

UNIVERSITY OF NAIROBI
SCHOOL OF MATHEMATICS
SMA 408 - COMPLEX ANALYSIS II
REVISION PROBLEMS - MAPPINGS

Q1. Find the fixed points of the mapping
(i) $w = (5z+4)/(z+5)$, (ii) $f(z) = \frac{3iz+1}{z-3}$

Q2. Find the bilinear transformation that maps the points $z = 1, i, -1$ to the points $w = i, 0, -i$.

Q3. Find the image of $|z-2i| = 2$ under the mapping $w = \frac{1}{z}$.

Q4. Given triangle T in the z plane with vertices at $i, 1-i, 1+i$. Determine the triangle T' into which T is mapped under the transformation s
(a) $w = 3z + 4 - 2i$ (b) $w = iz + 2 - i$

Q5 (a) Show that by means of the transformation $w = 1/z$ the circle C given by $|z-3| = 5$ is mapped into the circle $|w + 3/16| = 5/16$.
(b) Into what region is the interior of C mapped.

Q6. Determine the equations of the curve in the w plane into which the straight line $x+y=1$ is mapped under the transformation (a) $w=z^2$ (b) $w=1/z$

Q7. Consider the transformation $w=f(z)$, where $f(z)$ is analytic at z_0 , $f'(z_0) \neq 0$. Prove that under this transformation the tangent to any curve C in the z -plane passing through z_0 is rotated through the angle $\arg f'(z_0)$.

Q8. Show that under the transformation $w = z + \frac{1}{z}$, the circle $r=c$ maps into the ellipse $u = (c + \frac{1}{c}) \cos \theta$, $v = (c - \frac{1}{c}) \sin \theta$. Discuss the case $c=1$.

Answers

Q1 (i) $2, -2$ (ii) $3i \pm 2$

Q2. $w = \frac{iz+1}{-iz+1}$

Q3. $v + \frac{1}{4} = 0$

Q6. (a) $u^2 + 2v = 1$

(b) $u^2 + 2uv + 2v^2 = u + v$

END.