Forecasting Constituents of the Minimum Volatility Index

John Gilheany

November 6, 2017

Contents

Chapter 1

Opening Comments

I woud like to thank David Kane and Michael Parzen for their contribution and help on my thesis. This would not have been possible without their assistance.

Chapter 2

Abstract

The low-risk anomaly has created opportunities for arbitrage in the financial markets. As Baker et al. discuss in "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly," low-volatility and low-beta portfolios outperform and high-volatility and high-beta portfolios by a factor of several times due to benchmarking and lottery-preferences. The iShares MSCI USA Minimum Volatility (USMV) is an ETF tracking a minimum volatility index that was used to find data and will be used for trading arbitrage. Frazzini et al. discuss arbitrage opportunities by quantitative focused funds like AQR in "Betting Against Beta", and this thesis explores a more advanced type of index front-running as a potential arbitrage opportunity. Data was collected from USMV from its inception in October 2011, and from EUSA, the parent ETF of USMV, from the same period until December 2016. 52-week trailing beta, 52-week trailing volatility, lagged price/book, and current index membership were calculated, and a regression model was run to quantify the relationship between current index membership and these four variables. In the model, a probabilities of index membership were calculated and an optimal cutoff was calculated to which the model would be 95% accurate of its findings of a stock to be in or out of USMV, given the historical data. Backtesting with prior data showed with a model accuracy of 95%, arbitrage opportunities of X% could be collected after each rebalancing.

Chapter 3

Introduction

The iShares MSCI USA Minimum Volatility (USMV) Exchange Traded Fund (ETF) is designed to track the investment results of the MSCI Minimum Volatility USA index, which is composed of stocks with a lower volatility than the general market. This can provide investors with exposure to a portfolio with less risk than many alternatives, and historically has declined less in value than the broader market during economic downturns. The ETF is comprised of 189 holdings, and is rebalanced two times per year. The purpose of this dissertation is to create a logistic regression model that can accurately predict which stocks will be added or removed from this ETF before rebalancing occurs, and understand what factors are involved. The model will take into account volatility attributes of each stock, as well as others potentially significant predictor variables from prior studies. An accurate model will allow for arbitrage investment opportunities.

3.1 Exchange Traded Funds

An Exchange Traded Fund (ETF) is a collection of stocks and/or bonds in a single portfolio, that is traded on a major exchange just like a stock is (http://www.investopedia.com/terms/e/etf.asp). As a result, the price of an ETF fluctuates on a regular basis. Exchange Traded Funds generally have more liquidity and less fees when compared to other alternatives instruments like mutual funds. Owning an ETF can allow investors to minimize risk, since owning an ETF is comparable to owning a little bit of many different stocks. This diversification comes at lower costs and less effort for investors as well.

ETFs can also track an index, commodity, bonds, or basket of all of the above. Unlike an ETF, which is publicly traded, an index is not. The goal of the USMV ETF is to track the MSCI Minimum Volatility USA index, and this is more complicated than it seems. In addition to tracking this index, the ETF aims to mirror returns of the index and any difference is called tracking error. Many times, the tracking error is often very small, and can be around a tenth of a percent. This error can come from indices being market capitalization weighted, meaning that for each price fluctuations of each stock lead to the weighting being changed by a ratio of its market cap against the market cap of all stocks in the index (http://www.investopedia.com/articles/exchangetradedfunds/09/tracking-error-etf-funds.asp). With these stocks weightings in the index constantly changing and people buying in and out of ETFs constantly, it is hard to track performance entirely accurately. However, ETFs very closely follow indices, as their tracking errors are generally quite small. Thus, although ETF data is not the same as index data, they are very similar.

3.2 iShares MSCI Min Vol USA ETF

The iShares MSCI Min Vol USA ETF (USMV) is a Blackrock-managed ETF that tracks the investment results of the MSCI Minimum Volatility USA index. The MSCI Minimum Volatility USA index constituents come from the MSCI USA Index, which are roughly comprised of the top 600 US stocks by market cap. This minimum volatility index is intended to have a lower beta, lower volatility, lower cap bias, and contain more stocks with less risk than its parent index, which contains US mid-cap and large-cap stocks. The index is rebalanced twice a year, on the last trading days of May and November. The index typically has around 180 constituents, with an average of 20 new additions and 14 deletions every 6 months when rebalancing occurs. Over the last five, years, the number of additions has ranged from 12 to 25, while the deletions have been between 10 and 19. Changes to the index are usually announced nine trading days before they are set to take place.

Using the Barra Open Optimizer, USMV creates a minimum variance portfolio of low risk stocks, as a subset from its parent index of USA large-cap and mid-cap stock. Using this estimated security covariance matrix, the MSCI Minimum Volatility Index is the product of the lowest absolute volatility, considering the constraints. Moreover, these additions are simply a relabeling of existing stocks in the parent index, and do not include new additions to the parent index. The low-risk stocks chosen to be in USMV are determined by a set of constraints, like maintaining a certain sector or country weight relative to the parent index.

There are many specific constraints to this index. The first is that an individual stock cannot exceed 1.5% or 20 times the weight of the stock in the parent index. The minimum weight of a security in the index is also capped at 0.05%. USMV also aims to keep the weight of specific countries within a 5% range of the weight in the parent index, or 3 times the weight of the country in the parent index. Sector weights of USMV also cannot deviate more than 5% from the sector weights in the parent index. One way turnover of the index is also maxed at 10%. Thus, taking into account these constraints, the Barra Open Optimizer creates the lowest absolute volatility portfolio possible (https://seekingalpha.com/article/3964639-understanding-ishares-msci-usa-minimum-volatility-etf)

3.3 Purpose

As mentioned, the purpose of this thesis is to create a model to that will predict rebalancing of stocks in the Min Vol index, and thus the USMV ETF, before it actually happens. There is significant price movement whenever a stock is added or removed from a large ETF, like USMV. When a stock is added to the index, the ETF will buy large amounts of that stock, increasing the demand, and consequently market price for that stock. If the stock is bought in advance of this large purchase, then the investor can enjoy pretty immediate price appreciation in the stock. Moreover, if a stock is removed from the Min Vol index, the USMV ETF will sell all current holdings of the stock, which would increase the supply of the stock, driving down market price of the stock. If one were to short this stock before that happened, he/she can also profit from that event.

A phenomena known as ETF front-running has been around for a long time and is similar to what this paper hopes to accomplish, but is one step behind. ETF front-running involves traders buying or selling stocks in advance of ETF managers after they announce an exit or entrance of a position (https://seekingalpha.com/article/165877-how-traders-are-front-running-etfs). There is typically a slight lag between an announcement of an ETF to add or remove a position, and the actual purchase or sale of this position. By acting quickly, traders can scalp profit by buying a stock before an ETF does, and selling it to them at a slight profit, or short-selling a stock before an ETF exits the position, and then buying it back at the lower price. The thesis will take this one step farther, and try predict the stock addition or deletion before announcement. This will allow traders to similar front-run the index, but they will do so before the market is able to react, leading to larger profit opportunities.

3.4 Model

These goals of this paper will be achieved by creating a logistic regression model, which will be transformed to calculate a probability of a stock being in the out of the index. The predictor variables will include 52-week trailing volatility, 52-week trailing beta, price/book ratio, and whether or not the stock was in the index 6 during the previous rebalancing. These attributes were chosen after looking at the historical literature and understanding of the minimum volatility index.

Chapter 4

Literature Review

In this portion, the Low-Risk Anomaly will be introduced, and some arbitrage opportunities that take advantage of it will be analyzed.

4.1 Low-Risk Anomaly

In this section the Low-Risk Anomaly will be discussed in detail. This challenges a widely regarded financial principle that investing in higher-risk stocks will generally result in higher expected return. Portfolios of high-risk stocks and low-risk stocks were constructed and rebalanced regularly to reflect these characteristics, and the low-risk portfolios outperformed the high-risk portfolios by a factor of several times, over long-periods of time including 1929-2015 and 1968-2008. This may raise the question why people do not just invest in low-risk stocks then, since they are inherently safer and return more.

One the explanations is the need for money managers to be compared to a benchmark, such as an index like the S&P500. This is reasonable, as many fund managers are charging fees to manage money, and need a way to prove themselves and their abilities. By outperforming an index, a fund manager is able to generate alpha, and presumably raise money money or change higher fees. By underperforming an index, the fund managers has a hard time justifying fee charged to clients, since they could just invest in the index passively for little to no fee. Thus, much of the risk for them is relative, coming from potentially underperforming a benchmark. Moreover, with this doubling of assets under active management from 30% to 60% in 1968-2008, the low-risk anomaly intensified. Another metric of fund manager performance is through the "information ratio" (IR), or the expected return difference between the manager and the expected return of the S&P 500, divided by the volatility of this return difference. The goal of an investment manager is to maximize this number, best as possible, through picking stocks that will outperform the market. These help to create a greater demand for higher-risk stocks, while discouraging investments in low-beta, low-volatility stocks, and ultimately increases the market's appetite for risky stocks with high reward potential, driving up price and driving down expected return.

In addition to the focus on relative rather than absolute risk, money managers also often focus on single period returns, often as short as a month, which aim to remove the effects of compounding. This helps differentiate each manager's stock picking abilities, but ignores the real-world and significance of compounding. Humans also have a predisposition for the lottery affect, which is increased interest in stocks with high skew - that is high upside potential. Risk can also be decomposed into macro and micro-effects - that is, looking for and comparing the risk-return characteristics of stocks with different risk profiles, but similar country and industry risks. It turned out that the micro-effects, those that were stock-specific, were statistically significant at generating alpha, while the macro-effects, those that were country and industry-specific was not. These literature reviews also examine both volatility and beta as a measure of risk, and suggest that beta is not an adequate risk metric. In fact, though beta and volatility are obviously very correlated, beta

appears to be more related to this low-risk anomaly that volatility. This can have significant implications on the significance of the predictor variables in the logistic regression model in this paper. In the following section, these findings will be highlighted and discussed in more detail.

4.1.1 High Returns from Low Risk, By Pim van Vliet and Jan De Koning

Assuming an efficient market, one of the most widely accepted tenants of investing is that one will be receive higher reward for taking on more risk. Presumably, if this were not true, nobody would partake in higher risk investments. Some risks to consider when making an investment include those with respect to the market, liquidity, credit, inflation, and FX (https://www.getsmarteraboutmoney.ca/invest/investing-basics/understanding-risk/types-of-investment-risk/). When quantifying risk for a company, one can look at the standard deviation of the stock price. By looking at the historical dispersion of data from the mean, or normal returns, one find the stock's volatility. These calculations are based only on the price fluctuations of the stock, and no other external factors (http://www.investopedia.com/ask/answers/041415/what-are-some-common-measures-risk-used-risk-management.asp). Volatility is also one of the best indicators of bankruptcy. The other common form of risk measurement in finance is beta, which measures the stock's price volatility with regard to the market. The market is typically a benchmarked index relevant to the stock; for a large-cap US stock, the associated index could be the S&P 500. Beta is calculated by taking the covariance of the stock returns and the market returns, then dividing by the variance of the market. This yields a coefficient, which can be interpreted: a beta value of 1 indicates the stock price and market move together identically, a beta value of less than 1 indicates the stock is less volatile than the market, and a beta value of greater than 1 indicates the stock is more volatile than the market. Beta values can be negative as well, and hold the same interpretation as positive beta values, with the difference being that the stock price and market move in opposite directions. For example, a beta of -1 would mean the stock and market have the same volatility and changes in price, but that the performance is inversely related. Thus, the first sentence of this paragraph can have many meanings, but can be interpreted as saying that stocks with higher beta or volatility will have higher expected return.

In this book, Jan de Koning and Pim Van Vliet set out to investigate the question: Do high-volatility stocks return more than low-volatility stocks? The authors constructed high-volatility and low volatility portfolios, and compared the two over a 86-year time span, from 1929-2015. Over this period, low volatility stocks outperformed high volatility stocks by a factor of 18, excluding inflation and transaction costs. If both portfolios started off with $100 in 1929, the low-volatility portfolio end value would be worth $395,000, while the high-volatility portfolio would be worth just $21,000. In reality these values would both be much smaller if the costs of trading the stocks and inflation were considered, but excluding them is reasonable as the effect on both portfolios would be pretty similar. The low-volatility portfolio returned 10.2% annually whereas the high-volatility stocks returned just 6.4% annually. This annual difference of 3.8% is striking, and when considering compounding over an 86-year period, explains why the low-volatility portfolio's ending value was over 18 times that of the high-volatility portfolio. In this study, the annualized volatility of the low volatility portfolio was 13%, and the annualized volatility of the high volatility portfolio was around 2.5 times that, at 36%.

These findings present an anomaly in the field of finance, and beg the question of why anybody would invest in high-volatility stocks in the first place. Moreover, one may wonder how it is possible that a portfolio of low-volatility stocks can outperform high-volatility stocks over a long period of time. This anomaly has several explanations.

It is important to understand that a higher volatile stock or portfolio will move in greater magnitude than the underlying market. This holds true for both downside and upside scenarios. When the market increases in value, a high volatility stock will increase in excess of this. For example, if the market increases by 20% in one year, a high volatility portfolio would reasonably appreciate by more than 20% over the same period. However, when the market declines, a high volatility portfolio will decrease in excess of this amount. For instance, if the market decreases by 20% in one year, a high volatility portfolio would reasonably depreciate by more than 20% over the same period. Lower volatility portfolios would react in similar fashion, just with a lesser magnitude with respect to the market. With this in mind, one way the low volatility portfolio is able to outperform

the higher volatility portfolio is by losing less during times of financial stress. The authors conveniently started their analysis right at the beginning of the most severe economic depression in American history, the Great Depression. The Great Depression began in 1929 and eroded away around 80% of the market's value by the time the recovery began in 1932 (http://www.history.com/topics/1929-stock-market-crash). With this in mind, by 1932 the high volatility portfolio declined by over 80%, while the low volatility portfolio declined by less than 80%. More specifically, the high volatility portfolio shrunk from $100 in value to just $5, while the low volatility portfolio shrunk from $100 in value to $30. Thus, the results of this paper can be taken with a grain of salt, since although the portfolios each started off with the same amount of money in 1929, the low volatility portfolio was worth 6 times as much as the high volatility portfolio just four years later. However, this is an expected consequence of the high volatility portfolio, so these results should not be discredited. With this being said, since the low volatility portfolio was able to lose less money during market downturns, it is able to grow capital more effectively than the high volatility portfolio. To illustrate this, the portfolio values can be considered in 1932. As Benjamin Graham, a famous value investor, once noted the mathematical fact that "once you lose 95% of your money, you have to gain 1,900% just to get back where you started" in his classic book "The Intelligent Investor" (The Intelligent Investor). Likewise, when the high volatility portfolio lost six times as much value as the low volatility portfolio, it would have to outperform the low volatility portfolio by significantly more than 600% in order to return to the same value.

Thus, it seems very counterintuitive that fund managers and investors do not only invest in low volatility stocks, as it appears that these stocks will outperform high volatility stocks in the long run. Part of understanding why this is not a commonplace investment strategy for many comes from interpreting what risk is defined as by the financial community. Risk is not necessarily analogous to volatility to them, or even to losing money. For many fund managers, risk comes from underperforming a benchmark. David Blitz, the head of quantitative equity research at Robeco, discussed the need for benchmarking the performance of investment managers. Many managers, especially of actively managed funds, command handsome compensation in exchange for their investment acumen and diligence. Many hedge funds have a "Two and Twenty" compensation structure, where the managers charge a 2% fee on total assets under management and take and additional 20% of profits (http://www.investopedia.com/terms/t/two_and_twenty.asp). Given the large amount of fees clients are paying, it is reasonable to believe they expect to receive a greater return than if they had invested in a passive, market tracking index. These investment professionals need to prove to their bosses and clients that they are above average in their job. For example, if a hedge fund manager returns 10% in one year, but the market returns 10% that same year, the client will be upset, as they are now receiving a smaller return, given the hefty fees. In this case a $100 investment in a market tracking index, would return approximately $10 pre-tax, given there are little to no management fees. However, $100 invested in a hedge fund with a "Two and Twenty" structure would yield the same initial $10 pre-tax return, but the client would pay 2% of $100 ($2) in management fees, and 20% of the $10 profits ($2) in additional compensation. This would leave the client with a total of $6 return, or 6%, after all of the fees are paid out. As a result, given the fee structure and client demands, active money managers are often compared to a market benchmark like the S&P 500. More importantly, they are expected to outperform these benchmarks substantially. Benchmarking also helps add perspective to a manager's returns. If a manager returns 20% in one year when the market returns 10%, he/she will have many happy clients, but if the manager returns 20% in the same year that the market returns 30%, clients will not be as pleased. This is a concept known as "relative" risk.

Many investors, like individuals saving for retirement, primarily care about absolute risk, which is the total amount of money that is gained or lost due to overall stock movements, with regard to the starting amount of money invested. They will check their portfolio's total performance without much concern for the exact market return. If their portfolio gains 20% in one year, they will be content, even if the market increases 30% in the same period. For these investors, the horizon is long-term so the short-term performance isn't as important for them as it is for fund managers who may be trying to raise more capital or justify high fees from clients. Volatility, in itself, captures these changes in the price of a stock, and is an absolute risk measurement. Many institutional investors do not look at risk on an absolute level, as a retiree or mom and pop investor may, but instead look at the risk of a portfolio with respect to market or some other widely accepted benchmark. For these investors, the risk is not so much about losing money, rather is more centered around lagging the market or their peers. Investing is very much a relative game. To further illustrate this idea, for a fund manager if a portfolio drops 20% while the market drops 40%, this is seen as a much better

outcome than if a portfolio goes up 20% while the market goes up 40%. In the former, the manager lost money, but outperformed the market by 20%. In the latter, the manager made money, but lagged the market by 20%. This concept can be hard to fully grasp due to the natural bias towards focus on absolute risk. Several other papers have tried to explain this seemingly misunderstood phenomena, including Karceski in 2002, where is was noted that an extrapolation bias could cause mutual fund managers to care more about overperforming in a bull market, than underperforming in a bear market (https://www.jstor.org/stable/3595012).

Another explanation of the low-volatility anomaly is an increased focus on returns over short-term periods by many researchers and investors. By focusing on "single period returns", which in most academic studies is just a one-month period, the significance of compounding is removed. This is more of an arithmetic way to calculate returns, where each month's return can be averaged, for example. Mathematically, this is not the correct way to calculate a return since it does remove the affect of compounding, but is common in a way to compare fund managers. Moreover, when done over very short time periods (like a year), the effect of compounding is not as significant as it is for very long-term periods. To illustrate this, the following scenario can be considered: in one month a portfolio worth $100 drops 50% to $50, then the next month increases 50% to $75. The investment return is dependent on how one divides the time period. Looking at it on a monthly basis, even though the portfolio lost $25 in value, the net return would be -50% +50%, or 0% (with focus on single period returns, not accounting for compounding). However, looking at it on a longer term basis, the net return was -25%, as the $100 portfolio ended up losing $25 in value. By not fully including the magic "return upon return" effect of compounding, the high-volatility portfolio in this book performs more than 6% better per year.

In addition to the reasons mentioned, there are several psychological reasons why some investors are not attracted to low-risk stocks. Eric Falkenstein, a renowned author in the low volatility investing realm, wrote that "envy is at the root of the investment paradox." Some investors simply don't recognize the significance of compounding returns. Many others do, but are unable to utilize the paradox due to relative risk and career pressures. Analysts who choose big winners are more likely to get recognized and promoted than those who pick safer stocks with lower upside potential; funds that pick the right high risk stocks that turn out to be major home-runs will see more reward as an increase in AUM, and consequently an uptick in management fees. Moreover, some people do not invest in low risk stocks because they have less of an appeal than high risk stocks, where investors think they can make a lot of money easily and quickly. Even the news will have a bias towards reporting about and covering stocks that have become big winners. One famous big winner is Amazon - a $5,000 investment in 1997 would be worth $2,400,000 today, or an increase 49,000% (https://www.forbes.com/sites/robertberger/2017/05/20/amazons-49000-return-a-test-for-value-investors/#7b8efe2049cf). With all the excitement and reporting to this day on big winners like Amazon, many people forget about the number of similar technology companies that failed and are worth nothing today. These high-risk stocks are more "sexy" and have a "lottery ticket" element that attracts investors with the appeal of a big payday.

4.1.2 Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly, by Malcolm Baker, Brendan Bradley, and Jeffrey Wurgler

In the paper, the authors performed a study similar to that done by van Vliet and De Koning, except using 41 years of CRSP data, ranging from January 1968 – December 2008. Once again, it is important to note the ranges of dates used. The Great Recession began in 2007 after the bursting of the subprime mortgage bubble, leading to the collapse of several large, investment banks, and the government bailout of many others. This caused market indices like the Dow to drop from a high in 2007 of 14,164.43 to 8,776.39 by December 2008, representing a decrease in over 38% during that period (https://www.thebalance.com/stock-market-crash-of-2008-3305535). Thus just as van Vliet and De Koning started their sample right before the Great Depression to help amplify their results, Baker, Bradley, and Wurgler ended their sample after the Great Recession. Though this may help amplify the results, this does not take away from the significance of the findings.

Low-volatility, high-volatility, low-beta, and high-beta portfolios were constructed using the top 1,000 stocks by market capitalization, and then calculating each stock's five-year trailing volatility or beta. A dollar investment in the low-volatility portfolio in 1968 appreciated to $59.55 by 2008, or $10.12 in real terms when

accounting for inflation. On the other hand, the highest-volatility portfolio went from a dollar in value to 58 cents from 1968-2008, with a real value of just around 10 cents when considering inflation. Thus the low-volatility portfolio outperformed the high-volatility portfolio by over 100 times both in terms of nominal and real value. When using beta as a measure for risk, the finding was very similar. In the lowest-beta portfolio, a dollar grew to $60.46 in nominal value throughout the 41-year period, or $10.28 in real value after considering inflation. The highest-beta portfolio grew from a dollar to $3.77 in nominal terms, or $0.64 in real terms after accounting for the effects of inflation. Thus, the low-beta portfolio outperformed the low-beta portfolio by a factor of 16, in nominal and real terms. One interesting observation here is though both the low-beta and low-volatility portfolios outperformed the high-beta and high-volatility portfolios, respectively, the discrepancy was far more pronounced in the case of the the volatility portfolios. Another significant observation was that investors who owned either the high-beta or high-volatility portfolio in 1968, would have lost money, when accounting for the effects of inflation by 2008. Another important thing to note is that this was just the case for large-cap companies, as these portfolios were constructed from the top 1,000 stocks in terms of market cap. The fact that this anomaly was observed for large-cap companies is quite impressive, given there are generally a lesser degree of mispricings in that realm than with small-cap companies, because many small-cap stocks are not big enough for institutional investors to purchase. Thus, this anomaly would be larger if done for small-cap stocks instead. In addition, the portfolio end values assumed no transaction costs, which in reality should be considered. The high-beta and high-volatility portfolios cost more to rebalance on a monthly level, as was done in the paper, than the low-beta and low-volatility portfolios, indicating this anomaly is actually more pronounced that initially reported. Baker et al. noted that while high-beta portfolios outperformed the low-beta portfolios in up markets and underperformed the low-risk portfolios in down markets, that the low-beta anomaly persisted in both situations. On a market-adjusted basis, low-beta consistently generates high alpha. This was consistent with prior research (https://www.jstor.org/stable/2331255?seq=1#page_scan_tab_contents).

These findings are not new or revolutionary, and have been observed in many previous academic articles. Black, Jensen, and Scholes evaluated some of the assumptions and effectiveness of the Capital Asset Pricing Model (CAPM) in 1972 (http://citeseerx.ist.psu.edu/viewdoc/download;jsessionid=61A366AB71D4D50283E0DCF0CBBC9FB5?doi=10.1.1.665.9887&rep=rep1&type=pdf). The CAPM is a model that quantifies the relationship between risk and expects return for a stock, and considers investors should be compensated for the time value of their money and risk they are taking. This is calculated as the risk free rate plus the beta times the difference between the risk free rate and expected market return (http://www.investopedia.com/terms/c/capm.asp). This will give the expected return of the stock, and anything in excess of this will be considered alpha. Black, Jensen, and Scholes were able to find that expected excess return, alpha, but not strictly proportional to Beta, as it is in the CAPM. In 1975, Haugen and Heins similarly find the there is little support for the idea that risk premiums have manifested themselves in realized rates of return (https://www.jstor.org/stable/2330270?seq=1#page_scan_tab_contents). In fact, they point out that the relationship between risk and return is much flatter than it is in the CAPM. In 1992 Fama and French famously declared that beta was dead after finding a flat relationship between beta and return (http://faculty.som.yale.edu/zhiwuchen/Investments/Fama-92.pdf). Many additional papers and research has added evidence for disproving the Capital Asset Pricing Model (CAPM), and even suggest that beta may not be the correct measure of risk, and that the relationship between risk and return is not what many people believe it to be. However, other models relating risk and return, have had difficulty gaining acceptance and widespread usage in the finance industry. This paper is somewhat unique, in that Baker et al. does not try to prove what has already been shown in several academic works of literature, but instead offer several explanations for why this discrepancy exists.

One theory that is explored in detail is an investor's irrational preference for high-volatility stocks. Many investors have a natural preference for lotteries, even though there is a general aversion towards loss. If a stock has a positive skew, which is defined as a larger probability of a large positive payoff than probability of a small payoff, investors typically are very interested. Though skew is not the same as volatility, in their paper in 2010, Boyer, Mitton, and Vorkink make a strong case for how expected skewness is a proxy for volatility through their findings that expected skewness assists in explaining the observation that stocks with high idiosyncratic volatility have low expected returns (https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1541002). Another idea is representativeness, or that Bayes' rule and probability theory are often not natural

to people, even the most seasoned investment professionals. One example of this is selectively looking at a few speculative investments that have turned out to be massive successes, without considering the numerous failures. As mentioned earlier, the news will focus on Amazon's great success over the past twenty years, but will not focus as much on all of the tech stocks that became worthless after the bubble burst. By not separating and considering all winners and losers, the average investor may be inclined to overpay for a riskier stock. Overconfidence has also been tied to a preference for volatile stocks; optimists are generally more aggressive than pessimists. In a study, people were asked questions and how certain they were of their responses and it appeared many did not have an understanding of probability (http://web.mit.edu/curhan/www/docs/Articles/biases/3_J_Experimental_Psychology_Human_Perception_552_(Fischoff).pdf). Estimating the heat of a candle flame is very difficult, so for someone to say they are 80% sure of their answer is impressive, yet hard to believe. This can apply when people are asked for a confidence interval of the population of a city. Many times the person would be confident and give a very narrow range of people. This same concepts applies when valuing stocks and evaluating certain investment opportunities.

Moreover, as mentioned previously, the need for benchmarking, especially among institutional investors, is also believed to heavily play into the anomaly. In fact, in the first year of this study, institutional investors managed 30% of all money, but by 2008, the final year of the study, this figure increased to 60%. With this doubling of assets under active management, the anomaly being discussed intensified. This still begs the question as to why institutional investors do not buy more low-volatility stocks, and the answer has to do with benchmarking. One explanation involves how the typical fund manager is measured for their performance. One common metric is the "information ratio" (IR) which is the expected return difference between the manager and the expected return of the S&P 500, divided by the volatility of this return difference (tracking error) (http://www.investopedia.com/terms/i/informationratio.asp). The goal of an investment manager is to maximize this number, best as possible, through picking stocks. In 2009, Sensoy showed that over 61% of U.S. mutual fund managers are benchmarked against the S&P 500, while over 94% are benchmarked to some U.S. index benchmark (https://fisher.osu.edu/sites/default/files/performance-evaluation-jfe.pdf). Moreover, SEC rules require mutual funds to compare their performance to some benchmark (https://www.sec.gov/reportspubs/investor-publications/investorpubsinwsmfhtm.html). This intuitively makes sense, as it allows investors to assess the skill and ability of managers in an unbiased way, and also allows fund managers a chance to differentiate themselves. However, this makes institutional investors, who are managing the majority of the money in the US, less likely to buy low-volatility stocks, leading to higher prices and lower expected returns for the high-volatility stocks and further exacerbating the low-risk anomaly.

Investment managers without leverage will try to find mispriced stocks with a beta very close to market risk (beta of 1), overweighting positive-alpha stocks while underweighting negative-alpha stocks. When comparing the Sharpe ratio of large cap stocks for a low-volatility portfolio, it was found to be quite high at 0.38. However, IR was a very low 0.08, showing this would be very tough for a fund manager to invest in. To provide a comparison, during 1968-2008, the top value strategy portfolios had an IR of 0.51, and top momentum strategy portfolios had an IR of 0.64. This is extremely high compared to the IR of low-volatility stocks in this period, which ranged from 0.08 to 0.17, showing these constructed low-beta portfolios would being used by any fund manager. While Beta and volatility are undoubtedly very correlated, this paper showed that beta is more related to the anomaly than volatility, especially with large cap stocks, which is what most fund managers disproportionately focus their investments in.

Overall, irrational investor preference for lotteries and high volatility stocks, as well as investment managers' focus on benchmarks and IR, flatten and eventually invert the relationship between risk and return in the long-run. Moreover, it has been shown by Baker et al., and prior observations that the anomaly intensified with the increase in assets under active management of fund managers in the U.S. This reasons together have led to the findings that low-beta and low-volatility stocks have outperformed high-beta, high-volatility stocks from 1968-2008, in part due to combining great returns with low downturns. Investor preference for "lotteries" and a bias of overconfidence creates a higher demand for higher-volatility securities, and the need to benchmark creates a greater demand for higher risk stocks, while discouraging investments in low-beta, low-volatility stocks. This understandably, increases the market's appetite for risky stocks with high reward potential, driving up price and driving down expected return. These reasons appear perennial, so the anomaly is unlikely to cease to exist in the near future.

4.1.3 The Low-Risk Anomaly: A Decomposition into Micro and Macro Effects

The low-beta anomaly, which as discussed, relates to the outperformance of low-beta stocks, can be broken up into micro and macro effects. The micro effects include picking low-beta stocks, while keeping country and industry risk the same, while the macro effects involve picking low-beta countries or industries, while keeping stock-specific risk the same. In this paper, the micro effects were recorded by creating portfolios of equity longs and shorts, holding forecasted country and industry risk constant. The macro effects were observed by constructing long-short portfolios of various countries and industries, holding forecasted stock-specific risk constant. Studying a number of stocks within 29 industries and 31 different developed countries, the macro and micro effects were observed, and both together were shown to play an important role in the low risk anomaly.

Looking at 31 developed countries including Canada, France, Germany, Japan, and Singapore, the paper worked to decompose the low risk anomaly into country and stock specific effects. Similar to the industry findings, country-beta was able to predict stock betas to a certain extent, but not as well as historical stock betas were. Looking only at country betas yielded around half the risk reduction and two-thirds the risk adjusted return improvement, as compared to stock betas. This study implies that predicting risk of individual stocks is in itself very hard when only given data on country or industry risk, but when given all the data can have much more predictive power.

It was found that using industry beta to predict future stock betas was possible, but not as effective as just using historical stock betas. Industry beta information without stock information does improve risk-adjusted returns, just not to the same extent as it does with stock information. The paper also goes into detail trying to isolate pure industry effects and pure stock effects. Pure industry effects are the average differences between high and low beta industries, while holding constant stock risk. Pure stock risk is the opposite of that, calculating the average difference between high beta and low beta stocks, keeping industry risk constant. In the end, finding low-risk portfolios using selection of low risk stocks keeping industry constant was around four times more effective than using industries and keeping stock risk constant. Using the historical betas of both together, however, has more predictive power than either one alone.

Micro-selection of stocks, holding country and industry risks constant, was shown to be able to significantly reduce risk without a significant decrease in return. In some cases, high-risk stocks within particular industries were able to be distinctly identified, and they typically had similar returns when compared to low-risk stocks in the same industry and country. Macro-selection, holding stock-specific risks constant, was shown to lead to increases in return with small differences in risk especially with regards to the country chosen. High-risk countries were found to have distinctly lower returns than low-risk countries. These findings hold significant investment opportunity, due to the implication that people seeking arbitrage opportunities through mispricing of macro-effects like industry and sector, or through ETFs might not be as successful as exploiting the risk reduction opportunities stemming from micro-effects. While both the micro and macro-effects led to higher CAPM alpha by reducing risk and increasing returns, only the micro effects were found to be significant, whereas the macro-effects of country selection and industry selection were found to not be statistically significant. There is more of an arbitrage opportunity exploiting the micro-effects of individual stock selection than the macro effects. Even though the macro-effects have many limitations in practice, micro-effects are also limited due to leverage restrictions and benchmark mandates.

4.1.4 Understanding Defensive Equity, by Robert Novy-Marx

Defensive equity strategies generally will aim at including more safe/defensive stocks than risky/aggressive stocks, often with respect to a stock's volatility or beta. This strategy has been becoming very popular as of late, due in part to equity markets that have suffered two recent, severe downturns, negative nominal returns in the first decade of 2000, and literature proving a weak or negative relationship between risk and return. Lots of prior literature has looked at the relationship of performance relative to size and value, like some of the papers discussed above, but many have not done a deeper dive into the effect of specific factors, such as profitability. In fact, it has been shown that high profitability is the most significant predictor of low volatility, inherently causing these defensive equity strategies to indirectly tilt towards profitability,

in addition to more known factors like size and value. Size is very important, as small growth stocks are typically underweighted in these strategies, while large value stocks are overweighted. Likewise, value stocks are typically overweighted, while growth stocks are underweighted. Thus, the performance of defensive equity strategies can be explained through accounting for size, value, and profitability.

In terms of performance, defensive equity strategies, which are defined as low-volatility or low-beta in nature, have outperformed more aggressive strategies. This was already shown by van Vliet and De Koning in their research between 1929-2008, but this study analyzes a different timeframe. In this instance, low-volatility portfolios outperformed high-volatility portfolios from 1968-2015, and low-beta portfolios outperformed high-beta portfolios from 1968-2015. The paper also looks at the characteristics of these portfolios is more detail by analyzing log-likelihood ratios that a stock selected at random is of a given style, like size or value, relative to the unconditional probability of being that style. The findings were very telling. On average, low-volatility stocks were 30 times larger than high-volatility stocks. As a result, though the high-volatility names made up half the total number of stocks, they contributed to just 9% of the total market capitalization when compared with low volatility stocks. Moreover, when looking at the average returns across the portfolios of various volatilities, there seemed to be a relatively flat, slightly increased return with increasing volatility. The high-volatility/high-beta portfolio had a significant negative alpha, while the low-volatility/low-beta portfolio had a significant positive alpha.

Many of the volatile stocks tend to be small, unprofitable growth stocks, which can help explain the relationship of the defensive strategy performance to size, value, and profitability. In fact, since 1968-2015, the portfolio of high-volatility stocks almost mirrors the performance of the portfolio of unprofitable, small growth stocks. Thus, the outperformance of defensive stocks from 1968-2015 have delivered significant alphas, and can be explained when accounting for size, value, and profitability. In fact, profitability itself is so significant, that the case can be made that this alpha may be due in large part due to excluding unprofitable, small growth stocks. ## Low-Risk Anomaly Applications In this section, applications utilizing the Low-Risk Anomaly will be discussed. One example includes betting against Beta (BAB), which is a strategy used by quantitative hegde funds like AQR. Betting against Correlation (BAC) is a similar strategy in that it decomposes the effects of beta into two separate factors for a more concentrated investment. Moreover, index additions and deletions have proven to be instrumental in influencing the price of affected stocks, indicating that if a model can predict these movements accruately beforehand, there is an additional considerable arbirtrage opportunity.

4.1.5 Betting Against Beta, by Andrea Frazzini and Lasse Heje Pedersen

One of the observations from prior literature looking at the Low-Risk Anomaly suggests that Beta is not a great measure of risk, and is more related to the anomaly than volatility is. AQR, a successful quantitative-focused hedge fund, employs a strategy called Betting Against Beta (BAB), which is a simple method of statistical arbitrage generated by shorting high-beta stocks and longing low-beta stocks (http://www.investopedia.com/articles/investing/082515/how-aqr-places-bets-against-beta.asp). As discussed in some of the works previously, the premise behind this arbitrage opportunity is that high-beta stocks are overpriced while low-beta stocks are underpriced. The theory is that the stocks will eventually return to this equilibrium point, called the security market lined (SML). While the prices approach this median, the investor can capture this spread as an arbitrage opportunity.

The Capital Asset Pricing Model (CAPM) calculates the expected equity return given certain levels of risk. Any excess above this risk-adjusted return is the Sharpe Ratio, or alpha that is generated by the stock. Investors try to maximize this number, and one way to do it is by levering up, or using borrowed capital to invest. By paying a fixed interest rate on the borrowed capital, assuming the return is more than the interest rate, an investor can increase the return on their invested equity. As one can imagine, this is more risky, as returns in both the upward and downward direction are magnified. As a result, many investment managers like mutual funds are legally constrained on the amount of leverage they can apply to a portfolio. Due to this, many fund managers must overweight high-beta stocks to improve overall returns. This leads to a tilting towards beta, and a flattening of this SML in relation to CAPM. This leads to a pricing anomaly which firms

like AQR can take advantage of. By creating a market neutral strategy, and shorting high-beta stocks while longing low-beta stocks, they can capture this opportunity.

In this paper, a real-world resembling model is created with leverage and margin constraints in 55,600 stocks from 20 global stock, bond, credit, and futures markets. Some agents in this model cannot use any leverage, and some have limited margin constraints, much like many investors and fund managers. As mentioned, many mutual funds, pension funds, and individual investors are constrained by the amount of leverage they can take on, such that instead of investing in a portfolio yielding the highest Sharpe Ratio, they are forced to overweight portfolios with higher-risk stocks. This suggests fund managers hold high-beta stocks to a lower risk adjusted return standard than low-beta stocks, which would require leverage. Thus, if one cannot leverage or has significant leverage constraints, then this agent will overweight riskier securities. The model in this paper was able to empirically show this in the equities, bonds, and futures markets. This was done by sorting portfolios by betas, and realizing alphas and Sharpe Ratios declining with increases in portfolio-beta.

Presumably, if one could lever up without constraint, the investor would underweight high-beta assets and overweight low-beta assets. BAB factors help explain this better. A BAB factor is a portfolio longing low-beta securities (leveraged to a beta of 1), shorting high-beta assets (deleveraged to a beta of 1), and is market neutral. The model in the paper predicts that this portfolio will have a positive return, that increases with the spread in the betas and tightness of leverage constraints. Thus, longing low-beta and shorting high-beta yields significant, and positive risk-adjusted returns. This was observed in the model by looking at U.S., developing, and international equity markets and observing that the BAB factor yielded a Sharpe ratio that was double its value effect, and 40% greater than momentum. The BAB factor had very high risk-adjusted returns, and during four twenty-year periods between 1926 and 2012, produced significant positive returns. This generally held across other asset classes, including credit and treasury bond markets.

When a leverage constraint was met or surpassed, and the agent needs to deleverage, the BAB factor portfolio experiences negative returns, but its expected future returns still increased. This was once again shown with a time series with spreads of various funding constraints. Another central idea of the model was that increased funding liquidity risk compresses betas toward one. This was proven by looking at the volatility of funding constraints as funding liquidity risk; the end result was that the dispersion of betas when funding liquidity risk is high, and was much lower than when funding liquidity risk is low. In other words, tightening of funding constraints leads to a lower BAB factor.

Finally, the model showed that investors that are more constrained are forced to overweight riskier securities, while investors without such constraints can overweight low-risk securities. Studying a number of stock portfolios from constrained investors, most fund managers and individual investors' portfolios have a beta greater than one. However, many private equity firms that perform a leveraged buyouts are traditionally able to purchase firms with a beta below one, and apply leverage, allowing them to utilize this anomaly since they have lower leverage constraints than their public market counterparts. Great investor Warren Buffett even bets against beta, as many of his investments are leveraged, low-beta stocks. By having these constraints, though, the typical investor is forced to hold on to riskier, high-beta stocks, leading to effective of the BAB factor.

4.1.6 Betting Against Correlation: Testing Theories of the Low-Risk Effect, by Cliff Asness, Andrea Frazzini, Niels Joachim Gormsen, and Lasse Heje Pedersen

As mentioned previously, the "low-risk effect" is the idea that lower-risk or lower-volatility stocks, tend to generate a higher alpha than higher-risk or higher-volatility stocks. In trying to understand the reason this anomaly occurs, Asness et al. consider two possible explanations. The first looks at at whether this is caused by leverage constraints, or measurement using systematic risk. The second focuses on the behavioral effects, or idiosyncratic risks. One of the main issues with prior research, is that many low-risk factors are correlated and interrelated, making it hard to completely isolate certain factors or effects. In this paper, global data was used, with a couple new factors meant to control for existing factors.

"Financial Intermediaries and the Cross-Section of Asset Returns" by Adrian, Etula, and Muir in 2014, showed a link between return to the Betting Against Beta (BAB) factor and financial intermediary leverage (http://faculty.som.yale.edu/tylermuir/documents/FINANCIALINTERMEDIARIESANDCROSSSECTIONASSETRETURNS.2013.pdf). Many of these factors, including BAB, generally exhibited the "low-risk effect" and are were consequently very difficult to completely distinguish from one another. Thus, Asness, Frazzini, Gormsen, and Pedersen decided to attempt just that, by breaking down BAB into two separate factors: betting against correlation (BAC) and betting against volatility (BAV). This is done because beta itself can be decomposed into the stock's correlation with the market times its own volatility, divided by the market's overall volatility. BAC is accomplished through longing stocks with a low correlation to the market, and shorting those with a high correlation to the market, while trying to match the volatilities of both the long and short portfolios. BAV is achieved in a similar manner, except instead of longing and shorting correlation, volatility is longed and shorted while correlation is kept constant.

To address the behavioral explanation, the paper looks at some prior factors from observations made by Ang, Hodrick, Xing, and Zhang in their paper "The Cross-Section of Volatility and Expected Returns" (http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2006.00836.x/abstract). They found stocks with low idiosyncratic volatility (IVOL) had a greater risk-adjusted return, and in 2009 found that a low maximum return (LMAX), a measure of idiosyncratic skewness, is associated with greater risk-adjusted returns (http://web.ics.purdue.edu/~zhang654/jfe2009.pdf). This paper being discussed kept the focus on the LMAX and IVOL, but added another factor, scaled MAX (SMAX), which longs stocks with a low MAX return divided by ex-ante volatility, and consequently shorts stocks with a high MAX return divided by ex-ante volatility. This allows focus on the lottery demand, holding volatility relatively constant and only focusing on the distribution of the returns. Margin debt held by investors, and investor sentiment were also noted.

Overall, 58,415 stocks from the MSCI World Index from 24 different countries between January 1926 and December 2015 were covered. BAV and BAC ended up being very successful in controlling for the other factors that could influence the "low-risk effect." The BAV findings are in line with prior findings, and can be explained through behavioral effects like the lottery preference and leverage aversion. For all stocks, the BAC factor produced a significant six-factor alpha that was nearly independent of the other low-risk factors studied. This was partially due to the leverage aversion which indicates correlation changes in beta should be priced in. In terms of explaining the behavioral side with factors, SMAX was the only truly great, resilient measure used. The rest generally had higher turnover, and were consequently very susceptible to microstructure noise. SMAX attained positive risk-adjusted returns in the U.S. but negative risk-adjusted returns globally, which was seen with some other idiosyncratic risk factors. The paper showed that systematic low-risk factor generally tended to outperform behavioral risk factors, especially when considering turnover and time period length. All in all, the low-risk effect was believed to be driven by multiple factor effects, meaning both leverage constraints and the demand for lottery could play a role in influencing this. However, leverage constraint effects were a bit stronger, especially internationally. By cleaning up these factors, this paper is able to provide clearer explanations for previously observed results.

4.1.7 Price Response to Factor Index Additions and Deletions, by Joop Huij and Georgi Kyosev

One of the driving fundamental assumptions of finance is a flat demand curve for stocks, where risk is the main driver and each stock has a perfect substitute. However, this concept has been questioned for the past few years, with literature picking up on stocks showing supply shocks and quantifying how this affects their market price. Literature has shown several instances where large block sales of stock has negatively affected its price. This was often due to information contamination, which is new, significant information about the company in the market. This information often reflects fundamental changes in the company, and if it is negative, will understandably trigger block sales. Thus, the price change is less due to the supply shock, and moreso due to the fundamental change in the company's value (like a scandal or earnings report).

However, interesting patterns, that have not yet been fully explained, emerge from observations regarding S&P 500 company addition and deletions. When companies are added or removed from the index, it is

purely mechanical, and usually not due to some drastic fundamental change in the company. Assuming the market is efficient, the demand for stocks should not change due to being added or removed from an index, but several studies have shown that it does. Harris and Gurel (1986) (http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.1986.tb04550.x/full), Shleifer (1986) (http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.1986.tb04518.x/abstract), Beneish and Whaley (1996) (http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.1996.tb05231.x/full), Chen, Noronha, and Singal (2004) (http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2004.00683.x/full) all show how new additions to the S&P come with higher than normal returns for that company. Though they agree on the price movement, the studies tend to disagree on the precise reason for this price movement; some possible explanations include compensation for providing liquidity, better monitoring for investors when a company is added to a reputable and large index, and higher analyst coverage leading to more information and analysis available on the company. One primary concern is whether or not index reshuffling is an information-free event - that is, whether a company being added or removed adds information to the market about the company.

In this paper, the authors look at factor index rebalancing for an information-free event. Factor indices are part of a parent index of many other stocks, and are constructed in a mechanical way that is publicly available and usually based on ranking stocks off a particular ratio of characteristic. Looking at the MSCI Minimum Volatility (USMV) index, returns were recorded for the stocks that had been added/dropped. It was found that the cumulative return from announcement to the effective day was 1.07% for stocks added with a significant t-statistic of 7.16, with 62% of the stocks exhibiting a positive cumulative abnormal return. Of the 1.07% increase, 0.63% of it was gained the following day, indicating that a large part of the increase is due to greater demand from index funds. 0.31% of the return is lost five days after the rebalancing, but generally the price tends to stabilize afterwards after ten days. Thus, 68% of the price increase is permanent, while the other 32% is temporary and lost after a few days. This can be due to a number of reasons including a liquidity premium charged by the stock's owner or arbitrage activity. Average trading volume was also significantly more for stocks that were recently added to the index. For the days between the announcement and actual addition of the stock, the average trading volume was 30% higher than normal, with a significant t-statistic of 3.81. Moreover, there is a 74% increase in volume for the day prior to the actual adding of the security. A very similar phenomena occurs with stocks set to be dropped from the USMV Index; from the announcement of a stock being dropped to the day before it is actually deleted from the index, the total cumulative abnormal return is -0.91%, and a total of -0.57% comes the day right before. After the stocks are deleted, 64% have a negative return the following day, and only 0.49% of the -0.91% is regained after three weeks. Trading volume also spikes 46% on the day prior to removal from the index. After three weeks, it returns back to within 1% of the normal trading volume.

These findings imply that once a security is added to a factor index, the demand curve shifts to the right, moving the equilibrium. The trading volume change is likely due to index funds buying or selling massive amounts of the stocks that will be added or removed. Moreover, it was found that the amount of the return is also directly related to the weighting of the volume of stocks entering or leaving the factor index. All in all, these findings suggest an index arbitrage opportunity if the index additions or deletions can be predicted.

Chapter 5

Data Collection and Summary Statistics

5.1 EUSA and USMV Data Compilation

Data was downloaded from www.ishares.com for EUSA (iShares MSCI USA Equal Weighted ETF) and USMV (iShares Edge MSCI Min Vol USA ETF), from Oct 31, 2011 to December 31, 2016. As mentioned, iShares are a type of ETF managed by BlackRock that track the MSCI Minimum Volatility Index. iShares contains the month end data for the two ETFs of interest for each constituents. It included characteristics of stock, including: ticker, company name, asset class, weight of the stock relative to the entire index, price per share, number of shares, market value of the position, notional value of the position, sector, sedol number, isin number, exchange that the stock is listed on, and the month end date for the data. On the website, iShares had data for the positions and constituents of each ETF, for the last trading day of every month. Each month-end data set was individually downloaded, then aggregated to create the two separate raw data sets. The data was then cleaned.

5.2 EUSA and USMV Data Cleaning

After having a quick overview of the data, there were many issues with each respective data set that needed to be fixed before the analysis could begin. As USMV is a subset of EUSA, the issues were very similar, and those that existed in USMV, generally existed in USMV as well. The issues could be broke down into 3 main types: erroneous listed stock exchanges, problematic listed tickers, and price discrepancies due to issues like stock splits. Moreover, cash and cash related assets were removed from the data, as this dissertation focuses only on the stocks.

5.2.1 Non-US Exchanges

Looking at the unique exchanges of the data, it was observed that there were many foreign exchanges like the Swiss Exchange and the Mexican Exchange, which did not make sense, given the ETF constituents are supposed to be listed on US-based exchanges. These could be broke up into two more groups: companies that were incorretly listed overseas and are actually listed on US exchanges, and companies that also are actually listed on US exchanges but instead had their overseas exchange tickers listed.

The first type of error was from companies that are listed on either the NYSE and NASDAQ in reality, but were curiously listed on an foreign exchange instead in the data, but still had their US ticker used. One

example was BAC, Bank of America, which is listed on the NYSE, but was listed on the Swiss Stock Exchange in the data set. The price for BAC in the data set corresponded to the price of BAC in the NYSE, although it was listed on the Swiss Exchange. Moreover, BAC did not corresponded to Bank of America on the Swiss Exchange. Thus, after several checks, it could be concluded that BAC in the dtaa set was incorrectly listed on the Swiss Exchange, and should have been listed on the NYSE instead. Since the ticker would still be able to be read into WRDS, these cases were left as is, and no changes were made.

The second type of error was from companies listed on foreign exchanges that are listed on a US exchange as well, but their non-US ticker used. One example of this was Aflac, Inc. which was listed by its ticker "8686" on the Tokyo stock exchange. This was immediately a red flag due to the numbers in the ticker. This numeric ticker corresponded to Aflac, Inc. on the Tokyo exchange, but when checking the recorded price of the stock for corresponding dates, it matched up with the Aflac, Inc. stock on the NYSE, with ticker "AFL". Thus, when this happened, each company was treated on a case-by-case basis. In this case, since the stock price corresponded to AFL, the ticker name was changed from "8686" to "AFL". This would ensure the data could be properly read in from WRDS.

Overall, even with these numerous errors, it was a good sign because it implied that the data was generally correct (no internationally listed companies), but just recorded incorrectly. Thus, after making these changes, it was safe to assume the data was for the most part accurate.

5.2.2 Unrecognized Tickers

Another general type of error in the data occured when the ticker was not read into WRDS, causing all the prices for that ticker and company to be "NA". This was evaluated, once again, on a case-by-case basis, by observing which tickers WRDS did not recognize, and looking at the company name to understand why. Sometimes, the issue was very obvious. One example of a clear discrepancy was when the ticker had an asterisk at the end of it. After careful digging, the asterisk did not seem to mean anything, and it is unclear why some tickers contained it. One example was "AAPL*". This caused issues for reading the data in from WRDS, because that ticker was not read in as "AAPL" due to the asterisk. This was fixed by simply removing the asterik from the ticker name.

Another example of the ticker not being read in properly was when it contained numbers. Alflac was an example that was mentioned previously, but another one that applied here was "AG4" which was the ticker for Allergan. Since NYSE and NASDAQ tickers do not contain numbers, this was a clear red flag. After some research, it appeared AG4 is the ticker for Allergan on the Deutsche Boerse AG Stock Exchange. However, the prices corresponded to Allergan's on the NYSE. Thus, the ticker was changed to the ticker used for Allergan on the NYSE - AGN. Overall, though each category is unique, there has been a lot of overlap, and often times correcting one type of error would fix other errors too. For example here, many tickers that include numbers will not be read in, and this is usually because the ticker corresponds with the same company but on a foreign exchange.

5.2.3 Price Discrepancies

The general methodology to ensure a change in ticker was appropriate was to check the price of the stock at a specific date, in the EUSA data set, and then comparing it to the new ticker being assigned. If the price matched, the change was made. If the price did not match up, and was very different, research was performed to see if a stock-split might be the cause of this. If there was no evidence of a stock-split, then the stock further analyzed to see what the issue was. In addition to looking and when prices did not match up with tickers and companies for certain dates, monthly returns were calculated for each stock during the times they were in the index, and any abnormal returns (magnitude greater than 30% in one month) were look at manually. One example of this was Netflix's stock 7:1 stock split in 2015. The monthly data showed drastic fall in price from 656.94 on 2015-05-29 to a 114.31 on 2015-07-31, in just one month. This amounts to recorded loss of 82.5%. Since this surpassed the threshold set, it was look at in more detail. After some research, it was shown there was in fact a 7:1 stock split, so the price of the stock on 2015-07-31 was adjusted

to 800.17, and the appropriate calculations were done. Thus, in this case, the ticker was left alone, but just the price was adjusted.

Tickers that could not be determined were removed. In the end, the ticker named "1015736" and Orchard Supply Hardware Stores were removed from the data set. These together accounted for less than 0.2% of the data from one month-end date.

5.3 EUSA and USMV Data Overview

To get a sense of the EUSA data, summary statistics are shown below:

```
##      ticker                            name             asset.class
##  CB     :  101   3M CO                  :   63   Cash        :    0
##  AGN    :   67   ABBOTT LABORATORIES    :   63   Equity      :38598
##  NLSN   :   64   ACCENTURE PLC          :   63   Money Market:    0
##  A      :   63   ACTIVISION BLIZZARD INC:   63
##  AAP    :   63   ADOBE SYSTEM INC       :   63
##  AAPL   :   63   ADVANCE AUTO PARTS INC :   63
##  (Other):38177   (Other)                :38220
##      weight           price            shares        market.value
##  Min.   :0.0000   Min.   :  0.56   Min.   :     0   Min.   :2.000e+03
##  1st Qu.:0.0536   1st Qu.: 35.07   1st Qu.:   798   1st Qu.:5.289e+07
##  Median :0.1208   Median : 55.09   Median :  1546   Median :8.282e+07
##  Mean   :0.1629   Mean   : 71.91   Mean   :  3365   Mean   :1.610e+08
##  3rd Qu.:0.1636   3rd Qu.: 83.52   3rd Qu.:  3181   3rd Qu.:1.410e+08
##  Max.   :4.6773   Max.   :953.00   Max.   :133289   Max.   :6.804e+09
##                   NA's   :46
##  notional.value                          sector         sedol
##  Min.   :     40.5   Financials            :6825   2000019:   63
##  1st Qu.:  54595.6   Consumer Discretionary:6595   2002305:   63
##  Median :  74215.6   Information Technology:5487   2005973:   63
##  Mean   :  89651.9   Industrials           :4687   2008154:   63
##  3rd Qu.: 116297.5   Health Care           :4067   2011602:   63
##  Max.   :2106050.4   Energy                :3208   2018175:   63
##  NA's   :22878       (Other)               :7729   (Other):38220
##             isin
##  AN8068571086:   63
##  BMG0450A1053:   63
##  BMG0692U1099:   63
##  BMG169621056:   63
##  BMG3223R1088:   63
##  BMG491BT1088:   63
##  (Other)     :38220
##                                                 exchange
##  New York Stock Exchange Inc.                      :27499
##  NASDAQ                                            : 9238
##  Boerse Berlin                                     :  427
##  Deutsche Boerse Ag                                :  394
##  Bolsa Mexicana De Valores (Mexican Stock Exchange):  266
##  (Other)                                           :  658
##  NA's                                              :  116
##       date
##  Min.   :2011-10-31
```

```
##  1st Qu.:2013-02-28
##  Median :2014-06-30
##  Mean   :2014-06-12
##  3rd Qu.:2015-09-30
##  Max.   :2016-12-30
##
```

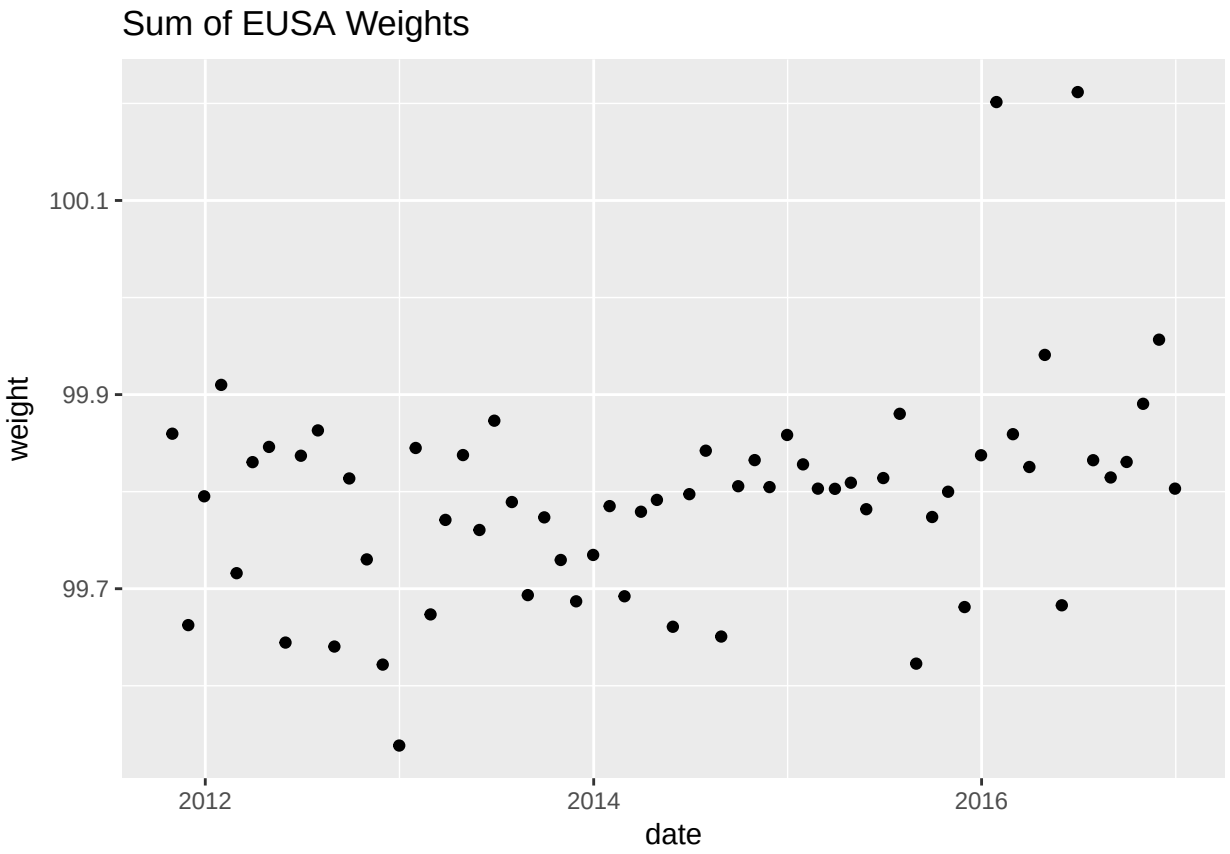
To get a sense of the USMV data, summary statistics are shown below:

```
##      ticker                                name            asset.class
##  CB     :  81   ABBOTT LABORATORIES          :  63   Cash        :   0
##  ABT    :  63   ALTRIA GROUP INC             :  63   Equity      :9229
##  ACGL   :  63   ARCH CAPITAL GROUP LTD       :  63   Money Market:   0
##  ADP    :  63   AT&T INC                     :  63
##  AMT    :  63   AUTOMATIC DATA PROCESSING INC:  63
##  AZO    :  63   AUTOZONE INC                 :  63
##  (Other):8833   (Other)                      :8851
##      weight           price            shares          market.value
##  Min.   :0.0002   Min.   :  0.56   Min.   :       6   Min.   :     1138
##  1st Qu.:0.2790   1st Qu.: 48.52   1st Qu.:   60888   1st Qu.:  5109636
##  Median :0.6016   Median : 71.52   Median :  233116   Median : 18493183
##  Mean   :0.6810   Mean   : 93.10   Mean   :  462436   Mean   : 30802130
##  3rd Qu.:1.0191   3rd Qu.:100.79   3rd Qu.:  567713   3rd Qu.: 39743039
##  Max.   :2.8287   Max.   :953.00   Max.   :15574666   Max.   :240885300
##                   NA's   :6
##  notional.value                          sector         sedol
##  Min.   :    22212   Health Care           :1631   2002305:  63
##  1st Qu.: 19047994   Financials            :1548   2005973:  63
##  Median : 39501565   Information Technology:1482   2065308:  63
##  Mean   : 52545119   Consumer Staples      :1222   2065955:  63
##  3rd Qu.: 70790821   Consumer Discretionary:1053   2073390:  63
##  Max.   :240885300   Utilities             : 616   2077905:  63
##  NA's   :5026        (Other)               :1677   (Other):8851
##             isin                                exchange
##  BMG0450A1053:  63   New York Stock Exchange Inc.:7010
##  BMG3223R1088:  63   NASDAQ                      :1890
##  BMG7496G1033:  63   Deutsche Boerse Ag          :  58
##  US00206R1023:  63   Spot Regulated Market - Bvb :  58
##  US0028241000:  63   Boerse Berlin               :  32
##  US02209S1033:  63   (Other)                     : 164
##  (Other)     :8851   NA's                        :  17
##       date
##  Min.   :2011-10-31
##  1st Qu.:2013-04-30
##  Median :2014-09-30
##  Mean   :2014-08-13
##  3rd Qu.:2015-11-30
##  Max.   :2016-12-30
##
```

5.4 EUSA and USMV Data Check

Thus, after cleaning all the data, a check was performed to test how accurate the data set actually was. This was done by comparing the weighted-returns from the index constructed from the data to the actual ETF returns on a monthly basis. ### Weights The first thing to claculate and check were the weights of EUSA and USMV for all stocks on a monthly basis. If the data were perfect, these should add up to 1. However, as some tickers and cash were removed, and given tracking error between the ETF and index, this was not expected. However, something very close to 1 was expected. The monthly change in weights for EUSA is shown below.

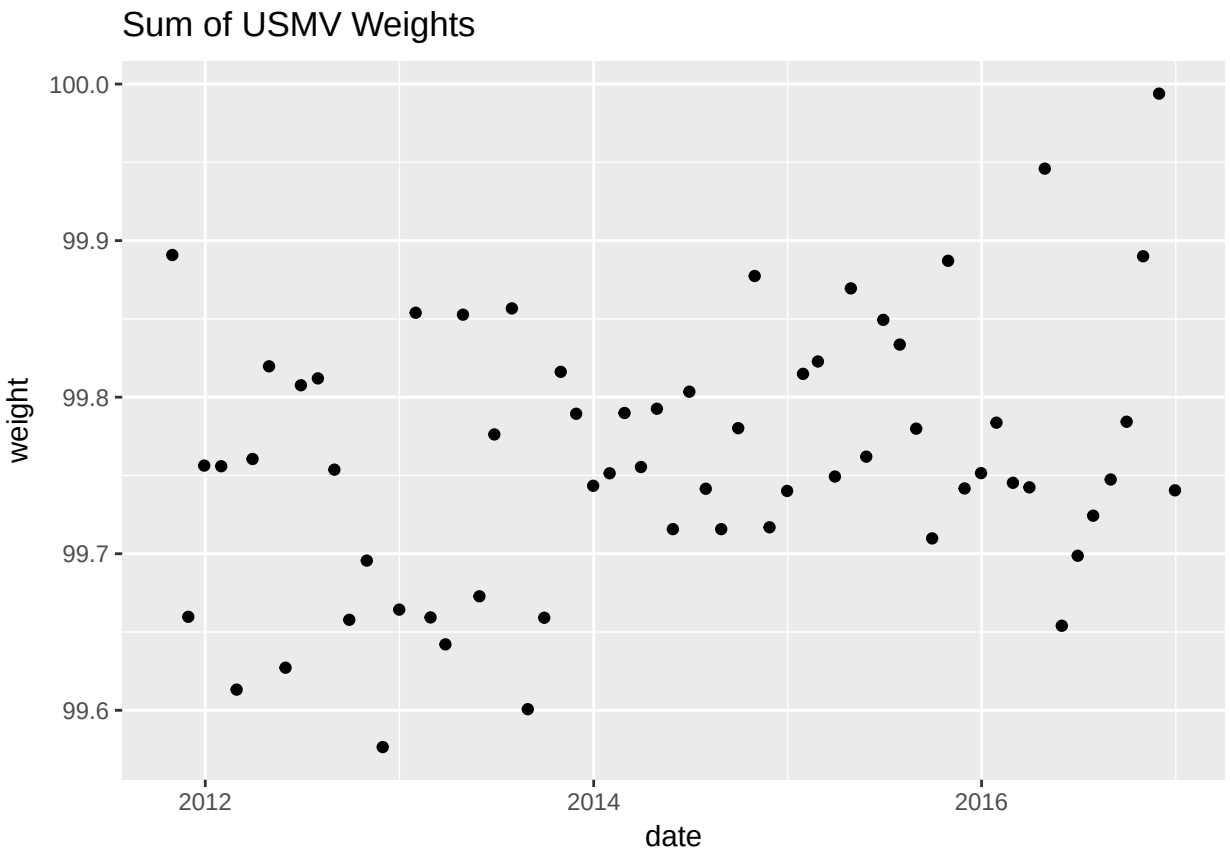
```
##       date                weight
##  Min.   :2011-10-31   Min.   : 99.54
##  1st Qu.:2013-02-14   1st Qu.: 99.73
##  Median :2014-05-30   Median : 99.80
##  Mean   :2014-05-30   Mean   : 99.79
##  3rd Qu.:2015-09-15   3rd Qu.: 99.84
##  Max.   :2016-12-30   Max.   :100.21
```



As shown in the scatterplot above for EUSA, the weights are very close to 100%, generally within 0.2%. The minimum weight is 99.54%, while the largest weight is 100.21%. The mean weight is 99.79%. The monthly change in weights for USMV is shown below.

```
##       date                weight
##  Min.   :2011-10-31   Min.   :99.58
##  1st Qu.:2013-02-14   1st Qu.:99.72
##  Median :2014-05-30   Median :99.76
##  Mean   :2014-05-30   Mean   :99.76
##  3rd Qu.:2015-09-15   3rd Qu.:99.81
```

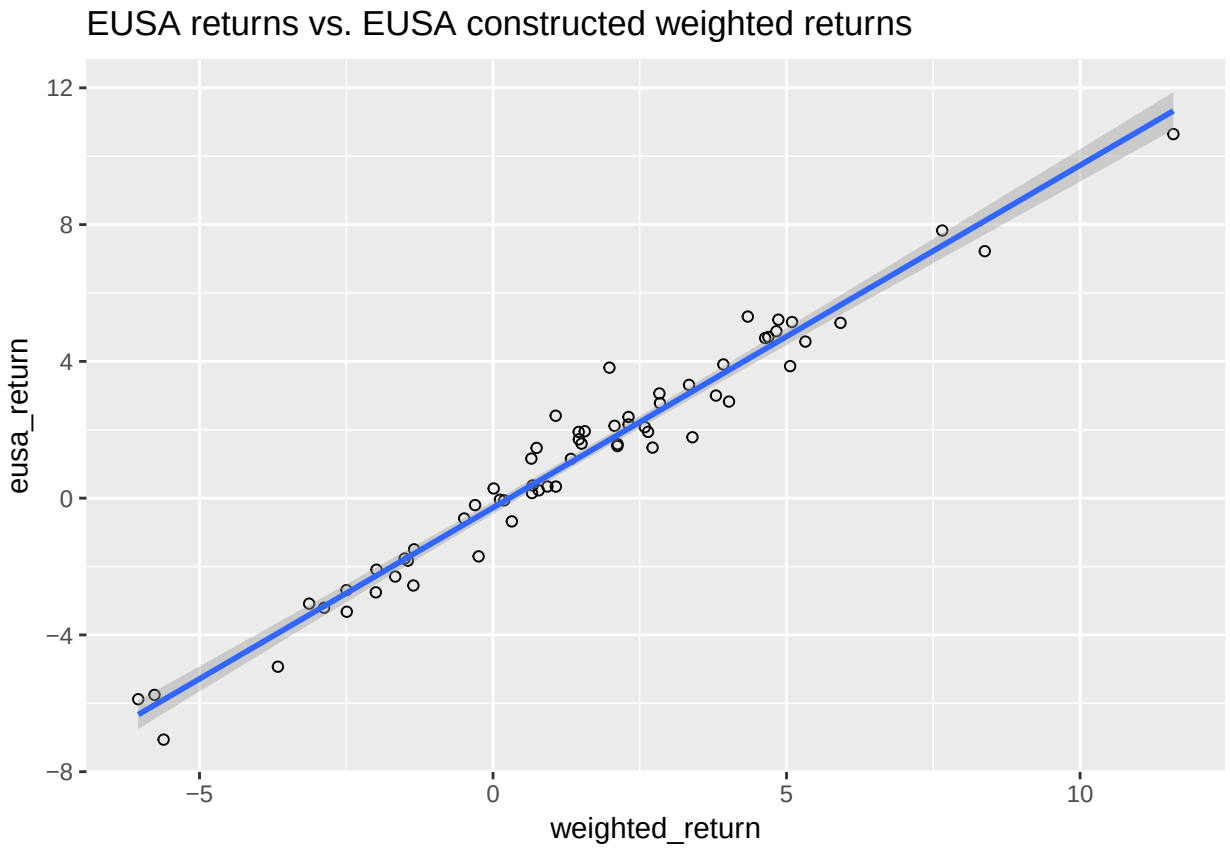
```
##  Max.   :2016-12-30   Max.   :99.99
```



As shown in the scatterplor above, the weights for USMV are very close to 100%, and no value exceeds 100%. The minimum weight is 99.58%, while the largest weight is 99.99%. The mean weight is 99.76%. Overall, these suggest the data is trustable.

5.4.1 Comparing actual ETF returns to constructed ETF returns for EUSA and USMV

Before taking the data completely at face value, some additional checks were performed. This was accomplished by comparing the weighted returns of the constructed index we had for our data (looking at each constituent's monthly return, multiplied by its weight), and comparing it to the actual ETF return. Thus, this provided a way to check how the weighted returns compared to the ETF returns for both EUSA and USMV. Though perfect correlation was not expected, a figure of at least 98% correlation between the weighted returns calculated and the ETF returns, on a monthly basis, was hoped for. The results for EUSA are shown below.

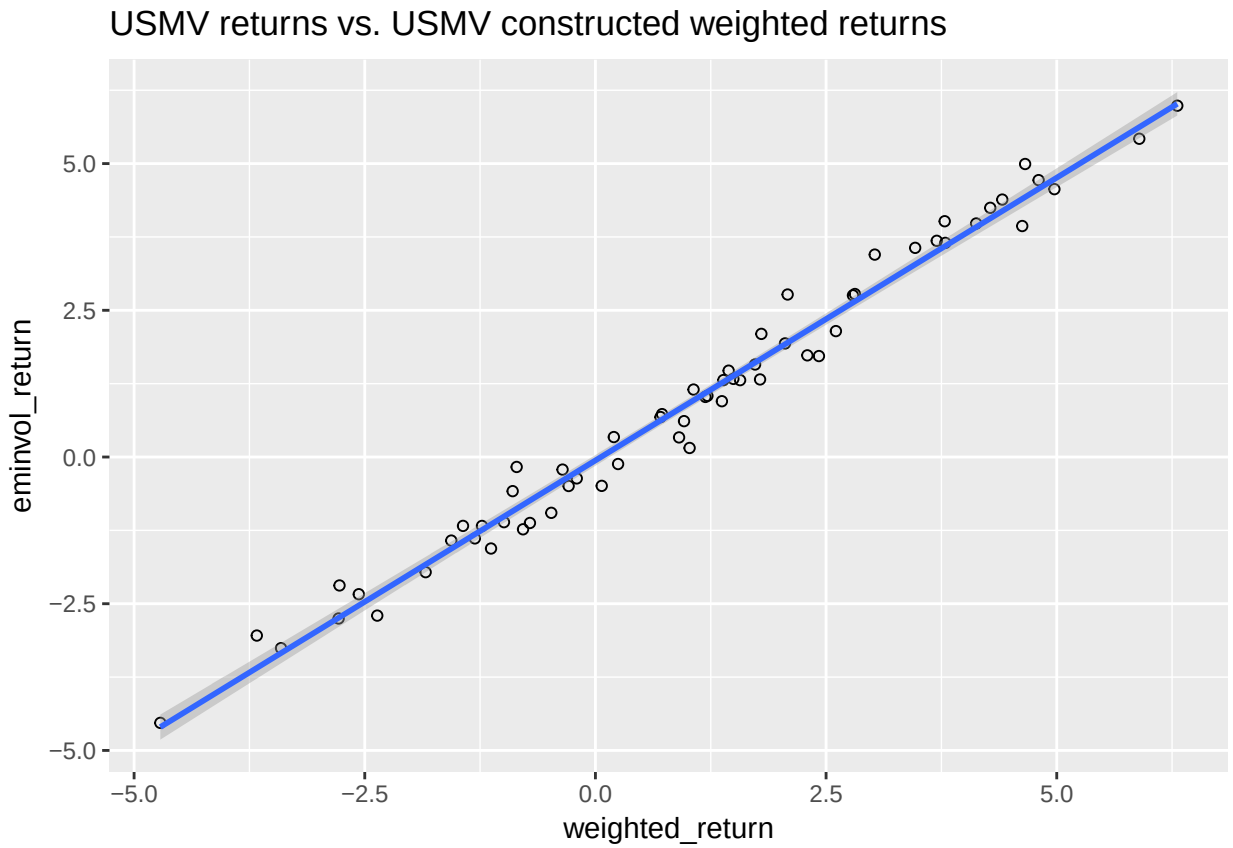



```
##                          [,1]
## Delt.1.arithmetic 0.9805503
```

As shown, the returns have a correlation greater than 0.98.

Shown below is the data for USMV.

```
# USA data
library(ggplot2)
data(usa)
data(minvol)
data(returns2)
ggplot(returns2, aes(x=weighted_return, y=eminvol_return)) +
    geom_point(shape=1) +  geom_smooth(method=lm) + ggtitle("USMV returns vs. USMV constructed weighted
```



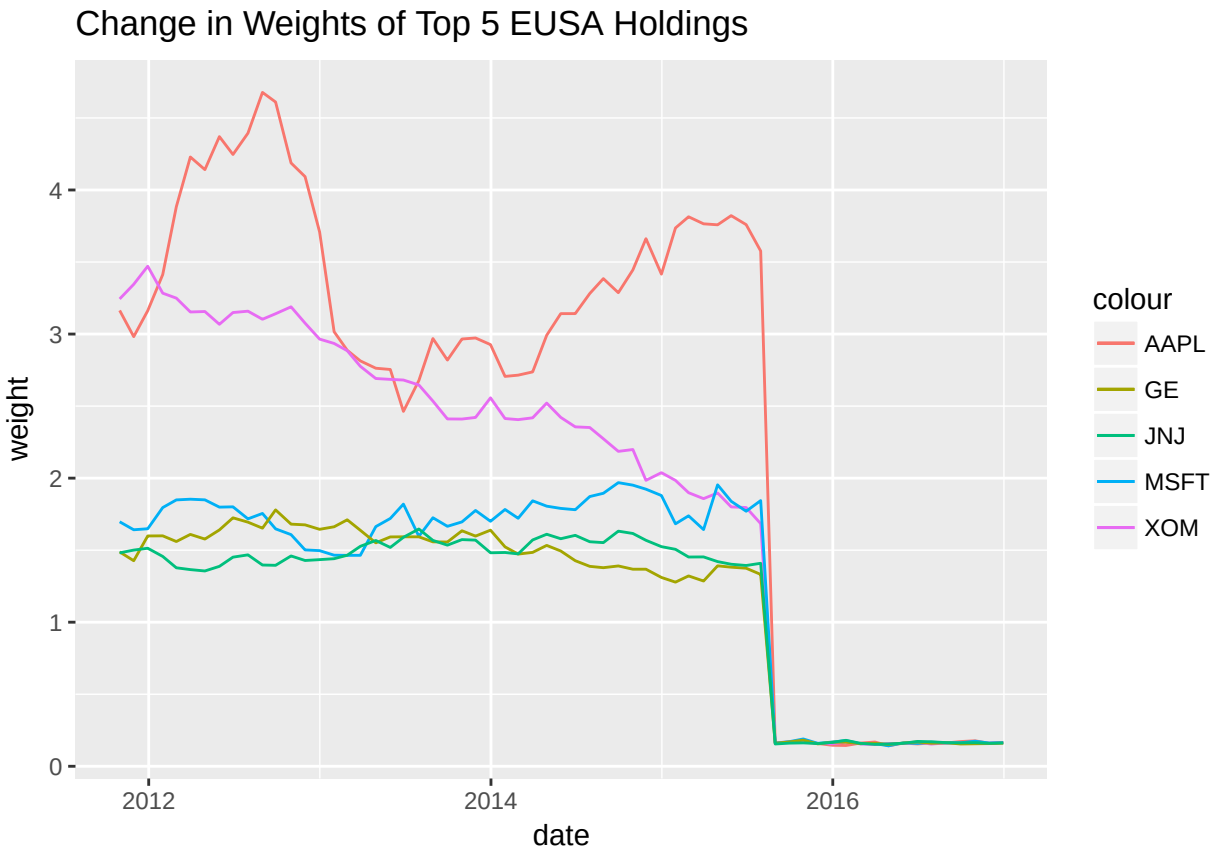

```
# Correlation between USMV returns and USMV constructed weighted returns
cor(returns2$eminvol_return, returns2$weighted_return)
```

```
##                         [,1]
## Delt.1.arithmetic 0.9906848
```

The correlation is 0.99.

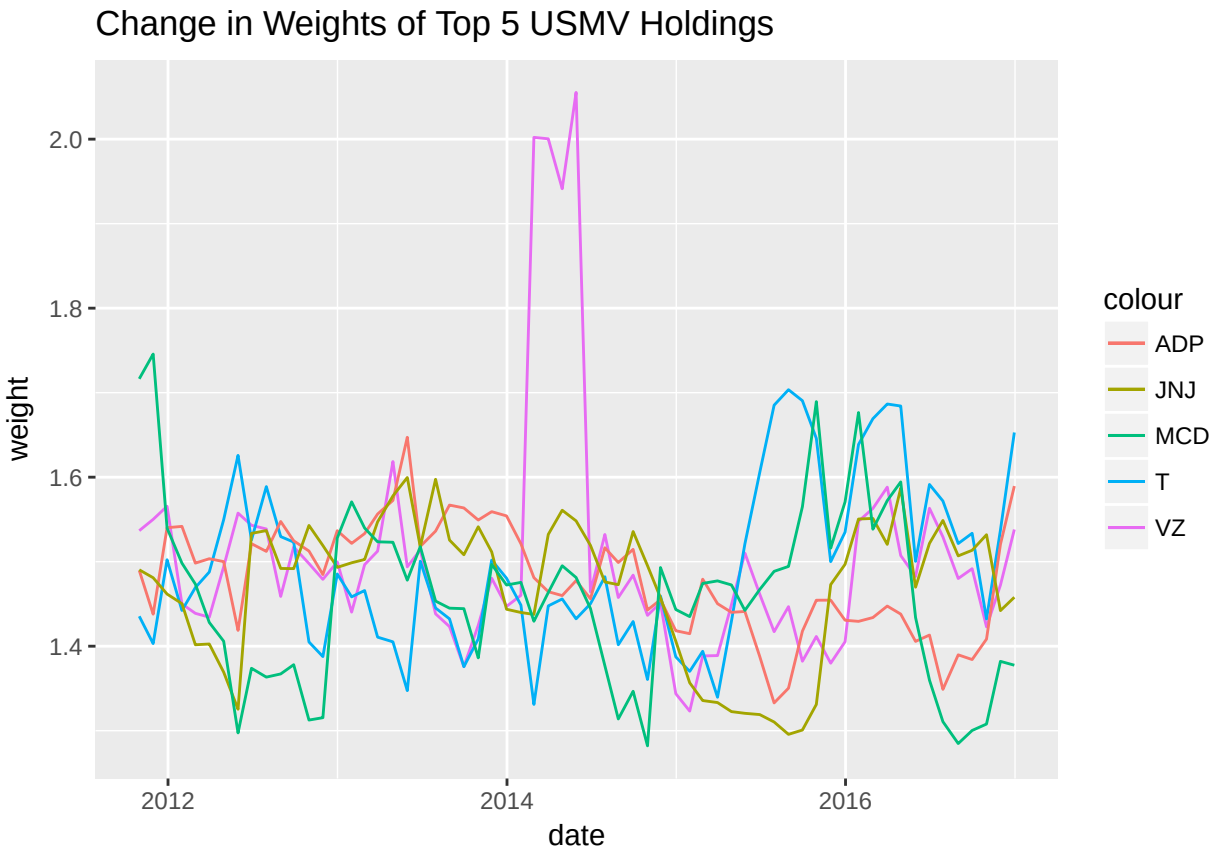
5.4.2 Change in 5 largest holdings by average weight for EUSA and USMV

The next thing we want to see is how the top 5 largest holdings, by average weight, in each index have changed in weighting over time. For EUSA, the 5 largest holdings were AAPL, XOM, MSFT, GE, and JNJ. Their change in weights are shown below.



Shown above, for EUSA, are some very interesting findings. The weights of the 5 companies are all very high, then suddenly all spike. Verifying this in the data, showed that for all 5 companies, holdings dropped significantly between 2015-07-31 and 2015-08-31. The reason for this is not entirely clear, but the general ETF started performing poorly around this time too. In July of 2015 the price per share was 45.20, then it dropped to 42.60 the following month, and dropped again to 40.50 in August 2015. Perhaps these large companies were doing poorly, and MSCI decided to try underweighting them.

For USMV, the 5 largest holdings were VZ, T, ADP, JNJ, and MCD. Their change in weights are shown below. As we can see below, with the exception of Verizon, the holdings generally remain between 1 and 1.6 percent of the overall portfolio.


Change in Weights of Top 5 USMV Holdings
weight
2.0
1.8
1.6
1.4
2012
2014
2016
date
colour
ADP
JNJ
MCD
T
VZ

Chapter 6

Data Analysis

6.1 Sector Weights

Sector weights were calculated over time for both EUSA and USMV. Plots were made and displayed to compare the relative sector weights of EUSA and USMV. This was done to get a sense of what industries may be inherently more "low risk," as well as making sure the data is resilient, and each sector weighting is within 5% as specified by the Barra Optimizer.

Sector Weight Summary Statistics:

```
data(usa_percent)
```

```
## Warning in data(usa_percent): data set 'usa_percent' not found
```

```
data(minvol_percent)
```

```
## Warning in data(minvol_percent): data set 'minvol_percent' not found
```

```
## Summary statistics of EUSA sector weights
head(usa_percent)
```

```
##               sector_name sector_count total     percent       date
## 1 Cash and/or Derivatives            2   631 0.003169572 2017-01-05
## 2  Consumer Discretionary          106   631 0.167987322 2017-01-05
## 3        Consumer Staples           43   631 0.068145800 2017-01-05
## 4                  Energy           44   631 0.069730586 2017-01-05
## 5              Financials           84   631 0.133122029 2017-01-05
## 6             Health Care           71   631 0.112519810 2017-01-05
```

```
tail(usa_percent)
```

```
##                sector_name sector_count total     percent       date
## 827 Information Technology           79   585 0.135042735 2011-10-31
## 828              Materials           33   585 0.056410256 2011-10-31
## 829            Real Estate            0   585 0.000000000 2011-10-31
## 830         S-T Securities            1   585 0.001709402 2011-10-31
## 831     Telecommunications           12   585 0.020512821 2011-10-31
## 832              Utilities           35   585 0.059829060 2011-10-31
```

```
summary(usa_percent)
```

```
##                   sector_name  sector_count          total
```

```
##  Cash and/or Derivatives: 64   Length:832         Length:832
##  Consumer Discretionary : 64   Class :character   Class :character
##  Consumer Staples       : 64   Mode  :character   Mode  :character
##  Energy                 : 64
##  Financials             : 64
##  Health Care            : 64
##  (Other)                :448
##     percent             date
##  Min.   :0.00000   Min.   :2011-10-31
##  1st Qu.:0.01426   1st Qu.:2013-02-21
##  Median :0.06820   Median :2014-06-14
##  Mean   :0.07692   Mean   :2014-06-14
##  3rd Qu.:0.12236   3rd Qu.:2015-10-07
##  Max.   :0.19293   Max.   :2017-01-05
##
```

```
## Summary statistics of USMV sector weights
head(minvol_percent)
```

```
##               sector_name sector_count total    percent       date
## 1 Cash and/or Derivatives            2   186 0.01075269 2017-01-05
## 2  Consumer Discretionary           18   186 0.09677419 2017-01-05
## 3        Consumer Staples           24   186 0.12903226 2017-01-05
## 4                  Energy            4   186 0.02150538 2017-01-05
## 5              Financials           21   186 0.11290323 2017-01-05
## 6             Health Care           33   186 0.17741935 2017-01-05
```

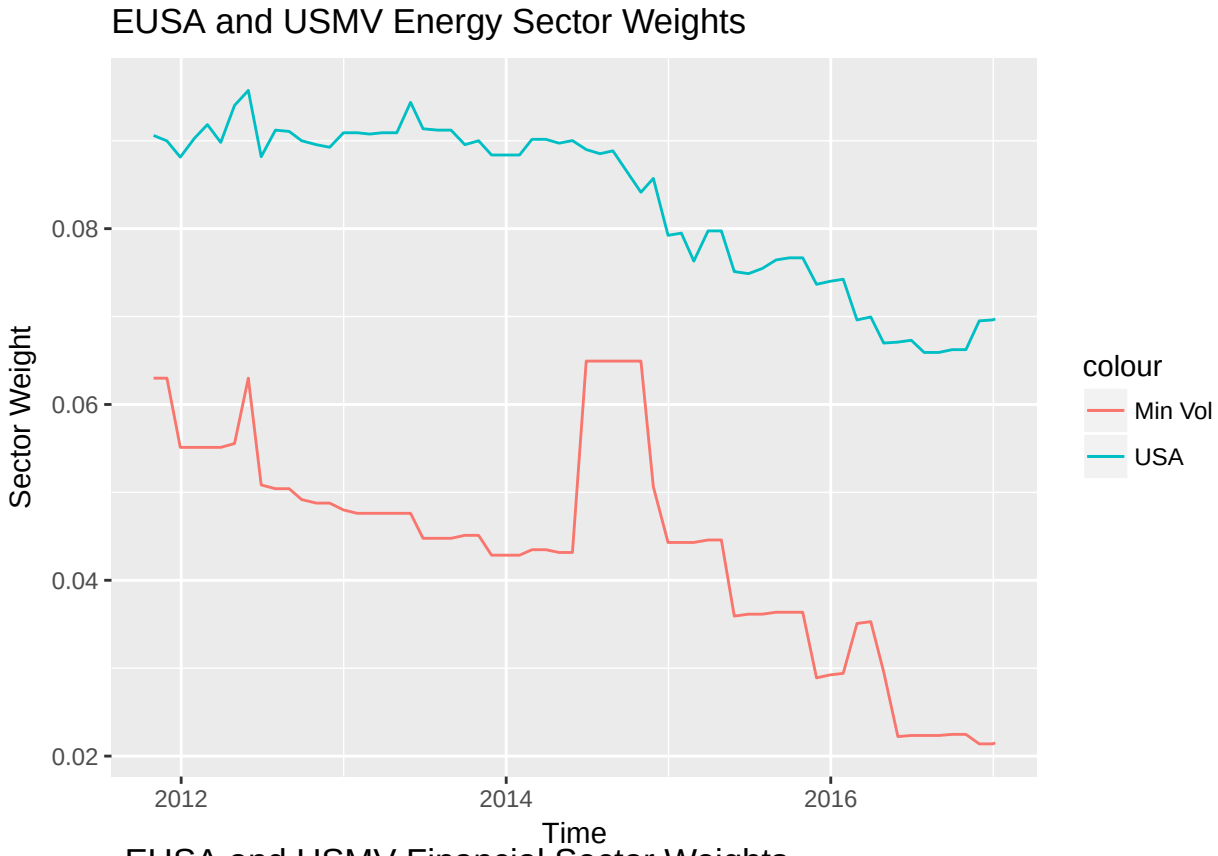
```
tail(minvol_percent)
```

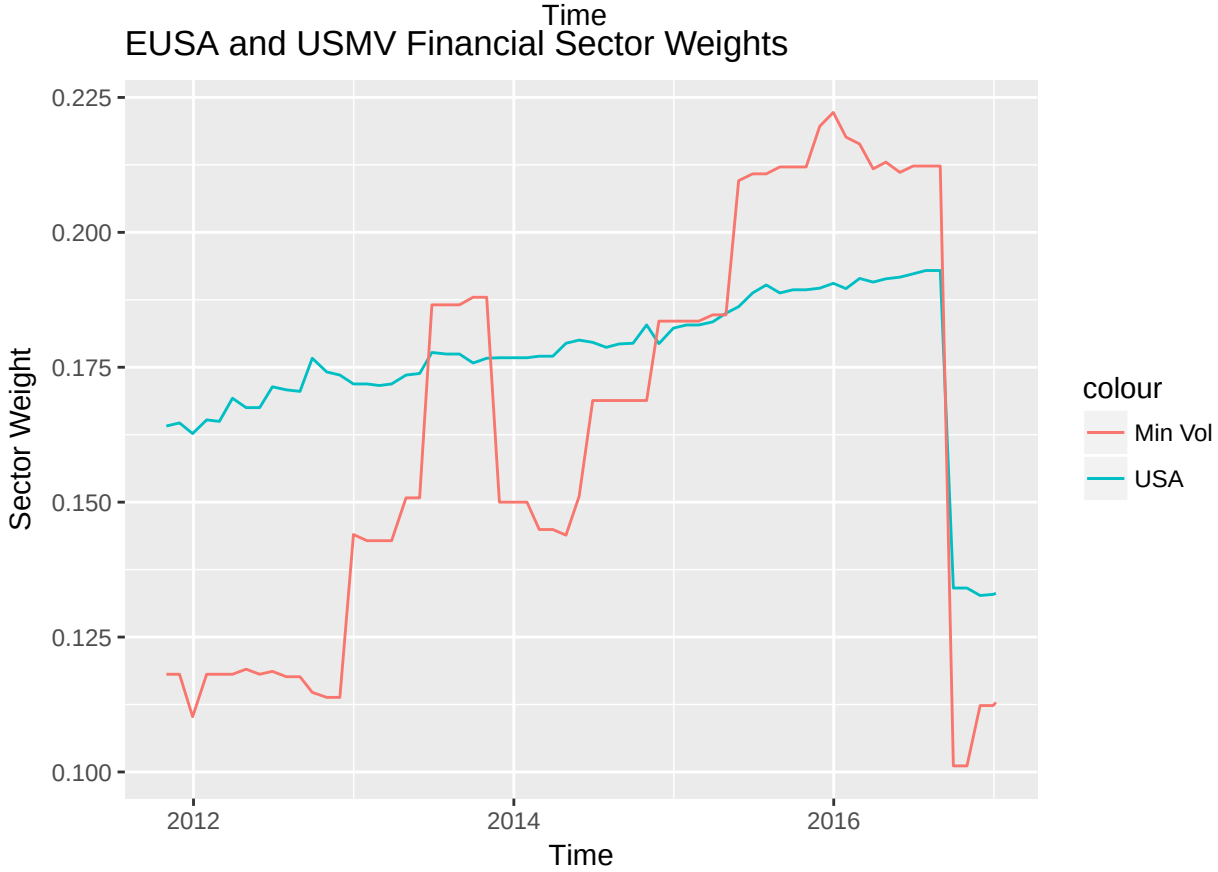
```
##                sector_name sector_count total     percent       date
## 827 Information Technology           19   127 0.149606299 2011-10-31
## 828             Materials            4   127 0.031496063 2011-10-31
## 829           Real Estate            0   127 0.000000000 2011-10-31
## 830         S-T Securities            1   127 0.007874016 2011-10-31
## 831     Telecommunications            7   127 0.055118110 2011-10-31
## 832             Utilities            9   127 0.070866142 2011-10-31
```

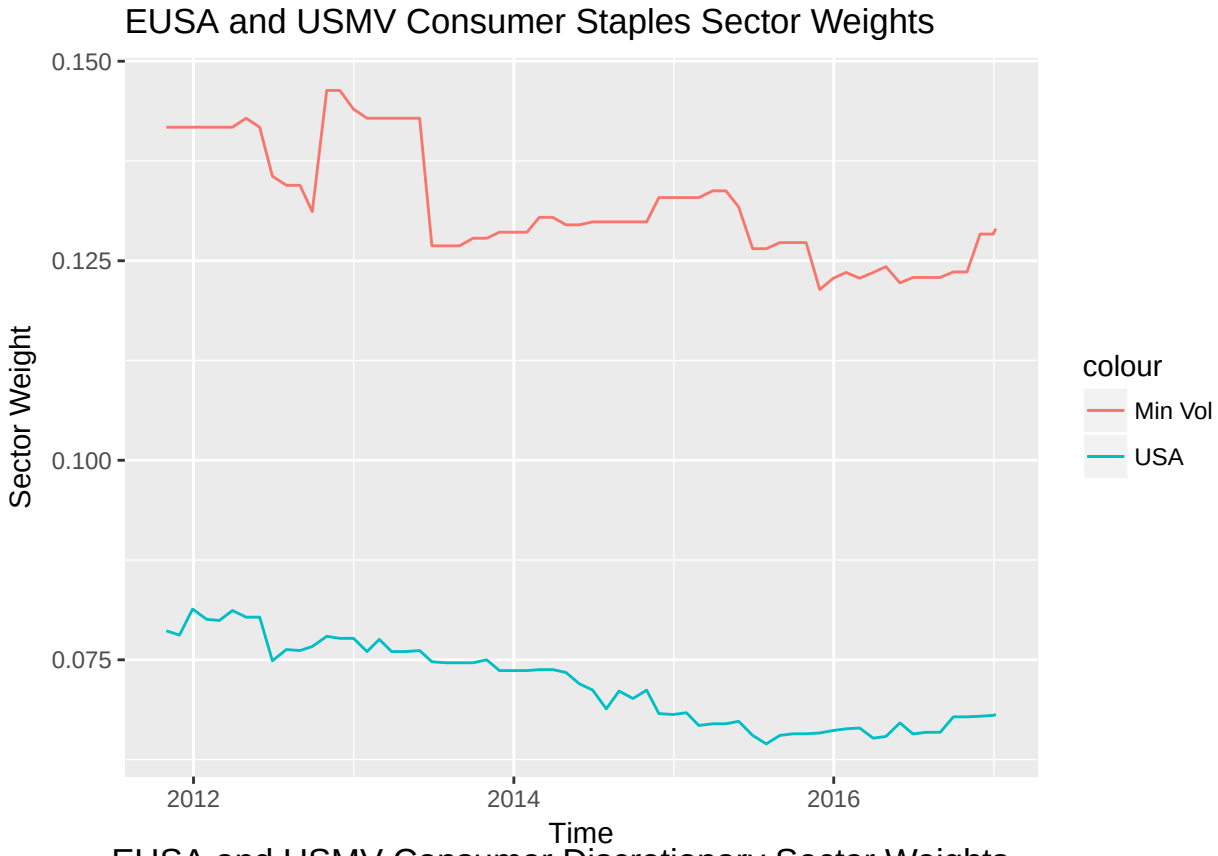
```
summary(minvol_percent)
```

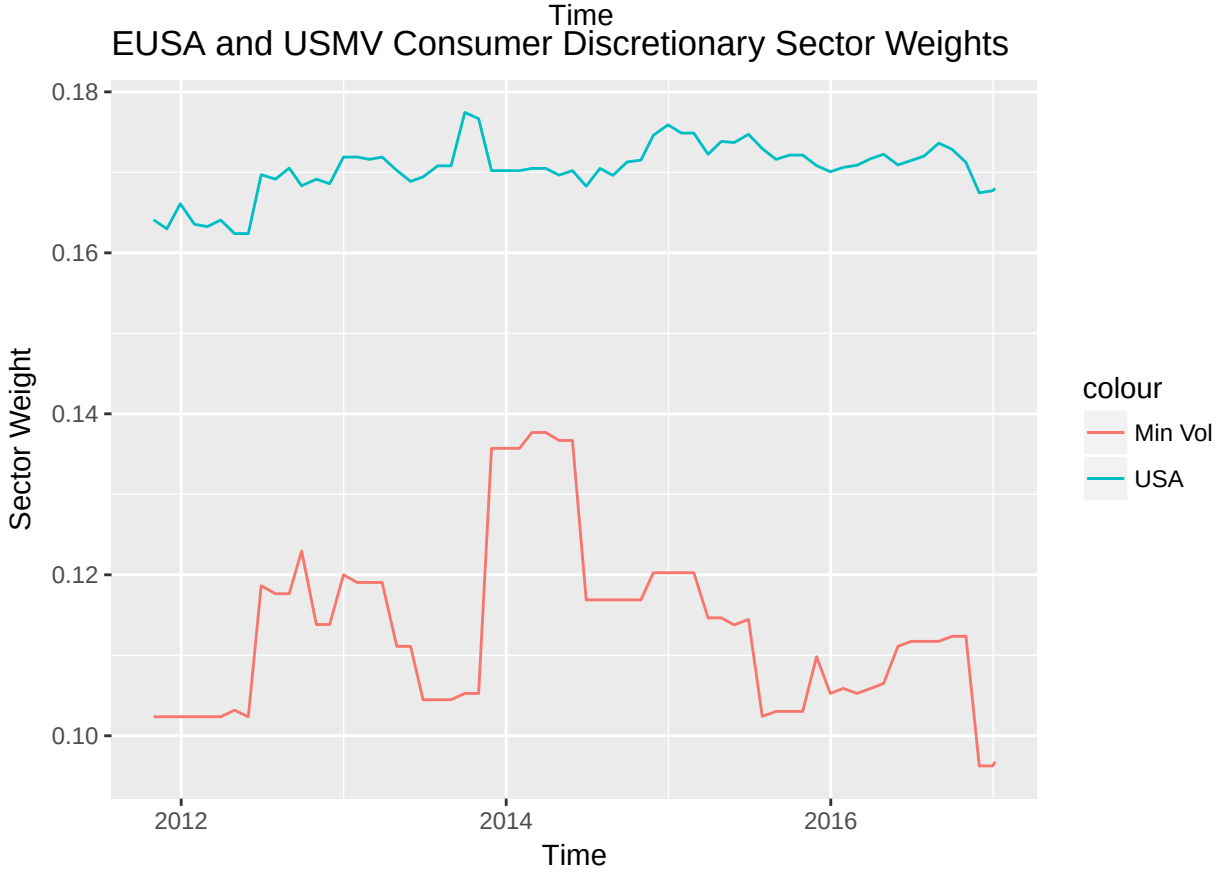
```
##                   sector_name  sector_count          total
##  Cash and/or Derivatives: 64   Length:832         Length:832
##  Consumer Discretionary : 64   Class :character   Class :character
##  Consumer Staples       : 64   Mode  :character   Mode  :character
##  Energy                 : 64
##  Financials             : 64
##  Health Care            : 64
##  (Other)                :448
##     percent             date
##  Min.   :0.00000   Min.   :2011-10-31
##  1st Qu.:0.02235   1st Qu.:2013-02-21
##  Median :0.06349   Median :2014-06-14
##  Mean   :0.07692   Mean   :2014-06-14
##  3rd Qu.:0.13043   3rd Qu.:2015-10-07
##  Max.   :0.22222   Max.   :2017-01-05
##
```

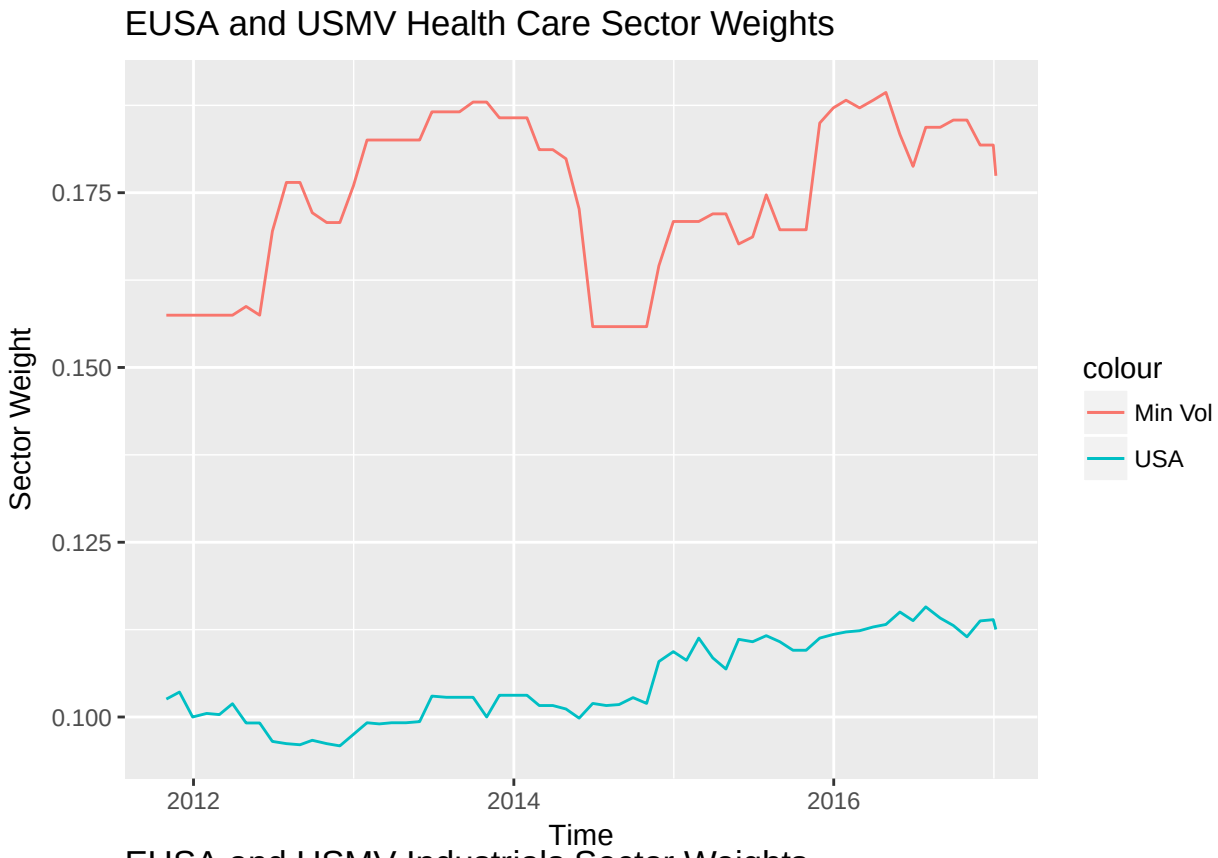
Sector Weights for EUSA and USMV:

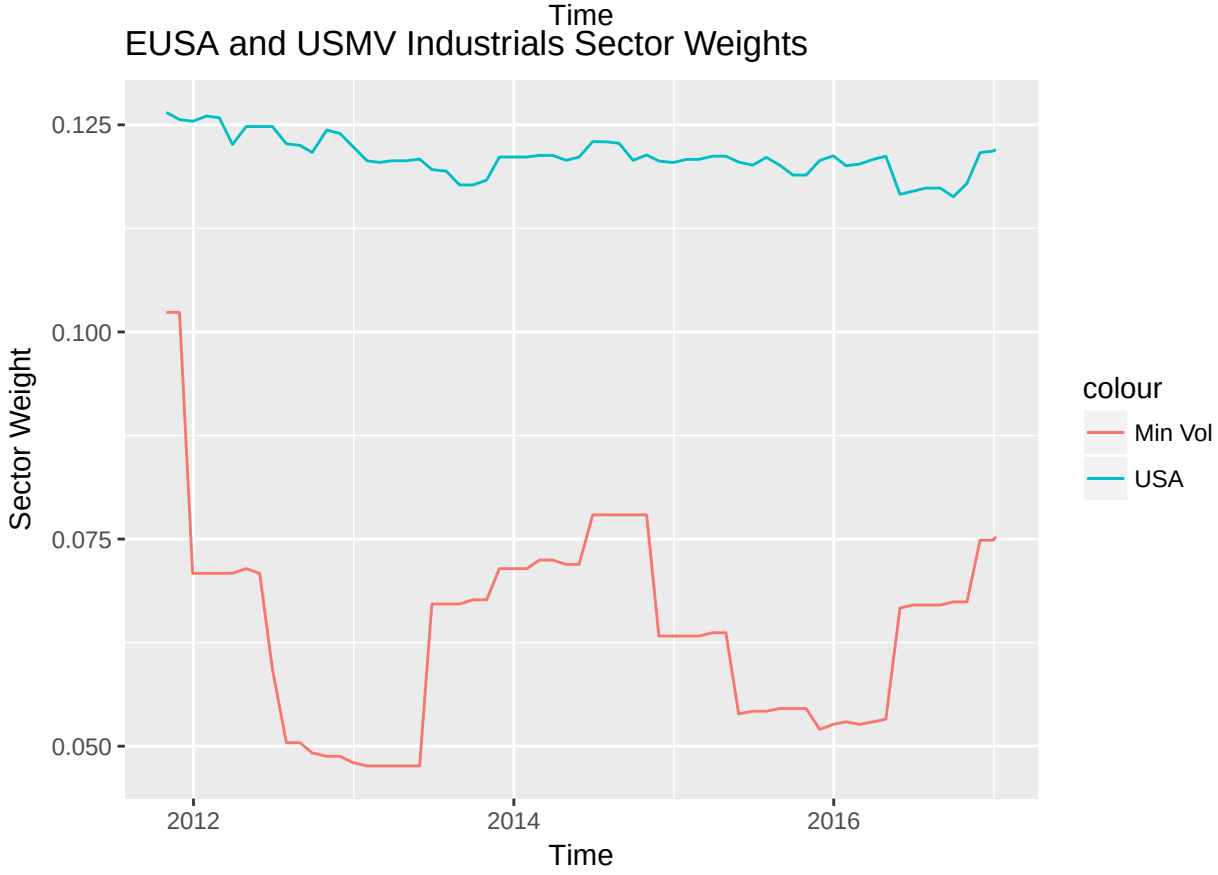

EUSA and USMV Energy Sector Weights
Sector Weight
0.08
0.06
0.04
0.02
2012
2014
2016
Time
colour
Min Vol
USA

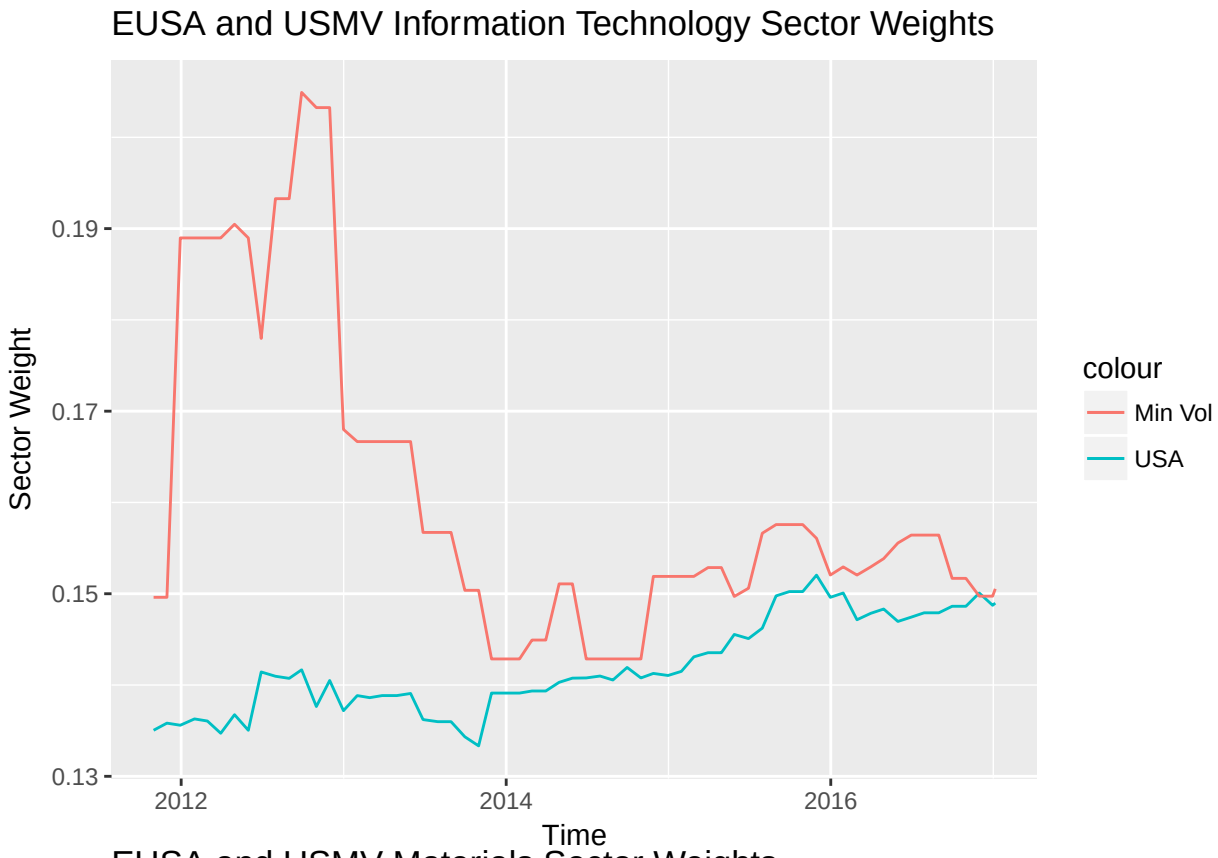

EUSA and USMV Financial Sector Weights
Sector Weight
0.225
0.200
0.175
0.150
0.125
0.100
2012
2014
2016
Time
colour
Min Vol
USA

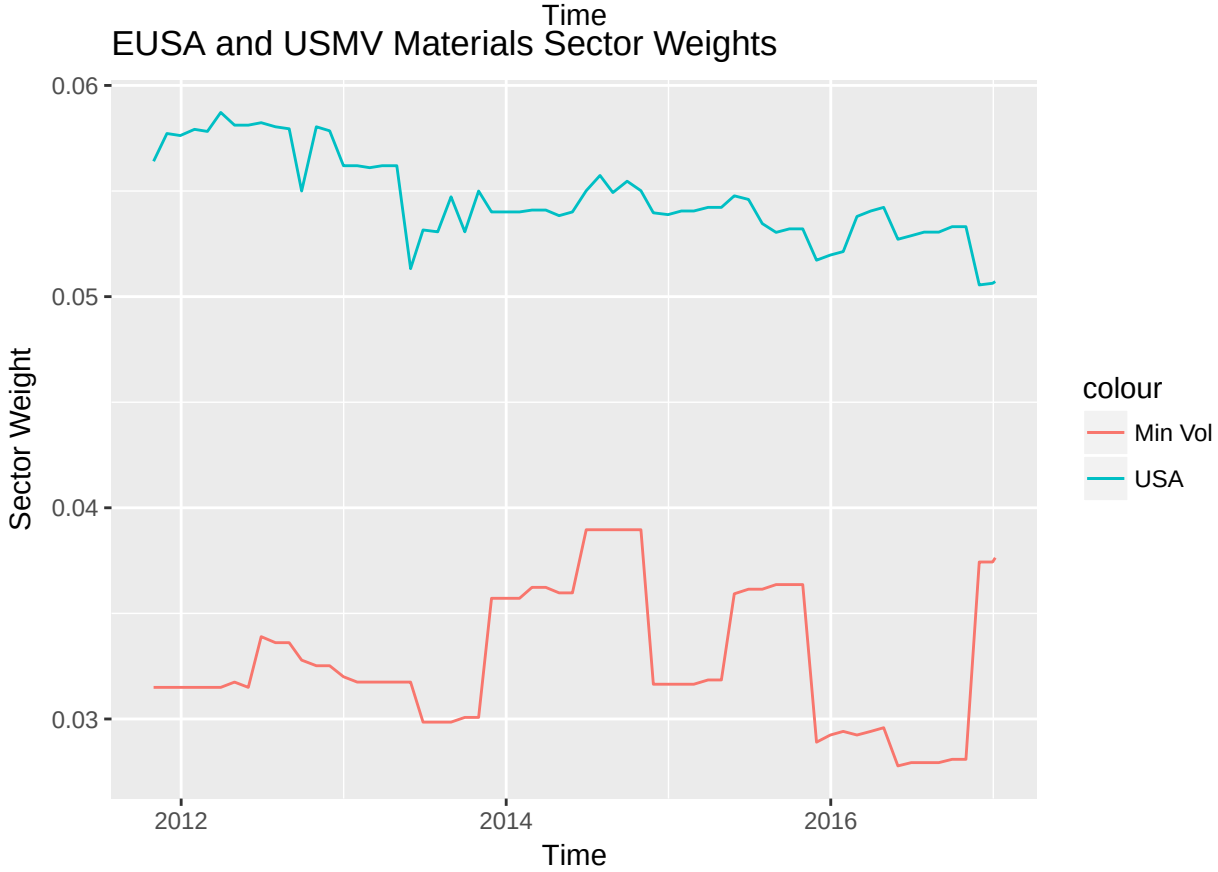

EUSA and USMV Consumer Staples Sector Weights
0.150
0.125
0.100
0.075
Sector Weight
2012
2014
2016
Time
colour
Min Vol
USA

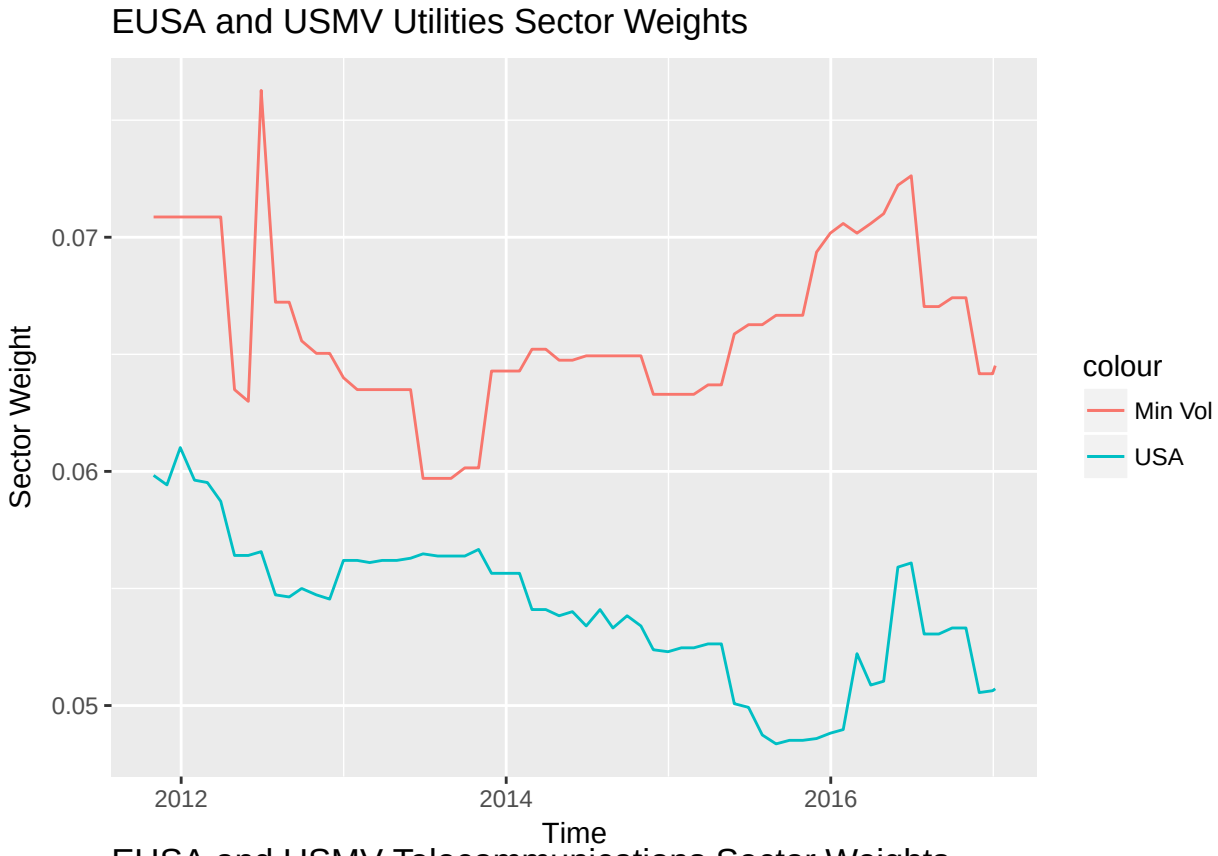

EUSA and USMV Consumer Discretionary Sector Weights
0.18
0.16
0.14
0.12
0.10
Sector Weight
2012
2014
2016
Time
colour
Min Vol
USA

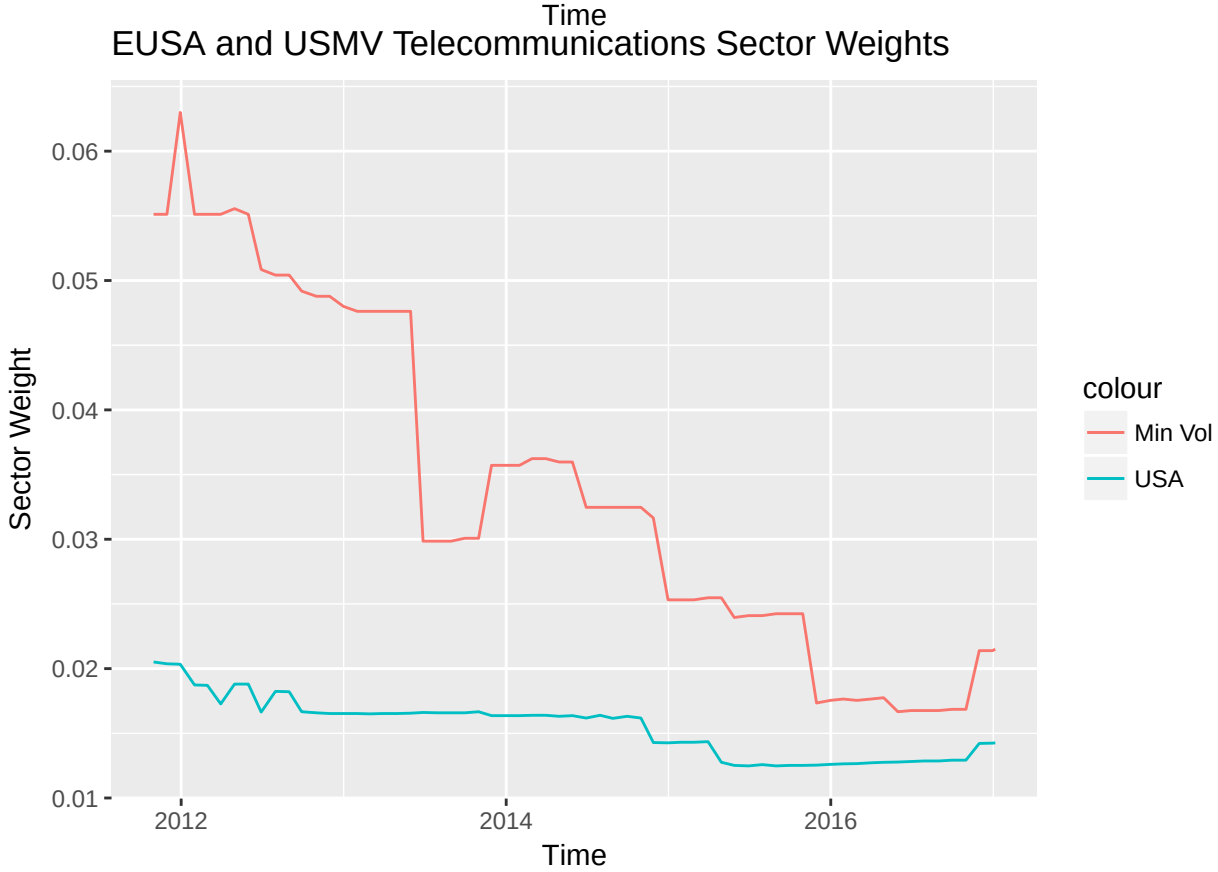

EUSA and USMV Health Care Sector Weights
Sector Weight
0.175
0.150
0.125
0.100
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Industrials Sector Weights
Sector Weight
0.125
0.100
0.075
0.050
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Information Technology Sector Weights
Sector Weight
0.19
0.17
0.15
0.13
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Materials Sector Weights
Sector Weight
0.06
0.05
0.04
0.03
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Utilities Sector Weights
Sector Weight
0.07
0.06
0.05
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Telecommunications Sector Weights
Sector Weight
0.06
0.05
0.04
0.03
0.02
0.01
2012
2014
2016
Time
colour
Min Vol
USA

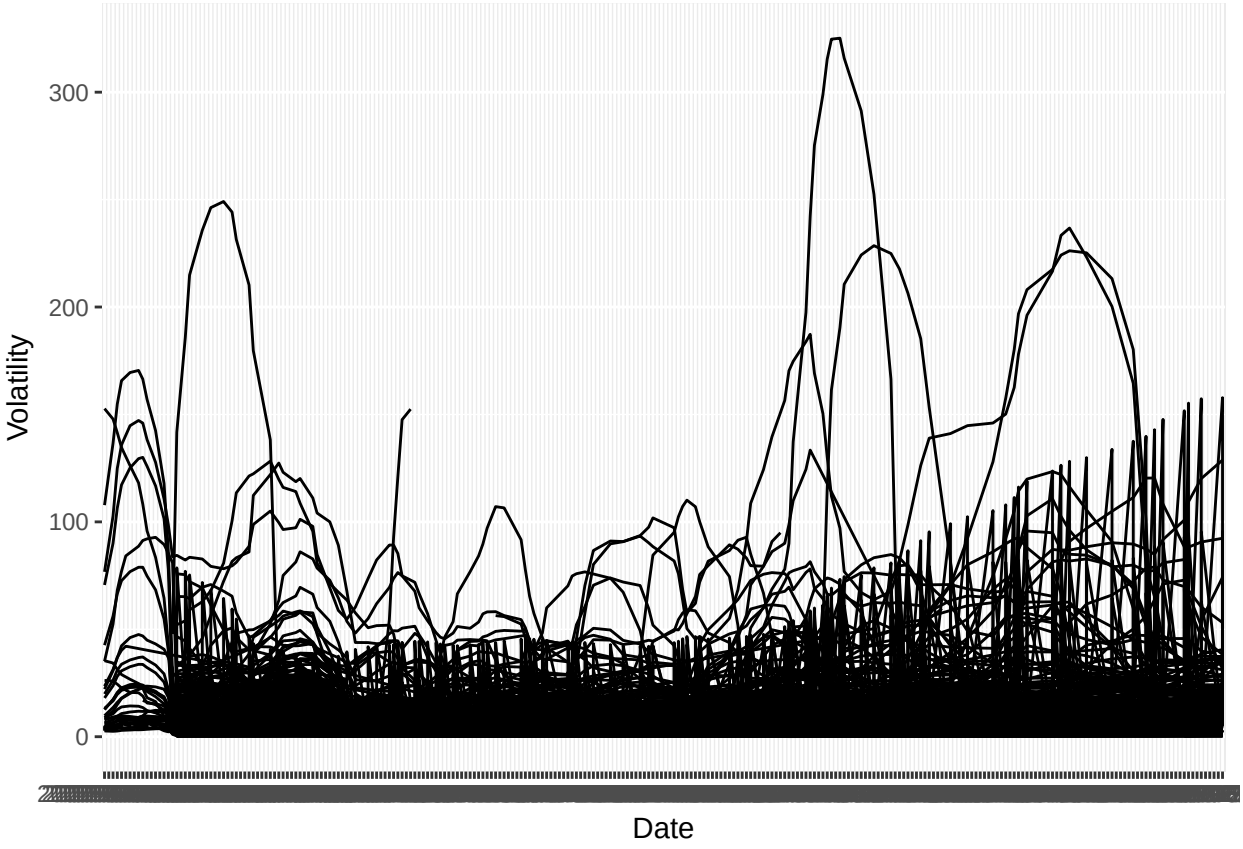
6.2 EUSA Constituent Trailing Volatilities

Data was collected from the past 10 years of the EUSA index. The data was collected from 12/31/2006 to 12/30/2016, from Wharton Research Data Services (WRDS) for the 908 historical constituents of the USA Equal Weight (EUSA) index, of which USMV is derived. Each tickers' 252-day (annual) trailing volatility was calculated and a month end spagetti plot was produced.

```
##         Date Ticker Volatility
## 1 2008-01-31      A   2.289449
## 2 2008-02-29      A   2.357208
## 3 2008-03-31      A   2.660299
## 4 2008-04-30      A   3.017754
## 5 2008-05-30      A   3.028868
## 6 2008-06-30      A   2.921781
```

```
##             Date Ticker Volatility
## 82257 2016-07-29    ZTS   2.718338
## 82258 2016-08-31    ZTS   3.150629
## 82259 2016-09-30    ZTS   3.363564
## 82260 2016-10-31    ZTS   3.372530
## 82261 2016-11-30    ZTS   3.436391
## 82262 2016-12-30    ZTS   3.664053
```

```
##          Date           Ticker        Volatility
##  2014-09-30:  774   GE     :  432   Min.   :  0.0492
##  2014-10-31:  774   LSI    :  246   1st Qu.:  2.4407
##  2014-11-28:  774   UA     :  234   Median :  4.3057
##  2014-12-31:  773   CBS    :  228   Mean   :  6.9432
##  2014-06-30:  771   LEN    :  228   3rd Qu.:  7.4648
##  2014-07-31:  771   MKC    :  228   Max.   :325.1242
##  (Other)   :77625   (Other):80666
```



6.3 EUSA Constituent Trailing Betas

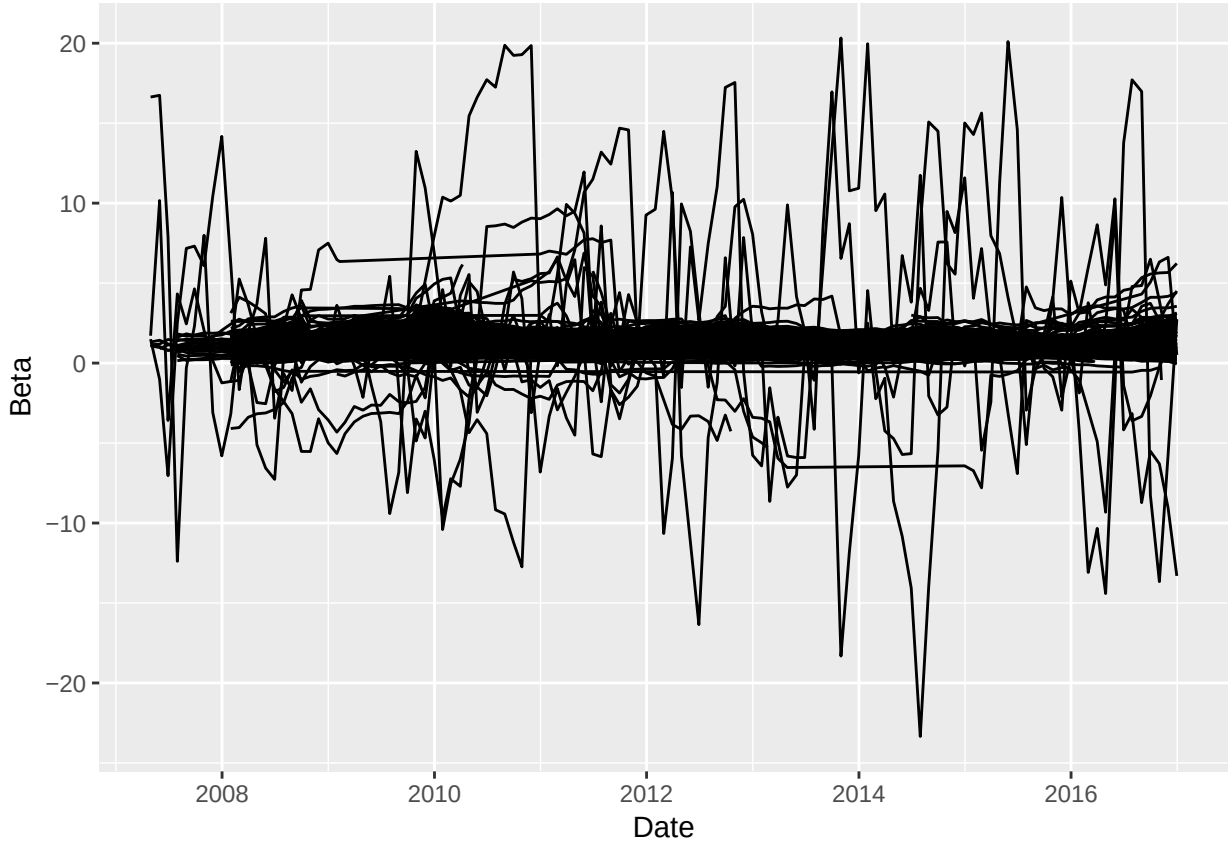
Data was collected from the past 10 years of the EUSA index. The data was collected from 12/31/2006 to 12/30/2016, was collected from WRDS for the 908 historical constituents of the USA Equal Weight (EUSA) index, of which USMV is derived. Each tickers' 252-day (annual) trailing beta was calculated and a month end spagetti plot was produced.

```
##           Date Ticker      Beta
## 1         <NA>   <NA>        NA
## 271 2008-01-31      A 0.9538067
## 291 2008-02-29      A 0.9473060
## 311 2008-03-31      A 0.9376670
## 333 2008-04-30      A 0.9588268
## 354 2008-05-30      A 0.9680630
```

```
##              Date Ticker      Beta
## 879817 2016-07-29    ZTS 1.0224673
## 902806 2016-08-31    ZTS 1.0302957
## 923805 2016-09-30    ZTS 0.9779760
## 944802 2016-10-31    ZTS 0.9830767
## 965801 2016-11-30    ZTS 0.9220281
## 986798 2016-12-30    ZTS 0.9549808
```

```
##       Date               Ticker              Beta
##  Min.   :2007-04-30   Length:78532       Min.   :-23.3438
##  1st Qu.:2010-04-30   Class :character   1st Qu.:  0.8204
```

```
##  Median :2012-07-31   Mode  :character   Median :  1.0523
##  Mean   :2012-07-21                      Mean   :  1.0993
##  3rd Qu.:2014-10-31                      3rd Qu.:  1.3295
##  Max.   :2016-12-30                      Max.   : 20.3256
##  NA's   :1                               NA's   :1
```



6.4 EUSA Constituent Price to Book Ratios

Data was collected from the past 10 years of the EUSA index. The data was collected from 12/31/2006 to 12/30/2016, from WRDS for the 908 historical constituents of the USA Equal Weight (EUSA) index, of which USMV is derived. Each tickers' Price to Book ratio was calculated in two ways, to ensure accuracy.

```
##    gvkey       Date Year Ticker Total_Assets BV_per_share
## 1 126554 2007-10-31 2007      A    7.554e+09       8.7405
## 2 126554 2008-10-31 2008      A    7.437e+09       7.3114
## 3 126554 2009-10-31 2009      A    7.612e+09       7.2397
## 4 126554 2010-10-31 2010      A    9.696e+09       9.3256
## 5 126554 2011-10-31 2011      A    9.057e+09      12.4371
## 6 126554 2011-10-31 2011      A    9.049e+09           NA
##   Shares_Outstanding Total_Liabilities Market_Value Share_Price Book_Value
## 1          370000000         4.320e+09  13634500000       36.85 3233985000
## 2          350000000         4.878e+09   7766500000       22.19 2558990000
## 3          346148000         5.106e+09   8563701500       24.74 2506007676
## 4          346144000         6.460e+09  12045811200       34.80 3228000486
## 5          346382000         4.741e+09  12840380700       37.07 4307987572
## 6                 NA                NA  12840380700          NA         NA
```

```
##       PBR1     PBR2
## 1 4.216006 4.216006
## 2 3.034986 3.034986
## 3 3.417269 3.417269
## 4 3.731663 3.731663
## 5 2.980598 2.980598
## 6       NA       NA
```

```
##       gvkey       Date Year Ticker Total_Assets BV_per_share
## 17182 13721 2014-12-31 2014    ZTS    6.607e+09       2.6151
## 17183 13721 2014-12-31 2014    ZTS    6.588e+09           NA
## 17184 13721 2015-12-31 2015    ZTS    7.913e+09       2.1472
## 17185 13721 2015-12-31 2015    ZTS    7.913e+09           NA
## 17186 13721 2016-12-31 2016    ZTS    7.649e+09       3.0171
## 17187 13721 2016-12-31 2016    ZTS    7.649e+09           NA
##       Shares_Outstanding Total_Liabilities Market_Value Share_Price
## 17182          501328000         5.270e+09  21572143800       43.03
## 17183                 NA                NA  21572143800          NA
## 17184          497400000         6.822e+09  23835408000       47.92
## 17185                 NA                NA  23835408000          NA
## 17186          492855000         6.150e+09  26382528200       53.53
## 17187                 NA                NA  26382528200          NA
##       Book_Value     PBR1     PBR2
## 17182 1311022853 16.45444 16.45444
## 17183         NA       NA       NA
## 17184 1068017280 22.31744 22.31744
## 17185         NA       NA       NA
## 17186 1486992820 17.74220 17.74220
## 17187         NA       NA       NA
```

```
##      gvkey                Date           Year          Ticker
##  Min.   :  1045   2013-12-31:1283   Min.   :2006   ACGL   :   33
##  1st Qu.:  7146   2012-12-31:1277   1st Qu.:2009   AET    :   33
##  Median : 14824   2014-12-31:1275   Median :2011   AFL    :   33
##  Mean   : 50895   2011-12-31:1265   Mean   :2011   AIZ    :   33
##  3rd Qu.: 65556   2015-12-31:1253   3rd Qu.:2014   AMG    :   33
##  Max.   :294524   2010-12-31:1222   Max.   :2016   ANTM   :   33
##                   (Other)   :9612   NA's   :33     (Other):16989
##   Total_Assets        BV_per_share      Shares_Outstanding
##  Min.   :0.000e+00   Min.   :-1489600   Min.   :0.000e+00
##  1st Qu.:3.912e+09   1st Qu.:       8   1st Qu.:1.010e+08
##  Median :9.538e+09   Median :      15   Median :1.973e+08
##  Mean   :4.725e+10   Mean   :    5311   Mean   :4.654e+08
##  3rd Qu.:2.632e+10   3rd Qu.:      27   3rd Qu.:4.344e+08
##  Max.   :2.573e+12   Max.   :16297416   Max.   :2.906e+10
##  NA's   :2568        NA's   :9284       NA's   :9188
##  Total_Liabilities    Market_Value        Share_Price
##  Min.   :0.000e+00   Min.   :3.545e+06   Min.   :   0.027
##  1st Qu.:2.135e+09   1st Qu.:4.445e+09   1st Qu.:  25.985
##  Median :6.310e+09   Median :8.851e+09   Median :  43.180
##  Mean   :4.451e+10   Mean   :2.166e+10   Mean   :  57.339
##  3rd Qu.:1.865e+10   3rd Qu.:1.942e+10   3rd Qu.:  67.955
##  Max.   :2.341e+12   Max.   :6.266e+11   Max.   :1466.060
##  NA's   :7785        NA's   :2764        NA's   :10064
##    Book_Value             PBR1              PBR2
```

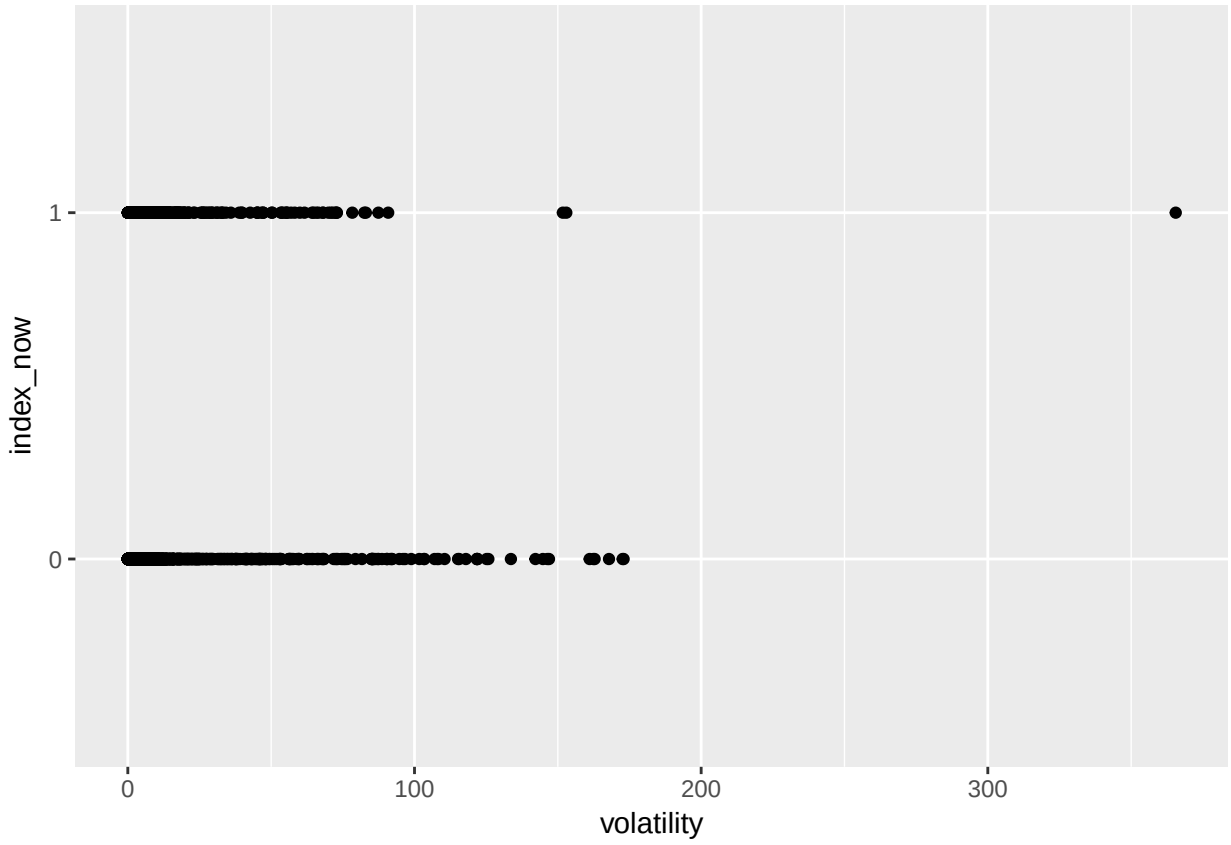
```
##  Min.   :-8.615e+10   Min.   :-687.634   Min.   :-687.634
##  1st Qu.: 1.313e+09   1st Qu.:   1.595   1st Qu.:   1.595
##  Median : 3.121e+09   Median :   2.626   Median :   2.626
##  Mean   : 8.170e+09   Mean   :   4.503   Mean   :   4.503
##  3rd Qu.: 7.376e+09   3rd Qu.:   4.416   3rd Qu.:   4.416
##  Max.   : 2.416e+11   Max.   :1575.000   Max.   :1575.000
##  NA's   :9284         NA's   :10079      NA's   :10079
```

Chapter 7

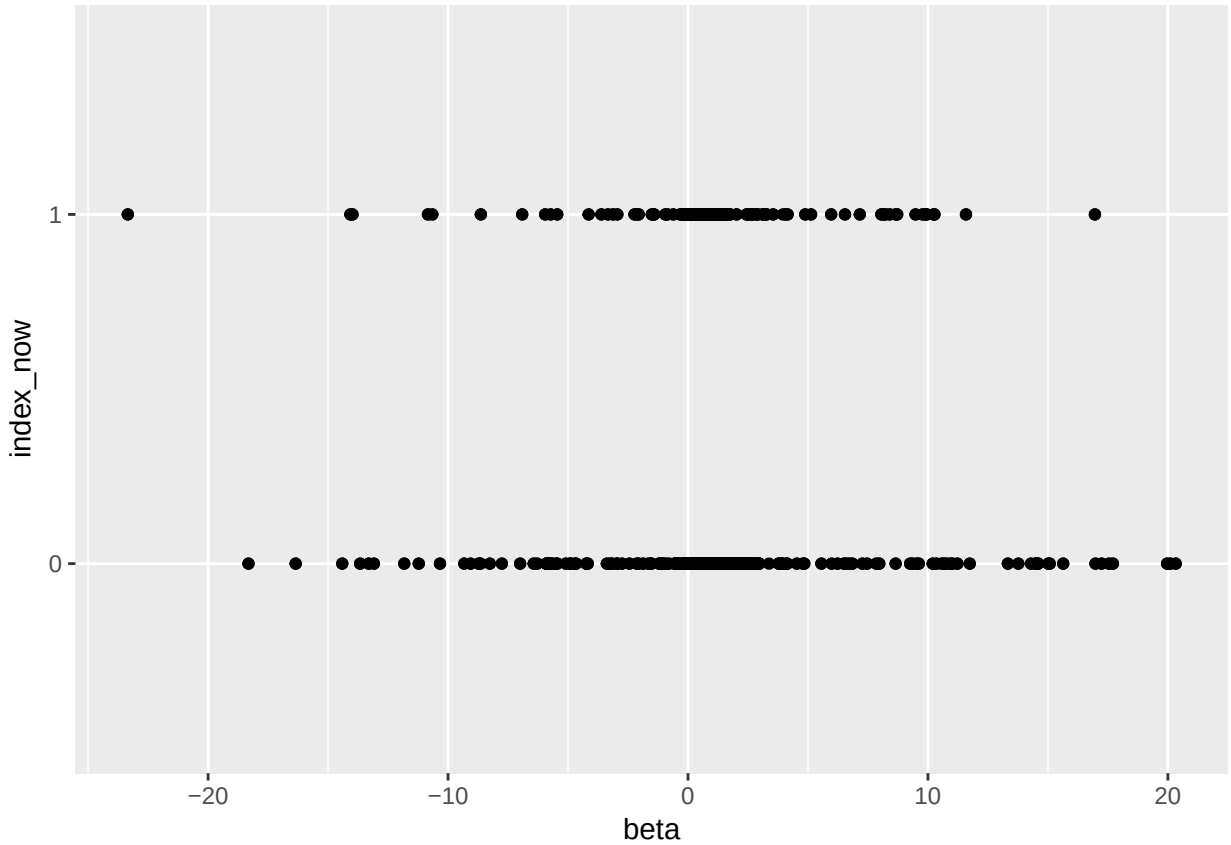
Data Distribution

The distribution of the predictor variable, if the stock is in the index now, with respect to each response variable is shown below.

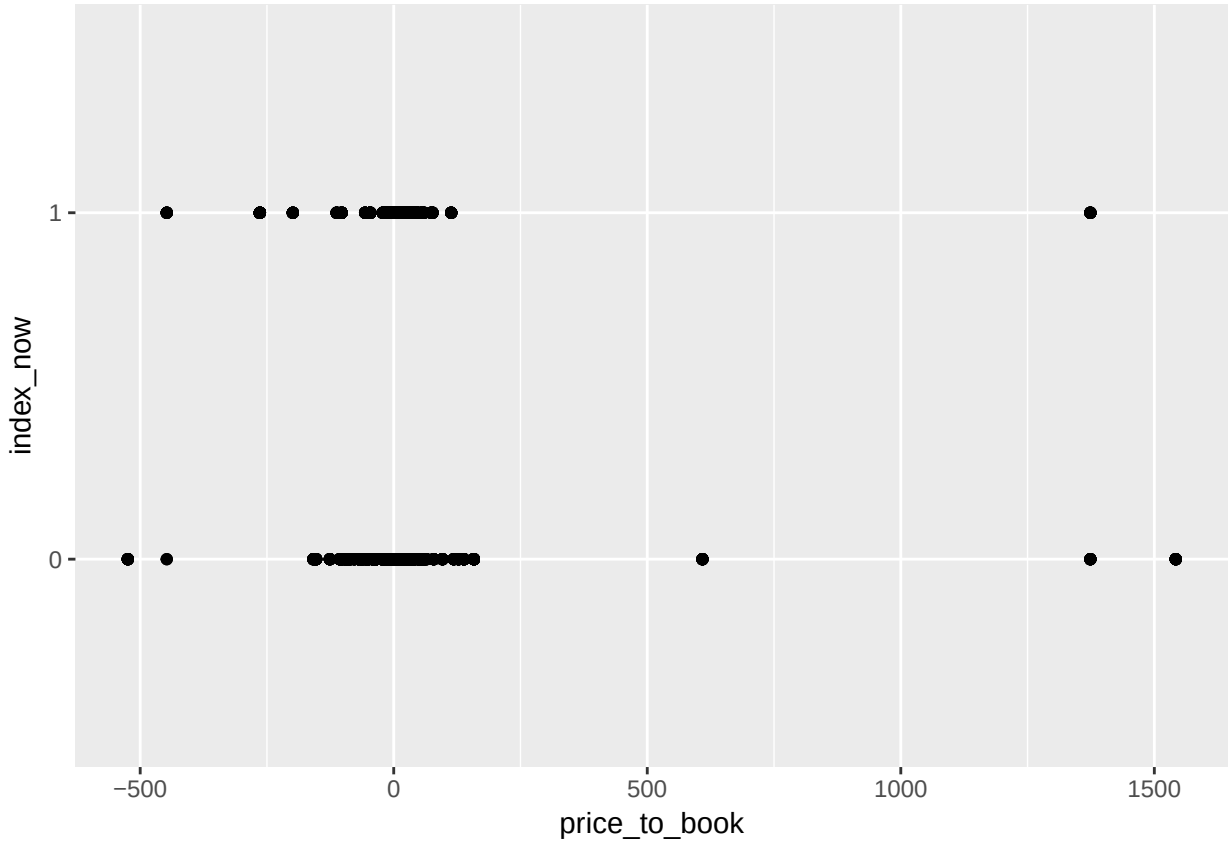
7.1 Index now vs. Trailing Volatility



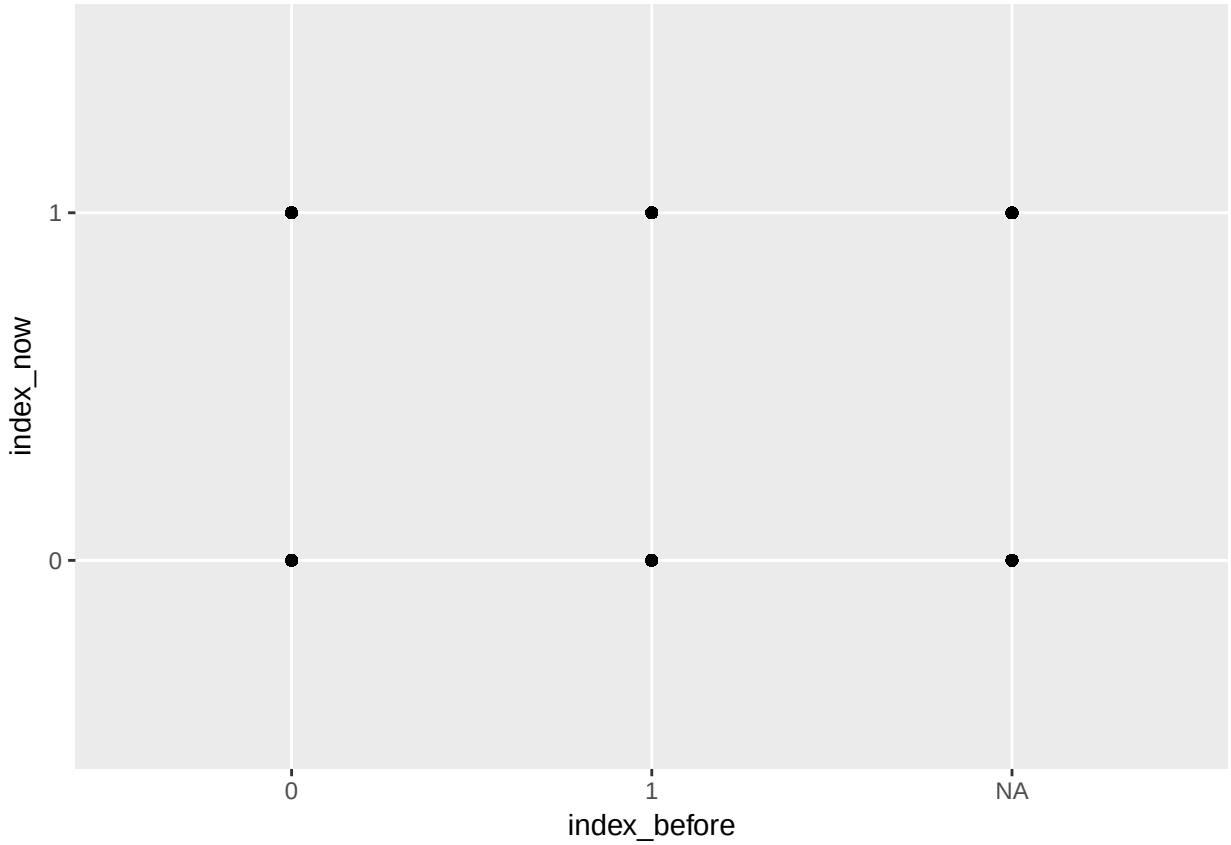
7.2 Index now vs. Trailing Beta



7.3 Index now vs. Price to Book Ratio



7.4 Index now vs. Index 6 months ago



Chapter 8

Model

Once the final data set was created and cleaned, with a number of response variables including trailing beta, trailing vol, and price to book value, and the associated outcome, which was measured by whether or not a stock was in the Min Vol index or not (1 if in, 0 if not in).

A snippet of the final data set is shown below

```
head(trainingData)
```

```
## # A tibble: 6 x 8
## # Groups:   ticker [6]
##         date ticker      beta volatility price_to_book weight index_now
##       <date> <fctr>     <dbl>      <dbl>         <dbl>  <dbl>    <fctr>
## 1 2013-09-30     CL 1.0451272  0.6023338    14.8676397 0.7228         1
## 2 2013-05-31    MKC 4.1521567  1.7358281     5.0811086 0.9495         1
## 3 2014-12-31    SJM 0.7885774  1.9425497     1.9548413 0.0497         1
## 4 2014-07-31    SPG 0.4961165  0.9387595     4.5435197 0.3643         1
## 5 2014-08-29     RE 0.7246354  0.6597441     0.7347133 0.5964         1
## 6 2016-03-31    EXR 0.6859407  1.7823979     2.7733161 0.0890         1
## # ... with 1 more variables: index_before <fctr>
```

```
tail(trainingData)
```

```
## # A tibble: 6 x 8
## # Groups:   ticker [6]
##         date ticker      beta volatility price_to_book weight index_now
##       <date> <fctr>     <dbl>      <dbl>         <dbl>  <dbl>    <fctr>
## 1 2012-08-31    MOS 1.3932857  1.0350886      1.692923      0         0
## 2 2013-01-31    HON 1.2007977  0.7540130      3.902511      0         0
## 3 2014-04-30   GILD 1.5797310  3.4051551     10.079184      0         0
## 4 2014-10-31     UA 1.4545868  0.9678980      3.496854      0         0
## 5 2016-10-31   XLNX 1.0518379  0.3432995      4.645901      0         0
## 6 2015-12-31     XL 0.7457147  0.8823093      1.719251      0         0
## # ... with 1 more variables: index_before <fctr>
```

```
summary(trainingData)
```

```
##       date                ticker           beta
##  Min.   :2012-01-31   MKC    :   86   Min.   :-23.3438
##  1st Qu.:2013-07-31   CB     :   52   1st Qu.:  0.7327
##  Median :2014-10-31   ADP    :   45   Median :  0.9262
```

```
##  Mean   :2014-10-01   K      :   45   Mean   :  0.9541
##  3rd Qu.:2015-11-30   SBAC   :   45   3rd Qu.:  1.1567
##  Max.   :2016-12-30   XEL    :   45   Max.   : 17.5440
##                       (Other):10880
##    volatility        price_to_book          weight       index_now
##  Min.   :  0.01073   Min.   :-524.357   Min.   :0.0000   0:5490
##  1st Qu.:  0.53315   1st Qu.:   1.726   1st Qu.:0.0000   1:5708
##  Median :  0.90939   Median :   2.954   Median :0.0526
##  Mean   :  1.81575   Mean   :   4.742   Mean   :0.3455
##  3rd Qu.:  1.57163   3rd Qu.:   4.957   3rd Qu.:0.6123
##  Max.   :152.96132   Max.   :1542.215   Max.   :2.7535
##
##  index_before
##  0:5988
##  1:5210
##
##
##
##
##
```

Given the nature of the data, a logit regression will be ran. Looking at all of the historical data and stock various characteristics, this would model the log odds of a stock being in the minimum volatility index as a combination of the linear predictors mentioned. Several models will be run in a panel, including one by certain months, and one by the entire pool of data.

8.1 Model 1: Entire Data Set (Monthly)

The first logit model that will be run is for the entire pool of monthly data.

8.1.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the USMV index should approximately be the same. Checking this, we can see that this is not the case. However, just around 24% of the data is from stocks that ereurrently in the index, so there is a class bias. As a result, we must sample the observations in approximately equal proportions to get better models.

```
table(monthly_final$index_now)
```

```
##
##     0     1
## 28639  8994
```

8.1.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's for the trainingData (development sample) in equal proportions. In doing so, we will put rest of the inputData not included for training into testData (validation sample). As a result, the size of development sample will be smaller that validation, which is okay, because, there are large number of observations.

```
# Create Training Data
input_ones <- monthly_final[which(monthly_final$index_now == 1), ]  # all 1's
```

```
input_zeros <- monthly_final[which(monthly_final$index_now == 0), ]  # all 0's
set.seed(100)  # for repeatability of samples
input_ones_training_rows <- sample(1:nrow(input_ones), 0.7*nrow(input_ones))  # 1's for training
input_zeros_training_rows <- sample(1:nrow(input_zeros), 0.7*nrow(input_ones))  # 0's for training. Pic
training_ones <- input_ones[input_ones_training_rows, ]
training_zeros <- input_zeros[input_zeros_training_rows, ]
trainingData <- rbind(training_ones, training_zeros)  # row bind the 1's and 0's
# Create Test Data
test_ones <- input_ones[-input_ones_training_rows, ]
test_zeros <- input_zeros[-input_zeros_training_rows, ]
testData <- rbind(test_ones, test_zeros)  # row bind the 1's and 0's
# Remove NA values in index_before
testData <- subset(testData, !is.na(index_before))
trainingData <- subset(trainingData, !is.na(index_before))
```

Now we can check class bias to see if it is more balanced. It is very close to being evenly weighted now.

```
table(trainingData$index_now)
```

```
##
##    0    1
## 5490 5708
```

8.1.3 Logistic Regression Model

Now the model can be run:

```
# Model 1
logit1 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=trainingData, family=

# Summary of Model 1
summary(logit1)
```

```
##
## Call:
## glm(formula = index_now ~ volatility + beta + price_to_book +
##     index_before, family = binomial(link = "logit"), data = trainingData)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -3.0173  -0.4514   0.1744   0.1891   2.6170
##
## Coefficients:
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -1.8759922  0.0592045 -31.687   <2e-16 ***
## volatility    -0.0043563  0.0054966  -0.793    0.428
## beta          -0.3127464  0.0371146  -8.426   <2e-16 ***
## price_to_book  0.0003945  0.0006032   0.654    0.513
## index_before1  6.1554851  0.1126370  54.649   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
```

```
##     Null deviance: 15519  on 11197  degrees of freedom
## Residual deviance:  4772  on 11193  degrees of freedom
## AIC: 4782
##
## Number of Fisher Scoring iterations: 6
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit1))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1532029     0.9956532     0.7314354     1.0003946   471.2953929
```

```
## Probability
(exp(coef(logit1))) / (1+(exp(coef(logit1))))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1328499     0.4989109     0.4224445     0.5000986     0.9978827
```

Looking at the monthly data is not a true representation of the results, because the index is rebalanced once every six months - not once a month.

8.1.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.86 - 0.0044 x vol - 0.31 x beta + 0.00039 x price_to_book + 6.16 x index_before
$\frac{p}{1-p}$ = exp(-1.86 - 0.0044 x vol - 0.31 x beta + 0.00039 x price_to_book + 6.16 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 0.996 times smaller, given a one unit increase in volatility. This response variable is not statistically significant.
- Beta: The odds ratio of being added to the index is 0.731 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 1.0051 times greater, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 410.261 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

8.1.5 Sanity Check

To take a sample stock to understand the model, we can look at a stock that was not in the USMV index on 12-30-2016, as see how accurate our model would be in predicting the probability of this stock being in the index. We can take AAL (American Airlines), which had a beta of 1.6312867, volatility of 0.8067945, price to book ratio of 4.6943413, and was not in the USMV index 6 months ago. This stock ended up not being in the minimum volatility index on 12-30-2016, so we would expect a probability to be relatively low.
- Odds Ratio:
$\frac{p}{1-p}$ = exp(-3.094 - 0.0032 x 0.8067945 - 0.25 x 1.6312867 + 0.00051 x 4.6943413 + 6.017 x 0)
$\frac{p}{1-p}$ = 0.03013677

- Probability:
p = (exp(-3.094 - 0.0032 x 0.8067945 - 0.25 x 1.6312867 + 0.00051 x 4.6943413 + 6.017 x 0) / (1+exp(-3.094 - 0.0032 x 0.8067945 - 0.25 x 1.6312867 + 0.00051 x 4.6943413 + 6.017 x 0)))
p = 0.02925511

The odds of AAL being in the index on 12-30-2016 is 0.03013677, and this translates to a probability of 2.93%. As expected, already knowing that the stock was not in the index, this low probability seems reasonable.
To further understand the model, we can look at a stock that was was in the USMV index on 12-30-2016, as see how accurate our model would be in predicting the probability of this stock being in the index. We can take AAPL (Apple), which had a beta of 1.0099644, volatility of 0.6118842, price to book ratio of 4.7037726, and it was in the USMV index 6 months ago. This stock ended up being in the minimum volatility index on 12-30-2016, so we would expect a probability to be relatively high
- Odds Ratio:
$\frac{p}{1-p}$ = exp(-3.094 - 0.0032 x 0.6118842 - 0.25 x 1.0099644 + 0.00051 x 4.7037726 + 6.017 x 1)
$\frac{p}{1-p}$ = 14.45369
- Probability:
p = (exp(-3.094 - 0.0032 x 0.6118842 - 0.25 x 1.0099644 + 0.00051 x 4.7037726 + 6.017 x 1) / (1+exp(-3.094 - 0.0032 x 0.6118842 - 0.25 x 1.0099644 + 0.00051 x 4.7037726 + 6.017 x 1)))
p = 0.9352905

The odds of AAL being in the index on 12-30-2016 is 14.45369, and this translates to a probability of 93.53%. As expected, already knowing that the stock was in the index, this high probability seems reasonable.

8.1.6 Model Quality

To test the quality of the model, several tests were done:
Predictive Power
The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.74.

```
library(InformationValue)
optCutOff <- optimalCutoff(testData$index_now, predicted)[1]
```

*VIF**_
Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
vif(logit1)
```

```
##    volatility          beta price_to_book  index_before
##      1.016126      1.018653      1.000327      1.005611
```

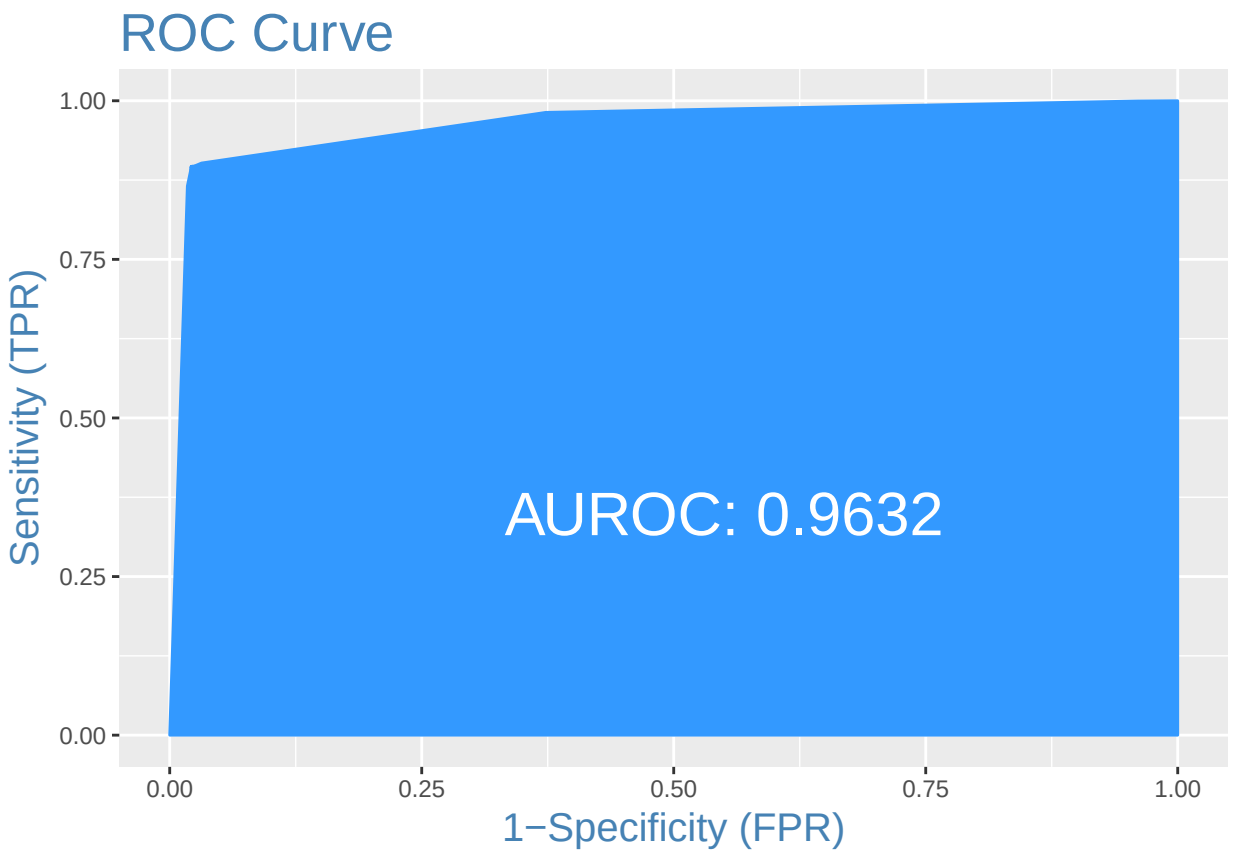
Misclassification Error
Misclassification error is the percentage mismatch of predicted vs actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 3.1%, which is quite low, and thus good.

```
predicted <- plogis(predict(logit1, testData))
misClassError(testData$index_now, predicted)
```

```
## [1] 0.0309
```

ROC
Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 96.3%.

```
plotROC(testData$index_now, predicted)
```



Concordance
Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 97.2% is a good quality model.

```
Concordance(testData$index_now, predicted)
```

```
## $Concordance
## [1] 0.9724405
##
## $Discordance
## [1] 0.02755952
##
## $Tied
```

```
## [1] -4.510281e-17
##
## $Pairs
## [1] 47632140
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 89.6%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 97.9%.

```
sensitivity(testData$index_now, predicted, threshold = optCutOff)
```

```
## [1] 0.8956805
```

```
specificity(testData$index_now, predicted, threshold = optCutOff)
```

```
## [1] 0.9789284
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData$index_now, predicted, threshold = optCutOff)
```

```
##       0    1
## 0 19001  256
## 1   409 2198
```

8.2 Model 2: November Model

Since the index is rebalanced twice a year (once in November and once in May), it makes sense to look at a model for each of these individual months. Thus, a subset of the data was taken for November, and the same procedures done at with Model 1.

```
# Subset data for dates from November only
november_final <- filter(monthly_final, date == "2011-11-30" | date == "2012-11-30"| date == "2013-11-2
# Remove NA values from set
november_final <- subset(november_final, !is.na(index_before))
```

8.2.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the USMV index should approximately be the same. Checking this, we can see that this is not the case. However, just around 26% of the data is from stocks that ereurrently in the index, so there is a class bias. As a result, we must sample the observations in approximately equal proportions to get a better model.

```
table(november_final$index_now)
```

```
##
##    0    1
## 2161  750
```

8.2.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's for the trainingData (development sample) in equal proportions. In doing so, we will put rest of the inputData not included for training into testData (validation sample). As a result, the size of development sample will be smaller that validation, which is okay, because, there are large number of observations.

```
# Create Training Data
input_ones2 <- november_final[which(november_final$index_now == 1), ]  # all 1's
input_zeros2 <- november_final[which(november_final$index_now == 0), ]  # all 0's
set.seed(100)  # for repeatability of samples
input_ones_training_rows2 <- sample(1:nrow(input_ones2), 0.7*nrow(input_ones2))  # 1's for training
input_zeros_training_rows2 <- sample(1:nrow(input_zeros2), 0.7*nrow(input_ones2))  # 0's for training.
training_ones2 <- input_ones2[input_ones_training_rows2, ]
training_zeros2 <- input_zeros2[input_zeros_training_rows2, ]
trainingData2 <- rbind(training_ones2, training_zeros2)  # row bind the 1's and 0's
# Create Test Data
test_ones2 <- input_ones2[-input_ones_training_rows2, ]
test_zeros2 <- input_zeros2[-input_zeros_training_rows2, ]
testData2 <- rbind(test_ones2, test_zeros2)  # row bind the 1's and 0's
```

Now we can check class bias to see if it is more balanced. It is evenly weighted now, with each being represented by 525 observations.

```
table(trainingData2$index_now)
```

```
##
##   0   1
## 525 525
```

8.2.3 Logistic Regression Model

Now the model can be run:

```
# Model 2
logit2 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=trainingData2, family=

# Summary of Model 2
summary(logit2)
```

```
##
## Call:
## glm(formula = index_now ~ volatility + beta + price_to_book +
##     index_before, family = binomial(link = "logit"), data = trainingData2)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.98863  -0.51069  -0.04623   0.27163   2.05545
##
## Coefficients:
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -1.4577528  0.1834842  -7.945 1.94e-15 ***
## volatility     0.0604954  0.0330935   1.828 0.067548 .
## beta          -0.4945911  0.1331544  -3.714 0.000204 ***
## price_to_book -0.0001288  0.0017214  -0.075 0.940368
```

```
## index_before1  5.0806517  0.2776866  18.296  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1455.61  on 1049  degrees of freedom
## Residual deviance:  585.48  on 1045  degrees of freedom
## AIC: 595.48
##
## Number of Fisher Scoring iterations: 6
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit2))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.2327588     1.0623627     0.6098202     0.9998712   160.8788646
```

```
## Probability
(exp(coef(logit2))) / (1+(exp(coef(logit2))))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1888113     0.5151192     0.3788126     0.4999678     0.9938225
```

Looking at the November model will be helpful for someone looking to predict index rebalancing between June and October.

8.2.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.46 + 0.061 x vol - 0.49 x beta - 0.00013 x price_to_book + 5.08 x index_before
$\frac{p}{1-p}$ = exp(-1.46 + 0.061 x vol - 0.49 x beta - 0.00013 x price_to_book + 5.08 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 1.063 times greater, given a one unit increase in volatility. This response variable is statistically significant, at an alpha level of 0.1.
- Beta: The odds ratio of being added to the index is 0.61 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 0.99 times smaller, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 160.88 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

8.2.5 Sanity Check

Will do later, if useful.

8.2.6 Model Quality

To test the quality of the model, several tests were done:
Predictive Power

The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.95.

```
library(InformationValue)
optCutOff2 <- optimalCutoff(testData2$index_now, predicted2)[1]
```

*VIF**_
Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
vif(logit2)
```

```
##    volatility          beta price_to_book  index_before
##      1.107794      1.106221      1.000799      1.003120
```

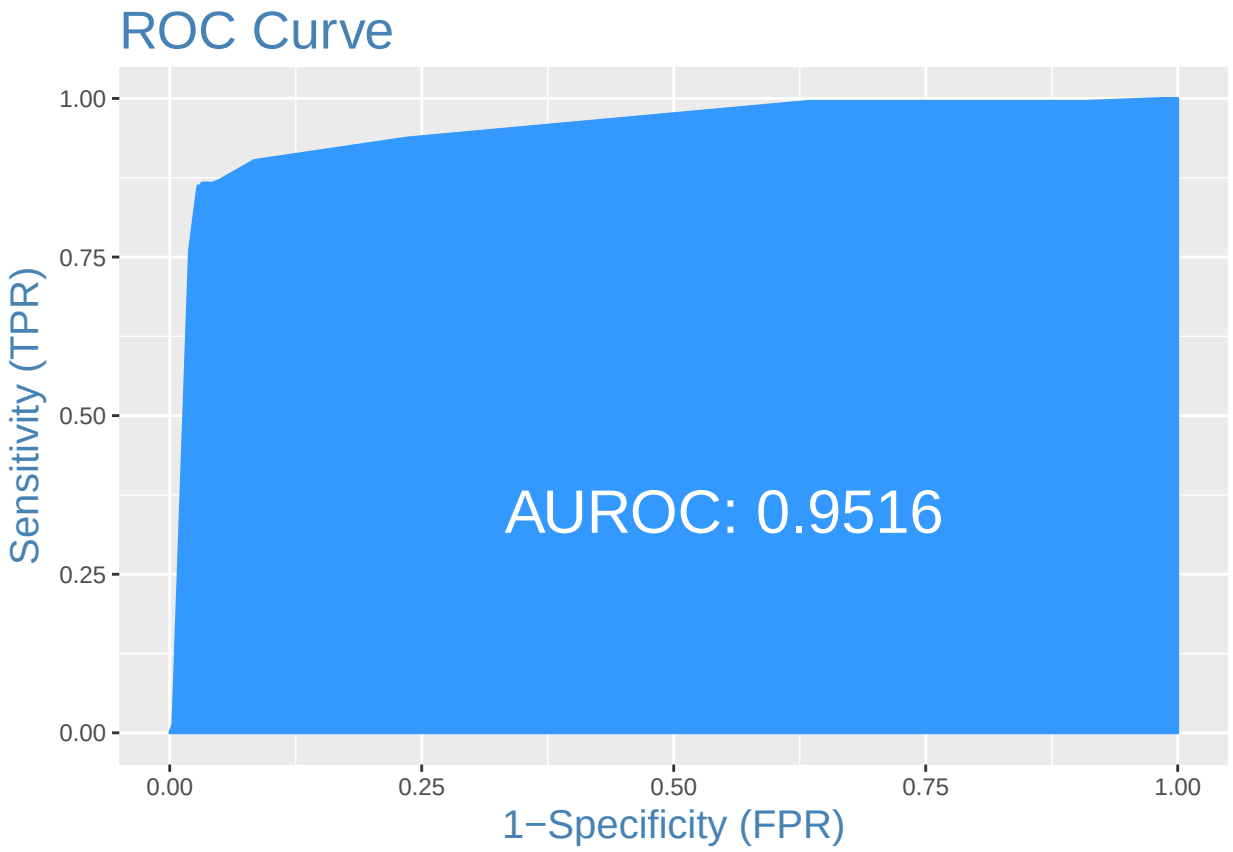
Misclassification Error
Misclassification error is the percentage mismatch of predicted vs actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 4.4%, which is quite low, and good.

```
predicted2 <- plogis(predict(logit2, testData2))
misClassError(testData2$index_now, predicted2)
```

```
## [1] 0.0435
```

ROC
Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 95.2%.

```
plotROC(testData2$index_now, predicted2)
```



Concordance
Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 95.5% is a good quality model.

```
Concordance(testData2$index_now, predicted2)
```

```
## $Concordance
## [1] 0.9558843
##
## $Discordance
## [1] 0.04411573
##
## $Tied
## [1] -6.938894e-18
##
## $Pairs
## [1] 368100
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 85.8%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 97.3%.

```
sensitivity(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
## [1] 0.8577778
```

```
specificity(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
## [1] 0.9731051
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
##      0   1
## 0 1592  32
## 1   44 193
```

8.3 Model 3: May Model

Since the index is rebalanced twice a year (once in November and once in May), it makes sense to look at a model for each of these individual months. Thus, a subset of the data was taken for May, and the same procedures done at with Model 1.

```
# Subset data for dates from May only
may_final <- filter(monthly_final, date == "2012-05-31" |  date == "2013-05-31"| date == "2014-05-30" |
# Remove NA values from set
may_final <- subset(may_final, !is.na(index_before))
```

8.3.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the USMV index should approximately be the same. Checking this, we can see that this is not the case. However, just around 24% of the data is from stocks that ereurrently in the index, so there is a class bias. As a result, we must sample the observations in approximately equal proportions to get a better model.

```
table(may_final$index_now)
```

```
##
##    0    1
## 2188  705
```

8.3.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's for the trainingData (development sample) in equal proportions. In doing so, we will put rest of the inputData not included for training into testData (validation sample). As a result, the size of development sample will be smaller that validation, which is okay, because, there are large number of observations.

```
# Create Training Data
input_ones3 <- may_final[which(may_final$index_now == 1), ]  # all 1's
input_zeros3 <- may_final[which(may_final$index_now == 0), ]  # all 0's
set.seed(100)  # for repeatability of samples
input_ones_training_rows3 <- sample(1:nrow(input_ones3), 0.7*nrow(input_ones3))  # 1's for training
```

```
input_zeros_training_rows3 <- sample(1:nrow(input_zeros3), 0.7*nrow(input_ones3))  # 0's for training.
training_ones3 <- input_ones3[input_ones_training_rows3, ]
training_zeros3 <- input_zeros3[input_zeros_training_rows3, ]
trainingData3 <- rbind(training_ones3, training_zeros3)  # row bind the 1's and 0's
# Create Test Data
test_ones3 <- input_ones3[-input_ones_training_rows3, ]
test_zeros3 <- input_zeros3[-input_zeros_training_rows3, ]
testData3 <- rbind(test_ones3, test_zeros3)  # row bind the 1's and 0's
```

Now we can check class bias to see if it is more balanced. It is evenly weighted now, with each being represented by 493 observations.

```
table(trainingData3$index_now)
```

```
##
##   0   1
## 493 493
```

8.3.3 Logistic Regression Model

Now the model can be run:

```
# Model 3
logit3 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=trainingData3, family=

# Summary of Model 3
summary(logit3)
```

```
##
## Call:
## glm(formula = index_now ~ volatility + beta + price_to_book +
##     index_before, family = binomial(link = "logit"), data = trainingData3)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -3.04816  -0.38611   0.00159   0.13885   2.34011
##
## Coefficients:
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -1.744382   0.249805  -6.983 2.89e-12 ***
## volatility    -0.039892   0.028585  -1.396 0.162842
## beta          -0.642378   0.165440  -3.883 0.000103 ***
## price_to_book -0.012360   0.007939  -1.557 0.119498
## index_before1  7.013927   0.497441  14.100  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1366.89  on 985  degrees of freedom
## Residual deviance:  327.87  on 981  degrees of freedom
## AIC: 337.87
##
## Number of Fisher Scoring iterations: 7
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit3))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1747529     0.9608932     0.5260398     0.9877159  1112.0132792
```

```
## Probability
(exp(coef(logit3))) / (1+(exp(coef(logit3))))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1487572     0.4900283     0.3447091     0.4969100     0.9991015
```

Looking at the May model will be helpful for someone looking to predict index rebalancing between December and April.

8.3.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.74 - 0.04 x vol - 0.64 x beta - 0.012 x price_to_book + 7.014 x index_before
$\frac{p}{1-p}$ = exp(-1.74 - 0.04 x vol - 0.64 x beta - 0.012 x price_to_book + 7.014 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 0.96 times smaller, given a one unit increase in volatility. This response variable is not statistically significant.
- Beta: The odds ratio of being added to the index is 0.52 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 0.99 times smaller, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 1112.01 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

8.3.5 Sanity Check

Will do later if useful.

8.3.6 Model Quality

To test the quality of the model, several tests were done:
Predictive Power
The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.98.

```
library(InformationValue)
optCutOff3 <- optimalCutoff(testData3$index_now, predicted3)[1]
```

*VIF**_

Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
vif(logit3)
```

```
##    volatility          beta price_to_book  index_before
##      1.124595      1.161319      1.006899      1.074031
```

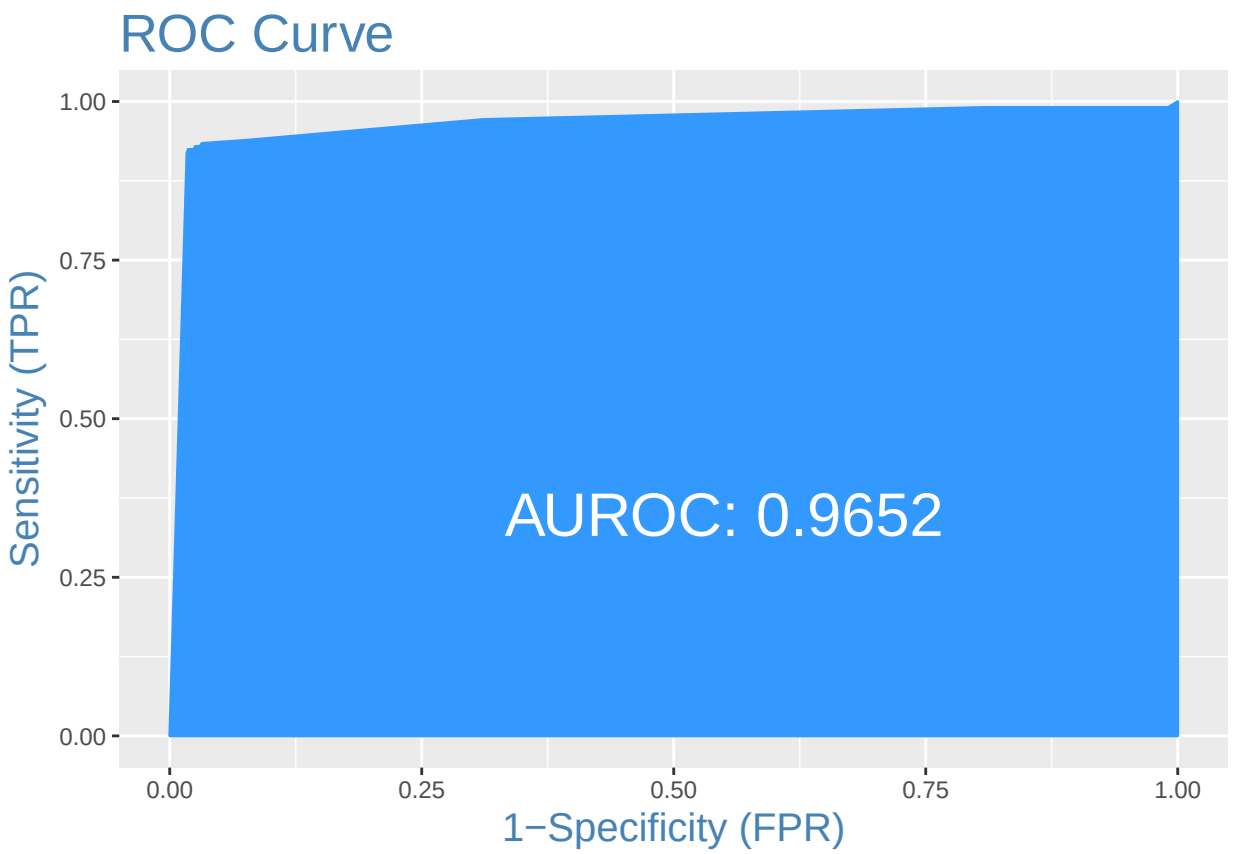
Misclassification Error
Misclassification error is the percentage mismatch of predicted vs actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 2.6%, which is quite low, and good.

```
predicted3 <- plogis(predict(logit3, testData3))
misClassError(testData3$index_now, predicted3)
```

```
## [1] 0.0262
```

ROC
Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 96.5%.

```
plotROC(testData3$index_now, predicted3)
```



Concordance

Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 97.3% is a good quality model.

```
Concordance(testData3$index_now, predicted3)
```

```
## $Concordance
## [1] 0.9732621
##
## $Discordance
## [1] 0.02673791
##
## $Tied
## [1] -3.469447e-18
##
## $Pairs
## [1] 359340
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 92.0%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 98.3%.

```
sensitivity(testData3$index_now, predicted3, threshold = optCutOff3)
```

```
## [1] 0.9198113
```

```
specificity(testData3$index_now, predicted3, threshold = optCutOff3)
```

```
## [1] 0.9828909
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData3$index_now, predicted3, threshold = optCutOff3)
```

```
##      0   1
## 0 1666  17
## 1   29 195
```

8.4 Model 4: Total Rebalancing (November & May) Model

Since the index is rebalanced twice a year (once in November and once in May), it makes sense to look at a model for both of these months. Thus, a subset of the data was taken for May and November, by combining the data sets from Model 2 and Model 3.

```
# Subset data for dates from May only
both_final <- rbind(may_final, november_final)
```

8.4.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the USMV index should approximately be the same. Checking this, we can see that this is not the case. However, just around 25% of the data is from stocks that ereurrently in the index, so there is a class bias. As a result, we must sample the observations in approximately equal proportions to get a better model.

```
table(both_final$index_now)
```

```
##
##    0    1
## 4349 1455
```

8.4.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's for the trainingData (development sample) in equal proportions. In doing so, we will put rest of the inputData not included for training into testData (validation sample). As a result, the size of development sample will be smaller that validation, which is okay, because, there are large number of observations.

```
# Create Training Data
input_ones4 <- both_final[which(both_final$index_now == 1), ]  # all 1's
input_zeros4 <- both_final[which(both_final$index_now == 0), ]  # all 0's
set.seed(100)  # for repeatability of samples
input_ones_training_rows4 <- sample(1:nrow(input_ones4), 0.7*nrow(input_ones4))  # 1's for training
input_zeros_training_rows4 <- sample(1:nrow(input_zeros4), 0.7*nrow(input_ones4))  # 0's for training.
training_ones4 <- input_ones4[input_ones_training_rows4, ]
training_zeros4 <- input_zeros4[input_zeros_training_rows4, ]
trainingData4 <- rbind(training_ones4, training_zeros4)  # row bind the 1's and 0's
# Create Test Data
test_ones4 <- input_ones4[-input_ones_training_rows4, ]
test_zeros4 <- input_zeros4[-input_zeros_training_rows4, ]
testData4 <- rbind(test_ones4, test_zeros4)  # row bind the 1's and 0's
```

Now we can check class bias to see if it is more balanced. It is evenly weighted now, with each being represented by 1018 observations.

```
table(trainingData4$index_now)
```

```
##
##    0    1
## 1018 1018
```

8.4.3 Logistic Regression Model

Now the model can be run:

```
# Model 4
logit4 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=trainingData4, family=

# Summary of Model 3
summary(logit4)
```

```
##
## Call:
```

```
## glm(formula = index_now ~ volatility + beta + price_to_book +
##     index_before, family = binomial(link = "logit"), data = trainingData4)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.76973  -0.46138   0.00366   0.21208   2.16144
##
## Coefficients:
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -1.840874   0.124092 -14.835  < 2e-16 ***
## volatility     0.002945   0.009558   0.308    0.758
## beta          -0.309455   0.071228  -4.345  1.4e-05 ***
## price_to_book -0.001894   0.001547  -1.224    0.221
## index_before1  5.886884   0.241988  24.327  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2822.50  on 2035  degrees of freedom
## Residual deviance:  931.66  on 2031  degrees of freedom
## AIC: 941.66
##
## Number of Fisher Scoring iterations: 6
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit4))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1586787     1.0029492     0.7338469     0.9981080   360.2808848
```

```
## Probability
(exp(coef(logit4))) / (1+(exp(coef(logit4))))
```

```
##   (Intercept)    volatility          beta price_to_book index_before1
##     0.1369480     0.5007362     0.4232478     0.4995265     0.9972321
```

Looking at this model will be helpful for someone looking to predict index rebalancing, generally, for both months.

8.4.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.84 + 0.003 x vol - 0.31 x beta - 0.0019 x price_to_book + 5.89 x index_before
$\frac{p}{1-p}$ = exp(-1.84 + 0.003 x vol - 0.31 x beta - 0.0019 x price_to_book + 5.89 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 1.0029 times greater, given a one unit increase in volatility. This response variable is not statistically significant.
- Beta: The odds ratio of being added to the index is 0.73 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 0.99 times smaller, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 360.28 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

8.4.5 Sanity Check

Will do later if useful.

8.4.6 Model Quality

To test the quality of the model, several tests were done:
Predictive Power
The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.77.

```
library(InformationValue)
optCutOff4 <- optimalCutoff(testData4$index_now, predicted4)[1]
```

*VIF**_
Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
vif(logit4)
```

```
##    volatility          beta price_to_book  index_before
##      1.022471      1.033581      1.007330      1.022806
```

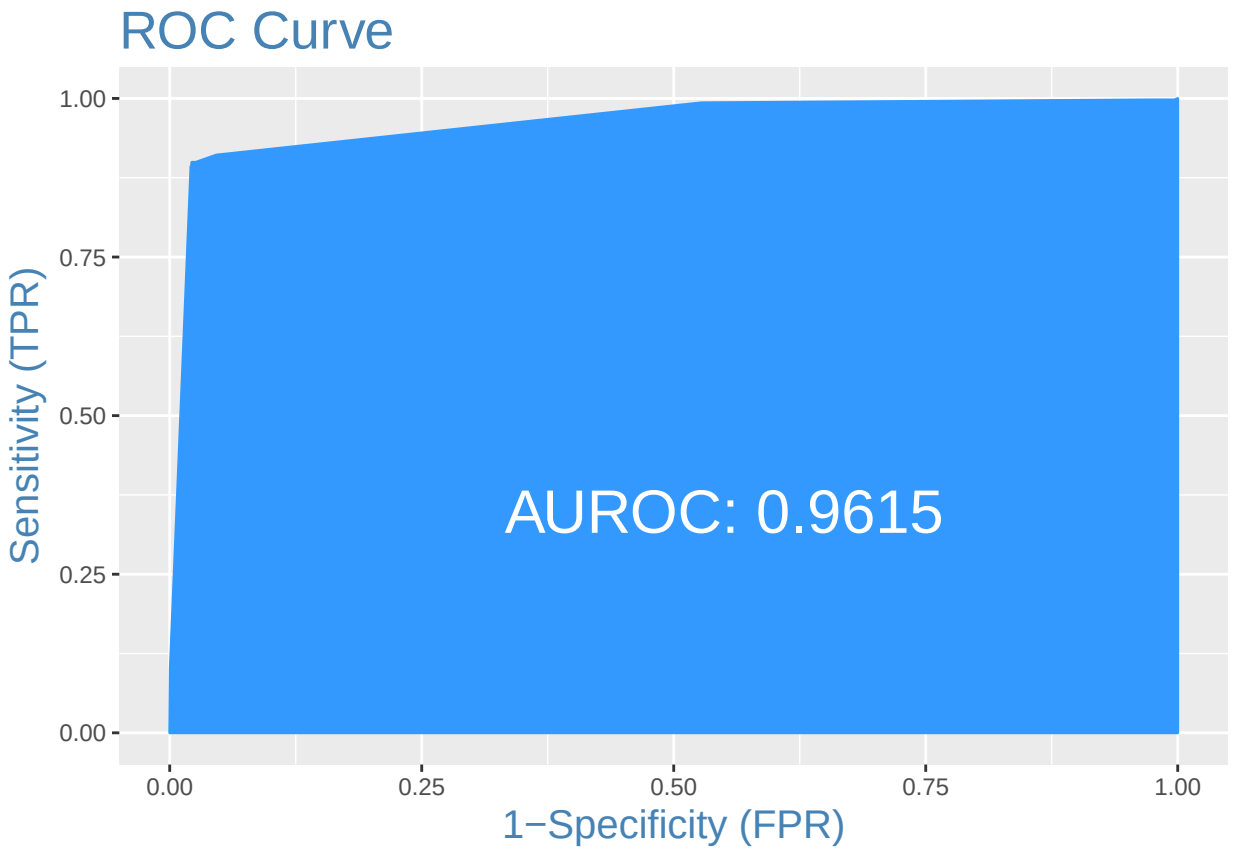
Misclassification Error
Misclassification error is the percentage mismatch of predicted vs actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 3.2%, which is quite low, and good.

```
predicted4 <- plogis(predict(logit4, testData4))
misClassError(testData4$index_now, predicted4)
```

```
## [1] 0.0321
```

ROC
Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 96.2%.

```
plotROC(testData4$index_now, predicted4)
```



Concordance
Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 97.1% is a good quality model.

```
Concordance(testData4$index_now, predicted4)
```

```
## $Concordance
## [1] 0.9714409
##
## $Discordance
## [1] 0.02855912
##
## $Tied
## [1] -4.857226e-17
##
## $Pairs
## [1] 1455647
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 89.9%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 97.8%.

```
sensitivity(testData4$index_now, predicted4, threshold = optCutOff4)
```

```
## [1] 0.8993135
```

```
specificity(testData4$index_now, predicted4, threshold = optCutOff4)
```

```
## [1] 0.9780847
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData4$index_now, predicted4, threshold = optCutOff4)
```

```
##      0   1
## 0 3258  44
## 1   73 393
```

Chapter 9

Conclusion

All in all, the 4 models were comprable in terms of statistical resiliency and predictive power. Each model may have a different usage, based of each model's strengths and weaknesses, and what goals the investor has in mind for the model. For example, an investor looking to capture an arbitrage opportunity in November, might be best suited in looking at the November specific model. Someone looking for abritrage opportunities throughout the year during both rebalances might look at the combined May and November model.

9.1 Side by Side Model Comparison

```
## Warning: 'rbind_list' is deprecated.
## Use 'bind_rows()' instead.
## See help("Deprecated")
```

```
## `mutate_each()` is deprecated.
## Use `mutate_all()`, `mutate_at()` or `mutate_if()` instead.
## To map `funs` over a selection of variables, use `mutate_at()`
```

```
## # A tibble: 10 x 6
##             term       key   `1`   `2`   `3`   `4`
##  *         <chr>     <chr> <dbl> <dbl> <dbl> <dbl>
##  1   (Intercept)  estimate -1.88 -1.46 -1.74 -1.84
##  2   (Intercept) std.error  0.06  0.18  0.25  0.12
##  3          beta  estimate -0.31 -0.49 -0.64 -0.31
##  4          beta std.error  0.04  0.13  0.17  0.07
##  5 index_before1  estimate  6.16  5.08  7.01  5.89
##  6 index_before1 std.error  0.11  0.28  0.50  0.24
##  7 price_to_book  estimate  0.00  0.00 -0.01  0.00
##  8 price_to_book std.error  0.00  0.00  0.01  0.00
##  9    volatility  estimate  0.00  0.06 -0.04  0.00
## 10    volatility std.error  0.01  0.03  0.03  0.01
```

As seen, each model gave out pretty similar coefficient values for the various response variables. Beta ranged between -0.31 and -0.64, index_before ranged from 5.08 to 7.01, price to book ranged from 0.00 to -0.01, and volatility ranged between -0.04 and 0.06.

Chapter 10

Discussion

After comparing all of the models, it makes sense to discuss the applications of these various models to the real world, and to finance.

10.1 Understanding of Relationships

Through these models, we can get a better understanding of the relationships between the predictor variables, and whether or not the stock is in the Min Vol index. In general, each model suggested an increase in beta will reduce the likelihood of a stock being in the min vol index, with all else held constant. This makes sense, as beta is one measure of risk and volatility. Moreover, it is a widely used metric in finance, so it is not surpising that it is a statistically significant variable. Moreover, the most significant variable was whether or not the stock was in the index before. This makes a lot of sense, as a stock currently in the index presumably has many min vol characteristics from before, that must be signficantly altered if it were to be removed. Moreover, stocks that were in the index previously were many times more likely to be in the index currently, than stocks that had previously not been in the index. This variable was also statistically significant. Surpisingly, volatility was not statistically signficant, though the index itself is called the "Minimum Volatility" Index. Moreover, price to book was also an insignficant variable, which does make sense. Each model was able to quantify these relationships, and help us better understand what

10.2 Arbitrage

Each model was able to take various attributes of a stock, and calculate a probability for it currently being in the index. Using the optimal cutoffs, we were able to get a sense of the probability value that would be significant in determining when a stock would be in or out of the index. For example, at a cutoff of 0.9, this would tell us that we could reasonably expect stocks with a probability of over 90% to be in the index, and stocks with less than a 90% probability to not be in the index. With this information, there are many different arbitrage opportunities. One could long stocks currently not in the index that have a probability greater than the optimal cutoff for that model. This would represent the stocks with the greatest chance of being added to the index, that are currently not in the index. If correct, prior studies would suggest that the stock price would consequently increase from this happening. Moreover, one could short stocks that are currently in the index, that have a probability value less than the cutoff. This could lead to an arbritrage opportunity if the stock is removed from the index, as one is short it.

Chapter 11

Bibliography

Vliet, P. V., & Koning, J. D. (2017). High returns from low risk: a remarkable stock market paradox.

Asness, C. S., Frazzini, A., Gormsen, N. J., & Pedersen, L. H. (2016). Betting Against Correlation: Testing Theories of the Low-Risk Effect.

Huij, J., & Kyosev, G. (2016). Price Response to Factor Index Additions and Deletions.

Baker, M., Bradley, B., & Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. Financial Analysts Journal, 67(1), 40-54.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. Journal of Financial Economics, 111(1), 1-25.

Baker, M., Bradley, B., & Taliaferro, R. (2014). The low-risk anomaly: A decomposition into micro and macro effects. Financial Analysts Journal, 70(2), 43-58.