^2\bar{\alpha}$ in eqs. 6.8 and 6.11. Therefore if the confinement scale of SU(N) is smaller than that of QCD then the direct production rate becomes completely negligible relative to the indirect production mechanism. The rate of Π production is the same as that found earlier in Eq. 6.5, but the branching ratio to two photons is modified:

$$\text{Br}(\Pi \to \gamma\gamma) \simeq \frac{3NQ^4\alpha^2}{\frac{7}{3}N\alpha_s^2 + \frac{3}{2}C_N\alpha_N^2 + 3NQ^4\alpha^2}, \tag{6.13}$$

where, as before, we have neglected the small contribution of $\Pi \to Z\gamma/ZZ$ to the total width, and also the contribution from $\Pi \to hh$. In this regime somewhat larger values of Q can be accommodated, such as $Q = 5/6$ for $N = 2$ [5].

Notice that in the $\Lambda_N < \Lambda_{\text{QCD}}$ regime the size of the mass splittings between the excited states becomes small as $\frac{4}{3}\bar{\alpha}_s + C_N\bar{\alpha}_N + Q^2\bar{\alpha} \to C_N\bar{\alpha}_N + (Q-Q_q)^2\bar{\alpha}$ in Eq. 6.8. We therefore expect no effective width enhancement due to the excited state decays at the LHC in the small Λ_N regime. Of course a larger effective width is still possible if there are several nearly degenerate scalar quirk states, which, as briefly mentioned earlier, can arise if χ transforms nontrivially under $SU(2)_L$.

Other signatures

While the two photon decay channel of the bound state should be the most important signature, the dominant decay is expected to be via $\Pi \to gg$ and $\Pi \to \mathcal{H}\mathcal{H}$. The former process is expected to lead to dijet production while the latter will be an invisible decay.

[4] If $\bar{\alpha}_N$ is sufficiently large, one can potentially have direct resonance production comparable or even dominating indirect production (such a scenario has been contemplated recently in [136, 137]). Naturally at such large $\bar{\alpha}_N$ the perturbative calculations become unreliable, and one would have to resort to non-perturbative techniques such as lattice computations.

[5] Although it is perhaps too early to speculate on the possible role of χ in a more elaborate framework, we nevertheless remark here that particles fitting its description are required for spontaneous symmetry breaking of extended Pati–Salam type unified theories [156].

The dijet cross section is easily estimated:

$$\sigma(pp \to jj) \approx \begin{cases} 2.6N \times \text{Br}(\Pi \to gg)\ \text{pb} & \text{at 13 TeV} \\ 0.5N \times \text{Br}(\Pi \to gg)\ \text{pb} & \text{at 8 TeV} \end{cases} . \tag{6.14}$$

The limit from 8 TeV data is $\sigma(pp \to jj) \lesssim 2.5$ pb [157, 158]. If gluons dominate the Π decays (i.e. $\text{Br}(\Pi \to gg) \approx 1$) then this experimental limit is satisfied for $N \le 5$. For sufficiently large α_N the invisible decay can be enhanced, thereby reducing $\text{Br}(\Pi \to gg)$. In this circumstance the bound on N from dijet searches would weaken.

The invisible decays $\Pi \to \mathscr{H}\mathscr{H}$ are not expected to lead to an observable signal at leading order for much of the parameter space of interest[6]. However, the bremsstrahlung of a hard gluon from the initial state: $pp \to \Pi g \to \mathscr{H}\mathscr{H} g$ can lead to a jet plus missing transverse energy signature. Current data are not expected to give stringent limits from such decay channels, however this signature could become important when a larger data sample is collected. Note though that the rate will become negligible in the limit that α_N becomes small. Also, in the small Λ_N regime, where the bound state is formed from $\chi\bar{q}$ and $\chi^\dagger q$, the two-body decay $\Pi \to g\gamma$ (jet + photon) will also arise as in this case the scalar quirk pair is not necessarily in the color singlet configuration. The decay rate at leading order is substantial:

$$\frac{\Gamma(\Pi \to j\gamma)}{\Gamma(\Pi \to \gamma\gamma)} = \frac{8\alpha_s}{3\alpha Q^2} . \tag{6.15}$$

Nevertheless, we estimate that this is still consistent with current data [161], but would be expected to become important when a larger data sample is collected.

Another important signature of the model will be the $pp \to \Pi \to Z\gamma$ and $pp \to \Pi \to ZZ$ processes. The rates of these decays, relative to $\Pi \to \gamma\gamma$, are estimated to be:

$$\begin{aligned} \frac{\Gamma(\Pi \to Z\gamma)}{\Gamma(\Pi \to \gamma\gamma)} &= 2\tan^2\theta_W, \\ \frac{\Gamma(\Pi \to ZZ)}{\Gamma(\Pi \to \gamma\gamma)} &= \tan^4\theta_W. \end{aligned} \tag{6.16}$$

If χ transforms nontrivially under $SU(2)_L$ then deviations from these predicted rates arize along with the tree-level decay $\Pi \to W^+W^-$.

[6] Scalar quirk loops can mediate hueball decays into gluons and other SM bosons [139, 159, 160]. The decay rate is uncertain, depending on the non-perturbative hueball dynamics. However, if the hueballs are able to decay within the detector then they can lead to observable signatures including displaced vertices. This represents another possible collider signature of the model.

Conclusions

We have considered a charged scalar particle χ of mass around 375GeV charged under both $SU(3)_c$ and a new confining gauge interaction (assigned to be $SU(N)$ for definiteness). These interactions confine $\chi^\dagger\chi$ into non-relativistic bound states whose decays into photons can explain the 750 GeV diphoton excess observed at the LHC. Taking the new confining group to be SU(2), we found that the diphoton excess required χ to have electric charge approximately $Q \sim [\frac{1}{2}, 1]$. An important feature of our model is that the exotic particle χ has a mass much greater than the $SU(N)$-confinement scale Λ_N. In the absence of light $SU(N)$-charged matter fields this makes the dynamics of this new interaction qualitatively different to that of QCD: pair production of the scalars and the subsequent formation of the bound state dominates over direct bound state resonance production (at least in the perturbative regime where $\Lambda_N \lesssim \Lambda_{\text{QCD}}$). Since χ is a Lorentz scalar, decays of $\chi^\dagger\chi$ bound states to lepton pairs are naturally suppressed, and thus constraints from dilepton searches at the LHC can be ameliorated. This explanation is quite weakly constrained by current searches and data from the forthcoming run at the LHC will be able to probe our scenario more fully. In particular, dijet, mono-jet, di-Higgs and jet + photon searches may be the most promising discovery channels.

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A
Mathematical notation

Throughout the paper we choose to label representations by their dimension, which we typeset in bold. Fields are labelled by their transformation properties under the Lorentz group and the SM gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. All spinors are treated as two-component objects transforming as either $(\mathbf{2}, \mathbf{1})$ (left-handed) or $(\mathbf{1}, \mathbf{2})$ (right-handed) under the Lorentz group, written as $\mathrm{SU}(2)_+ \otimes \mathrm{SU}(2)_-$. The left-handed spinors carry undotted spinor indices $\alpha, \beta, \ldots \in \{1, 2\}$, while the right-handed spinors carry dotted indices $\dot{\alpha}, \dot{\beta}, \ldots \in \{\dot{1}, \dot{2}\}$. Wherever possible we attempt to conform to the conventions of Ref. [88] when working with spinor fields (see appendix G for the correspondence to four-component notation and appendix J for SM-fermion nomenclature). For objects carrying a single spacetime index V_μ we define

$$V_{\alpha\dot{\beta}} = \sigma^{\mu}_{\alpha\dot{\beta}} V_\mu \quad \text{and} \quad \bar{V}_{\dot{\alpha}\beta} = \bar{\sigma}^{\mu}_{\dot{\alpha}\beta} V_\mu. \tag{A.1}$$

Note that in this notation

$$\Box = \partial_\mu \partial^\mu = \tfrac{1}{2}\mathrm{Tr}[\partial\bar{\partial}] = \tfrac{1}{2}\mathrm{Tr}[\bar{\partial}\partial], \tag{A.2}$$

and we will often just use $\Box$ to represent the contraction of two covariant derivatives $D_\mu D^\mu$ where this is clear from context. For field-strength tensors, generically $X_{\mu\nu}$, we work with the irreducible representations (irreps) $X_{\alpha\beta}$ and $\bar{X}_{\dot{\alpha}\dot{\beta}}$, where

$$X_{\{\alpha\beta\}} = 2i[\sigma^{\mu\nu}]_\alpha^{\ \gamma}\epsilon_{\gamma\beta}X_{\mu\nu} \quad \text{and} \quad \bar{X}_{\{\dot{\alpha}\dot{\beta}\}} = 2i[\bar{\sigma}^{\mu\nu}]^{\dot{\gamma}}_{\ \dot{\beta}}\epsilon_{\dot{\alpha}\dot{\gamma}}X_{\mu\nu}, \tag{A.3}$$

or the alternate forms with one raised and one lowered index.

Indices for $\mathrm{SU}(2)_L$ (isospin) are taken from the middle of the Latin alphabet. These are kept lowercase for the fundamental representation for which $i, j, k, \ldots \in \{1, 2\}$ and the indices of the adjoint are capitalised $I, J, K, \ldots \in \{1, 2, 3\}$. Colour indices are taken from the beginning of the Latin alphabet and the same distinction between lowercase and uppercase letters is made. For both SU(2) and SU(3), a distinction between raised and lowered indices is maintained such that, for example, $(\psi^i)^\dagger = (\psi^\dagger)_i$ for an isodoublet field ψ. However, we often specialise to the case of only raised, symmetrised indices for SU(2), and use a tilde to denote a conjugate field whose $\mathrm{SU}(2)_L$ indices have been raised:

$$\tilde{\psi}^i \equiv \epsilon^{ij}\psi_j^\dagger. \tag{A.4}$$

We adopt this notation from the usual definition of $\tilde{H}$, and note that throughout the paper we freely interchange between $\tilde{\psi}^i$ and $\psi_i^\dagger$. For the sake of tidiness, we sometimes use parentheses $(\cdots)$ to indicate the contraction of suppressed indices. Curly braces are reserved to indicate symmetrised indices $\{\cdots\}$ and square brackets enclose antisymmetrised indices $[\cdots]$, but this notation is avoided when the permutation symmetry between indices is clear. We use τ^I and λ^A for the Pauli and Gell-Mann matrices, and normalise the non-abelian vector potentials of the SM such that

$$(W_{\alpha\dot{\beta}})^i_{\ j} = \frac{1}{2}(\tau^I)^i_{\ j}W^I_{\alpha\dot{\beta}} \quad \text{and} \quad (G_{\alpha\dot{\beta}})^a_{\ b} = \frac{1}{2}(\lambda^A)^a_{\ b}G^A_{\alpha\dot{\beta}}. \tag{A.5}$$

Flavour (or family) indices of the SM fermions are represented by the lowercase Latin letters $\{r, s, t, u, v, w\}$.

For the non-gauge degrees of freedom in the SM we capitalise isospin doublets (Q, L, H), while the left-handed isosinglets are written in lowercase with a bar featuring as a part of the name of the field ($\bar{u}$, $\bar{d}$, $\bar{e}$). The representations and hypercharges for the SM field content are summarised in Table A.1. Our definition of the SM gauge-covariant derivative is exemplified by

$$\bar{D}_{\dot{\alpha}\beta}Q_r^{\beta ai} = \left[\delta^a_b\delta^i_j(\bar{\partial}_{\dot{\alpha}\beta} + ig_1Y_Q\bar{B}_{\dot{\alpha}\beta}) + ig_2\delta^a_b(\bar{W}_{\dot{\alpha}\beta})^i_{\ j} + ig_3\delta^i_j(\bar{G}_{\dot{\alpha}\beta})^a_{\ b}\right]Q_r^{\beta bj}. \tag{A.6}$$

Note that the derivative implicitly carries $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_c$ indices [explicit on the

Field	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$Q^{\alpha a i}$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$(\mathbf{2}, \mathbf{1})$
$L^{\alpha i}$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{2}, \mathbf{1})$
$\bar{u}_a^{\alpha}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\mathbf{2}, \mathbf{1})$
$\bar{d}_a^{\alpha}$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$(\mathbf{2}, \mathbf{1})$
$\bar{e}^{\alpha}$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{2}, \mathbf{1})$
$(G_{\alpha\beta})^a{}_b$	$(\mathbf{8}, \mathbf{1}, 0)$	$(\mathbf{3}, \mathbf{1})$
$(W_{\alpha\beta})^i{}_j$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{3}, \mathbf{1})$
$B_{\alpha\beta}$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{3}, \mathbf{1})$
H^i	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1})$

Table A.1: The SM fields and their transformation properties under the SM gauge group G_{SM} and the Lorentz group. The final unbolded number in the 3-tuples of the G_{SM} column represents the $U(1)_Y$ charge of the field, normalised such that $Q = I_3 + Y$. For the fermions a generational index has been suppressed.

right-hand side of Eq. (A.6)] which are suppressed on the left-hand side to reduce clutter. Where appropriate we show these indices explicitly.

We represent the SM quantum numbers of fields as a 3-tuple $(\mathbf{C}, \mathbf{I}, Y)_L$, with $\mathbf{C}$ and $\mathbf{I}$ the dimension of the colour and isospin representations, Y the hypercharge of the field, and L an (often omitted) label of the Lorentz representation: S (scalar), F (fermion) or V (vector), although sometimes we use the irrep, *e.g.* $(\mathbf{2}, \mathbf{1})$. We normalise the hypercharge such that $Q = I_3 + Y$. Finally, for exotic fields that contribute to dimension-six operators at tree-level, we try and adopt names consistent with Table 3 of Ref. [**?**].

Definition of Symbols and Acronyms

D

DFT density functional theory

L

lipsum Lorem Ipsum, a special type of fudge
  dolor No idea why
  ibit Sounds right, doesn't it?

P

π (π) Greek letter pi, Π does this work?

R

radial distribution function ($g(r)$)
RDF radial distribution function

Index

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