UNIVERSITY EXAMINATION FOR THE 2021/2022 ACADEMIC YEAR,

BSC.AND BSC .MATH.

UNIT: ORDINARY DIFFERENTIAL EQUATIONS II.

COURSE CODE :SMA310 -ODE II .

EXAMINATION DATE:FRIDAY **20./05/2022.** **TIME: 2.00- 4.00PM**…………

Instructions: Answer Question **ONE** any other **TWO** questions**.**

.**QUESTION ONE .(30 marks)**

(a).Solve the initial value problem $X' = AX, X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $and\ A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}$ $[6\ marks]$

(b).Prove the following theorems:

(i). If $U_1,...,U_k$ are root vectors corresponding to the kth eigenvalue $\lambda,\ of$

A then $X(t) = e^{\lambda t}(U_k + tU_{k-1} + ... + U_1 \frac{t^{k-1}}{(k-1)!})$ is a solution of $X' = AX$

[4 marks]

(ii).If $X_1(t) = e^{\lambda t}U_1, X_2(t) = e^{\lambda t}(U_2 + tU_1),..., X_k(t) = e^{\lambda t}(U_k + tU_{k-1} + ... + \frac{U_1 t^{k-1}}{(k-1)!})$

Are solutions of $X' = AX, then\ X_1,..., X_k$ are linearly independent,i.e.

$C_1X_1 + C_2X_2 + ... + C_kX_k = 0$ for some arbitrary constants $C_i, s.$ $[4\ marks]$

(c).Find the real-valued fundamental solution.

$x_1' = -3x_1, x_2' = 3x_2 - 2x_3, x_3' = x_2 + x_3.$ $[6\ marks]$

(d).Use the diagonalization procedure to find the general solution,

$x_1' = x_1, x_2' = x_1 + 2x_2, x_3' = x_1 - x_3.$ [10 marks]

QUESTION TWO.(20 marks)

Find the general of the inhomogeneous system $X' = AX + F(t)$,

Where;

(i). $A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$ and $F(t) = \begin{bmatrix} 0 \\ Sin3x \end{bmatrix}$ $[10\ marks]$

(ii).$A = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ $and\ F(t) = \begin{bmatrix} 1 \\ Cot\ t \end{bmatrix}$ [10 marks]

QUESTION THREE.(20 MARKS)

.Reduce the third order ordinary differential equation

y'''-y''-4y'+4y=0 in the companion system of linear equations and hence solve Completely. [20 marks]

QUESTION FOUR.(20 MARKS)

Prove the theorem that. If $X(t)$ $is\ any\ solution\ of\ X' = AX$, then the following inequality holds for all t and t_0 .

$$||X(t)|| \leq \left|\left|X(t_0)\right|\right| exp(||A|||t - t_0|).$$

Hence apply the above theorem to the system;

$x_1' = -(Sin\ t)x_2 + 4$;

$x_2' = -x_1 + 2tx_2 - x_3 + e^t$;

$x_3' = 3(Cos\ t)x_1 + x_2 + \frac{1}{t}x_3 - 5t^2$.

[20 marks]

QUESTION FIVE .(20 MARKS)

Prove the theorem that if A(t) is analytic at t_0,the the power series expansion

$X(t) = \sum_{k=0}^{\infty} X_k(t - t_0)^k, \left|t - t_0\right|$<r is a solution of $X' = AX, X(t_0) = X_0$with

$$X_{k+1} = \frac{1}{k+1}\sum_{i=0}^{k} A_i X_{k-i}.$$

Hence or otherwise find the series solution of 2nd order I.V.P

$X'' + X = 0\,, X(0) = 1\ and\ X'(0) = 0.$ [$20\ marks$]