

Vignette

John Gilheany

2/19/2017

Literature Review

High Returns from Low Risk By Pim van Vliet and Jan De Koning

One of the most widely believed tenants of finance is the concept that with more risk comes more reward. However, looking at historical market returns, this does not appear to be the case. Over an 86-year period from 1929, low volatility stocks outperformed high volatility stocks by a factor of 18. If both portfolios started off with the same \$100, the low volatility portfolio end value would be \$395,000, while the high volatility portfolio would be worth just \$21,000. Low risk stocks returned 10.2% annually whereas the high risk stocks returned just 6.4% annually. This difference of 3.8% is striking, and presents an anomaly in the field of finance.

This begs the question of how a portfolio of lower volatility stocks can outperform higher volatility stocks over a long period of time. The primary way this occurs is that the low volatility portfolio loses less during times of financial stress. For example, in 1932 following the Great Depression, it was observed that the high volatility portfolio shrunk from \$100 in value to \$5 in value, while the low volatility portfolio shrunk from \$100 to \$30. Since the low volatility portfolio is able to lose less money, it is able to grow capital more effectively than the high volatility portfolio. In this example, the annualized volatility of the low risk portfolio was 13%, and the annualized volatility of the high risk portfolio was around 2.5 times that, at 36%. In addition to being more risky, the high volatility portfolio was outperformed by 18 times.

Thus, it seems very counterintuitive that fund managers and investors would not only invest in low risk stocks. Part of understanding this comes from interpreting what risk is defined as in the financial community. Risk is not necessarily defined as losing money, as it may be for an individual, but instead underperforming a benchmark. Volatility is also an important concept to understand. Volatility is an important measure of financial risk, as it comes from the price fluctuations of a stock or investment. Volatility is also one of the best indicators of bankruptcy. Taking some risk does pay off, as the relationship between risk and return starts off slightly positive before leveling off and becoming negative. Many researchers focus on short-term periods when analyzing stock returns instead of longer term for a couple of reasons. The first reason many focus on “single period returns”, which in most academic studies is just a one month period, is because this takes away the significance of compounding. The longer the investment period, the more risk one takes in hurting long term returns through compounding. By not fully including the magic “return upon return” effect of compounding, a high-risk portfolio in this book performs more than 6% better per year. For example, if in month a portfolio worth \$100 drops 50% to \$50, then the next month increases 50% to \$75, the investment return is dependent on how one divides the time period. Looking at it on a monthly basis, even though the portfolio lost \$25, the net return would be $-50\% + 50\%$, or 0%. Looking at it on a long term basis, the net return was -25% .

David Blitz, the head of quantitative equity research at Robeco, discusses this different perspective as somewhat due to the need to benchmark the performance of an investment manager. This is a concept known as “relative” risk. In the examples above, everything has been in respect to absolute risk - that is how much money is being gained or lost due to overall stock movements, with regard to the starting amount of money invested. Volatility, in itself, captures these changes in the price of a stock, and is an absolute risk measurement. Many institutional investors do not look at risk on an absolute level, as a retiree or mom and pop investor may, but instead look at the risk of a portfolio with respect to market or some other widely accepted benchmark. For these investors, the risk is not as much about losing money, rather is more about lagging the market or their peers. Investing is very much a relative game. If a portfolio drops 20% while

the market drops 40%, this is seen as a much better outcome than if a portfolio goes up 20% while the market goes up 40%. Thus, a portfolio that moves closely with the market has a very low relative risk. This risk can be calculated as volatility by looking at the relative price movements, instead of the absolute price movements.

Investment professionals focus on relative risk for a number of reasons, one of which is the fact that they are not managing their own money. They need to prove to their bosses and clients that they are above average in their job. If a particular benchmark cannot be beaten by these investors, clients may ask why pay for them to manage their money when they could put it in a low-fee or no-fee mutual fund. This is one of the reasons that institutional investors need to compare their performance to some benchmark. Thus, the focus for investors is return and relative risk. Adding low risk stocks to the portfolio causes relative risk to increase a lot, making it an unappealing investment because low absolute risk inherently causes high relative risk. A low risk portfolio only makes sense if absolute risk is what one cares about. Thus, for those investors who don't care about relative risk and just absolute risk, low risk stocks are a great investing opportunity.

In addition to the reasons mentioned, there are several additional reasons why some investors are not attracted to low-risk stocks. Eric Falkenstein, a renowned author in the low volatility investing realm, wrote that "envy is at the root of the investment paradox." Some investors don't recognize the significance of compounding returns. Others do, but are unable to utilize the paradox due to relative risk and career pressures. Analysts who choose big winners are more likely to get recognized than those who pick safer stocks with lower upside potential, and funds that pick the right high risk stocks also see more reward in an increase in AUM. Moreover, some people do not invest in low risk stocks because they have less appeal of high risk stocks, where they think they can make money easily and quickly. These high risk stocks are more "sexy" and have a "lottery ticket" element that attracts investors with the appeal of a big payday.

Betting Against Correlation: Testing Theories of the Low-Risk Effect

A recurring phenomena in finance, is the observation of the "low-risk effect." This is the idea that lower risk or lower volatility stocks, tend to have higher alpha than higher risk or higher volatility stocks. In trying to understand the reason this anomaly occurs, are two possible explanations. The first looks at whether this is caused by leverage constraints, meaning measurement using systematic risk. The second focuses on the behavioral effects, or idiosyncratic risks. One of the main issues with prior research, is that a lot of the low-risk factors are correlated and interrelated, making it hard to isolate certain factors or effects. In this paper, the global data was used, with a couple new factors meant to control for existing factors.

Previous studies, like one by Adrian, Etula, and Muir in 2014, showed a link between return to the BAB (Betting Against Beta) factor and financial intermediary leverage. Many of these factors, though, including BAB, generally exhibit the "low-risk effect" and are thus very hard to differentiate between. Thus, this paper decided to do just that, by breaking down BAB into two other factors: betting against correlation (BAC) and betting against volatility (BAV). BAC is accomplished through longing stocks with low correlation to the market, and shorting those with high correlation to the market, while trying to match the volatilities of both the long and short portfolios. BAV is achieved in a similar manner, except instead of longing and shorting correlation, volatility is used, and correlation is kept constant.

To address the behavioral explanation, the paper looks at some prior factors from studies done by Ang, Hodrick, Xing, and Zhang in 2006 and 2009. The first study found stocks with low idiosyncratic volatility (IVOL) have a greater risk-adjusted return, while the second found that a low maximum return (LMAX), a measure of idiosyncratic skewness, is associated with greater risk-adjusted returns. This paper kept the focus on LMAX and IVOL, but added another factor, scaled MAX (SMAX), which longs stocks with a low MAX return divided by ex ante volatility, and then shorts stocks with a high MAX return divided by ex ante volatility. This focuses on the lottery demand, holding volatility relatively constant and only focusing on the distribution of the returns. Margin debt held by investors, and investor sentiment were also noted.

In the paper, 58,415 stocks from the MSCI World Index, from 24 countries between January 1926 and December 2015 were covered. BAB and BAC ended up being very successful in controlling for the other factors that could influence the “low-risk effect.” For all stocks, the BAC factor produced a significant six-factor alpha that was nearly independent of the other low-risk factors studied. In terms of explaining the behavioral side with factors, SMAX was the only truly great, resilient measure used. The rest generally had higher turnover, and were consequently very susceptible to microstructure noise. SMAX attained positive risk-adjusted returns in the U.S. but negative risk-adjusted returns globally, which was seen with some other idiosyncratic risk factors. The paper showed that systematic low-risk factor generally tended to outperform behavioral risk factors, especially when considering turnover and time period length. All in all, the low-risk effect was believed to be driven by multiple factor effects, meaning both leverage constraints and the demand for lottery could play a role in effecting this. However, leverage constraint effects were a bit stronger, especially internationally.

Price Response to Factor Index Additions and Deletions

Some of the driving fundamental assumptions of finance is the flat demand curve for stocks, where risk is the main driver and each stock has a perfect substitute. However, this concept has been questioned for the past few years, with literature picking up on stocks with show supply shocks and checking how this affects their price. The literature has shown several instances where large block sales of stock has negatively affected its price. This was often due to information contamination, which is new, significant information about the company in the market. This information often reflects fundamental changes in the company, and if is is negative, will understandably trigger block sales. Thus, the price change is less due to the supply shock, and moreso due to the fundamental change in the company’s value (like a scandal or earnings report).

However, interesting patterns that have not yet been fully explained have been observed regarding S&P 500 company addition and deletions. When companies are added or removed from the index, it is purely mechanical, and usually not due to some drastic fundamental change in the company. Assuming the market is efficient, the demand for stocks should not change due to being added or removed from an index, but several studies have shown that it does. Harris and Gurel (1986), Shleifer (1986), Beneish and Whaley (1996), Chen, Noronha, and Singal (2004) all show how new additions to the S&P come with higher than normal returns for that company. Though they agree on the price movement, the studies tend to have a hard time agreeing on the reason for this price movement. Some possible explanations include compensation for providing liquidity, better monitoring for investors when a company is added to a reputable and large index, and higher analyst coverage leading to more information and analysis available on the company. One primary concern is whether or not index reshuffling is an information-free event - that is, whether a company being added or removed adds information to the market about the company.

In this paper, the authors look at factor index rebalancing for an information-free event. Factor indices are part of a parent index of many other stocks, and are constructed in a mechanical way that is publically available and usually based on ranking stocks off a particular ratio of characteristic. Looking at the MSCI Minimum Volatility index, stocks returns were recorded for the stocks that had been added/dropped. It was found that the cumulative return from announcement to the effective day was 1.07% for stocks added with a significant t-statistic of 7.16, with 62% of the stocks exhibiting a positive cumulative abnormal return. Of the 1.07% increase, 0.63% of it was gained the following day, indicating that a large part of the increase is from an increase in demand from index funds. 0.31% of the return is lost five days after the rebalancing, but generally the price tends to stabilize afterwards after ten days. Thus, 68% of the price increase is permanent, while the other 32% is temporary and lost after a few days. This can be due to a number of reasons including a liquidity premium charged by the stock’s owner or arbitrage activity. Average trading volume was also significantly more for stocks that were recently added to the index. Between the announcement and actual day of adding the stock, the average trading volume was 30% higher than normal, with a significant t-statistic of 3.81. Moreover, there is a 74% increase in volume for the day prior to the actual adding of the security. A very similar phenomena occurs with stocks set to be dropped from the MSCI Minimum Volatility Index. From the announcement of a stock being dropped to the day before it is actually deleted from the index, the

total cumulative abnormal return is -0.91%, and -0.57% of this comes the day right before. After the stocks are deleted, 64% of them have a negative return the following day, and only 0.49% of the -0.91% is regained after three weeks. Trading volume also spikes 46% on the day prior to removal from the index. After three weeks, it returns back to within 1% of the normal trading volume.

These findings imply that once a security is added to a factor index, the demand curve shifts to the right, moving the equilibrium. The trading volume change is likely due to index funds buying or selling massive amounts of the stocks that will be added or removed. Moreover, it was found that the amount of the return is also directly related to the weighting of the volume of stocks entering or leaving the factor index. All in all, these findings suggest an index arbitrage opportunity if the index additions or deletions can be predicted.

Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly

Low-beta and low-volatility stocks have outperformed high-beta, high-volatility stocks from 1968-2008, combining great returns with low downturns. This can be explained by behavioral models of security prices. First is the idea that investors have a preference for “lotteries” and a bias of overconfidence creates a higher demand for higher-volatility securities. Second is the idea that this arbitrage opportunity is very limited, as the need to benchmark creates a greater demand for higher risk stocks, while discouraging investments in low-beta, low-volatility stocks. Several other papers have tried to explain this seemingly misunderstood phenomena, including Karceski in 2002, where it was noted that an extrapolation bias could cause mutual fund managers to care more about overperforming in a bull market, than underperforming in a bear market. This understandably, increases the market’s appetite for risky stocks with high reward potential. This paper focuses on the distortions caused specifically by benchmarking.

In the paper, the authors used 41 years of CRSP data, ranging from January 1968 – December 2008. Using the top 1,000 stocks by market cap, the five year trailing volatilities were calculated and returns were tracked. A dollar investment from 1968 made its way to \$59.55, or \$10.12 in real terms when accounting for inflation. On the other hand, the highest volatility portfolio went from a dollar in value to 58 cents during the period, with a real value of just around 10 cents when considering inflation. When using beta as a measure for risk, the finding was very similar. In the lowest-beta portfolio, a dollar grew to \$60.46 in nominal value, or \$10.28 in real value after inflation. The highest-beta portfolio grew from a dollar to \$3.77 in nominal terms, or \$0.64 in real terms after inflation. This held true for large cap companies, but the discrepancy was even higher for smaller cap companies. Generally, the low-risk portfolios also grew much more steadily and constant, without many drawdowns. To add on to this as well, the portfolio values did not include transaction costs. The high-risk portfolios cost more to rebalance on a monthly level, as was done in the paper, than the low-risk portfolios, indicating this anomaly is more pronounced than initially reported. These findings are not novel, but this paper attempts to explain them in a new way. Many theories add evidence for disproving the Capital Asset Pricing Model (CAPM), and even suggest that beta may not be the correct measure of risk. Other models relating risk and return, however, have had difficulty gaining acceptance and widespread usage in the finance industry.

One theory explored in detail is an investor’s irrational preference for high-volatility stocks. First, many investors have a natural preference for lotteries, even though there is a general aversion towards loss. If a stock has a positive skew, that is a larger probability of a large positive payoff than probability of a small payoff, investors typically are very interested. Though skew is not the same as volatility, in their paper in 2010, Boyer, Mitton, and Vorkink make a strong case for how expected skewness is a proxy for volatility. Another idea is representativeness, or that Bayes’ rule and probability theory are often not natural to people. One example of this is, selectively looking at a few speculative investments that have turned out to be massive successes, without considering the numerous failures. Overconfidence has also been tied to a preference for volatile stocks; optimists are generally more aggressive than pessimists.

The need for benchmarking, especially among institutional investors, is also believed to heavily play into the anomaly. In fact, from the period when institutional investors managed 30% of all money to 60%, the anomaly

intensified. This still begs the question as to why institutional investors do not buy more low-volatility stocks, and the answer has to do with benchmarking. The typical fund manager is judged for his/her “information ratio” (IR) which is the expected return difference between the manager and the expected return of the S&P 500, divided by the volatility of this return difference (tracking error). The goal of the investment manager is to maximize this information ratio, best as possible, through picking stocks. In 2009, Sensoy showed that over 61% of U.S. mutual fund managers are benchmarked against the S&P 500, while over 94% are benchmarked to some U.S. index benchmark. Moreover, SEC rules require mutual funds to compare their performance to some benchmark. This intuitively makes sense, as it allows investors to assess the skill and ability of managers in an unbiased way, and also allows fund managers a chance to differentiate themselves.

Investment managers without leverage will try to find mispriced stocks with a beta very close to market risk (beta of 1), overweighting positive-alpha stocks while underweighting negative-alpha stocks. When comparing the Sharpe ratio of large cap stocks for a low-volatility portfolio, it was quite high at 0.38. However, IR was a very low 0.08, showing this would be very tough for a fund manager to invest in. While Beta and volatility are undoubtedly very correlated, the study showed that beta is more related to the anomaly than volatility, especially with large cap stocks, which is what most fund managers disproportionately focus their invests in. Both volatility and beta play a significant role in this anomaly in smaller cap stocks. Looking at the period from 1968-2008, the top value strategy portfolios had an IR of 0.51, and top momentum strategy portfolios had an IR of 0.64. This is extremely high compared to the IR of low-volatility stocks in this period, which ranged from 0.08 to 0.17.

Overall, irrational investor preference for lotteries and high volatility stocks, as well as investment managers’ focus on benchmarks and IR flatten or even invert the relationship between risk and return as we know it. This has been shown by the study, and prior observations that the anomaly intensified with the increase in AUM of fund managers in the U.S. These reasons appear perennial, so the anomaly will likely not be going away anytime soon. One other factor to this is compounding of the low-volatility portfolio. Since it suffers fewer drawdowns, this means the portfolio is able to experience more stable and upward growth. # Betting Against Beta # MSCI Min Vol Index Information # The Low-Risk Anomaly: A Decomposition into Micro and Macro Effects # Data Analysis Data was collected from data on ishares.com, and compiled from 10/31/2011 to 1/5/2017.

```
# Load Packages and View the Data
```

```
library(mscdata)
```

```
library(ggplot2)
```

```
data(usa)
```

```
data(minvol)
```

```
## Summary statistics of EUSA Data
```

```
head(usa)
```

```
##   X Ticker      Name Asset.Class Weight.... Price Shares Market.Value
## 1 1 <NA> 1015736      <NA>      0.1950 26.63 10991 292690000
## 2 2 MMM 3M CO      Equity      0.4706 79.02 160 12643000
## 3 3 MMM 3M CO      Equity      0.4659 81.04 154 12480000
## 4 4 MMM 3M CO      Equity      0.4626 81.73 6872 561649000
## 5 5 MMM 3M CO      Equity      0.4709 86.71 7211 625266000
## 6 6 MMM 3M CO      Equity      0.4556 87.60 6672 584467000
##   Notional.Value      Sector SEDOL      ISIN
## 1      NA Industrials      <NA>      <NA>
## 2      NA Industrials 2595708 US88579Y1010
## 3      NA Industrials 2595708 US88579Y1010
## 4      NA Industrials 2595708 US88579Y1010
## 5      NA Industrials 2595708 US88579Y1010
## 6      NA Industrials 2595708 US88579Y1010
##           Exchange      Date
## 1      <NA> 2012-12-31
```

```
## 2 New York Stock Exchange Inc. 2011-10-31
## 3 New York Stock Exchange Inc. 2011-11-30
## 4 New York Stock Exchange Inc. 2011-12-30
## 5 New York Stock Exchange Inc. 2012-01-31
## 6 New York Stock Exchange Inc. 2012-02-29
```

```
tail(usa)
```

```
##           X Ticker           Name Asset.Class Weight.... Price Shares
## 39325 39325     ZTS ZOETIS INC CLASS A      Equity      0.1649 51.10   2377
## 39326 39326     ZTS ZOETIS INC CLASS A      Equity      0.1695 52.01   2397
## 39327 39327     ZTS ZOETIS INC CLASS A      Equity      0.1600 47.80   2472
## 39328 39328     ZTS ZOETIS INC CLASS A      Equity      0.1608 50.38   2847
## 39329 39329     ZTS ZOETIS INC CLASS A      Equity      0.1706 53.53   3015
## 39330 39330     ZTS ZOETIS INC CLASS A      Equity      0.1690 53.93   3015
##           Market.Value Notional.Value      Sector  SEDOL      ISIN
## 39325      121465000      121464.7 Health Care B95WG16 US98978V1035
## 39326      124668000      124668.0 Health Care B95WG16 US98978V1035
## 39327      118162000      118161.6 Health Care B95WG16 US98978V1035
## 39328      143432000      143431.9 Health Care B95WG16 US98978V1035
## 39329      161393000      161393.0 Health Care B95WG16 US98978V1035
## 39330      162599000      162599.0 Health Care B95WG16 US98978V1035
##           Exchange      Date
## 39325 New York Stock Exchange Inc. 2016-08-31
## 39326 New York Stock Exchange Inc. 2016-09-30
## 39327 New York Stock Exchange Inc. 2016-10-31
## 39328 New York Stock Exchange Inc. 2016-11-30
## 39329 New York Stock Exchange Inc. 2016-12-30
## 39330 New York Stock Exchange Inc. 2017-01-05
```

```
summary(usa)
```

```
##           X           Ticker           Name
## Min.      :    1   CB      : 102   3M CO      :    64
## 1st Qu.: 9833   NLSN      :   65   ABBOTT LABORATORIES      :    64
## Median :19666   A        :   64   ACCENTURE PLC      :    64
## Mean    :19666   AAP      :   64   ACTIVISION BLIZZARD INC:    64
## 3rd Qu.:29498   ABC      :   64   ADOBE SYSTEM INC      :    64
## Max.    :39330   (Other):38939   ADVANCE AUTO PARTS INC :    64
##           NA's      :   32   (Other)      :38946
##           Asset.Class      Weight....      Price      Shares
## Cash      :   66   Min.      :-0.3905   Min.      : 0.56   Min.      : -174670
## Equity    :39240   1st Qu.: 0.0542   1st Qu.: 35.08   1st Qu.:    805
## Money Market: 23   Median : 0.1236   Median : 55.21   Median :   1563
## NA's      :    1   Mean    : 0.1626   Mean    : 72.05   Mean    :   3566
##           3rd Qu.: 0.1638   3rd Qu.: 83.77   3rd Qu.:   3199
##           Max.    : 4.6773   Max.    :953.00   Max.    : 873628
##           NA's      :47
##           Market.Value      Notional.Value      Sector
## Min.      :0.000e+00   Min.      : -174670   Financials      :6909
## 1st Qu.:5.321e+07   1st Qu.:   55505   Consumer Discretionary:6703
## Median :8.418e+07   Median :   75582   Information Technology:5590
## Mean    :1.607e+08   Mean    :   91931   Industrials     :4765
## 3rd Qu.:1.435e+08   3rd Qu.:  118997   Health Care     :4138
## Max.    :6.804e+09   Max.    :2106050   Energy          :3253
```

```
## NA's :7 NA's :22930 (Other) :7972
## SEDOL ISIN
## 2000019: 64 AN8068571086: 64
## 2002305: 64 BMG0450A1053: 64
## 2005973: 64 BMG0692U1099: 64
## 2008154: 64 BMG169621056: 64
## 2011602: 64 BMG3223R1088: 64
## (Other):38920 (Other) :38952
## NA's : 90 NA's : 58
## Exchange
## New York Stock Exchange Inc. :27970
## NASDAQ : 9405
## Boerse Berlin : 427
## Deutsche Boerse Ag : 395
## Bolsa Mexicana De Valores (Mexican Stock Exchange): 266
## (Other) : 661
## NA's : 206
## Date
## 2015-06-30: 641
## 2015-08-31: 641
## 2015-05-29: 639
## 2015-09-30: 639
## 2015-10-30: 639
## 2015-11-30: 638
## (Other) :35493
```

```
## Summary statistics of USMV Data
head(minvol)
```

```
## X Ticker Name Asset.Class Weight.... Price Shares Market.Value
## 1 1 MMM 3M CO Equity 0.7488 79.02 244 19281
## 2 2 MMM 3M CO Equity 0.7592 81.04 244 19774
## 3 3 MMM 3M CO Equity 0.2892 109.35 94494 10332919
## 4 4 MMM 3M CO Equity 0.3000 117.43 59406 6976047
## 5 5 MMM 3M CO Equity 0.2999 113.58 57686 6551976
## 6 6 MMM 3M CO Equity 0.3095 119.41 52611 6282280
## Notional.Value Sector SEDOL ISIN
## 1 NA Industrials 2595708 US88579Y1010
## 2 NA Industrials 2595708 US88579Y1010
## 3 NA Industrials 2595708 US88579Y1010
## 4 NA Industrials 2595708 US88579Y1010
## 5 NA Industrials 2595708 US88579Y1010
## 6 NA Industrials 2595708 US88579Y1010
## Exchange Date
## 1 New York Stock Exchange Inc. 2011-10-31
## 2 New York Stock Exchange Inc. 2011-11-30
## 3 New York Stock Exchange Inc. 2013-06-28
## 4 New York Stock Exchange Inc. 2013-07-31
## 5 New York Stock Exchange Inc. 2013-08-30
## 6 New York Stock Exchange Inc. 2013-09-30
```

```
tail(minvol)
```

```
## X Ticker Name Asset.Class Weight.... Price
## 9498 9498 ZBH ZIMMER BIOMET HOLDINGS INC Equity 0.1391 104.95
## 9499 9499 ZTS ZOETIS INC CLASS A Equity 0.1002 30.89
```

##	9500	9500	ZTS	ZOETIS INC CLASS A	Equity	0.0962	29.81
##	9501	9501	ZTS	ZOETIS INC CLASS A	Equity	0.0972	29.15
##	9502	9502	ZTS	ZOETIS INC CLASS A	Equity	0.1021	31.12
##	9503	9503	ZTS	ZOETIS INC CLASS A	Equity	0.0992	31.66
##		Shares	Market.Value	Notional.Value	Sector	SEDOL	ISIN
##	9498	163220	17129939	17129939	Health Care	2783815	US98956P1021
##	9499	115890	3579836		NA Health Care	B95WG16	US98978V1035
##	9500	75069	2237807		NA Health Care	B95WG16	US98978V1035
##	9501	72889	2124714		NA Health Care	B95WG16	US98978V1035
##	9502	66567	2071565		NA Health Care	B95WG16	US98978V1035
##	9503	66458	2104060		NA Health Care	B95WG16	US98978V1035
##				Exchange	Date		
##	9498	New York	Stock Exchange Inc.	2017-01-05			
##	9499	New York	Stock Exchange Inc.	2013-06-28			
##	9500	New York	Stock Exchange Inc.	2013-07-31			
##	9501	New York	Stock Exchange Inc.	2013-08-30			
##	9502	New York	Stock Exchange Inc.	2013-09-30			
##	9503	New York	Stock Exchange Inc.	2013-10-31			

summary(minvol)

##	X	Ticker	Name
##	Min. : 1	CB : 82	ABBOTT LABORATORIES : 64
##	1st Qu.:2376	ABT : 64	ALTRIA GROUP INC : 64
##	Median :4752	ACGL : 64	ARCH CAPITAL GROUP LTD : 64
##	Mean :4752	ADP : 64	AT&T INC : 64
##	3rd Qu.:7128	AMT : 64	AUTOMATIC DATA PROCESSING INC: 64
##	Max. :9503	(Other):9134	AUTOZONE INC : 64
##		NA's : 31	(Other) :9119
##	Asset.Class	Weight....	Price
##	Cash : 66	Min. :-0.3451	Min. : 0.56
##	Equity :9413	1st Qu.: 0.2697	1st Qu.: 48.35
##	Money Market: 24	Median : 0.5922	Median : 71.56
##		Mean : 0.6730	Mean : 92.93
##		3rd Qu.: 1.0116	3rd Qu.:100.98
##		Max. : 2.8287	Max. :953.00
##		NA's :6	
##	Shares	Market.Value	Notional.Value
##	Min. : -20891956	Min. : 1138	Min. : -20891956
##	1st Qu.: 62755	1st Qu.: 5184586	1st Qu.: 18801242
##	Median : 240777	Median : 18738857	Median : 39482427
##	Mean : 525786	Mean : 31303694	Mean : 52633035
##	3rd Qu.: 590304	3rd Qu.: 40420826	3rd Qu.: 71639502
##	Max. : 38506545	Max. :240885300	Max. :240885300
##		NA's :8	NA's :5064
##	Sector	SEDOL	ISIN
##	Health Care :1664	2002305: 64	BMG0450A1053: 64
##	Financials :1569	2005973: 64	BMG3223R1088: 64
##	Information Technology:1510	2065308: 64	BMG7496G1033: 64
##	Consumer Staples :1246	2065955: 64	US00206R1023: 64
##	Consumer Discretionary:1071	2073390: 64	US0028241000: 64
##	Utilities : 628	(Other):9093	(Other) :9126
##	(Other) :1815	NA's : 90	NA's : 57
##	Exchange	Date	
##	New York Stock Exchange Inc.:7158	2016-11-30: 187	


```
## NASDAQ :1926 2016-12-30: 187
## Deutsche Boerse Ag : 58 2017-01-05: 186
## Spot Regulated Market - Bvb : 58 2016-05-31: 180
## Boerse Berlin : 32 2016-06-30: 179
## (Other) : 164 2016-07-29: 179
## NA's : 107 (Other) :8405
```

Next, sector weights were calculated over time for both EUSA and USMV. Plots were made by sector and displayed to compare the relative weights of EUSA and USMV.

```
data(usa_percent)
data(minvol_percent)
```

```
## Summary statistics of EUSA sector weights
head(usa_percent)
```

```
##          sector_name sector_count total    percent    date
## 1 Cash and/or Derivatives      2    631 0.003169572 2017-01-05
## 2 Consumer Discretionary     106    631 0.167987322 2017-01-05
## 3 Consumer Staples           43    631 0.068145800 2017-01-05
## 4 Energy                     44    631 0.069730586 2017-01-05
## 5 Financials                 84    631 0.133122029 2017-01-05
## 6 Health Care                71    631 0.112519810 2017-01-05
```

```
tail(usa_percent)
```

```
##          sector_name sector_count total    percent    date
## 827 Information Technology      79    585 0.135042735 2011-10-31
## 828 Materials                  33    585 0.056410256 2011-10-31
## 829 Real Estate                 0    585 0.000000000 2011-10-31
## 830 S-T Securities              1    585 0.001709402 2011-10-31
## 831 Telecommunications         12    585 0.020512821 2011-10-31
## 832 Utilities                  35    585 0.059829060 2011-10-31
```

```
summary(usa_percent)
```

```
##          sector_name sector_count    total
## Cash and/or Derivatives: 64 Length:832 Length:832
## Consumer Discretionary : 64 Class :character Class :character
## Consumer Staples       : 64 Mode  :character Mode  :character
## Energy                  : 64
## Financials              : 64
## Health Care             : 64
## (Other)                 :448
##    percent    date
## Min.   :0.00000 Min.   :2011-10-31
## 1st Qu.:0.01426 1st Qu.:2013-02-21
## Median :0.06820 Median :2014-06-14
## Mean   :0.07692 Mean   :2014-06-14
## 3rd Qu.:0.12236 3rd Qu.:2015-10-07
## Max.   :0.19293 Max.   :2017-01-05
##
```

```
## Summary statistics of USMV sector weights
head(minvol_percent)
```

```
##          sector_name sector_count total    percent    date
```

```
## 1 Cash and/or Derivatives      2   186 0.01075269 2017-01-05
## 2 Consumer Discretionary      18   186 0.09677419 2017-01-05
## 3 Consumer Staples           24   186 0.12903226 2017-01-05
## 4 Energy                      4   186 0.02150538 2017-01-05
## 5 Financials                  21   186 0.11290323 2017-01-05
## 6 Health Care                 33   186 0.17741935 2017-01-05
```

```
tail(minvol_percent)
```

```
##          sector_name sector_count total    percent    date
## 827 Information Technology      19   127 0.149606299 2011-10-31
## 828 Materials                   4   127 0.031496063 2011-10-31
## 829 Real Estate                 0   127 0.000000000 2011-10-31
## 830 S-T Securities              1   127 0.007874016 2011-10-31
## 831 Telecommunications          7   127 0.055118110 2011-10-31
## 832 Utilities                   9   127 0.070866142 2011-10-31
```

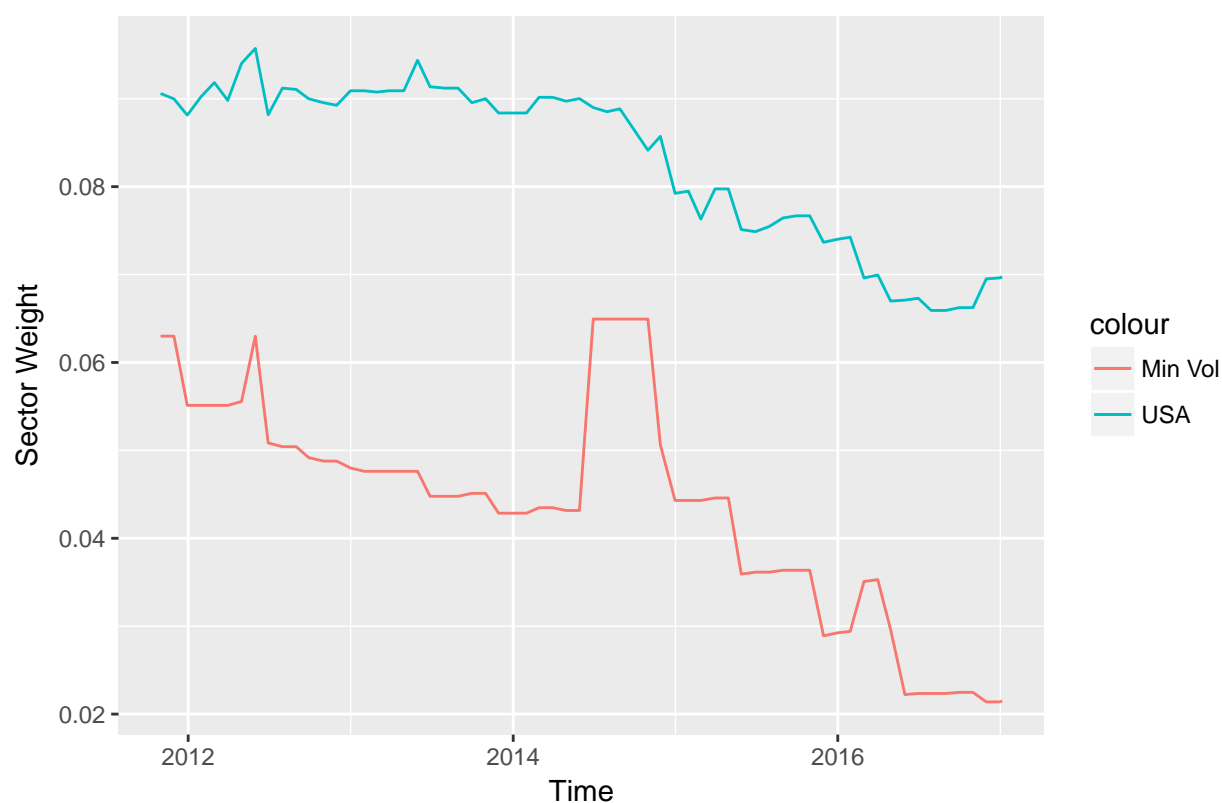
```
summary(minvol_percent)
```

```
##          sector_name sector_count    total
## Cash and/or Derivatives: 64 Length:832 Length:832
## Consumer Discretionary : 64 Class :character Class :character
## Consumer Staples       : 64 Mode  :character Mode  :character
## Energy                  : 64
## Financials              : 64
## Health Care             : 64
## (Other)                 :448
##      percent          date
## Min.   :0.00000 Min.   :2011-10-31
## 1st Qu.:0.02235 1st Qu.:2013-02-21
## Median :0.06349 Median :2014-06-14
## Mean   :0.07692 Mean   :2014-06-14
## 3rd Qu.:0.13043 3rd Qu.:2015-10-07
## Max.   :0.22222 Max.   :2017-01-05
##
```

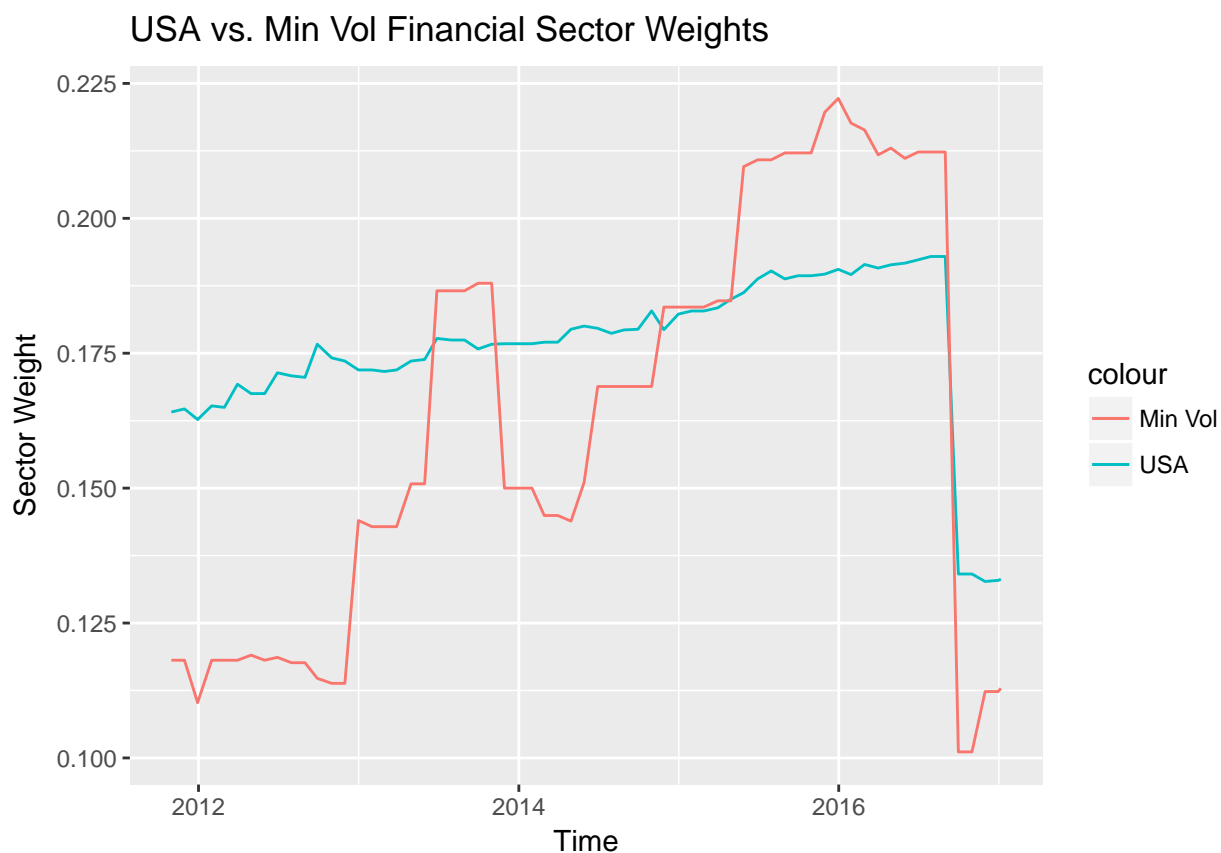
EUSA and USMV weightings by Sector

```
## Energy
Eng1 <- usa_percent[which(usa_percent$sector_name=="Energy"), ]
Eng2 <- minvol_percent[which(minvol_percent$sector_name=="Energy"), ]
ggplot(Eng1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Energy Sector Weights") + xlab("Time") + ylab("Sector Weight") +
geom_line(data = Eng2, aes(x=date, y=percent, colour="Min Vol"),show.legend = TRUE)
```

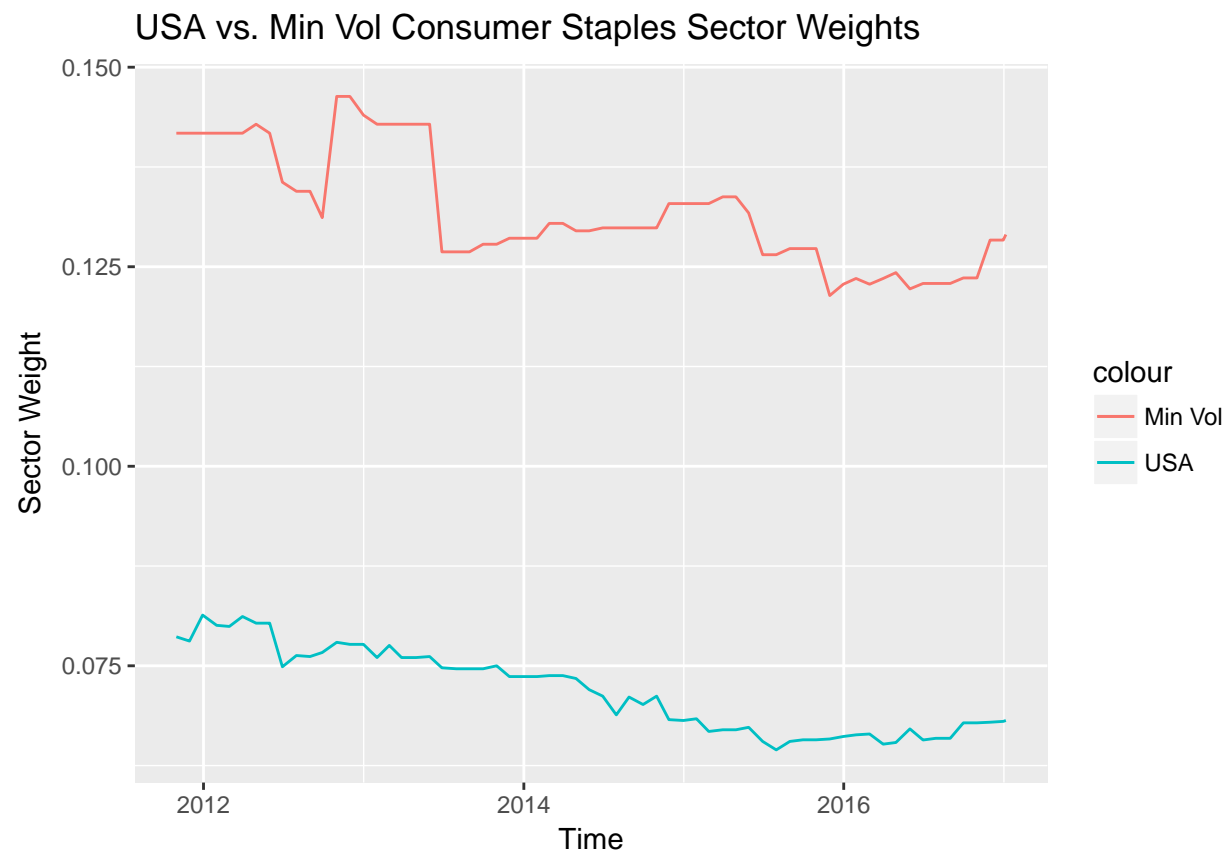
USA vs. Min Vol Energy Sector Weights



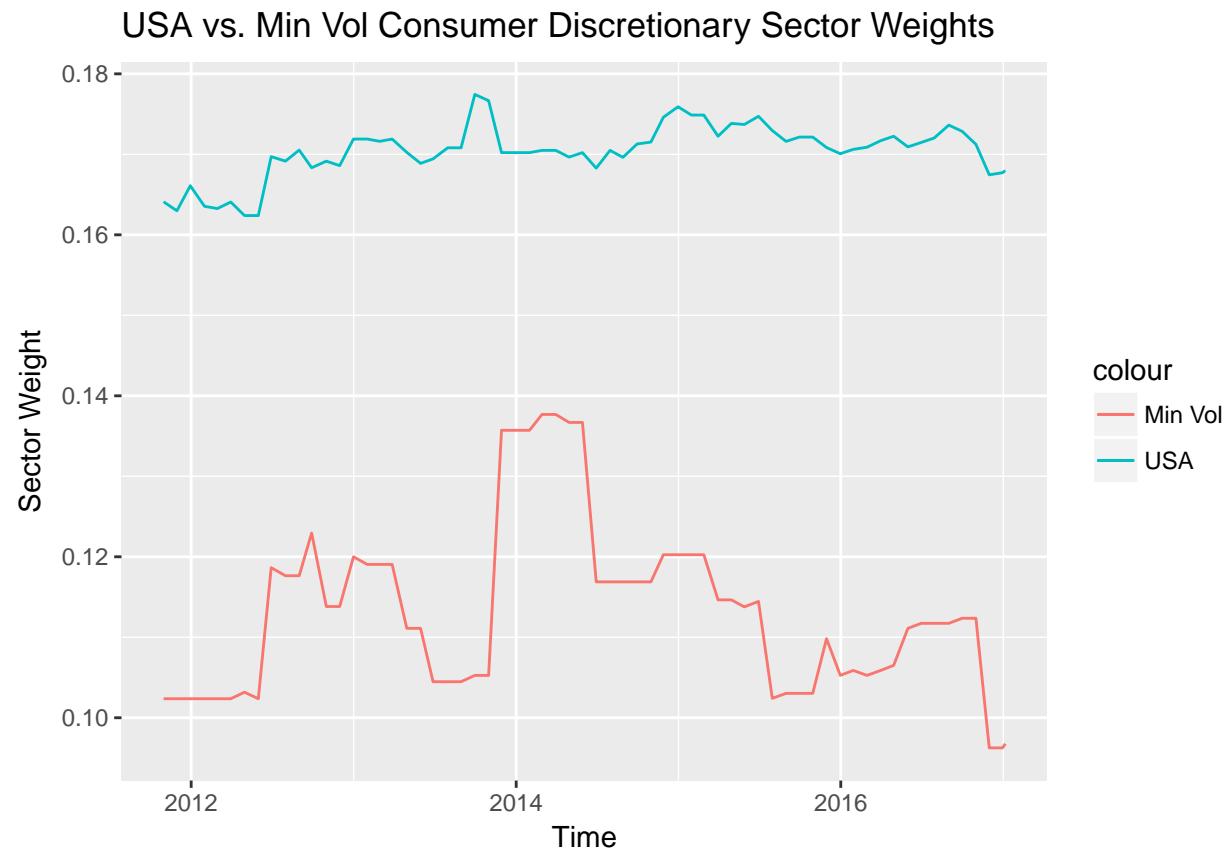
```
## Financials
Fin1 <- usa_percent[which(usa_percent$sector_name=="Financials"), ]
Fin2 <- minvol_percent[which(minvol_percent$sector_name=="Financials"), ]
ggplot(Fin1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Financial Sector Weights") + xlab("Time") +
ylab("Sector Weight") +
geom_line(data = Fin2, aes(x=date, y=percent, colour="Min Vol"),show.legend = TRUE)
```



```
## Consumer Staples
ConStap1 <- usa_percent[which(usa_percent$sector_name=="Consumer Staples"), ]
ConStap2 <- minvol_percent[which(minvol_percent$sector_name=="Consumer Staples"), ]
ggplot(ConStap1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Consumer Staples Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = ConStap2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```

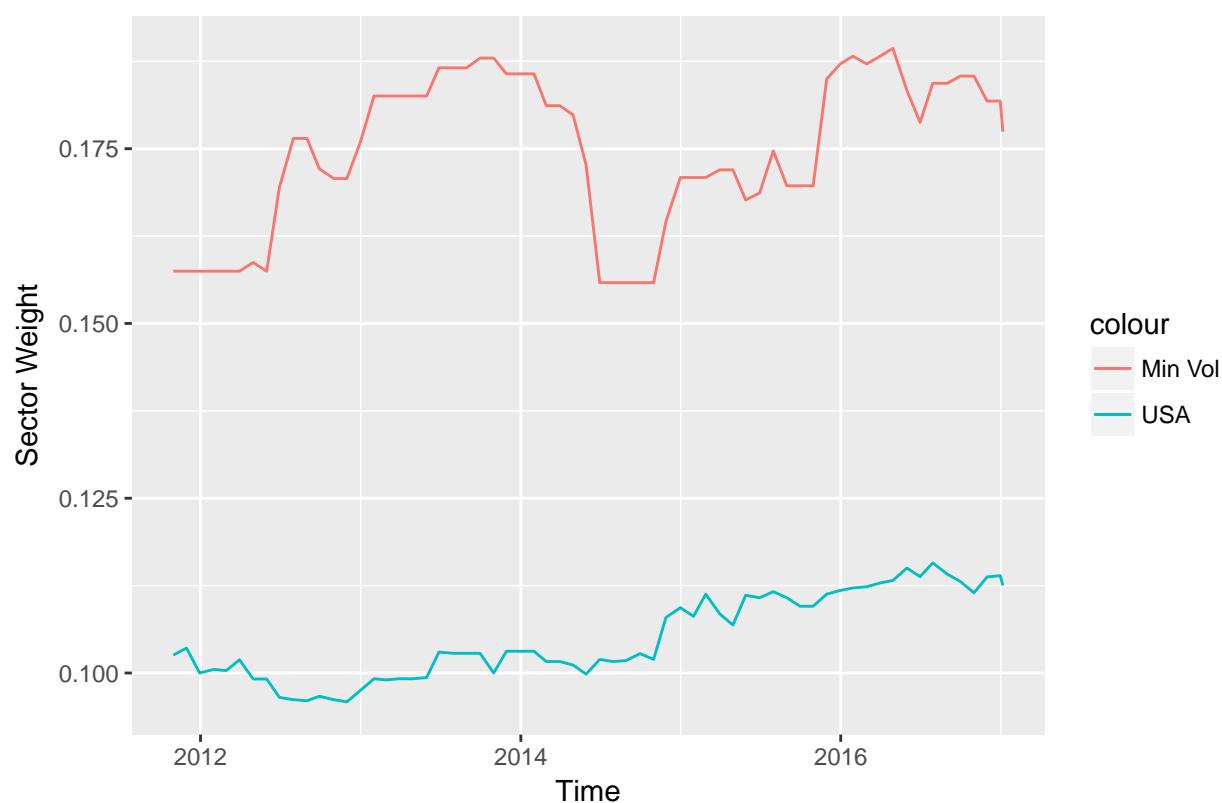


```
## Consumer Discretionary
ConDis1 <- usa_percent[which(usa_percent$sector_name=="Consumer Discretionary"), ]
ConDis2 <- minvol_percent[which(minvol_percent$sector_name=="Consumer Discretionary"), ]
ggplot(ConDis1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Consumer Discretionary Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = ConDis2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```

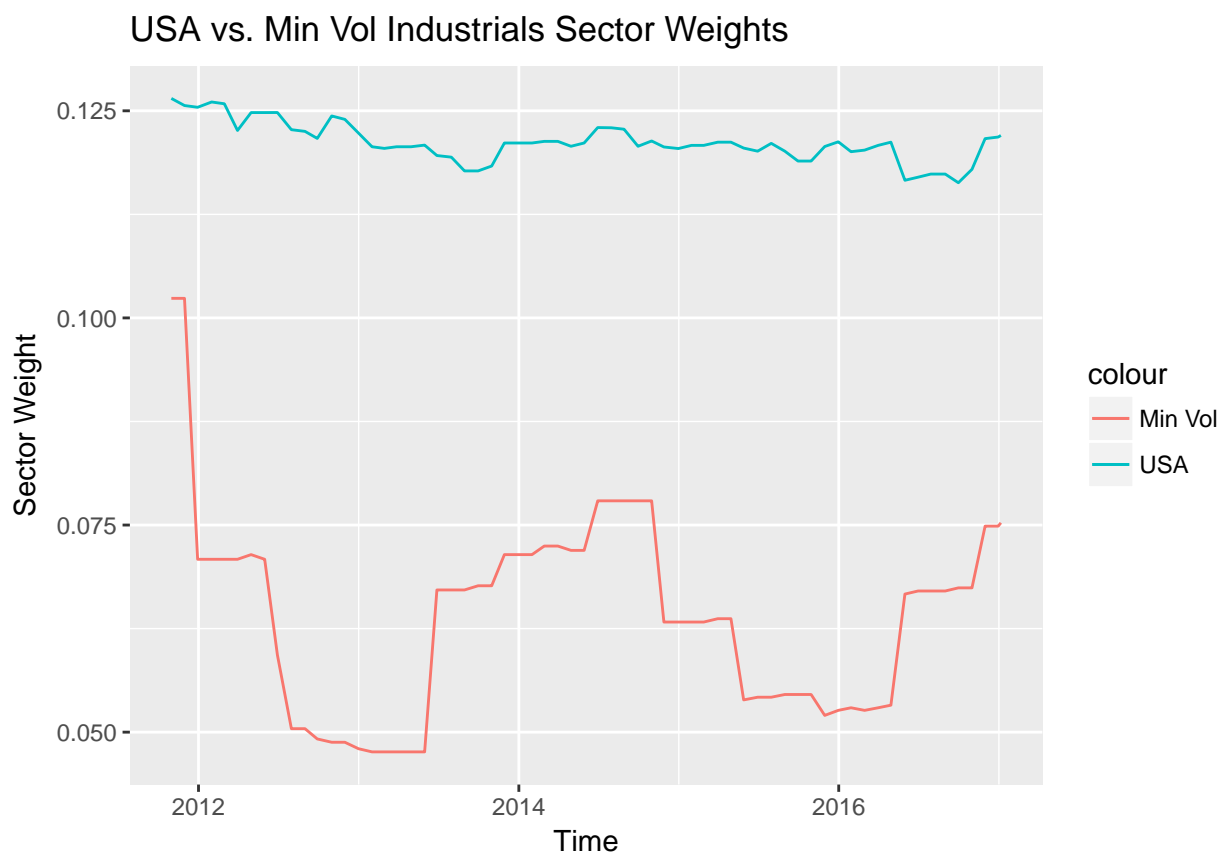


```
## Health Care
Health1 <- usa_percent[which(usa_percent$sector_name=="Health Care"), ]
Health2 <- minvol_percent[which(minvol_percent$sector_name=="Health Care"), ]
ggplot(Health1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Health Care Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = Health2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```

USA vs. Min Vol Health Care Sector Weights

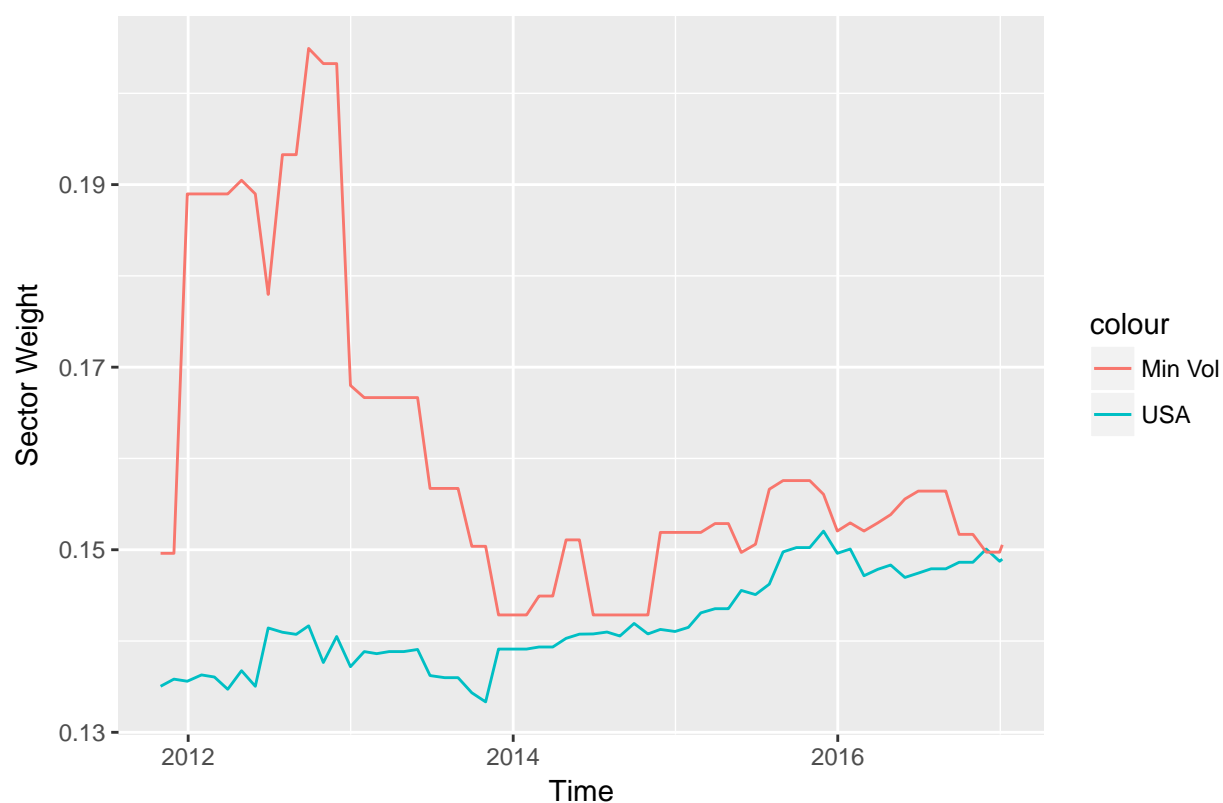


```
## Industrials
Ind1 <- usa_percent[which(usa_percent$sector_name=="Industrials"), ]
Ind2 <- minvol_percent[which(minvol_percent$sector_name=="Industrials"), ]
ggplot(Ind1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Industrials Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = Ind2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```

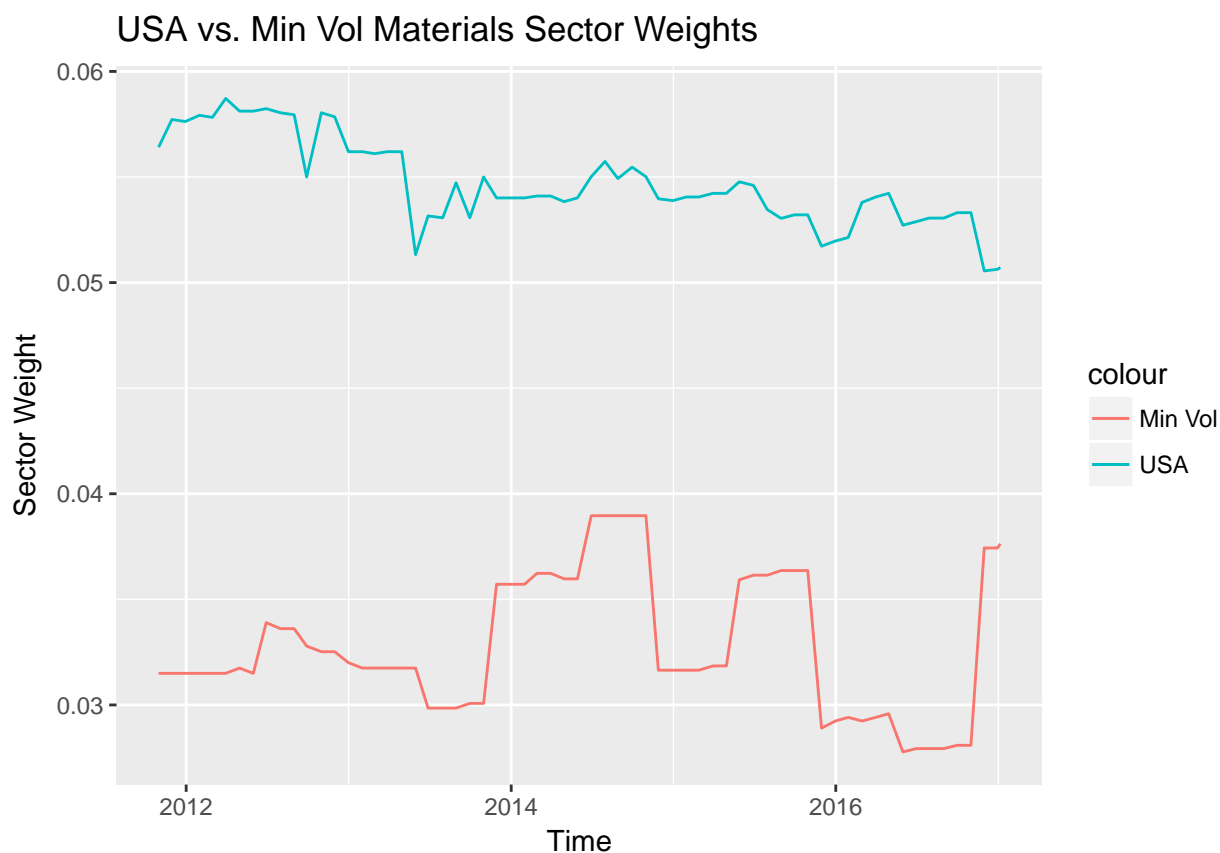


```
## Information Technology
IT1 <- usa_percent[which(usa_percent$sector_name=="Information Technology"), ]
IT2 <- minvol_percent[which(minvol_percent$sector_name=="Information Technology"), ]
ggplot(IT1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Information Technology Sector Weights") +
xlab("Time") + ylab("Sector Weight") + geom_line(data = IT2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```


USA vs. Min Vol Information Technology Sector Weights

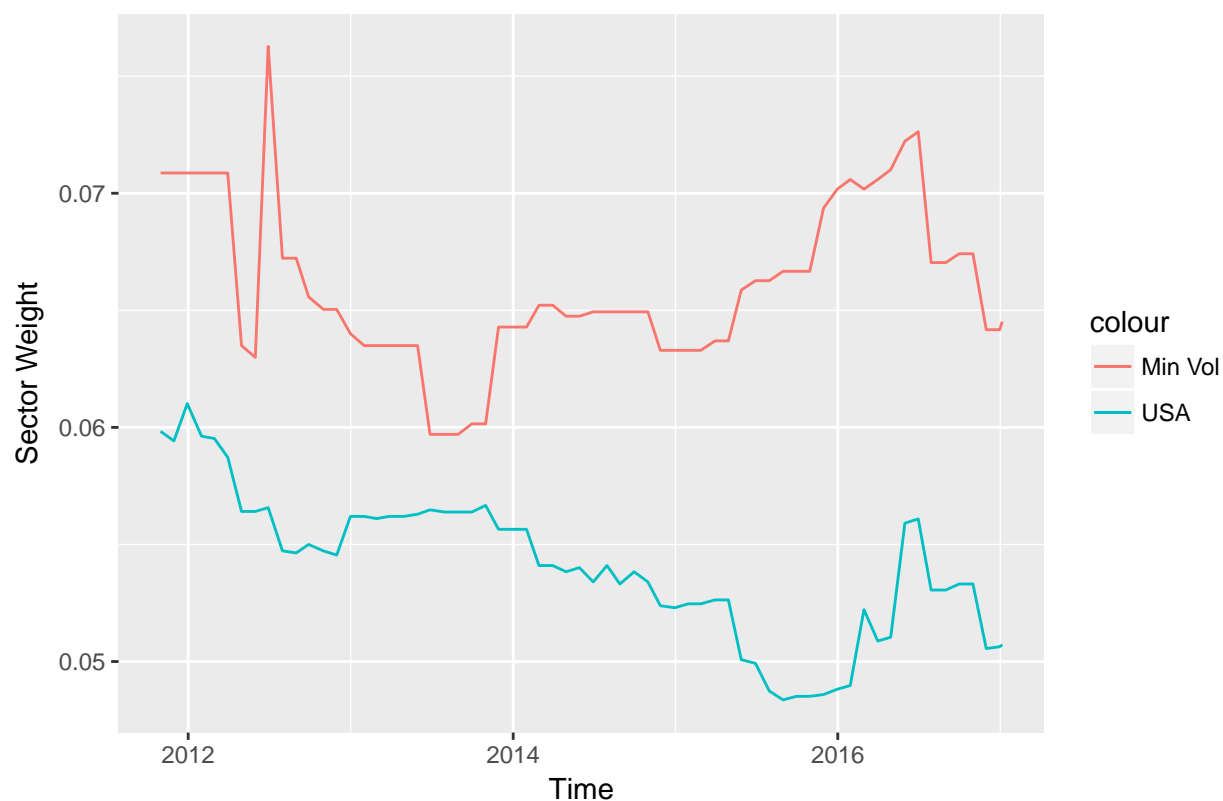


```
## Materials
Mat1 <- usa_percent[which(usa_percent$sector_name=="Materials"), ]
Mat2 <- minvol_percent[which(minvol_percent$sector_name=="Materials"), ]
ggplot(Mat1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Materials Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = Mat2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```



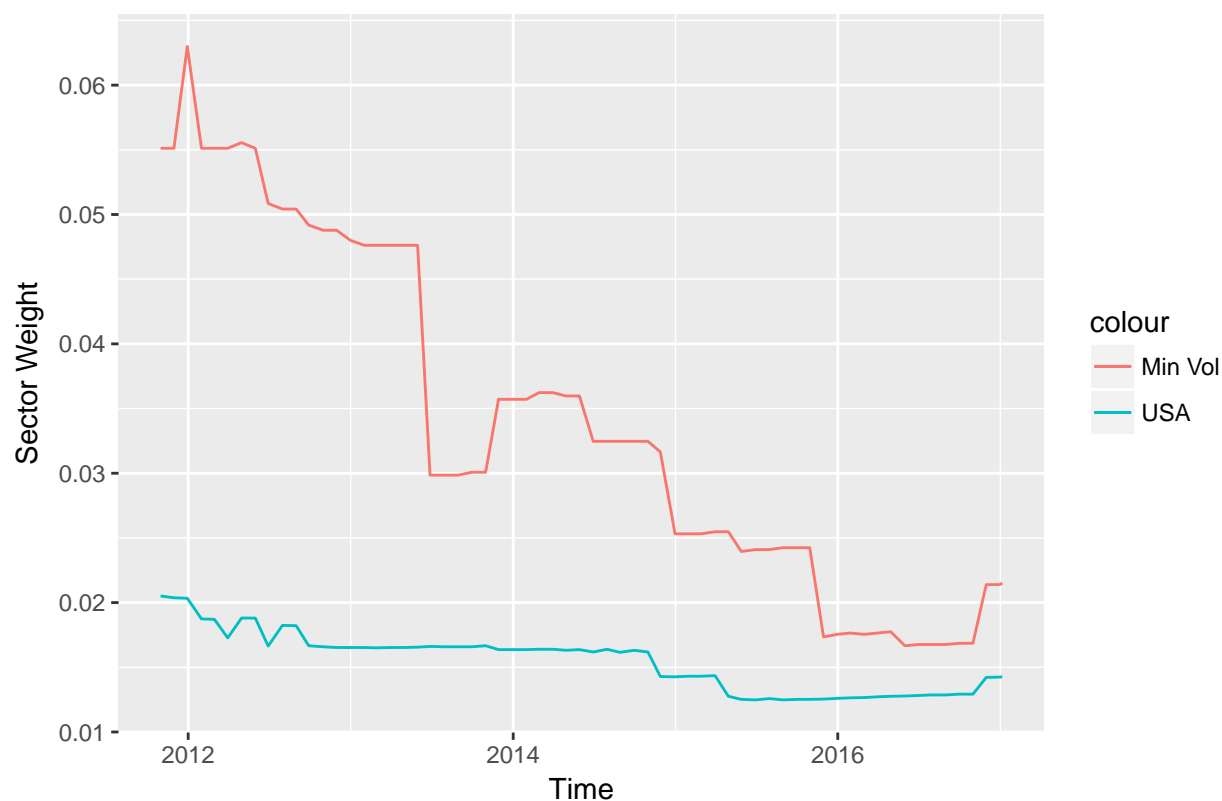
```
## Utilities
Util1 <- usa_percent[which(usa_percent$sector_name=="Utilities"), ]
Util2 <- minvol_percent[which(minvol_percent$sector_name=="Utilities"), ]
ggplot(Util1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Utilities Sector Weights") + xlab("Time") +
ylab("Sector Weight") + geom_line(data = Util2, aes(x=date, y=percent,
colour="Min Vol"),show.legend = TRUE)
```

USA vs. Min Vol Utilities Sector Weights



```
## Telecommunication Services
Telecom1 <- usa_percent[which(usa_percent$sector_name=="Telecommunications"), ]
Telecom2 <- minvol_percent[which(minvol_percent$sector_name=="Telecommunications"), ]
ggplot(Telecom1, aes(date, percent, colour = "USA")) + geom_line() +
ggtitle("USA vs. Min Vol Telecommunications Services Sector Weights") +
xlab("Time") + ylab("Sector Weight") + geom_line(data = Telecom2, aes(x=date,
y=percent, colour="Min Vol"),show.legend = TRUE)
```

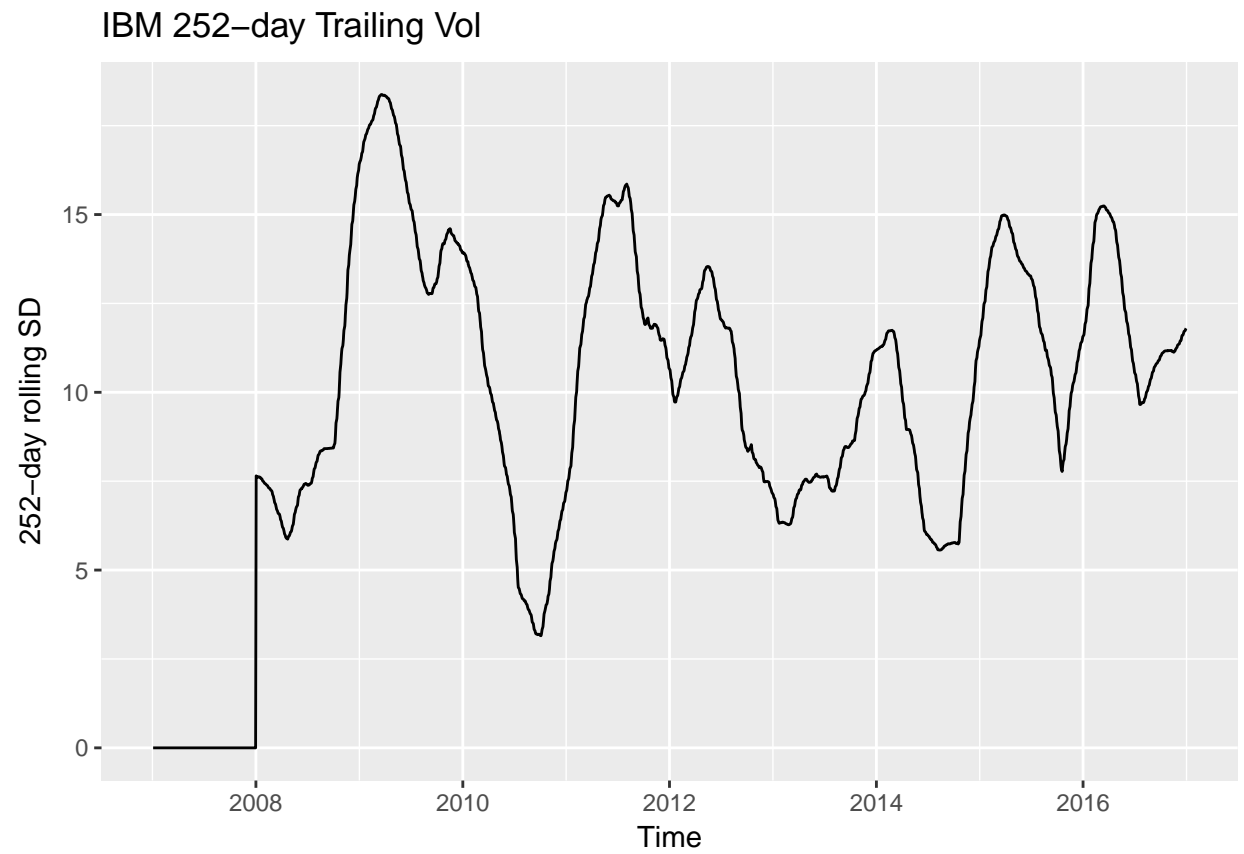
USA vs. Min Vol Telecommunications Services Sector Weights



Sample WRDS Data

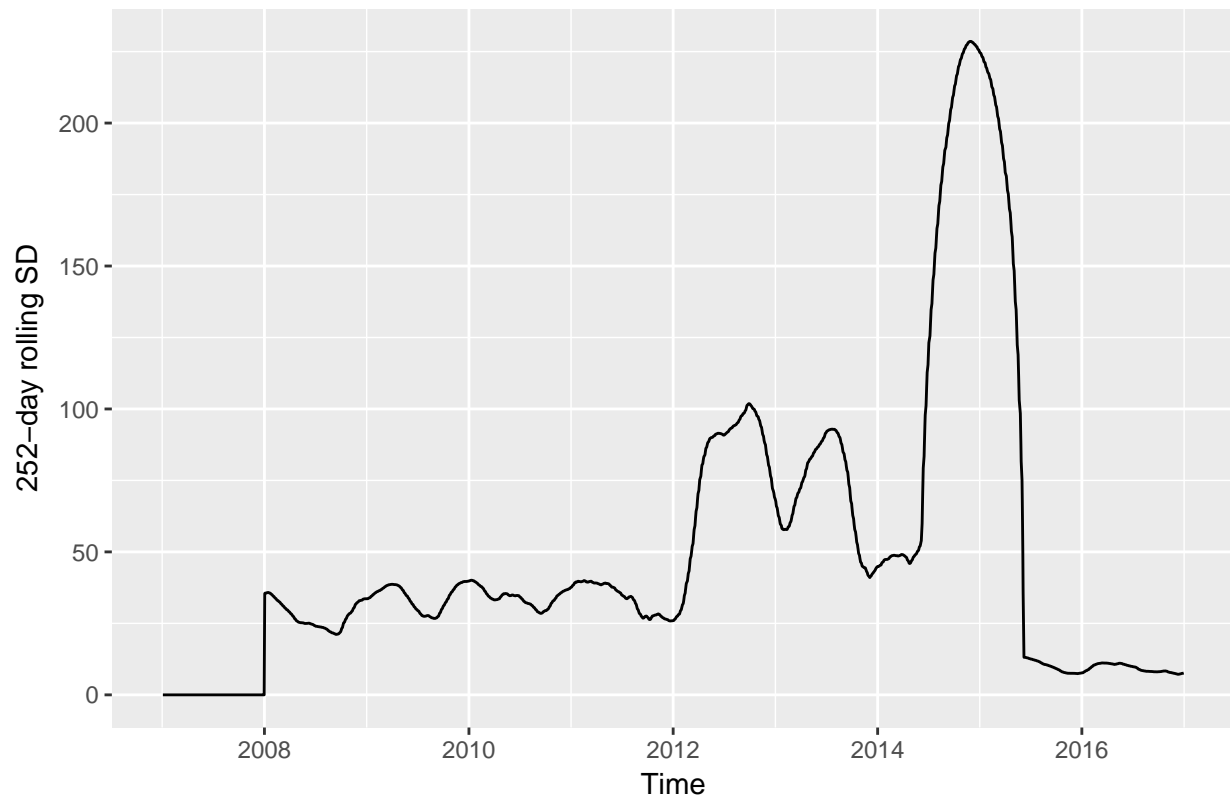
10-years of data, from 12/31/2006 to 12/31/2016, was collected from WRDS for IBM, Google, and Apple, to find their 252-day (annual) trailing volatility.

```
#IBM 252-day Trailing Vol from 12/31/2006 to 12/31/2016
ggplot(ibm, aes(Date, sd)) + ggtitle("IBM 252-day Trailing Vol") + geom_line() +
xlab("Time") + ylab("252-day rolling SD")
```



```
#Apple 252-day Trailing Vol from 12/31/2006 to 12/31/2016  
ggplot(aapl, aes(Date, sd)) + ggtitle("AAPL 252-day Trailing Vol") + geom_line() +  
xlab("Time") + ylab("252-day rolling SD")
```

AAPL 252-day Trailing Vol



```
#Microsoft 252-day Trailing Vol from 12/31/2006 to 12/31/2016  
ggplot(msft, aes(Date, sd)) + ggtitle("MSFT 252-day Trailing Vol") + geom_line() +  
xlab("Time") + ylab("252-day rolling SD")
```

MSFT 252-day Trailing Vol

