Forecasting Constituents of the MSCI Minimum Volatility Index Through Logistic Regression

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Preface

This thesis explores a way of predicting index constituents using logistic regression.

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Abstract

The low-risk anomaly has created opportunities for arbitrage in the financial markets. As Baker et al. discuss in “Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly,” low-volatility and low-beta portfolios outperform and high-volatility and high-beta portfolios by a factor of several times due to benchmarking and lottery-preferences. The iShares MSCI USA Minimum Volatility (USMV) is an ETF tracking a minimum volatility index that was used to find data and will be used for trading arbitrage. Frazzini et al. discuss arbitrage opportunities by quantitative focused funds like AQR in “Betting Against Beta”, and this thesis explores a more advanced type of index front-running as a potential arbitrage opportunity. Data was collected from USMV from its inception in October 2011, and from EUSA, the parent ETF of USMV, from the same period until December 2016. 52-week trailing beta, 52-week trailing volatility, lagged price/book, and current index membership were calculated, and a regression model was run to quantify the relationship between current index membership and these four variables. In the model, a probabilities of index membership were calculated and an optimal cutoff was calculated to which the model would be 95% accurate of its findings of a stock to be in or out of USMV, given the historical data. Backtesting with prior data showed with a model accuracy of 95%, arbitrage opportunities of X% could be collected after each rebalancing.

Chapter 1

Introduction

1.1 Background

The iShares Edge MSCI USA Minimum Volatility (USMV) Exchange Traded Fund (ETF) is designed to track the investment results of the MSCI Minimum Volatility USA index, which is composed of stocks with a lower volatility than the general market. This can provide investors with exposure to a portfolio with less risk compared to many alternatives, and historically has declined less in value than the broader market during economic downturns. The ETF is comprised of 189 holdings, and is rebalanced two times per year, with the intention of mirroring the changes made by the index. The purpose of this dissertation is to create a logistic regression model that can accurately predict which stocks will be added or removed from this ETF before rebalancing occurs, and to understand what factors are involved. The model will take into account volatility attributes of each stock, as well as others potentially significant predictor variables from prior studies. An accurate model will allow for arbitrage investment opportunities.

1.1.1 Exchange Traded Funds (ETFs)

An Exchange Traded Fund (ETF) is a collection of stocks and/or bonds in a single portfolio that is traded on a major exchange just like a stock is (Hayes, 2017). As a result, the price of an ETF fluctuates on a regular basis. Exchange Traded Funds generally have more liquidity and less fees when compared to other alternative instruments, like mutual funds. Owning an ETF can allow investors to minimize risk, since owning an ETF is comparable to owning a little bit of many different stocks. This diversification comes at lower costs and less effort for investors as well.

ETFs can also track an index, commodity, bond, or basket of all of the above. Unlike an ETF, which is publicly traded, an index is not. The goal of the USMV ETF is to track the MSCI Minimum Volatility USA index, and this is more complicated than it seems. In addition to tracking this index, the ETF aims to mirror returns of the index, and any difference between the ETF return and actual index return is called tracking error. The tracking error is often very small and can be around a tenth of a percent. This error can come from indices being market capitalization weighted, meaning that price fluctuations of each stock lead to the weighting being changed by a ratio of its market cap against the market cap of all stocks in the index (Fontinelle, 2009). With these stocks weightings in the index constantly changing and people buying in and out of ETFs constantly, it is hard to track performance entirely accurately. However, ETFs very closely follow indices, as their tracking errors are generally quite small. Thus, although ETF data is not the same as index data, the two are extremely comparable.

1.1.2 iShares MSCI Min Vol USA ETF

The iShares Edge MSCI Min Vol USA ETF (USMV) is a Blackrock-managed ETF that tracks the investment results of the MSCI Minimum Volatility USA index. The MSCI Minimum Volatility USA index constituents come from the MSCI USA Index, which are roughly comprised of the top 600 US stocks by market cap. This minimum volatility index is intended to have a lower beta, lower volatility, lower cap bias, and contain more stocks with less risk than its parent index, which contains US mid-cap and large-cap stocks. On the last trading days of May and November, the index is rebalanced twice a year. The index typically has around 180 constituents, with an average of 20 new additions and 14 deletions every 6 months when rebalancing occurs. Over the last five years, the number of additions has ranged from 12 to 25, while the deletions have been between 10 and 19. Changes to the index are usually announced nine trading days before they are set to take place (BlackRock, 2017).

Using the Barra Open Optimizer, USMV creates a minimum variance portfolio of low-risk stocks as a subset from its parent index of US large-cap and mid-cap stock. Using this estimated security covariance matrix, the MSCI Minimum Volatility Index is the product of the lowest absolute volatility, considering the constraints (MSCI, 2013). Moreover, these additions are simply a relabeling of existing stocks in the parent index and do not include new additions to the parent index. The low-risk stocks chosen to be in USMV are determined by a set of constraints, like maintaining a certain sector or country weight, relative to the parent index.

There are many specific constraints to this index. The first is that an individual stock cannot exceed 1.5% or 20 times the weight of the stock in the parent index. The minimum weight of a security in the index is also capped at 0.05%. USMV also aims to keep the weight of specific countries within a 5% range of the weight in the parent index, or 3 times the weight of the country in the parent index. Sector weights of USMV cannot deviate more than 5% from the sector weights in the parent index. One-way turnover of the index is also maxed at 10%. Thus, taking into account these constraints, the Barra Open Optimizer creates the lowest absolute volatility portfolio possible (Wynne, 2016).

1.1.3 Purpose

As mentioned, the purpose of this thesis is to create a model to that will predict rebalancing of stocks in the Min Vol index, and consequently the USMV ETF, before it actually happens. There is substantial price movement whenever a stock is added or removed from a large ETF, like USMV. When a stock is added to the index, the ETF will buy large amounts of that stock, increasing the demand, and consequently, the market price for that stock. If the stock is bought in advance of this large purchase, then the investor can enjoy pretty immediate price appreciation in the stock. Moreover, if a stock is removed from the Min Vol index, the USMV ETF will sell all current holdings of the stock, which will increase the supply of the stock, driving down market price of the stock. If one were to short this stock before that happened, he/she can also profit from that event.

A phenomena known as ETF front-running is similar to what this thesis hopes to accomplish, but is one step behind. ETF front-running occurs when traders buy or sell stocks in advance of ETF managers after they announce an exit or entrance of a position (MacDonald, 2009). There is typically a slight lag between an announcement of an ETF to add or remove a position and the actual purchase or sale of this position. By acting quickly, traders can scalp profit by buying a stock before an ETF does, and selling it to them later at a slight profit, or short-selling a stock before an ETF exits the position, and then buying it back at the lower price. This thesis will take things one step farther and try predict the stock addition or deletion before announcement. This will allow traders to similarly front-run the index, but they will do so before the market is able to react, leading to larger profit opportunities.

1.1.4 Logistic Regression Model

The goals of this paper will be achieved by creating a logistic regression model, which will be transformed to calculate a probability of a stock being in the out of the index. A logistic regression model will be used since the dependent variable is dichotomous. The predictor variables will include 52-week trailing volatility, 52-week trailing beta, price/book ratio, and whether or not the stock was in the index 6 months before, during the previous rebalancing. These attributes were chosen after looking at the historical literature and understanding of the minimum volatility index.

1.2 Literature Review

1.2.1 Overview of the Low-Risk Anomaly

The Low-Risk Anomaly stems from the observation that portfolios of low-risk stocks have higher risk-adjusted returns than portfolios of high-risk stocks. This idea challenges a widely regarded financial principle that investing in higher-risk stocks will generally result in higher expected return. In several studies, portfolios of high-risk stocks and low-risk stocks were constructed and rebalanced regularly to reflect these characteristics, and the low-risk portfolios outperformed the high-risk portfolios by a factor of several times, over long-periods of time including 1929-2015 (Vliet & Koning, 2016) and 1968-2008 (Baker, Bradley, & Wurgler, 2011).

One of the explanations is the need for money managers to be compared to a benchmark, such as an index like the S&P 500. This is reasonable, as many fund managers are charging fees to manage money, and need a way to prove themselves and their abilities. By outperforming an index, a fund manager is able to generate alpha, and presumably raise money money or change higher fees. By underperforming an index, the fund managers have a hard time justifying fee charges to clients, since they could just invest in the index passively for little to no fee. Thus, much of the risk for them is relative, coming from potentially underperforming on a benchmark. Moreover, with this doubling of assets under active management from 30% to 60% in 1968-2008, the low-risk anomaly intensified. Another metric of fund manager performance is through the "information ratio" (IR), or the expected return difference between the manager and the expected return of the S&P 500, divided by the volatility of this return difference. The goal of an investment manager is to maximize this number, best as possible, through picking stocks that will outperform the market. These help to create a greater demand for higher-risk stocks while discouraging investments in low-beta, low-volatility stocks, and ultimately increases the market's appetite for risky stocks with high reward potential, driving up price and driving down expected return.

In addition to the focus on relative rather than absolute risk, money managers also often focus on single period returns, often as short as a month, which aim to remove the effects of compounding. This helps differentiate each manager's stock picking abilities, but ignores the real-world and significance of compounding. Humans also have a predisposition for the lottery effect, which is increased interest in stocks with high skew - that is high upside potential. Risk can also be decomposed into macro and micro-effects - that is, looking for and comparing the risk-return characteristics of stocks with different risk profiles, but similar country and industry risks. It turned out that the micro-effects, those that were stock-specific, were statistically significant at generating alpha, while the macro-effects, those that were country and industry-specific was not. These literature reviews also examine both volatility and beta as a measure of risk and suggest that beta is not an adequate risk metric. In fact, though beta and volatility are obviously very correlated, beta appears to be more related to this low-risk anomaly than volatility. This can have significant implications on the significance of the predictor variables in the logistic regression model in this dissertation.

There are many real-world trading strategies that utilize this anomaly. One example includes betting against Beta (BaB), which is a strategy used by quantitative hedge funds like AQR. Betting against Correlation (BaC) is a similar strategy in that it decomposes the effects of beta into two separate factors for a more concentrated investment. Moreover, index additions and deletions have proven to be instrumental in influencing the price of affected stocks, indicating that if a model can predict these movements accurately beforehand, there is an additional considerable arbitrage opportunity.

1.2.2 Evidence of the Low-Risk Anomaly

Measures of Risk

Assuming an efficient market, one of the most widely accepted tenets of investing is that one will be receiving higher reward for taking on more risk. Presumably, if this were not true, nobody would partake in higher risk investments. Some risks to consider when making an investment include those with respect to the market, liquidity, credit, inflation, and FX (Ontario, 2017). When quantifying risk for a company, one can look at the standard deviation of the stock price. By looking at the historical dispersion of data from the mean, or normal returns, one finds the stock's volatility. These calculations are based only on the price fluctuations of the stock and no other external factors (Investopedia, 2015). Volatility is also one of the best indicators of bankruptcy. The other common form of risk measurement in finance is beta, which measures the stock's price volatility compared to that of the stock market. The market is typically a benchmarked index relevant to the stock; for a large-cap US stock, the associated index could be the S&P 500. Beta is calculated by taking the covariance of the stock returns and the market returns, then dividing by the variance of the market:

$$\beta_p = \frac{Cov(r_p, r_b)}{Var(r_b)}$$

Figure 1.1: Beta

This yields a coefficient, which can be interpreted: a beta value of 1 indicates the stock price and market move together identically, a beta value of less than 1 indicates the stock is less volatile than the market, and a beta value of greater than 1 indicates the stock is more volatile than the market. Beta values can be negative as well, and hold the same interpretation as positive beta values, with the difference being that the stock price and market move in opposite directions. For example, a beta of -1 would mean the stock and market have the same volatility and changes in price, but that the performance is inversely related. Thus, the first sentence of this paragraph can have many meanings, but can be interpreted as saying that stocks with higher beta or volatility will have higher expected return.

1929-2015

Jan de Koning and Pim Van Vliet set out to investigate the question: *Do high-volatility stocks return more than low-volatility stocks?* by constructing high-volatility and low-volatility portfolios, and comparing the two over a 86-year time span, from 1929-2015. Over this period, low-volatility stocks outperformed high-volatility stocks by a factor of 18, excluding inflation and transaction costs. If both portfolios started off with $100 in 1929, the low-volatility portfolio end value would be worth $395,000, while the high-volatility portfolio would be worth just $21,000. In reality, these values would both be much smaller if the costs of trading the stocks and inflation were considered, but excluding them is reasonable as the effect on both portfolios would be pretty similar. The low-volatility portfolio returned 10.2% annually whereas the high-volatility portfolio returned just 6.4% annually. This annual difference of 3.8% is striking, and when considering compounding over an 86-year period, helps explain why the low-volatility portfolio's value was over 18 times that of the high-volatility portfolio. In this study, the annualized volatility of the low-volatility portfolio was 13%, and the annualized volatility of the high-volatility portfolio was around 2.5 times that, at 36% (Vliet & Koning, 2016).

1968-2008

Baker et al. performed a study similar to that done by van Vliet and de Koning, but used 41 years of CRSP data, ranging from January 1968 – December 2008. It is important to note the ranges of dates used; the Great Recession began in 2007 after the bursting of the subprime mortgage bubble, leading to the collapse of several large, investment banks, and the government bailout of many others. This caused market indices, like the Dow, to drop from a high in 2007 of 14,164.43 to 8,776.39 by December 2008, representing a decrease in over 38% during that period (Amadeo, 2017). Thus, just as van Vliet and De Koning started their sample right before the Great Depression to help amplify their results, Baker, Bradley, and Wurgler ended their sample after the Great Recession. Though this may help amplify the results, this does not take away from the significance of the findings (Baker et al., 2011).

Baker et al. constructed low-volatility, high-volatility, low-beta, and high-beta portfolios using the top 1,000 stocks by market capitalization and then calculated each stock's five-year trailing volatility or beta. A dollar investment in the low-volatility portfolio in 1968 appreciated to $59.55 by 2008, or $10.12 in real terms, when accounting for inflation. On the other hand, the highest-volatility portfolio went from a dollar in value to just 58 cents from 1968-2008, with a real value of around 10 cents, when considering inflation. Thus the low-volatility portfolio outperformed the high-volatility portfolio by over 100 times both in terms of nominal and real value. When using beta as a measure for risk, the finding was very similar. In the lowest-beta portfolio, a dollar grew to $60.46 in nominal value throughout the 41-year period, or $10.28 in real value after considering inflation. The highest-beta portfolio grew from a dollar to $3.77 in nominal terms, or $0.64 in real terms after accounting for the effects of inflation. Thus, the low-beta portfolio outperformed the high-beta portfolio by a factor of 16, in both nominal and real terms. One interesting observation here is though both the low-beta and low-volatility portfolios outperformed the high-beta and high-volatility portfolios, respectively, the discrepancy was far more pronounced in the case of the the volatility portfolios. Another significant observation was that investors who owned either the high-beta or high-volatility portfolio in 1968 would have lost money when accounting for the effects of inflation by 2008. It is also important to note that this was just the case for large-cap companies, as these portfolios were constructed from the top 1,000 stocks in terms of market capitalization. The fact that this anomaly was observed for large-cap companies is quite impressive, given there are generally a lesser degree of mispricing in that realm than with small-cap companies, because many small-cap stocks are not big enough for institutional investors to invest in. Thus, this anomaly would be larger if the study had been done for small-cap stocks instead. In addition, the portfolio end values assumed no transaction costs, which in reality would have been considered. The high-beta and high-volatility portfolios also cost more to rebalance on a monthly level, as was done in the paper, than the low-beta and low-volatility portfolios, indicating this anomaly is actually more pronounced than initially reported. Baker et al. noted that while high-beta portfolios outperformed the low-beta portfolios in up-markets and underperformed the low-risk portfolios in down-markets, that the low-beta anomaly persisted in both situations. On a market-adjusted basis, low-beta consistently generates high alpha (Baker et al., 2011). This was consistent with prior research (Pettengill, Sundaram, & Mathur, 1995).

Critique of the Capital Asset Pricing Model

These findings are not new or revolutionary and have been observed in many previous academic articles. Black, Jensen, and Scholes evaluated some of the assumptions and effectiveness of the Capital Asset Pricing Model (CAPM) (Jensen, Black, & Scholes, 1972). CAPM is a model that quantifies the relationship between risk and expected return for a stock, and considers that investors should be compensated for the time value of their money and risk they are taking. This is calculated as the risk free rate plus the beta times the difference between the risk free rate and expected market return (Brennan, 1989):

$$\overline{r_a} = r_f + \beta_a(\overline{r_m} - r_f)$$

Where:

r_f = Risk free rate

β_a = Beta of the security

$\overline{r_m}$ = Expected market return

Figure 1.2: Capital Asset Pricing Model

This will give the expected return of the stock, and anything in excess of this will be considered alpha. Black, Jensen, and Scholes were able to find that expected excess return, alpha, was not strictly proportional to Beta, as it is mathematically in the CAPM. In 1975, Haugen and Heins similarly found that there was little support for the idea that risk premiums have manifested themselves in realized rates of return. In fact, they pointed out that the relationship between risk and return is much flatter than it is in the CAPM (Haugen & Heins, 1975). Twenty years later, Fama and French famously declared that beta was dead after finding a flat relationship between beta and stock returns (Fama & French, 1992). Many additional papers and researchers have added evidence for disproving the Capital Asset Pricing Model, and even suggest that beta may not be the correct measure of risk, meaning that the relationship between risk and return is not what many people believe it to be (Mullins, 1982). However, other models relating risk and return have had difficulty gaining acceptance and widespread usage in the finance industry.

1.2.3 Possible Explanations for the Low-Risk Anomaly

Compounding

To fully grasp the low-risk anomaly, it is important to understand that a higher volatile stock or portfolio will move in greater magnitude than the underlying market. This holds true for both downside and upside scenarios. When the market increases in value, a high-volatility stock will increase in excess of this. For example, if the market increases by 20% in one year, a high-volatility portfolio would reasonably appreciate by more than 20% over the same period. However, when the market declines, a high-volatility portfolio would be expected to decrease in excess of this amount. For instance, if the market decreases by 20% in one year, a high-volatility portfolio would reasonably depreciate by more than 20% over the same period. Lower-volatility portfolios would react in similar fashion, just with a lesser magnitude with respect to the market. With this in mind, one way the low-volatility portfolio is able to outperform the higher-volatility portfolio is by losing less during times of financial stress. Pim van Vliet and Jan De Koning conveniently started their analysis right at the beginning of the most severe economic depression in American history, the Great Depression (Vliet & Koning, 2016). The Great Depression began in 1929 and eroded away around 80% of the market's value by the time the recovery began in 1932 (History, 2010). With this in mind, by 1932 the high-volatility portfolio declined by over 80% while the low-volatility portfolio declined by less than 80%. More specifically, the high-volatility portfolio shrunk from $100 in value to just $5 while the low-volatility portfolio shrunk from $100 in value to $30. Thus, the results of this Vliet et al. could be taken with a grain of salt, since although the portfolios each started off with the same amount of money in 1929, the low-volatility portfolio was worth 6 times as much as the high-volatility portfolio just four years later. However, this is an expected consequence of the high-volatility portfolio, so these results should not be discredited. With this being said, since the low-volatility portfolio was able to lose less money during market downturns, it is able to grow, or compound, its capital more effectively than the high-volatility portfolio. To illustrate this, the portfolio values can be considered in 1932, when the high-volatility portfolio lost over 80% in value. As Benjamin Graham, a famous value investor, once noted the mathematical fact that "once you lose 95% of your money, you have to gain 1,900% just to get back where you started" (Graham, 1965). Similarly, since the high-volatility portfolio lost six times as much value as the low-volatility portfolio by 1932, it would have to outperform the low-volatility portfolio by significantly more than 600% in order to return to the same value.

Benchmarking

Thus, it seems very counterintuitive that fund managers and investors would not only invest in low-volatility stocks, as it appears that these stocks will outperform high-volatility stocks in the long-run. Part of understanding why this is not a commonplace investment strategy for many comes from interpreting what risk is defined as by the financial community. To investors, risk is not necessarily analogous to volatility, or even losing money. For many fund managers, risk comes from underperforming a benchmark. David Blitz, the head of quantitative equity research at Robeco, discussed the need for benchmarking the performance of investment managers (Vliet & Koning, 2016). Many managers, especially of actively managed funds, command handsome compensation in exchange for their investment acumen and diligence. Many hedge funds have a "Two and Twenty" compensation structure, where the managers charge a 2% fee on total assets under management and take an additional 20% of profits (Investopedia, 2017). Given the large amount of fees clients are paying, it is reasonable to believe that they expect to receive a greater return than if they had invested in a passive, market tracking ETF. These investment professionals need to prove to their bosses and clients that they are above average at their job. For example, if a hedge fund manager returns 10% in one year, but the market returns 10% that same year, the client will be upset, as they are now receiving a smaller return, given the hefty fees. In this case, a $100 investment in a market tracking index would return approximately $10 pre-tax, given there are little to no management fees. However, $100 invested in a hedge fund with a "Two and Twenty" structure would yield the same initial $10 pre-tax return, but the client would pay 2% of $100 ($2) in management fees, and 20% of the $10 profits ($2) in additional compensation. This would leave the client with a total of $6 return, or 6%, after all of the fees are paid out, and around half as much as if they had invested in a market tracking ETF. As a result, given the fee structure and client demands, active money managers are often compared to a market benchmark like the S&P 500; more importantly, they are expected to outperform these benchmarks substantially. Benchmarking also helps add perspective to a manager's returns. If a manager returns 20% in one year when the market returns 10%, he/she will have many happy clients. However, if the manager returns 20% in the same year that the market returns 30%, clients will not be as pleased. This is a concept known as "relative" risk.

Some investors, like individuals saving for retirement, primarily care about absolute risk, which is the total amount of money that is gained or lost due to overall stock movements, with regard to the starting amount of money invested. They will check their portfolio's total performance without much concern for the exact market return. If their portfolio gains 20% in one year, they will be content, even if the market increases 30% in the same period. For these investors, the horizon is long-term, so the short-term performance isn't as important for them as it is for fund managers who may be trying to raise more capital or justify high fees from clients. Volatility, in itself, captures these changes in the price of a stock, and is an absolute risk measurement. Many institutional investors do not look at risk on an absolute level, as a retiree or mom-and-pop investor may, but instead look at the risk of a portfolio with respect to the stock market or some other widely accepted benchmark. For these investors, the risk is not so much about losing money, rather it is more centered around lagging the market or their peers. Investing is very much a relative game. To further illustrate this idea, in relation to a fund manager, if a portfolio drops 20% while the market drops 40%, this is seen as a much better outcome than if a portfolio goes up 20% while the market goes up 40%. In the former, the manager lost money, but outperformed the market by 20%. In the latter, the manager made money but lagged the market by 20%. This concept can be hard to fully grasp due to the natural bias towards focus on absolute risk. Many have tried to explain this seemingly misunderstood phenomena, including Jason Karceski, who noted that an extrapolation bias could cause mutual fund managers to care more about outperforming in a bull market, than underperforming in a bear market (Karceski, 2002).

In fact, in 1968, institutional investors managed 30% of all money, but by 2008, the final year of the Baker et al. study, this figure increased to 60% (Baker et al., 2011). With this doubling of assets under active management, the low-risk anomaly intensified. In addition to being directly benchmarked against an index like the S&P 500, fund managers can also be compared using the "information ratio" (IR) which is the expected return difference between the manager and the expected return of the S&P 500, divided by the volatility of this return difference (tracking error) (Goodwin, 1998):

$$IR_P = \frac{\bar{R}_P - \bar{R}_B}{\hat{\sigma}_{P-B}}$$

$\bar{R}_P$ represents the return of the portfolio for the time period under measurement

$\bar{R}_B$ represents the return of the benchmark

$\hat{\sigma}_{P-B}$ is the standard deviation of the difference in returns between the portfolio and its benchmark

Figure 1.3: Information Ratio

The goal of an investment manager is to maximize this number, as best possible, through picking stocks. In fact, over 61% of U.S. mutual fund managers are benchmarked against the S&P 500, while over 94% are benchmarked to some U.S. index benchmark (Sensoy, 2009). Moreover, SEC rules require mutual funds to compare their performance to some benchmark (SEC, 2017). This intuitively makes sense, as it allows investors to assess the skill and ability of managers in an unbiased way, and also allows fund managers a chance to differentiate themselves. However, this makes institutional investors, who are managing the majority of the money in the United States, less likely to buy low-volatility stocks, leading to higher prices and lower expected returns for the high-volatility stocks and further exacerbating the low-risk anomaly.

Going back to Baker's findings, it is very hard for fund managers to attain a high IR with a low-risk portfolio. Part of this stems from the need for investment managers without leverage to find mispriced stocks with a beta very close to market risk (beta of 1), overweighting positive-alpha stocks while underweighting negative-alpha stocks. When comparing the Sharpe ratio of large cap stocks for a low-volatility portfolio, it was found to be quite high at 0.38. However, the Information Ratio was a very low 0.08, showing that this would be very tough for a fund manager to have. To provide a comparison, during 1968-2008, the top value strategy portfolios had an IR of 0.51, and top momentum strategy portfolios had an IR of 0.64. This is extremely high compared to the IR of low-volatility stocks in this period, which ranged from 0.08 to 0.17, showing these constructed low-beta portfolios would be unlikely to be used by any fund manager. While beta and volatility are undoubtedly very correlated, this shows that beta is more related to the anomaly than volatility, especially with large cap stocks, which is what most fund managers disproportionately focus their investments in (Baker et al., 2011).

Single-Period Returns

Another explanation of the low-volatility anomaly is an increased focus on returns over short-term periods by many researchers and investors. By focusing on "single period returns," which in most academic studies is just a one-month period, the significance of compounding is removed. This is more of an arithmetic way to calculate returns, where each month's return can be averaged, for example. Mathematically, this is not the correct way to calculate a return since it does remove the effect of compounding, but is a common way to compare fund managers. Moreover, when done over very short time periods (like a year), the effect of compounding is not as significant as it is for very long-term periods. To illustrate this, the following scenario can be considered: in one month a portfolio worth $100 drops 50% to $50, then the next month increases 50% to $75. The investment return is dependent on how one divides the time period. Looking at it on a monthly basis, even though the portfolio lost $25 in value, the net return would be -50% +50%, or 0% (with focus on single period returns, not accounting for compounding). However, looking at it on a longer-term basis, the net return was -25%, as the $100 portfolio ended up losing $25 in value. By not fully including the magic "return upon return" effect of compounding, the high-volatility portfolio discussed earlier performed more than 6% better per year (Vliet & Koning, 2016).

Psychological and Behavioral Factors

In addition to the reasons mentioned, there are several psychological reasons why some investors are not attracted to low-risk stocks. Eric Falkenstein, a renowned author in the low-volatility investing realm, wrote that "envy is at the root of the investment paradox" (Falkenstein, 2012). Some investors simply don't recognize the significance of compounding returns. Many others do but are unable to utilize the paradox due to relative risk and career pressures. Analysts who choose big winners are more likely to get recognized and promoted than those who pick safer stocks with lower upside potential; funds that pick the right high-risk stocks that turn out to be major home-runs will see more reward as an increase in AUM, and consequently an uptick in management fees. Moreover, some people do not invest in low-risk stocks because they have less of an appeal than high-risk stocks, where investors think they can make a lot of money easily and quickly. Even the news will have a bias towards reporting about and covering stocks that have become big winners. One famous big winner is Amazon: a \$5,000 investment in 1997 would be worth \$2,400,000 today, or an increase 49,000% (Berger, 2017). With all the excitement and reporting to this day on big winners like Amazon, many people forget about the number of similar technology companies that failed during the dot-com bubble and are worth nothing today. Thus, these high-risk stocks are more “sexy” and have a “lottery ticket” element that tempts investors with the appeal of a big payday.

Many investors have a natural preference for lotteries, even though there is a general aversion towards loss. If a stock has a positive skew, which is defined as a larger probability of a large positive payoff than probability of a small payoff, investors typically are very interested (Baker et al., 2011). Though skew is not the same as volatility, through their research, Boyer, Mitton, and Vorkink made a strong case for how expected skewness is a proxy for volatility through their findings that expected skewness assisted in explaining the observation that stocks with high idiosyncratic volatility have low expected returns (Boyer, Mitton, & Vorkink, 2009).

Another idea is representativeness, or that Bayes’ rule and probability theory are often not natural to people, even in the most seasoned investment professionals. One example of this is selectively looking at a few speculative investments that have turned out to be massive successes without considering the numerous failures. As mentioned earlier, the news will focus on Amazon’s great success over the past twenty years, but will not focus as much on all of the tech stocks that became worthless after the bubble burst. By not separating and considering all winners and losers, the average investor may be inclined to overpay for a riskier stock (Baker et al., 2011).

Overconfidence has also been tied to a preference for volatile stocks; optimists are generally more aggressive than pessimists. In a study, people were asked questions on how certain they were of their responses and it appeared many did not have an understanding of probability. Estimating the heat of a candle flame is very difficult, so for someone to say they are 80% sure of their answer is impressive, yet hard to believe. This can apply when people are asked for a confidence interval of the population of a city. In many instances, the person would be confident and give a very narrow range of people (Fischhoff, Slovic, & Lichtenstein, 1977). This same concept applies when valuing stocks and evaluating certain investment opportunities.

Overall, irrational investor preference for lotteries and high-volatility stocks, as well as investment managers’ focus on benchmarks and IR, flatten and eventually invert the relationship between risk and return in the long-run. Moreover, it has been shown by Baker et al., and prior observations that the anomaly intensified with the increase in assets under active management of fund managers in the U.S. These reasons together have led to the findings that low-beta and low-volatility stocks have outperformed high-beta, high-volatility stocks from 1968-2008, in part due to combining great returns with low downturns. Investor preference for “lotteries” and a bias of overconfidence creates a higher demand for higher-volatility securities, and the need to benchmark creates a greater demand for higher risk stocks, while discouraging investments in low-beta, low-volatility stocks. This understandably, increases the market’s appetite for risky stocks with high reward potential, driving up price and driving down expected return. These reasons appear perennial, so the anomaly is unlikely to cease to exist in the near future.

Profitability and Value

Defensive equity strategies generally aim at constructing a portfolio comprised of more low-risk stocks. This strategy has been becoming very popular as of late, in part due to equity markets that have suffered two recent, severe downturns, negative nominal returns in the first decade of 2000, and literature proving a weak or negative relationship between risk and return (Novy-Marx, 2014). Prior literature has looked at the relationship of performance relative to size and value, but many studies have not done a deeper dive into the effect of specific factors, such as profitability (Chan, Hamao, & Lakonishok, 1991). In fact, it has been shown that high profitability is the most significant predictor of low-volatility, inherently causing these defensive equity strategies to indirectly tilt towards profitability, in addition to the more known factors, like size and value. Size is very important, as small growth stocks are typically underweight in these strategies, while large value stocks are overweight. Likewise, value stocks are typically overweight, while growth stocks are underweight. Thus, the performance of defensive equity strategies can be explained by accounting for size, value, and profitability (Novy-Marx, 2014).

In terms of performance, defensive equity strategies, which are defined as low-volatility or low-beta in nature, have outperformed more aggressive strategies. This was already shown between 1929-2008 (Vliet & Koning, 2016) and in a separate study from 1968-2015 (Baker et al., 2011), but Novy-Marx analyzes a different timeframe. He looks at the characteristics of these low-risk portfolios in more detail by analyzing log-likelihood ratios that a stock selected at random is of a given style, like size or value, relative to the unconditional probability of being that style (Novy-Marx, 2014). The findings were very telling: on average, low-volatility stocks were 30 times larger than high-volatility stocks. As a result, though the high-volatility names made up half the total number of stocks, they contributed to just 9% of the total market capitalization when compared with low-volatility stocks. Moreover, when looking at the average returns across the portfolios of various volatilities, there seemed to be a relatively flat, slightly increased return with increasing volatility. The high-volatility/high-beta portfolio had a significant negative alpha, while the low-volatility/low-beta portfolio had a significant positive alpha (Novy-Marx, 2014).

Many of the volatile stocks tended to be small, unprofitable growth stocks which can help explain the relationship of the defensive strategy performance to size, value, and profitability. In fact, since 1968-2015, the portfolio of high-volatility stocks almost mirrored the performance of the portfolio of unprofitable, small growth stocks. Thus, the outperformance of defensive stocks from 1968-2015 have delivered significant alpha and can be explained when accounting for size, value, and profitability. In fact, profitability itself is so significant, that the case can be made that this alpha may come in large part from excluding unprofitable, small growth stocks (Novy-Marx, 2014).

1.2.4 Further Decomposition of the Low-Risk Anomaly into Micro and Macro Effects

The low-beta anomaly can be further broken up into micro and macro effects. The micro effects can be observed through picking low-beta stocks, and keeping country and industry risk the same, while the macro effects can be isolated by picking low-beta countries or industries, while keeping stock-specific risk the same. Baker et al. generates observable micro effects by creating portfolios of equity longs and shorts, holding forecasted country and industry risk constant. The macro effects were observed by constructing long-short portfolios of various countries and industries, holding forecasted stock-specific risk constant. Studying a number of stocks within 29 industries and 31 different developed countries, the macro and micro effects were observed and, both together, were shown to play an important role in explaining the low-risk anomaly (Baker, Bradley, & Taliaferro, 2014).

Looking at 31 developed countries including Canada, France, Germany, Japan, and Singapore, Baker et al. worked to decompose the low-risk anomaly into country and stock specific effects. Similar to the industry findings, country-beta was able to predict stock betas to a certain extent, but not as well as historical stock betas were. Looking only at country betas yielded around half the risk reduction and two-thirds the risk-adjusted return improvement, as compared to stock betas. This study implied that predicting risk of individual stocks is in itself very hard when only given data on country or industry risk, but when given all the data can have much more predictive power (Baker et al., 2014).

It was found that using industry beta to predict future stock betas was possible, but not as effective as just using historical stock betas. Industry beta information without stock information does improve risk-adjusted returns, just not to the same extent as stock information does. Baker et al. go into detail trying to isolate pure-industry effects and pure-stock effects. Pure-industry effects are the average differences between high and low-beta industries, while holding constant stock risk. Pure-stock risk is the opposite of that, calculating the average difference between high-beta and low-beta stocks, keeping industry risk constant. In the end, finding low-risk portfolios using selection of low-risk stocks and keeping industry constant was around four times more effective than using low-risk industries and keeping stock risk constant. However, once again, using the historical betas of both together, has more predictive power than either one alone (Baker et al., 2014).

Micro-selection of stocks, holding country and industry risks constant, was shown to significantly reduce risk without a significant decrease in return. In some cases, high-risk stocks within particular industries were able to be distinctly identified, and they typically had similar returns when compared to low-risk stocks in the same industry and country. Macro-selection, which involved holding stock-specific risks constant, was shown to lead to increases in return with small differences in risk, especially with regards to the country chosen. Stocks in high-risk countries were found to have distinctly lower returns than low-risk countries. These findings hold significant investment opportunity, due to the implication that people seeking arbitrage opportunities through mispricing of macro-effects like industry and sector, or through ETFs might not be as successful as exploiting the risk reduction opportunities stemming from micro-effects. While both the micro and macro-effects led to higher CAPM alpha by reducing risk and increasing returns, only the micro-effects were found to be statistically significant, whereas the macro-effects of country selection and industry selection were not found to be statistically significant. Thus, there is more of an arbitrage opportunity exploiting the micro-effects of individual stock selection than the macro-effects like country to industry. Even though the macro-effects have many limitations in practice, micro-effects are also limited due to leverage restrictions and benchmark mandates.

1.2.5 Real-World Applications of the Low-Risk Anomaly

Betting Against Beta (BaB)

Prior studies suggest that beta is not a great measure of risk and is more related to the anomaly than volatility is. AQR, a successful quantitative-focused hedge fund, employs a strategy called Betting Against Beta (BaB), which is a simple method of statistical arbitrage generated by shorting high-beta stocks and longing low-beta stocks (Investopedia, 2015). As discussed in some of the works previously, the premise behind this arbitrage opportunity is that high-beta stocks are overpriced while low-beta stocks are underpriced. The theory is that the stocks will eventually return to this equilibrium point, called the security market line (SML). While the prices approach this median, the investor can capture this spread as an arbitrage opportunity (Investopedia, 2015).

The Capital Asset Pricing Model calculates the expected equity return given certain levels of risk. Any excess above this risk-adjusted return is the Sharpe Ratio, or alpha that is generated by the stock. The Sharpe Ratio is calculated by taking the mean portfolio return and subtracting the risk free rate, then dividing by the standard deviation of portfolio returns (Investopedia, 2017):

$$= \frac{\bar{r}_p - r_f}{\sigma_p}$$

Where:

$\bar{r}_p$ = Expected porfolio return

r_f = Risk free rate

σ_p = Portfolio standard deviation

Figure 1.4: Sharpe Ratio

Investors try to maximize this number, and one way to do it is by levering up, or using borrowed capital to invest. By paying a fixed interest rate on the borrowed capital, assuming the return is more than the interest rate, an investor can increase the return on their invested equity. As one may imagine, this is more risky, as returns in both the upward and downward direction are magnified. As a result, many investment managers, like mutual funds, are legally constrained on the amount of leverage they can apply to a portfolio. Due to this, many fund managers must overweight high-beta stocks to improve overall returns. This leads to a tilting towards beta, and a flattening of this SML in relation to CAPM. This leads to a pricing anomaly which firms like AQR can take advantage of. By creating a market neutral strategy, and shorting high-beta stocks while longing low-beta stocks, they can capture this opportunity.

Andrea Frazzini and Lasse Heje Pedersen of AQR describe this strategy through a model in one of their studies (Frazzini & Pedersen, 2014). In this paper, a real-world resembling model is created with leverage and margin constraints in 55,600 stocks from 20 global stock, bond, credit, and future markets. Some agents in this model cannot use any leverage, and some have limited margin constraints, much like many investors and fund managers. As mentioned, many mutual funds, pension funds, and individual investors are constrained by the amount of leverage they can take on, such that instead of investing in a portfolio yielding the highest Sharpe Ratio, they are forced to overweight portfolios with riskier stocks. This suggests fund managers hold high-beta stocks to a lower risk adjusted return standard than low-beta stocks, which would require leverage. Thus, if someone cannot leverage or has significant leverage constraints, then they will overweight riskier securities. The model in this paper was able to empirically show this in the equities, bonds, and futures markets. This was done by sorting portfolios by betas, and realizing alphas and Sharpe Ratios declined with increases in portfolio-beta (Frazzini & Pedersen, 2014).

Presumably, if one could lever up without constraint, the investor would underweight high-beta assets and overweight low-beta assets. BaB factors help explain this. A BaB factor is a portfolio longing low-beta securities (leveraged to a beta of 1), shorting high-beta assets (deleveraged to a beta of 1), that is market neutral. Frazzini's model predicts that this portfolio will have a positive return that increases with the spread in the betas and tightness of leverage constraints. Thus, being long low-beta stocks and short high-beta stocks yields significant, and positive risk-adjusted returns. This was observed in the model by looking at U.S., developing, and international equity markets and noting that the BaB factor yielded a Sharpe ratio that was double its value effect, and 40% greater than momentum. The BaB factor had very high risk-adjusted returns, and during four twenty-year periods between 1926 and 2012, produced significant positive returns. This generally held across other asset classes, too, including credit and treasury bond markets (Frazzini & Pedersen, 2014).

When a leverage constraint was met or surpassed, and the agent needs to deleverage, the BaB factor portfolio experiences negative returns, but its expected future returns still increased. This was once again shown with a time series with spreads of various funding constraints. Another central idea of the model was that increased funding liquidity risk compresses betas toward one. Frazzini et al. proved this by looking at the volatility of funding constraints as funding liquidity risk; the end finding was that the dispersion of betas when funding liquidity risk is high, and was much lower than when funding liquidity risk is low. In other words, tightening of funding constraints leads to a lower BaB factor (Frazzini & Pedersen, 2014).

Finally, the model showed that investors who are more leverage-constrained are forced to overweight riskier securities, while investors without such constraints could overweight low-risk securities. Observing a number of stock portfolios from constrained investors showed most fund managers and individual investors' portfolios have a beta greater than one. However, many private equity firms that perform leveraged buyouts are traditionally able to purchase firms with a beta below one and apply leverage, allowing them to utilize this anomaly due to the the fact that they have lower leverage constraints than their public market counterparts. Famed investor Warren Buffett even bets against beta, as many of his investments are leveraged, low-beta stocks. By having these constraints, though, the typical public markets investor is forced to hold on to riskier, high-beta stocks, leading to effectiveness of the BaB factor (Frazzini & Pedersen, 2014).

Betting Against Correlation

In trying to understand the low-risk anomaly and how it could be applied, Asness et al. considered two possible explanations. The first looked at whether it was caused by leverage constraints, or measurement using systematic risk. The second focused on the behavioral effects, or idiosyncratic risks. One of the main issues with prior research is that many low-risk factors are correlated and interrelated, making it hard to completely isolate certain factors or effects. Adrian et al. showed a link between return of the BaB factor and financial intermediary leverage (Adrian, Etula, & Muir, 2014). Many of these factors, including BaB, generally exhibited the "low-risk effect" and were consequently very difficult to completely distinguish from one another.

Asness et al. attempted to clarify this by introducing a couple new factors meant to control for existing factors (Asness, Frazzini, Gormsen, & Pedersen, 2016). They broke down BaB into two separate factors: betting against correlation (BaC) and betting against volatility (BAV). This was done because beta itself is calculated by taking the stock's correlation with the market times its own volatility and dividing by the market's overall volatility. BaC is accomplished through longing stocks with a low correlation to the market, and shorting those with a high correlation to the market, while trying to match the volatilities of both the long and short portfolios. BAV is achieved in a similar manner, except through longing stocks with low-volatility and shorting high-volatility stocks while keeping correlation constant.

To address the behavioral explanation, Asness et al. looked at some prior factors from observations made by Ang et al. (Ang, Hodrick, Xing, & Zhang, 2006). They found that stocks with low idiosyncratic volatility (IVOL) had a greater risk-adjusted return, and in 2009 added to this by finding that a low maximum return (LMAX), a measure of idiosyncratic skewness, is associated with greater risk-adjusted returns (Ang, Hodrick, Xing, & Zhang, 2009). Asness et al. kept the focus on the LMAX and IVOL, but added another factor, scaled MAX (SMAX), which longs stocks with a low MAX return divided by ex-ante volatility, and consequently shorts stocks with a high MAX return divided by ex-ante volatility (Asness et al., 2016). This allows focus on the lottery demand, holding volatility relatively constant and only focusing on the distribution of the returns. Margin debt held by investors and investor sentiment were also noted.

Overall, 58,415 stocks from the MSCI World Index from 24 different countries between January 1926 and December 2015 were covered (Asness et al., 2016). BAV and BaC were found to be very successful in controlling for the other factors that could influence the "low-risk effect." The BAV findings are in line with prior findings, and can be explained through behavioral effects like the lottery preference and leverage aversion. For all stocks, the BaC factor produced a significant six-factor alpha that was nearly independent of the other low-risk factors studied. This was partially due to the leverage aversion, which indicates correlation changes in beta should be priced in. In terms of explaining the behavioral side with factors, SMAX was the only truly great, resilient measure used. The rest generally had higher turnover, and were consequently very susceptible to microstructure noise. SMAX attained positive risk-adjusted returns in the U.S. but negative risk-adjusted returns globally, which was seen with some other idiosyncratic risk factors. Asness et al. showed that systemic low-risk factors generally tended to outperform behavioral risk factors, especially when considering turnover and time period length. All in all, the low-risk effect was believed to be driven by multiple factor effects, meaning both leverage constraints and lottery demand could play a role in influencing this. However, leverage constraint effects were a bit stronger, especially internationally. By cleaning up these factors, clearer explanations for previously observed results were provided (Asness et al., 2016).

Stock Price Response to Index Rebalancing

One of the driving fundamental assumptions of finance is a flat demand curve for stocks, where risk is the main driver and each stock has a perfect substitute. However, this concept has been questioned for the past few years, with literature picking up on stocks showing supply shocks and quantifying how this affects their market price. Literature has shown several instances where large block sales of stock has negatively affected its price (Harris & Gurel, 1986). This was often due to information contamination, which is new, significant information about the company in the market. This information often reflects fundamental changes in the company, and, if it is negative, will understandably trigger block sales. Thus, the price change could be less due to the supply shock, and caused more by fundamental changes in the company's value, such as a scandal or earnings report release.

However, interesting patterns that have not yet been fully explained emerge from observations regarding S&P 500 company addition and deletions. When companies are added or removed from the index, it is often purely mechanical and usually not due to some drastic fundamental change in the company. Assuming the market is efficient, the demand for stocks should not change due to being added or removed from an index, but several studies have shown that it does (Shleifer, 1986). The studies agree that new additions to the S&P come with higher than normal returns for that company (Beneish & Whaley, 1996). Though they agree on the price movement, the studies tend to disagree on the precise reason for this price movement; some possible explanations include: compensation for providing liquidity, better monitoring for investors when a company is added to a reputable and large index, and higher analyst coverage leading to more information and analysis available on the company (Chen, Noronha, & Singal, 2004). One primary concern is whether or not index reshuffling is an information-free event - that is, whether a company being added or removed adds information about the company to the market.

Huji et al. looked at factor index rebalancing for an information-free event (Huij & Kyosev, 2016). Factor indices are part of a parent index of many other stocks and are constructed in a mechanical way that is publicly available and usually based on ranking stocks off a particular ratio of characteristic. The authors looked at one example, the MSCI Minimum Volatility index, and recorded returns for the stocks that had been added or dropped. It was found that the cumulative return from announcement to the effective day was 1.07% for stocks added with a significant t-statistic of 7.16, with 62% of the stocks exhibiting a positive cumulative abnormal return. Of the 1.07% increase, 0.63% of it was gained the following day, indicating that a large part of the increase was due to greater demand from index funds. 0.31% of the return was lost five days after the rebalancing, but generally the price tended to stabilize after around ten days. Thus, 68% of the price increase was long lasting while the other 32% was temporary and lost after a few days. This can be due to a liquidity premium charged by the stock's owner or arbitrage activity. Average trading volume was also significantly higher for stocks that were recently added to the index. For the days between the announcement and actual addition of the stock, the average trading volume was 30% higher than normal, with a significant t-statistic of 3.81. Moreover, there was a 74% increase in volume for the day prior to the actual addition of the security. A very similar phenomena occurs with stocks set to be dropped from the USMV Index; from the announcement of a stock being dropped to the day before it is actually deleted from the index, the total cumulative abnormal return was -0.91%, and a total of -0.57% came the day before. After the stocks were deleted, 64% had a negative return the following day, and only 0.49% of the -0.91% was regained after three weeks. Trading volume also spiked 46% on the day prior to removal from the index. After three weeks, it returned back to within 1% of the normal trading volume (Huij & Kyosev, 2016).

These findings imply that once a security is added to a factor index, the demand curve shifts to the right, moving the equilibrium. The trading volume change is likely due to index funds buying or selling massive amounts of the stocks that will be added or removed. Moreover, it was found that the amount of the return is also directly related to the weighting of the volume of stocks entering or leaving the factor index. All in all, these findings suggest an index arbitrage opportunity exists if index additions or deletions can be accurately predicted.

Chapter 2

Data Gathering Process

2.1 Data Aggregation

Data was collected for the iShares MSCI USA Equal Weighted ETF (EUSA), which tracks the parent index of the minimum volatiltiy index, and iShares Edge MSCI Min Vol USA ETF (USMV), which tracks the minimum volatility index, from Oct 31, 2011 to December 31, 2016 (BlackRock, 2017). The iShares data contained this information for the two ETFs of interest for each constituent on the last trading day of every month. This included characteristics of each stock, such as: ticker, company name, asset class, weight of the stock relative to the entire index, price per share, number of shares, market value of the position, notional value of the position, sector, sedol number, isin number, exchange that the stock is listed on, and the month end date for the data. Each month-end dataset was individually downloaded, then aggregated to create the two separate raw data sets - one for EUSA and one for USMV. The data was then cleaned.

2.2 Data Cleaning

After having a quick overview of the data, there were many issues with each respective dataset that needed to be fixed before the analysis could occur. Since USMV is a subset of EUSA, the issues were very similar, and those that existed in USMV, generally existed in USMV as well. The issues could be broken down into 3 main types: erroneous listed stock exchanges, problematic listed tickers, and price discrepancies due to issues like stock splits. Moreover, cash and cash related assets were removed from the data, as this dissertation focuses only on the stocks.

2.2.1 Non-US Exchanges

Looking at the unique exchanges of the data, it was observed that there were several foreign exchanges like the Swiss Exchange and the Mexican Stock Exchange. For example, in the EUSA data set, there were several foreign stock exchanges:

```r
load("~/thesis_final/data/usa.Rda")
# Print out unique exchanges in USA data set
head(unique(usa$exchange))
```

```
[1] New York Stock Exchange Inc.
[2] NASDAQ
[3] Bolsa Mexicana De Valores (Mexican Stock Exchange)
[4] Nyse Euronext - Euronext Paris
[5] Boerse Berlin
[6] Tokyo Stock Exchange
17 Levels: Boerse Berlin Boerse Duesseldorf ... Tokyo Stock Exchange
```

This did not make sense, given the ETF constituents are supposed to be US-focused, meaning they should be listed on US-based exchanges. Of the non-US exchange errors, they can be further broken up into two subgroups: companies that were incorretly listed overseas and are actually listed on US exchanges, and companies that also are actually listed on US exchanges but instead had their overseas exchange tickers listed.

Mislabeled Exchanges

The first type of error involved companies that are actually listed on either the NYSE and NASDAQ, but were listed on a foreign exchange in the data and still had their US ticker used. One example was BAC (Bank of America) which is listed on the NYSE, but was listed on the Swiss Stock Exchange in the dataset:

```
  ticker                 name weight price       exchange       date
1    BAC BANK OF AMERICA CORP 0.5812  6.83 Swiss Exchange 2011-10-31
2    BAC BANK OF AMERICA CORP 0.4643  5.44 Swiss Exchange 2011-11-30
3    BAC BANK OF AMERICA CORP 0.4732  5.56 Swiss Exchange 2011-12-30
4    BAC BANK OF AMERICA CORP 0.5823  7.13 Swiss Exchange 2012-01-31
5    BAC BANK OF AMERICA CORP 0.6235  7.97 Swiss Exchange 2012-02-29
6    BAC BANK OF AMERICA CORP 0.7375  9.57 Swiss Exchange 2012-03-30
```

The price for BAC in the data set corresponded to the price of BAC in the NYSE, even though it was listed on the Swiss Exchange; BAC also did not corresponded to Bank of America on the Swiss Exchange. Thus, after several checks, it could be concluded that BAC in the data set was incorrectly listed on the Swiss Exchange, and should have been listed on the NYSE instead. Since the ticker was still correct and would be read in properly in the later stages of this paper, these cases were left as is and no changes were made.

Mislabeled Tickers

The second type of error stemmed from companies listed on foreign exchanges that are also listed on a US exchange in reality, but had their non-US ticker used in the data set. One example of this was Aflac, Inc. which was listed by its ticker “8686” on the Tokyo stock exchange:

```
  ticker       name weight price              exchange       date
1   8686 AFLAC INC. 0.1779 45.09 Tokyo Stock Exchange 2011-10-31
2   8686 AFLAC INC. 0.1719 43.44 Tokyo Stock Exchange 2011-11-30
3   8686 AFLAC INC. 0.1699 43.26 Tokyo Stock Exchange 2011-12-30
4   8686 AFLAC INC. 0.1810 48.23 Tokyo Stock Exchange 2012-01-31
5   8686 AFLAC INC. 0.1698 47.25 Tokyo Stock Exchange 2012-02-29
6   8686 AFLAC INC. 0.1646 45.99 Tokyo Stock Exchange 2012-03-30
```

This immediately raised a red flag due to the numbers in the ticker. This numeric ticker corresponded to Aflac, Inc. on the Tokyo exchange, but when checking the recorded price of the stock for corresponding dates, it matched up with the Aflac, Inc. stock on the NYSE, with ticker "AFL". Thus, when this happened, each company was treated on a case-by-case basis. In this case, since the stock price corresponded to AFL, the ticker name was changed from "8686" to "AFL". This would ensure the data could be properly read in later on.

2.2.2 Unrecognized Tickers

The final general type of error in the data occured when the ticker incorrectly recorded in the data set. This was evaluated, once again, on a case-by-case basis, by observing which tickers were not recognize, and looking at the company name to understand why. Sometimes, the issue was very obvious. One example of a clear discrepancy was when the ticker had an asterisk at the end of it. After careful digging, the asterisk did not seem to mean anything, and it is unclear why some tickers contained it. One ticker was "AAPL*":

```
  ticker      name weight  price       date
1  AAPL* APPLE INC 3.1640 404.78 2011-10-31
2  AAPL* APPLE INC 2.9821 382.20 2011-11-30
3  AAPL* APPLE INC 3.1623 405.00 2011-12-30
4  AAPL* APPLE INC 3.4130 456.48 2012-01-31
5  AAPL* APPLE INC 3.8840 542.44 2012-02-29
6  AAPL* APPLE INC 4.2291 599.47 2012-03-30
```

This would cause future issues when reading the data in, because that ticker was not read in as "AAPL" due to the asterisk. This was fixed by simply removing the asterik from the ticker name.

Another example of the ticker not being read in properly was when it contained numbers. Alflac was an example that was mentioned previously, but another one that applied here was "AG4" which was the ticker for Allergan. Since NYSE and NASDAQ tickers do not contain numbers, this was a clear issue:

```
  ticker     name weight      exchange price       date
1    AG4 ALLERGAN 0.2160 Boerse Berlin 84.12 2011-10-31
2    AG4 ALLERGAN 0.2157 Boerse Berlin 83.72 2011-11-30
3    AG4 ALLERGAN 0.2275 Boerse Berlin 87.74 2011-12-30
```

```
4    AG4 ALLERGAN 0.2178 Boerse Berlin 87.91 2012-01-31
5    AG4 ALLERGAN 0.2126 Boerse Berlin 89.59 2012-02-29
6    AG4 ALLERGAN 0.2204 Boerse Berlin 95.43 2012-03-30
```

After some research, it found that AG4 is the ticker for Allergan on the Deutsche Boerse AG Stock Exchange, but the prices corresponded to that of Allergan's on the NYSE. Thus, the AG4 ticker was changed to the ticker used for Allergan on the NYSE - AGN. Overall, though each category is unique, there has been a lot of overlap, and often times correcting one type of error would fix other errors too. For example here, many tickers that include numbers would lead to obvious errors, and this was often times because the ticker corresponded with the right company, just on a foreign exchange.

2.2.3 Price Discrepancies

The general methodology to ensure a change in ticker was appropriate was to check the price of the stock at a specific date, in the USA data set, and then compare it to the new ticker being assigned. If the price matched, the change was made. If the price did not match up, and was very different, research was performed to see if a stock-split might be the cause of this. If there was no evidence of a stock-split, then the stock further analyzed to see what the issue was. In addition to looking at when prices did not match up with tickers and companies for certain dates, monthly returns were calculated for each stock during the times they were in the index, and any abnormal returns (magnitude greater than 30% in one month) were look at manually. One example of this was Netflix's stock 7:1 stock split in 2015. The monthly data showed drastic fall in price from 656.94 on 2015-05-29 to a 114.31 on 2015-07-31, in just one month. This amounts to recorded loss of 82.5%:

```
  ticker         name weight exchange  price       date
1   NFLX NETFLIX INC 0.1840   NASDAQ 624.06 2015-05-29
2   NFLX NETFLIX INC 0.1945   NASDAQ 656.94 2015-06-30
3   NFLX NETFLIX INC 0.2329   NASDAQ 114.31 2015-07-31
4   NFLX NETFLIX INC 0.1693   NASDAQ 115.03 2015-08-31
5   NFLX NETFLIX INC 0.1588   NASDAQ 103.26 2015-09-30
```

Since this surpassed the threshold set, it was look at in more detail. After some research, it was shown there was in fact a 7:1 stock split, so the price of the stock on 2015-07-31 was adjusted to 800.17, and the appropriate calculations were done. Thus, in this case, the ticker was left alone, but just the price was adjusted.

Tickers and stock names that could not be determined were removed. In the end, "1015736" and "Orchard Supply Hardware Stores" were removed from the data set. These together accounted for less than 0.2% of the data for a given month-end date.

2.3 Data Overview

To get a sense of the cleaned EUSA data, an overview is shown below:

```
Warning: package 'dplyr' was built under R version 3.4.2

  ticker                     name asset.class weight  price shares
1      A AGILENT TECHNOLOGIES INC      Equity 0.1090  37.07     79
2     AA                ALCOA INC      Equity 0.0981  10.76    245
3    AAP   ADVANCE AUTO PARTS INC      Equity 0.0388  65.07     16
4   AAPL                APPLE INC      Equity 3.1640 404.78    210
5    ABC   AMERISOURCEBERGEN CORP      Equity 0.0942  40.80     62
6    ABT      ABBOTT LABORATORIES      Equity 0.7018  53.87    350
  market.value notional.value                 sector   sedol         isin
1      2929000             NA            Health Care 2520153 US00846U1016
2      2636000             NA              Materials 2021805 US0138171014
3      1041000             NA Consumer Discretionary 2822019 US00751Y1064
4     85004000             NA Information Technology B011001 US0378331005
5      2530000             NA            Health Care 2795393 US03073E1055
6     18855000             NA            Health Care 2002305 US0028241000
                                          exchange       date
1                      New York Stock Exchange Inc. 2011-10-31
2                      New York Stock Exchange Inc. 2011-10-31
3                      New York Stock Exchange Inc. 2011-10-31
4 Bolsa Mexicana De Valores (Mexican Stock Exchange) 2011-10-31
5                      New York Stock Exchange Inc. 2011-10-31
6                      New York Stock Exchange Inc. 2011-10-31
```

Shown below are summary statistics for EUSA. Each name is representd 63 times, which is indicates no name is over represented in the dataset. Moreover, every row is an equity, which makes sense considering cash and cash assets were removed from at the beginning of the cleaning process. The weights of each stock in the EUSA ETF are also all between 0 and 4.6773%. There are no negative values and no stocks that appear to be incorrectly overweight. Moreover, each sector appears to be reasonably distributed, and the dates correctly range from 10-31-2011 to 12-30-2016.

```
                   name                asset.class        weight      
 3M CO                   :    63   Cash        :    0   Min.   :0.0000  
 ABBOTT LABORATORIES     :    63   Equity      :38598   1st Qu.:0.0536  
 ACCENTURE PLC           :    63   Money Market:    0   Median :0.1208  
 ACTIVISION BLIZZARD INC:    63                         Mean   :0.1629  
 ADOBE SYSTEM INC        :    63                         3rd Qu.:0.1636  
 ADVANCE AUTO PARTS INC :    63                         Max.   :4.6773  
 (Other)                 :38220                                           
                    sector          date           
 Financials             :6825   Min.   :2011-10-31  
 Consumer Discretionary:6595   1st Qu.:2013-02-28  
 Information Technology:5487   Median :2014-06-30  
 Industrials            :4687   Mean   :2014-06-12  
 Health Care            :4067   3rd Qu.:2015-09-30  
```

```
Energy                 :3208   Max.   :2016-12-30
(Other)                :7729
```

To get a sense of the cleaned USMV, an overview is shown below:

```
  ticker                   name asset.class weight  price shares
1    AAP ADVANCE AUTO PARTS INC      Equity 0.7986  65.07    316
2   AAPL              APPLE INC      Equity 0.0943 404.78      6
3    ABC AMERISOURCEBERGEN CORP      Equity 0.7479  40.80    472
4    ABT    ABBOTT LABORATORIES      Equity 1.5544  53.87    743
5   ACGL ARCH CAPITAL GROUP LTD      Equity 0.7767  35.97    556
6    ACN          ACCENTURE PLC      Equity 1.6452  60.26    703
  market.value notional.value                 sector   sedol         isin
1        20562             NA Consumer Discretionary 2822019 US00751Y1064
2         2429             NA Information Technology B011001 US0378331005
3        19258             NA            Health Care 2795393 US03073E1055
4        40025             NA            Health Care 2002305 US0028241000
5        19999             NA             Financials 2740542 BMG0450A1053
6        42363             NA Information Technology B4BNMY3 IE00B4BNMY34
                                     exchange       date
1                   New York Stock Exchange Inc. 2011-10-31
2 Bolsa Mexicana De Valores (Mexican Stock Exchange) 2011-10-31
3                   New York Stock Exchange Inc. 2011-10-31
4                   New York Stock Exchange Inc. 2011-10-31
5                                          NASDAQ 2011-10-31
6                   New York Stock Exchange Inc. 2011-10-31
```

Shown below are summary statistics for USMV Each name is representd 63 times, which is indicates no name is over represented in the dataset. Moreover, every row is an equity, which makes sense considering cash and cash assets were removed from at the beginning of the cleaning process. The weights of each stock in the USMV ETF are also all between 0.0002 and 2.8287%. There are no negative values and no stocks that appear to be incorrectly overweight. Moreover, each sector appears to be reasonably distributed, and the dates correctly range from 10-31-2011 to 12-30-2016.

```
                           name          asset.class       weight
ABBOTT LABORATORIES          :  63   Cash        :   0   Min.   :0.0002
ALTRIA GROUP INC             :  63   Equity      :9229   1st Qu.:0.2790
ARCH CAPITAL GROUP LTD       :  63   Money Market:   0   Median :0.6016
AT&T INC                     :  63                       Mean   :0.6810
AUTOMATIC DATA PROCESSING INC:  63                       3rd Qu.:1.0191
AUTOZONE INC                 :  63                       Max.   :2.8287
(Other)                      :8851
                   sector          date
Health Care            :1631   Min.   :2011-10-31
Financials             :1548   1st Qu.:2013-04-30
```

```
Information Technology:1482   Median :2014-09-30
Consumer Staples      :1222   Mean   :2014-08-13
Consumer Discretionary:1053   3rd Qu.:2015-11-30
Utilities             : 616   Max.   :2016-12-30
(Other)               :1677
```

2.4 Data Check

In addition to the basic checks above, further tests were performed to check how accurate and complete the data was. This was accomplished by comparing the weighted-returns from the indices constructed from ETF data to the actual ETF returns on a monthly basis.

2.4.1 Weights

Total weights of the constructed US Equal Weight and Minimum Volatility indices were calculated by adding up each constituent's weight on a monthly basis. If the data were perfect, these should add up to 1 for each month. However, given some tickers and cash were removed, and given tracking error between the ETF and index, this was not expected. The monthly change in weights for the constructed US Equal Weight Index is shown below:

```
     date                 weight
Min.   :2011-10-31   Min.   : 99.54
1st Qu.:2013-02-14   1st Qu.: 99.73
Median :2014-05-30   Median : 99.80
Mean   :2014-05-30   Mean   : 99.79
3rd Qu.:2015-09-15   3rd Qu.: 99.84
Max.   :2016-12-30   Max.   :100.21
```

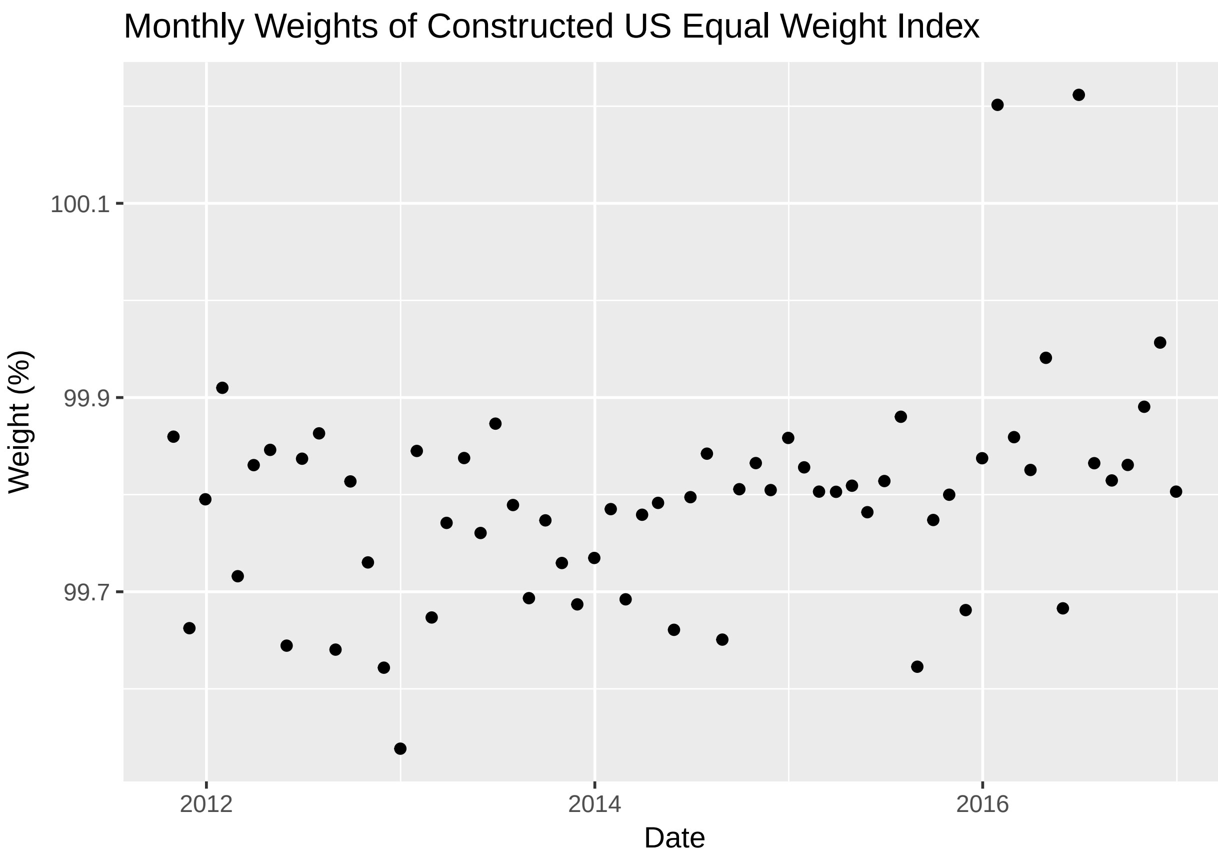


Figure 2.1: The weights for the constructed US Equal Weight Index are very close to 100%, with just two months exceeding 100%.The minimum weight is 99.54%, while the largest weight is 100.21%. The mean weight is 99.79%.

The monthly change in weights for the constructed Minimum Volatility Index is shown below:

```
     date                weight
Min.   :2011-10-31   Min.   :99.58
1st Qu.:2013-02-14   1st Qu.:99.72
Median :2014-05-30   Median :99.76
Mean   :2014-05-30   Mean   :99.76
3rd Qu.:2015-09-15   3rd Qu.:99.81
Max.   :2016-12-30   Max.   :99.99
```

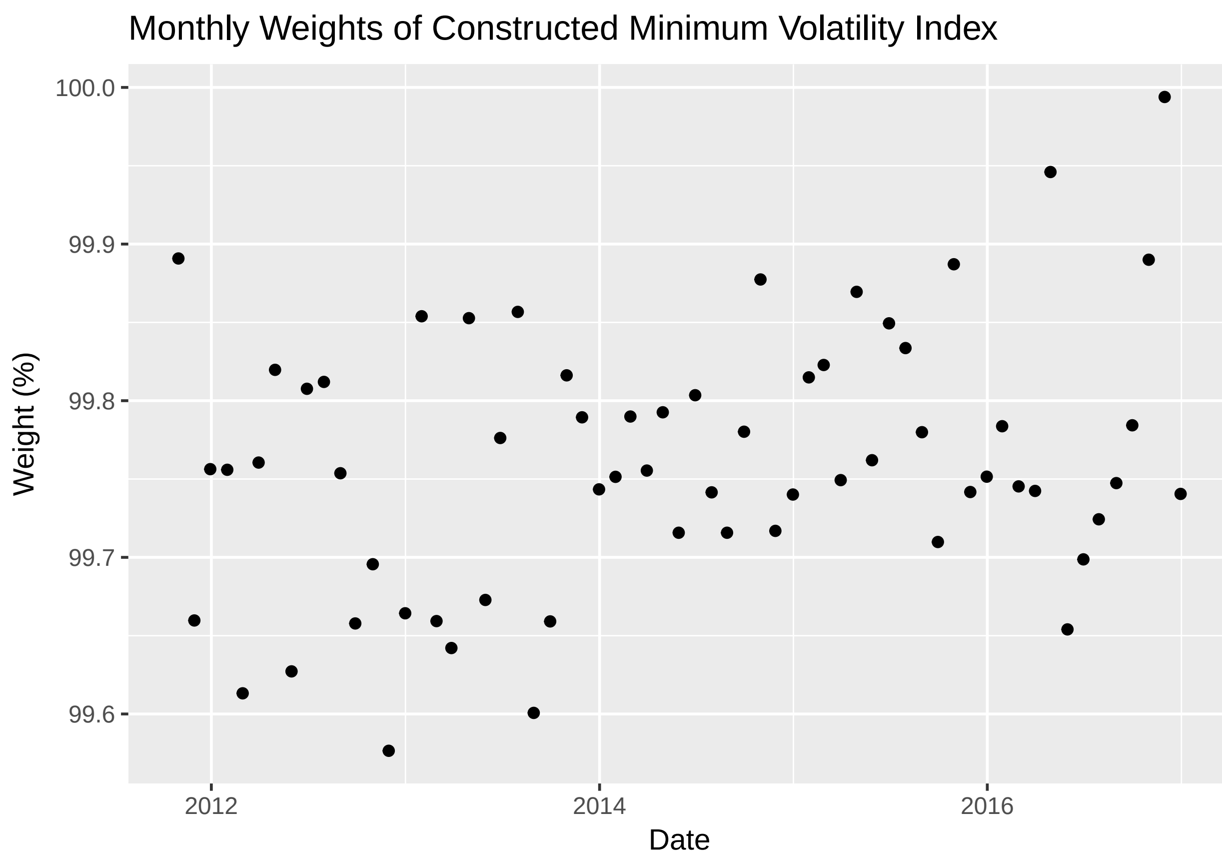


Figure 2.2: The weights for the constructed Minimum Volatility Index are very close to 100%, with no value exceeding 100%. The minimum weight is 99.58%, while the largest weight is 99.99%. The mean weight is 99.76%.

2.4.2 Comparing ETF returns to Constructed Index returns

Given the weights were very close to 100% for both constru on a monthly basis, an additional check was performed by comparing the weighted returns of the constructed indices to the actual ETFs that mirror them. The ETFs are traded in the stock market, so price daily information on each was widely available. Thus, this provided a way to check how the constructed weighted returns compared to the ETF returns for both the constructed US Equal Weight Index and Minimum Volatility Index. Correlations were calculated between the performance of the constructed US Equal Weight Index and EUSA, as well as between the performance of the constructed Minimum Volatility Index and USMV. The higher the correlation, the more accruate the data is.

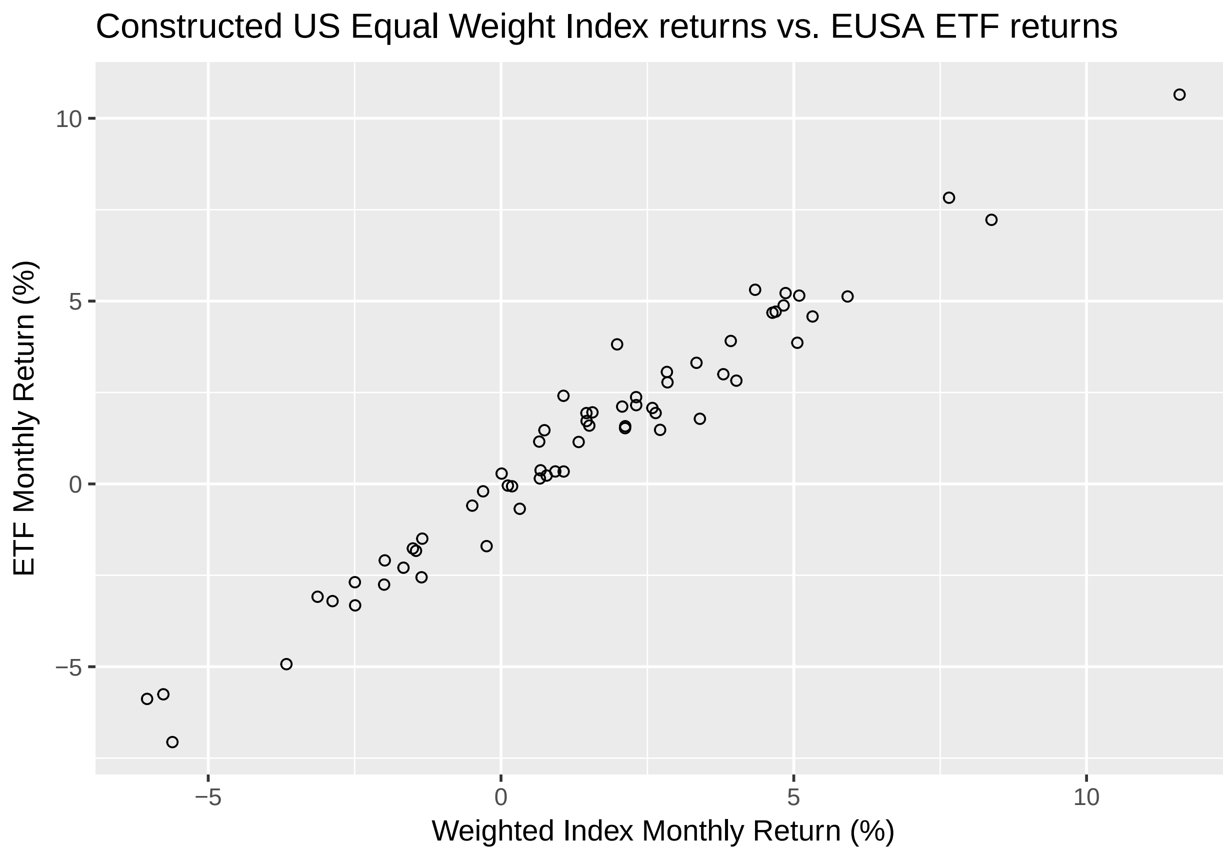


Figure 2.3: The correlation between the Constructed US Equal Weight Index monthly returns and EUSA ETF monthly returns is 98.06%.

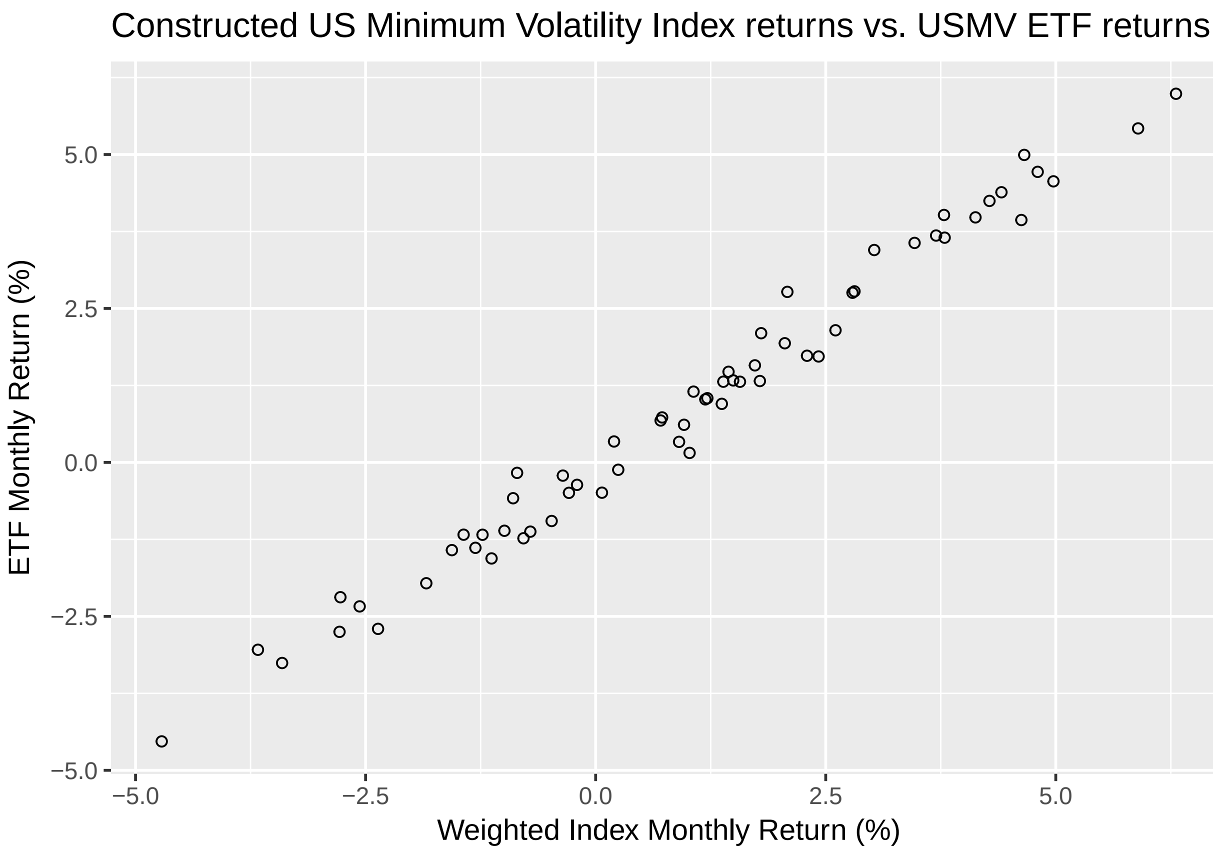


Figure 2.4: The correlation between the Constructed Minimum Volatility Index monthly returns and USMV ETF monthly returns is 99.07%.

Chapter 3

Data Analysis

3.1 Changes in Largest Holdings by Average Weight in Minimum Volatility Index

The top 5 largest holdings in the Minimum Volatility Index by average weight are Verizon, AT&T, Automatic Data Processing, Johnson & Johnson, and McDonald's.

Table 3.1: Top 5 Holdings in the Minimum Volatility Index over Time by Average Weight

Rank	Ticker	Average Weight
1	VZ	1.508046
2	T	1.493976
3	ADP	1.479657
4	JNJ	1.470513
5	MCD	1.458286

3.2 Sector Weights

Sector weights were calculated over time for both the US Equal Weight and the Minimum Volatility Index. This was done to get a sense of what industries may be inherently more "low-risk" as well as checking to make sure the data is resilient, and each sector weighting in the Minimum Volatility Index is within 5% of the US Equal Weight Index, as specified by the Barra Optimizer.

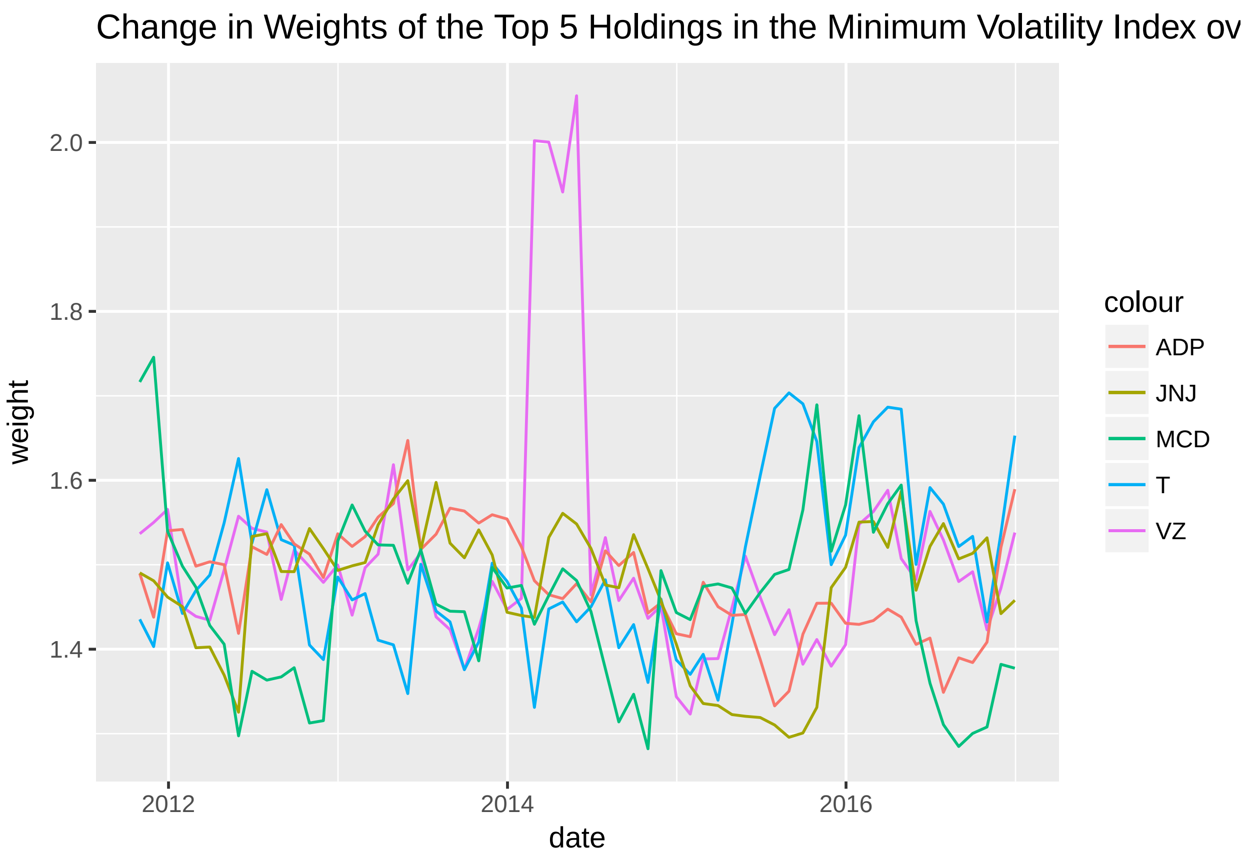
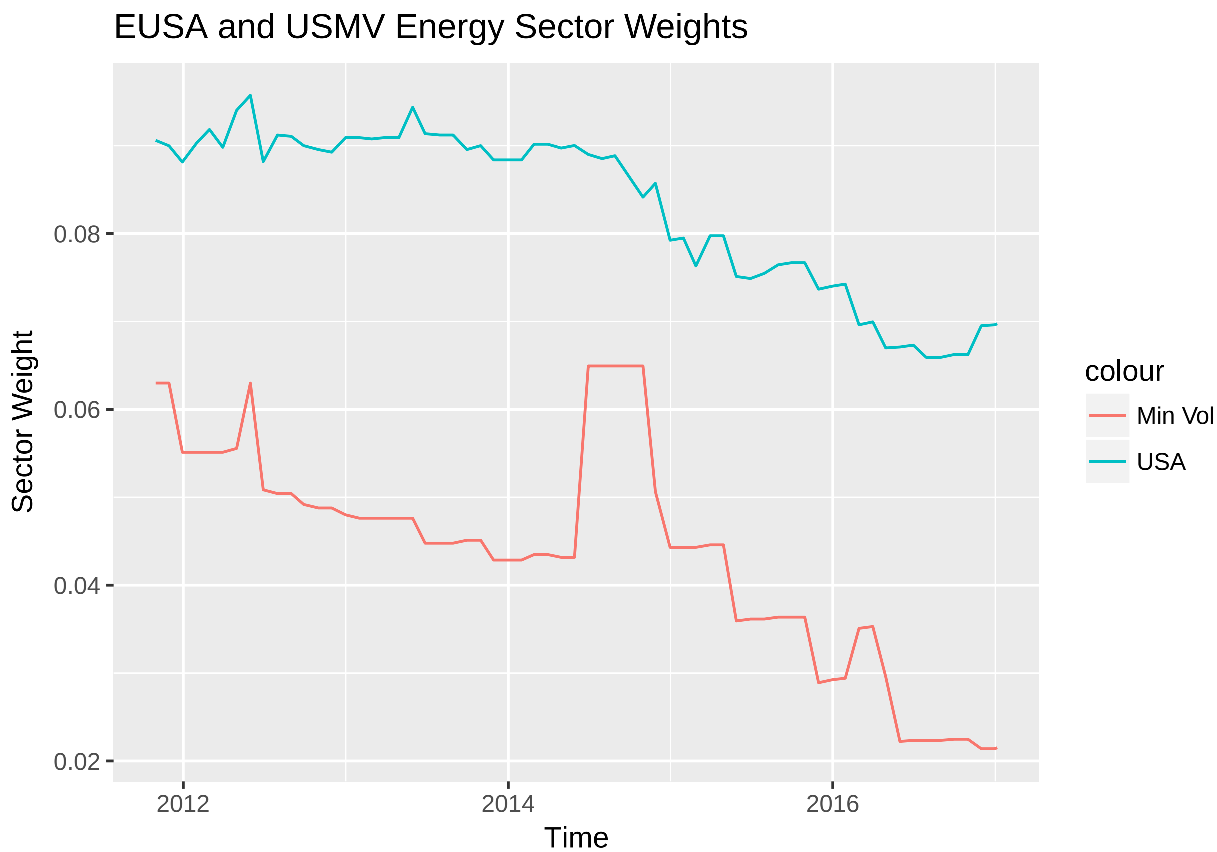
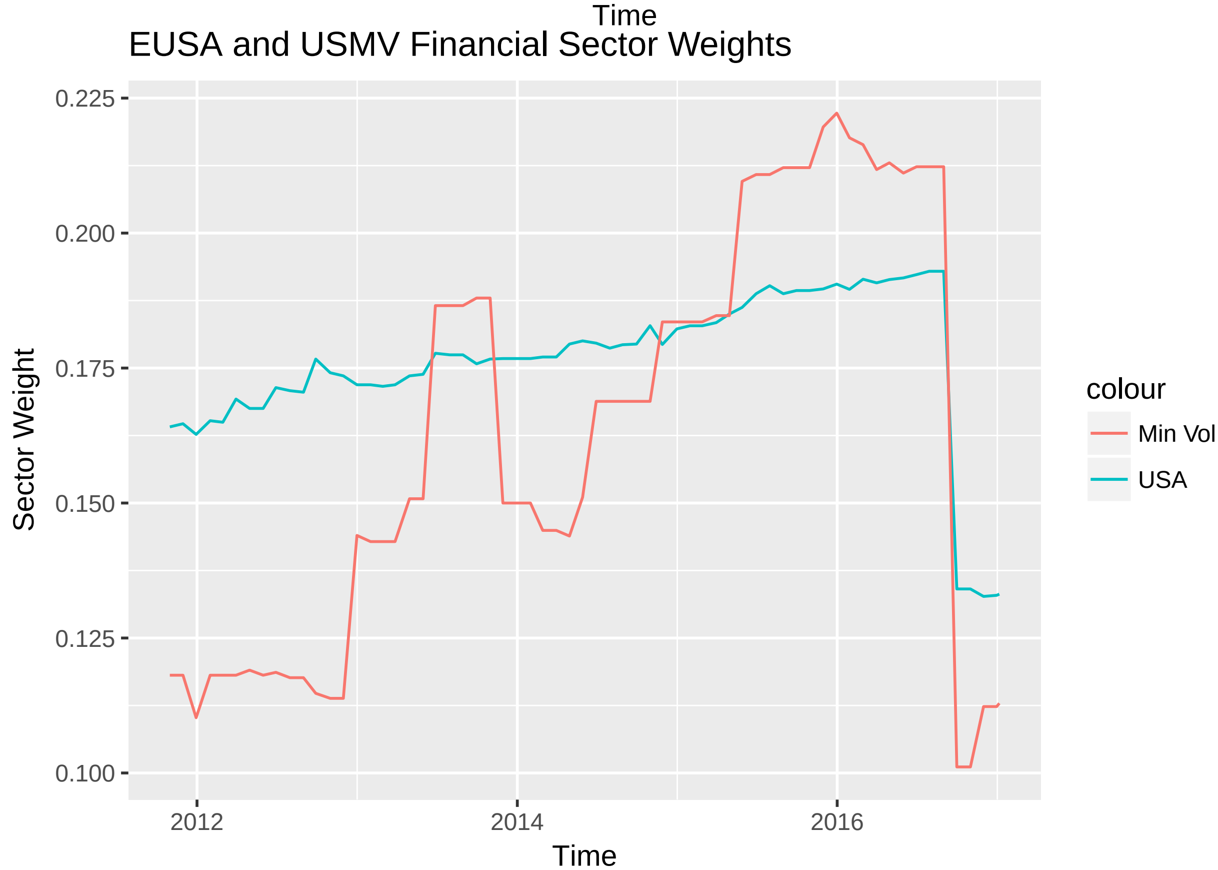
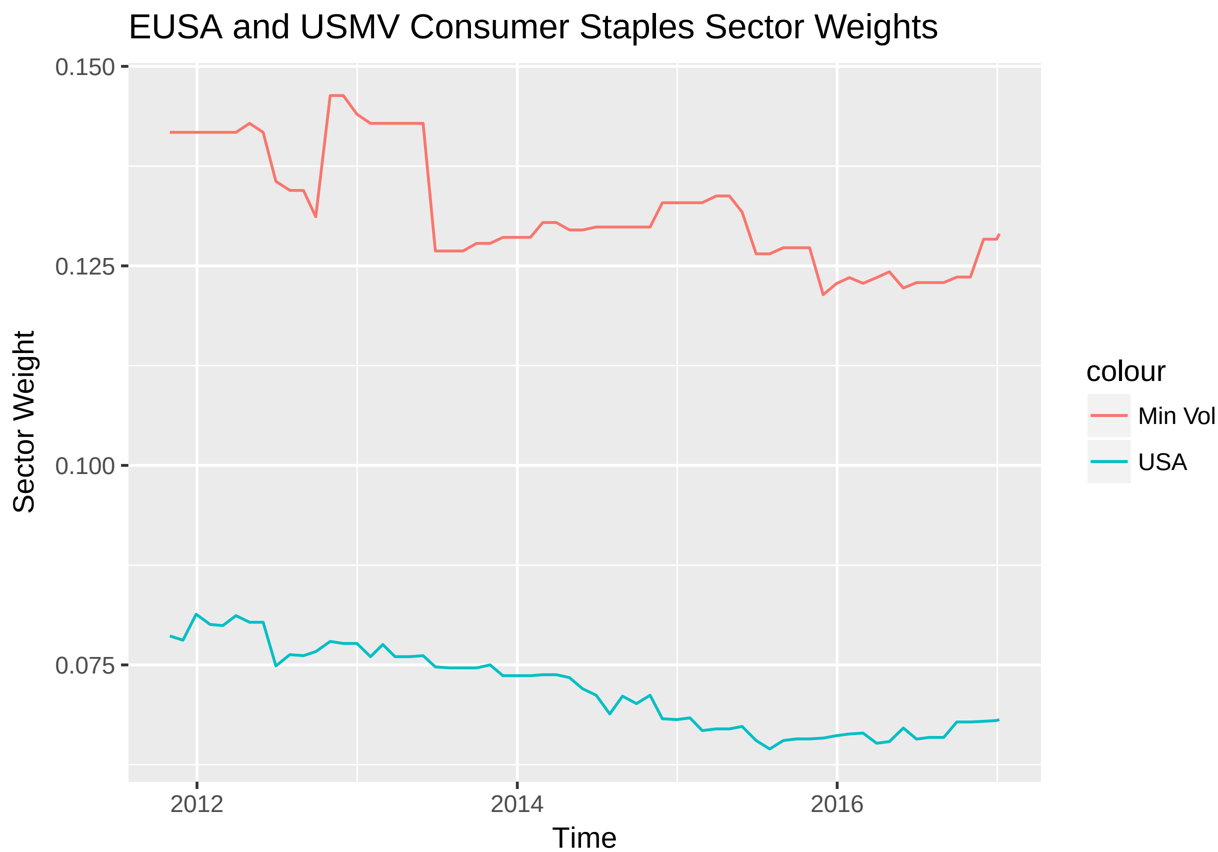
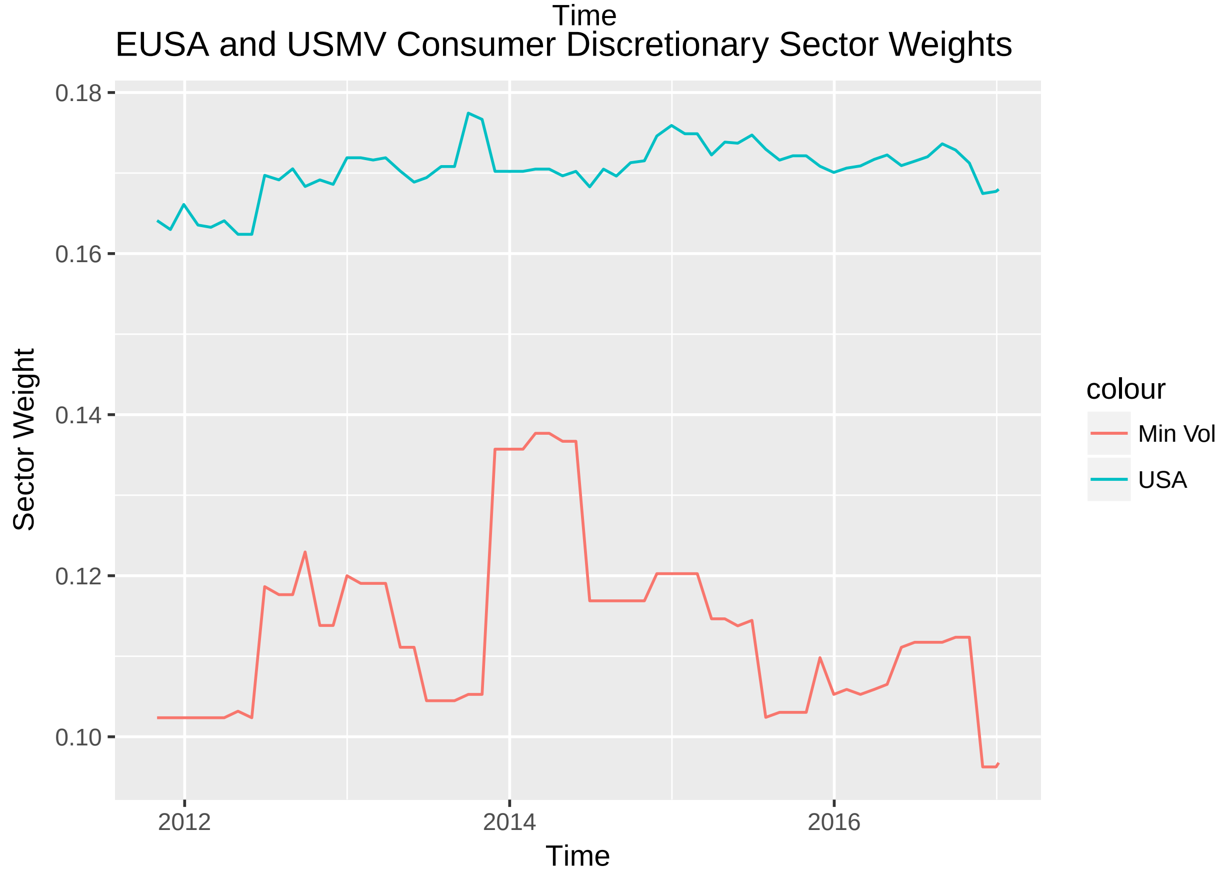


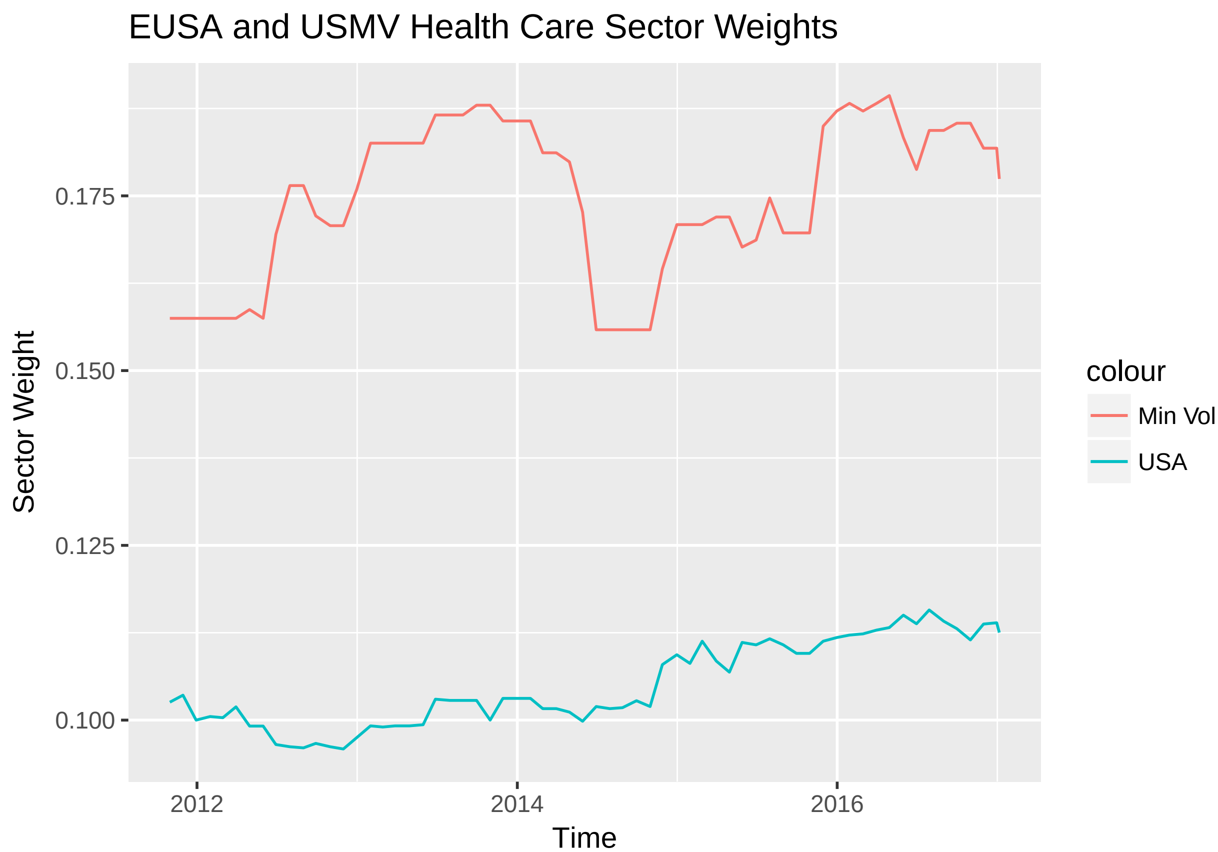
Figure 3.1: The 5 largest holdings of the Minimum Volatility Index are VZ, T, ADP, JNJ, and MCD. Since the index's inception, these stocks have generally remained between a weight of 1.3% and 1.7% of the overall portfolio, with the exception of Verizon, which reached around 2% in 2014.

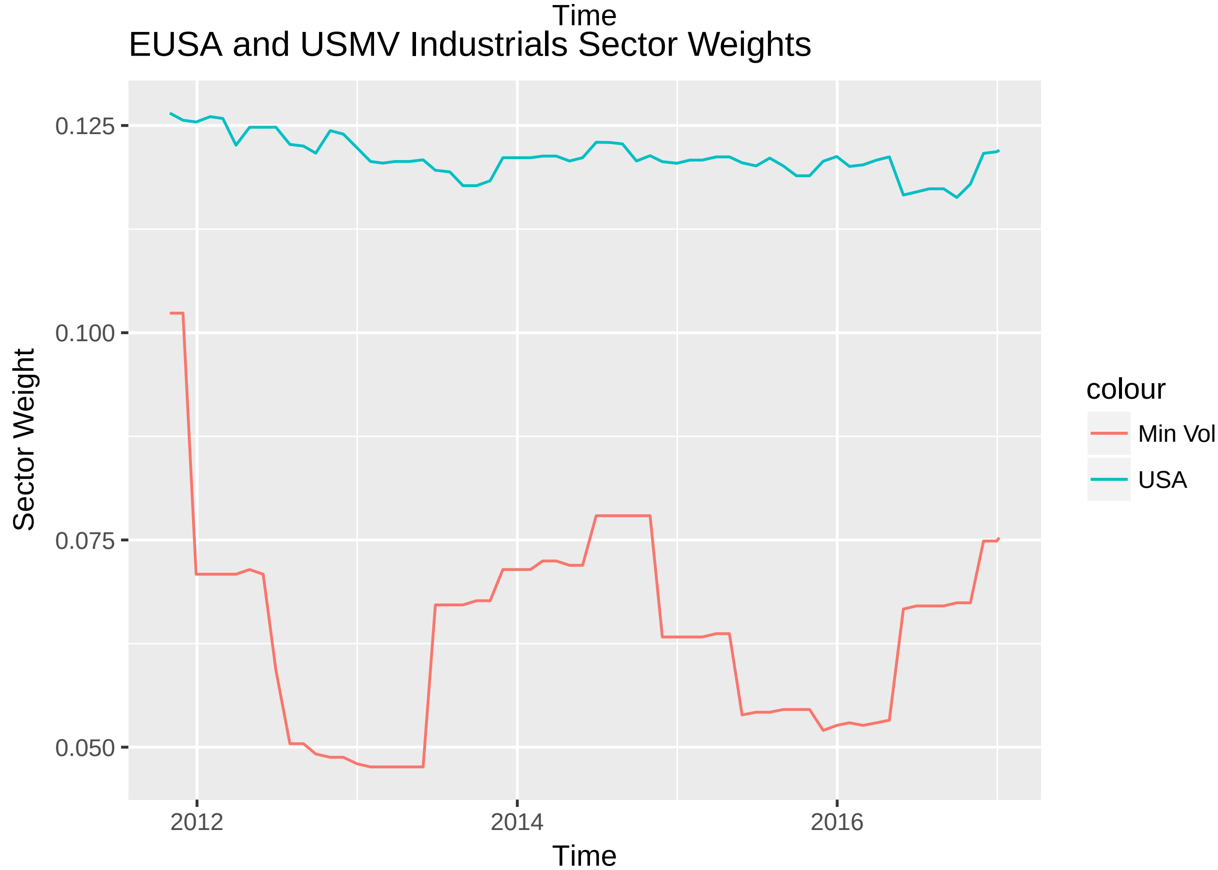

EUSA and USMV Energy Sector Weights
Sector Weight
0.08
0.06
0.04
0.02
2012
2014
2016
Time
colour
Min Vol
USA

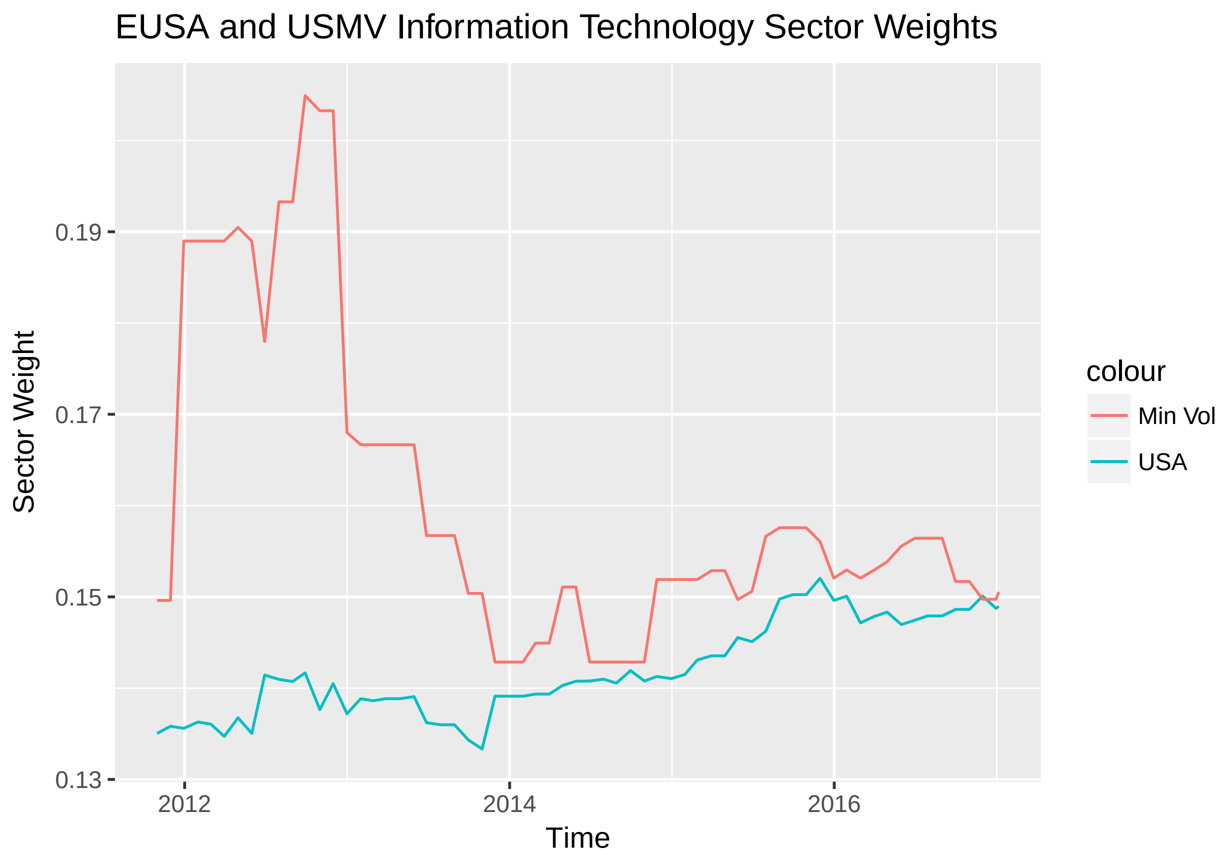

EUSA and USMV Financial Sector Weights
Sector Weight
0.225
0.200
0.175
0.150
0.125
0.100
2012
2014
2016
Time
colour
Min Vol
USA

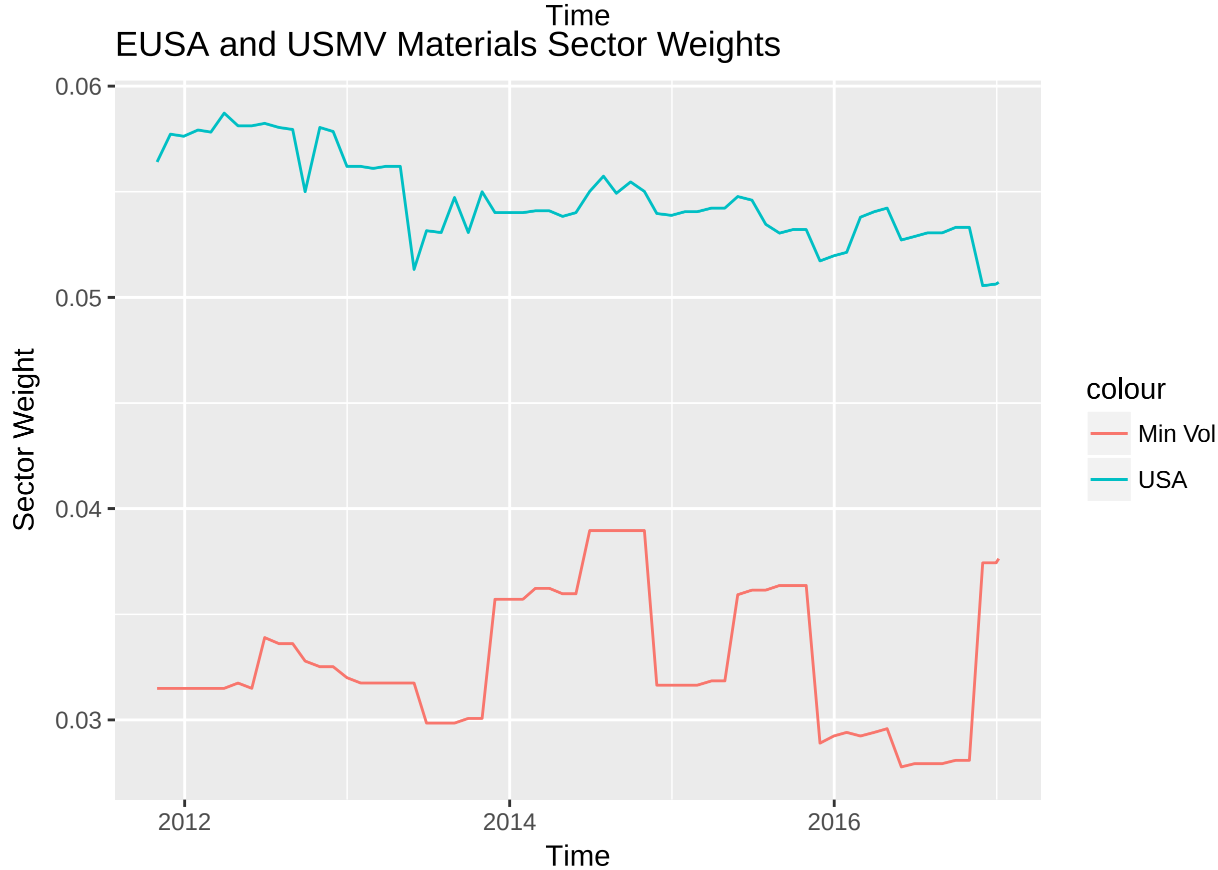

EUSA and USMV Consumer Staples Sector Weights
0.150
0.125
0.100
0.075
Sector Weight
2012
2014
2016
Time
colour
Min Vol
USA

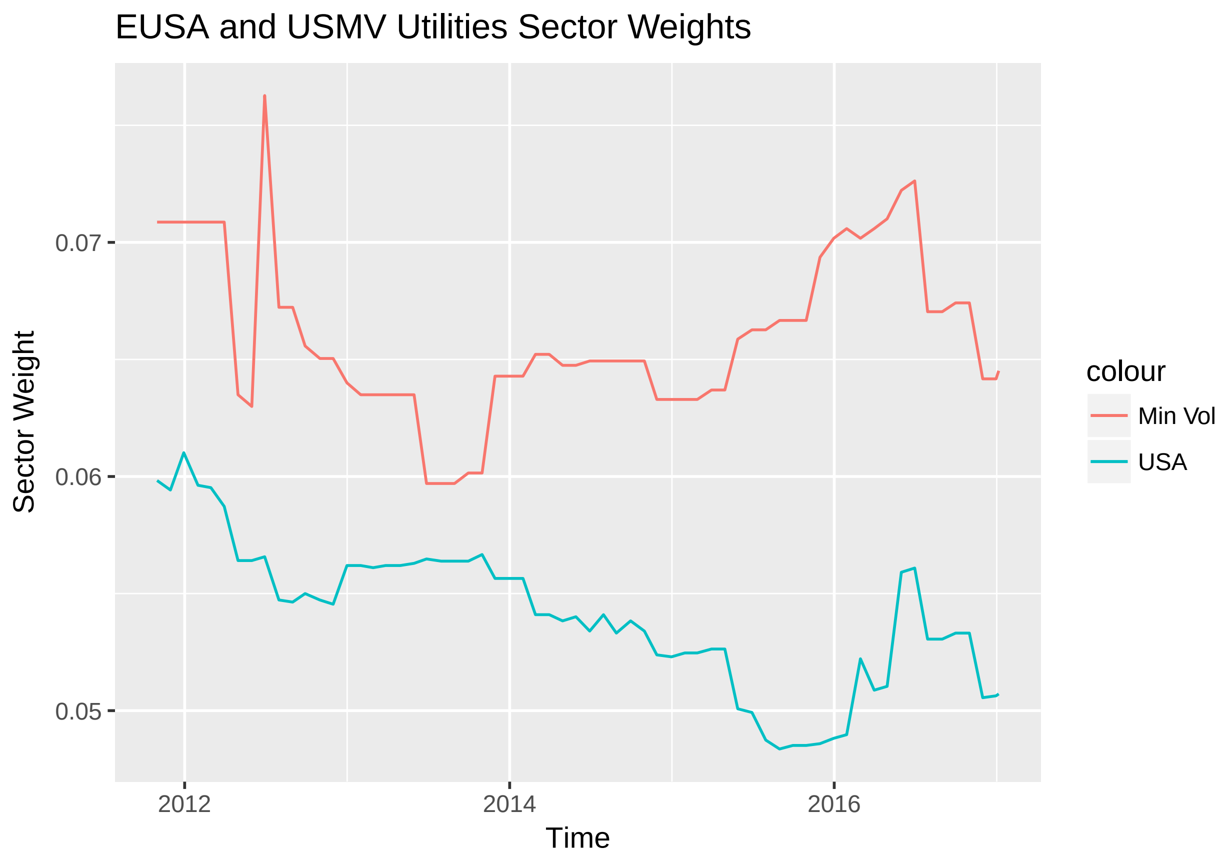

EUSA and USMV Consumer Discretionary Sector Weights
0.18
0.16
0.14
0.12
0.10
Sector Weight
2012
2014
2016
Time
colour
Min Vol
USA

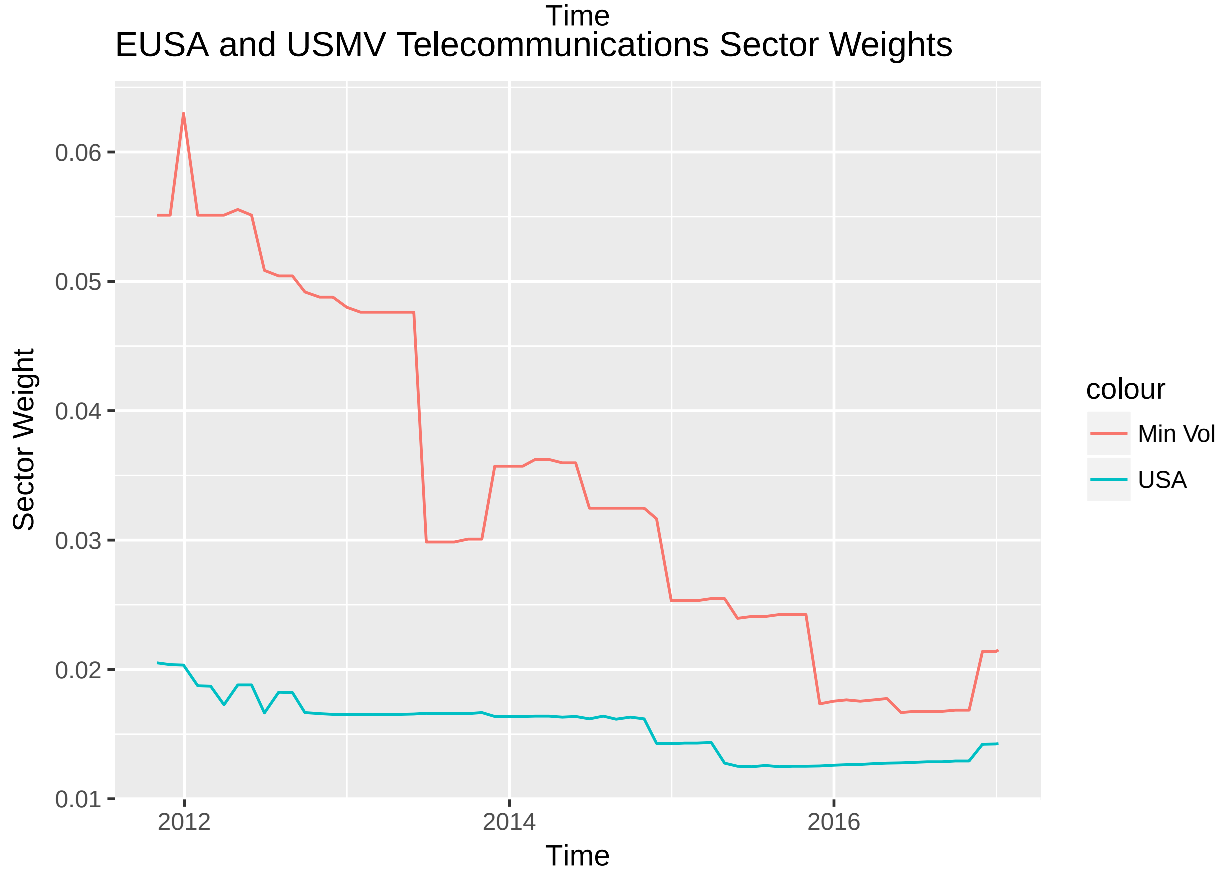

EUSA and USMV Health Care Sector Weights
Sector Weight
0.175
0.150
0.125
0.100
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Industrials Sector Weights
Sector Weight
0.125
0.100
0.075
0.050
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Information Technology Sector Weights
Sector Weight
0.19
0.17
0.15
0.13
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Materials Sector Weights
Sector Weight
0.06
0.05
0.04
0.03
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Utilities Sector Weights
Sector Weight
0.07
0.06
0.05
2012
2014
2016
Time
colour
Min Vol
USA


EUSA and USMV Telecommunications Sector Weights
Sector Weight
0.06
0.05
0.04
0.03
0.02
0.01
2012
2014
2016
Time
colour
Min Vol
USA

3.3 Trailing Volatilities

Data was collected from 10/31/2010 to 12/30/2016, from Wharton Research Data Services (WRDS) for the 908 historical constituents of the USA Equal Weight (EUSA) ETF, of which the Minimum Volatility Index is derived. Each tickers' 252-day (annual) trailing volatility was calculated, and a month end spagetti plot was produced for stocks in the Minimum Volatility Index, and the remainder of the stock comprising the US Equal Weight Index, to get a relative sense of each group's volatility attributes. The stocks comprising the Minimum Volatility Index generally had a lower volatility than those in the US Equal Weight Index.

```
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
0.00492  0.50030  0.87466  1.44738  1.56562 87.37030
```

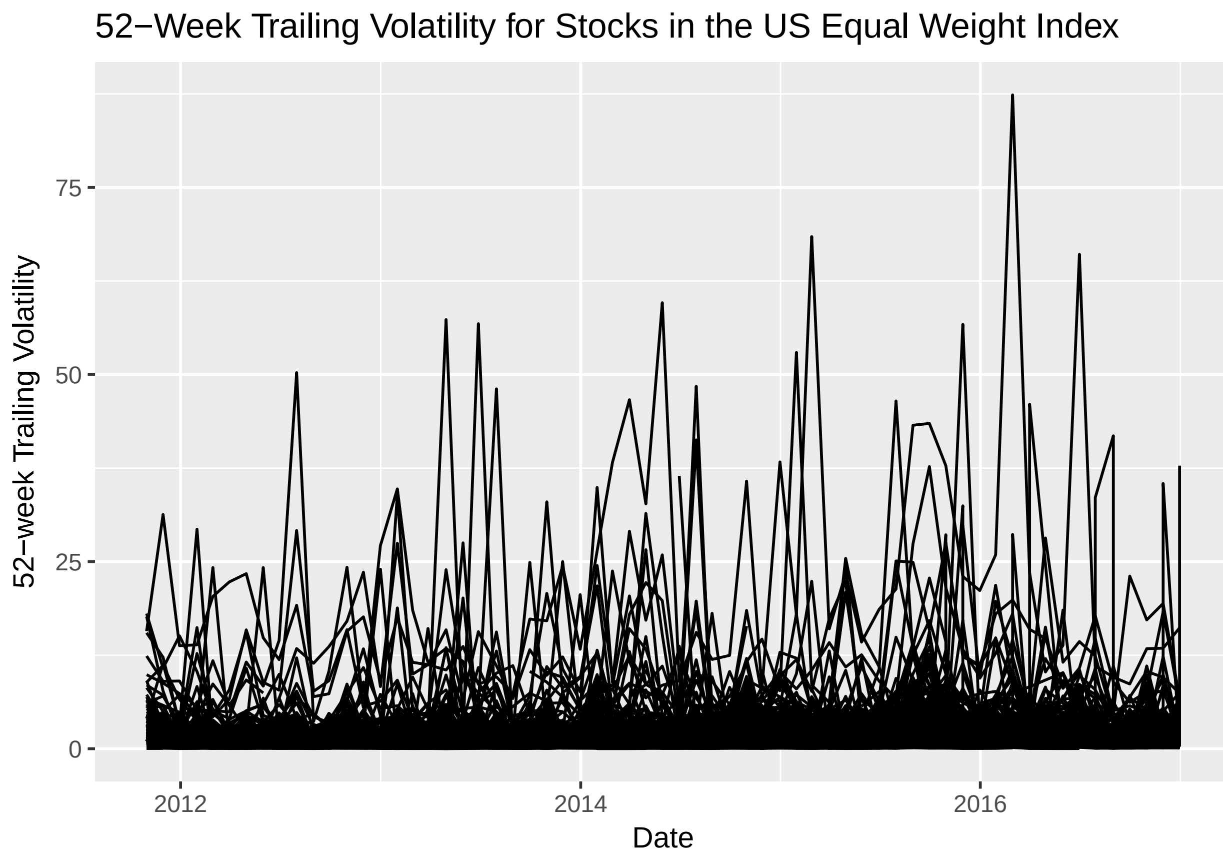


Figure 3.2: The 52-week trailing volatilities for stocks in the US Equal Weight Index ranged from 0.00492 to 87.37030, with a mean value of 1.44738.

```
   Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
0.03085  0.54767  0.91226  1.40784  1.51399 34.20974
```

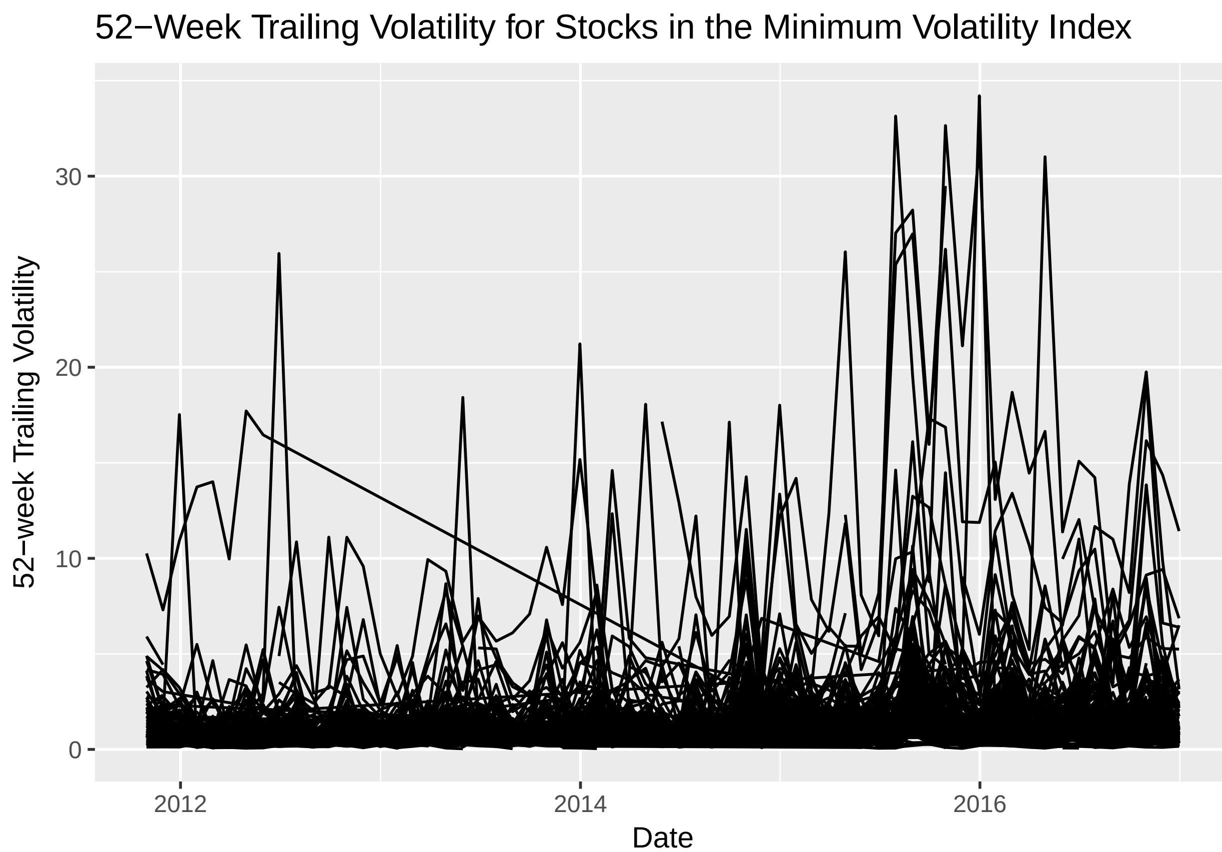


Figure 3.3: The 52-week trailing volatilities for stocks in the Minimum Volatility Index ranged from 0.03085 to 34.20974, with a mean value of 1.40784.

3.4 Trailing Betas

Data was collected from 10/31/2010 to 12/30/2016, from Wharton Research Data Services (WRDS) for the 908 historical constituents of the USA Equal Weight (EUSA) ETF, of which the Minimum Volatility Index is derived. Each tickers' 252-day (annual) trailing beta was calculated, and a month end spagetti plot was produced for stocks in the Minimum Volatility Index, and the remainder of the stock comprising the US Equal Weight Index, to get a relative sense of each group's volatility attributes. Generally, the stocks comprising the Minimum Volatility Index had a lower beta than those in the US Equal Weight Index.

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-4.9037  0.9316  1.1250  1.1505  1.3512  6.4952
```

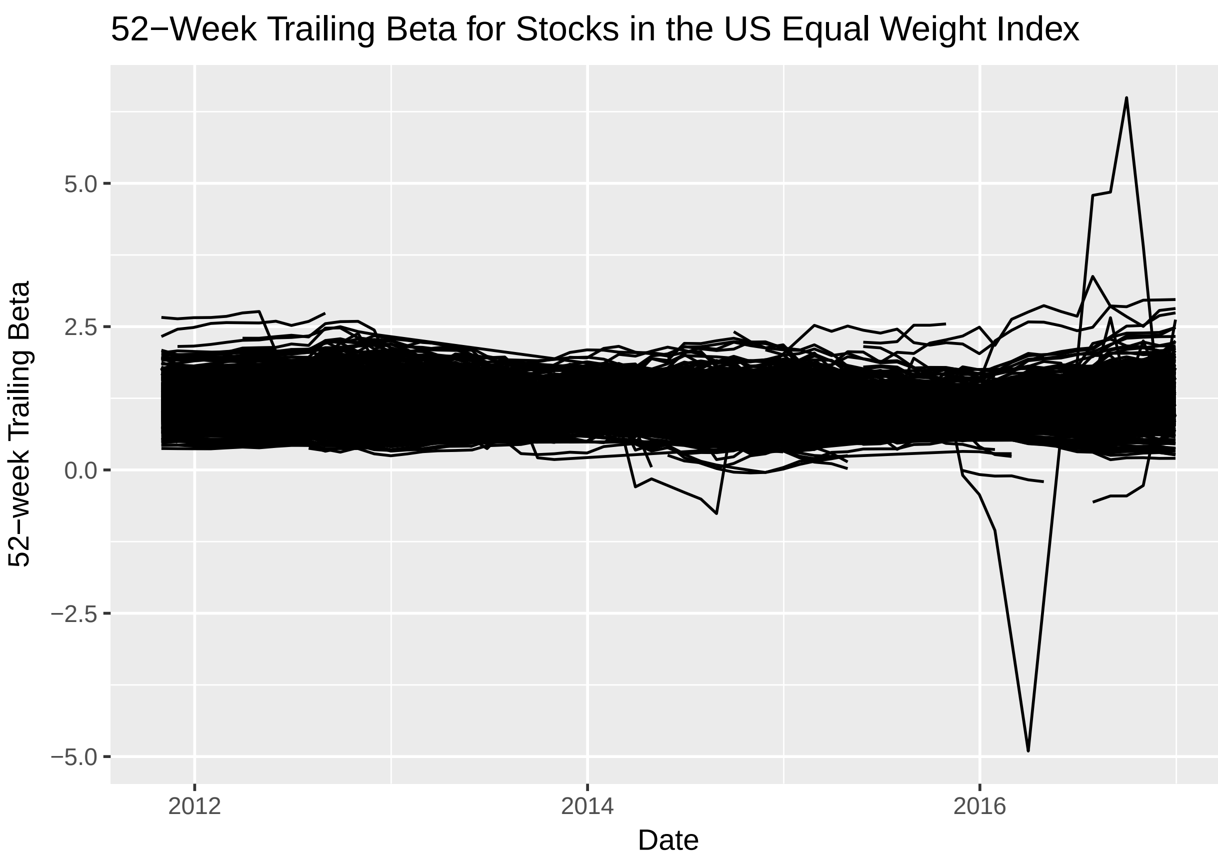


Figure 3.4: The 52-week trailing betas for stocks in the US Equal Weight Index ranged from -4.9037 to 6.4952, with a mean value of 1.1505

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.2473  0.6319  0.7912  0.7856  0.9330  3.2021
```

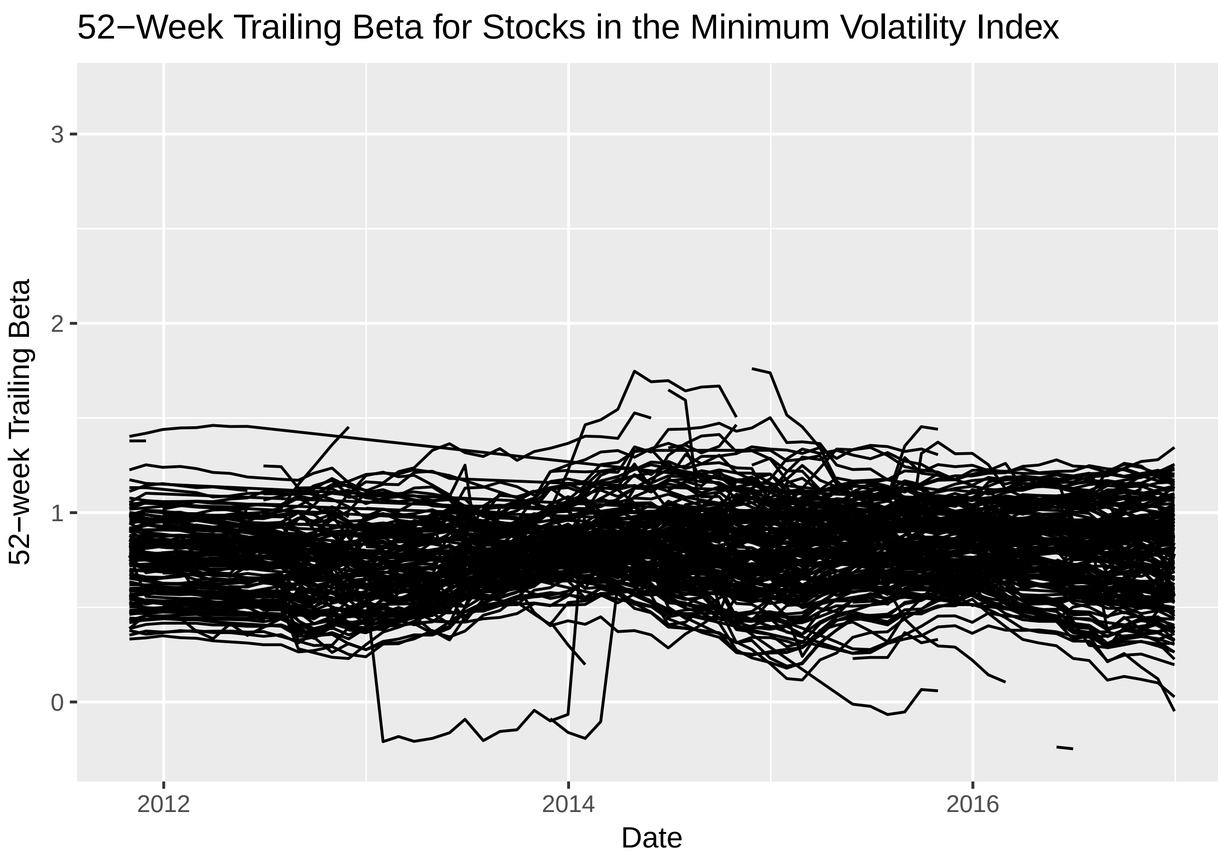


Figure 3.5: The 52-week trailing betas for stocks in the Minimum Volatility Index ranged from -0.2473 to 3.2021, with a mean value of 0.7856

3.5 Price to Book Ratios

Data was collected from 10/31/2010 to 12/30/2016, from Wharton Research Data Services (WRDS) for the 908 historical constituents of the USA Equal Weight (EUSA) ETF, of which the Minimum Volatility Index is derived. Each tickers' Price to Book ratio was calculated, and a month end spagetti plot was produced for stocks in the Minimum Volatility Index, and the remainder of the stock comprising the US Equal Weight Index, to get a relative sense of each group's volatility attributes. The stocks comprising the Minimum Volatility Index generally had a similar price to book ratio when compared those in the US Equal Weight Index.

```
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.2829  1.6559  2.6368  4.1138  4.3370 96.1338
```

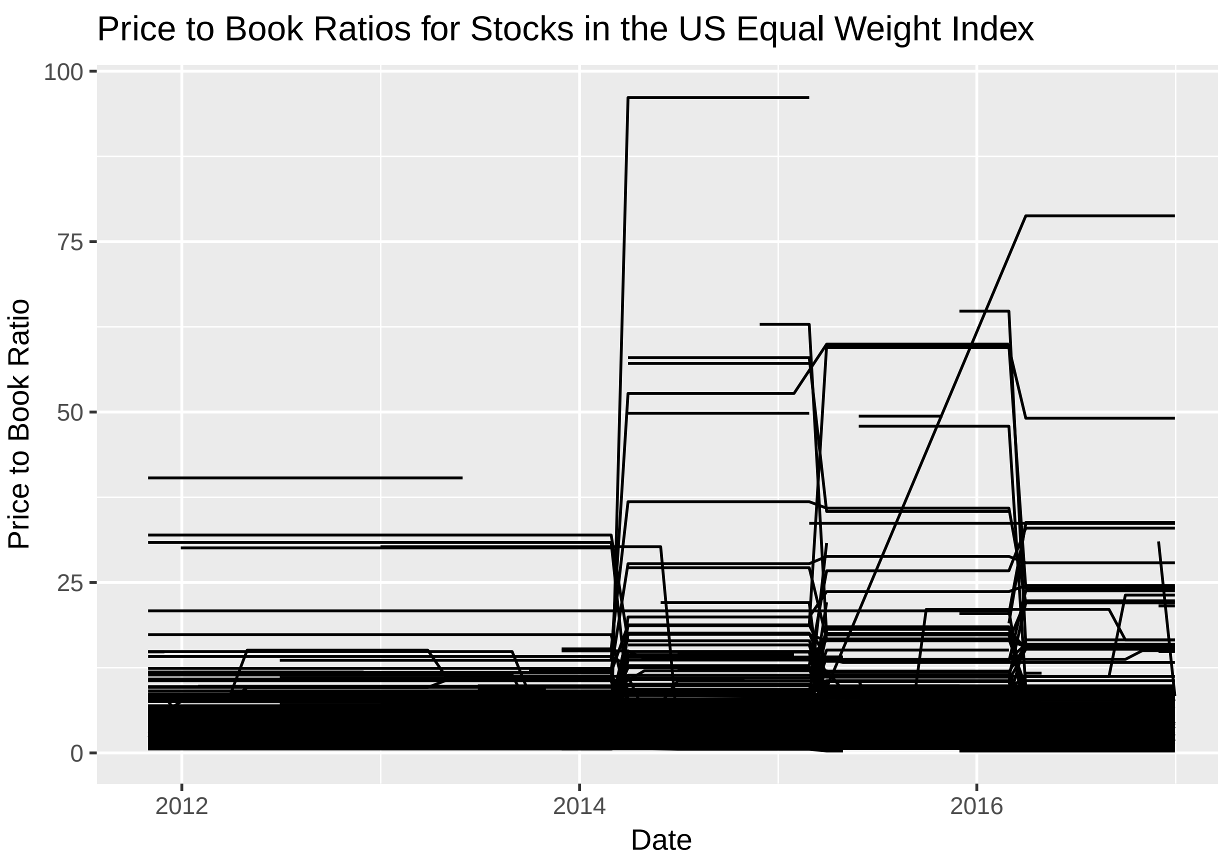


Figure 3.6: The price to book ratios for stocks in the US Equal Weight Index ranged from 0.2829 to 96.1338, with a mean value of 4.1138

```
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.2829  2.0196  3.2208  5.0907  5.5060 76.4406
```

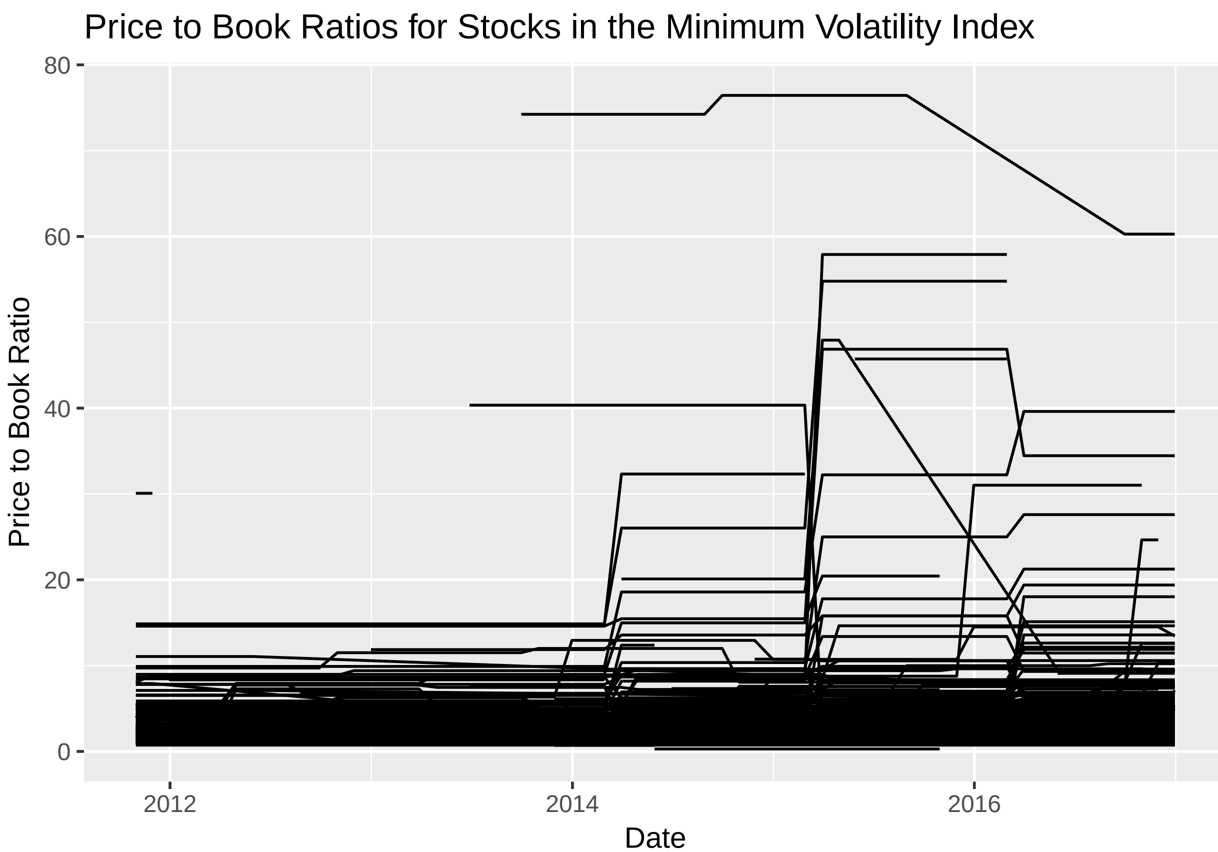


Figure 3.7: The price to book ratios for stocks in the Minimum Volatility Index ranged from 0.2829 to 76.4406, with a mean value of 5.0907

Chapter 4

Model

Once the final data set was created and cleaned, with a number of response variables including trailing beta, trailing volatility, price to book ratio, and the whether or not a stock was in the Minimum Volatility index or not (1 if in, 0 if not in). Given the nature of the data, a logistic regression was run. Looking at all of the historical data and stock various characteristics, this modeled the log odds of a stock being in the minimum volatility index as a combination of the linear predictors mentioned. Several models were ran, including one by certain months, and one by the entire pool of data, and the best one was chosen to test in further detail.

4.1 Model 1: May Model

Since the index is rebalanced twice a year (once in November and once in May), it makes sense to look at a model for each of these individual months. Since the dataset to test this model will be from May 2017, a subset of the data was taken for May.

```
# Subset data for dates from May only
may_final <- filter(monthly_data, date == "2012-05-31" |  date == "2013-05-31"| da
# Remove NA values from set
may_final <- subset(may_final, !is.na(index_before))
```

4.1.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the Min Vol index should approximately be the same. However, after checking this, it is clear that this is not the case, as, just around 24% of the data is from stocks that are currently in the index, meaning there is a class bias. As a result, observations must be sampled in approximately equal proportions to get a better model.

```
table(may_final$index_now)
```

```
  0    1
```

```
2105  670
```

4.1.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's in equal proportions, and using that development sample for the model. The unsampled data will be kept as validation sample to test the model later on. As a result, the size of development sample will be smaller that validation sample, which is not a problem, since there are large number of observations.

There is no more class bias, as the sample is now evenly weighted now, with each outcome being represented by 468 observations.

```r
table(trainingData1$index_now)
```

```
  0   1 
468 468 
```

4.1.3 Logistic Regression Model

```r
# Model 1
logit1 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=traini

# Summary of Model 1
summary(logit1)
```

```
Call:
glm(formula = index_now ~ volatility + beta + price_to_book + 
    index_before, family = binomial(link = "logit"), data = trainingData1)

Deviance Residuals: 
     Min        1Q    Median        3Q       Max  
-2.97967  -0.24913   0.00044   0.16385   2.64827  

Coefficients:
               Estimate Std. Error z value Pr(>|z|)    
(Intercept)    1.388773   0.590426   2.352   0.0187 *  
volatility    -0.055224   0.096908  -0.570   0.5688    
beta          -4.377319   0.675066  -6.484 8.92e-11 ***
price_to_book  0.002863   0.025520   0.112   0.9107    
index_before1  6.525377   0.427180  15.275  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1297.57  on 935  degrees of freedom
Residual deviance:  262.65  on 931  degrees of freedom
AIC: 272.65

Number of Fisher Scoring iterations: 7
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit1))
```

```
 (Intercept)    volatility          beta price_to_book index_before1
  4.00992632    0.94627359    0.01255898    1.00286711  682.23660573
```

```
## Probability
(exp(coef(logit1))) / (1+(exp(coef(logit1))))
```

```
 (Intercept)    volatility          beta price_to_book index_before1
  0.80039627    0.48619762    0.01240321    0.50071575    0.99853638
```

Looking at the May model will be helpful for someone looking to predict index rebalancing between December and April.

4.1.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.74 - 0.04 x vol - 0.64 x beta - 0.012 x price_to_book + 7.014 x index_before
$\frac{p}{1-p}$ = exp(-1.74 - 0.04 x vol - 0.64 x beta - 0.012 x price_to_book + 7.014 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 0.96 times smaller, given a one unit increase in volatility. This response variable is not statistically significant.
- Beta: The odds ratio of being added to the index is 0.52 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 0.99 times smaller, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 1112.01 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

4.1.5 Model Quality

To test the quality of the model, several tests were done:
Predictive Power
The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.85.

```
library(InformationValue)
predicted1 <- plogis(predict(logit1, testData1))
optCutOff1 <- optimalCutoff(testData1$index_now, predicted1)[1]
```

*VIF***__
Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
```

```
Attaching package: 'car'

The following object is masked from 'package:dplyr':

    recode
```

```
vif(logit1)
```

```
  volatility          beta price_to_book  index_before
    1.028972      1.146407      1.027968      1.150409
```

Misclassification Error
Misclassification error is the percentage mismatch of predicted vs actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 2.3%, which is quite low, and good.

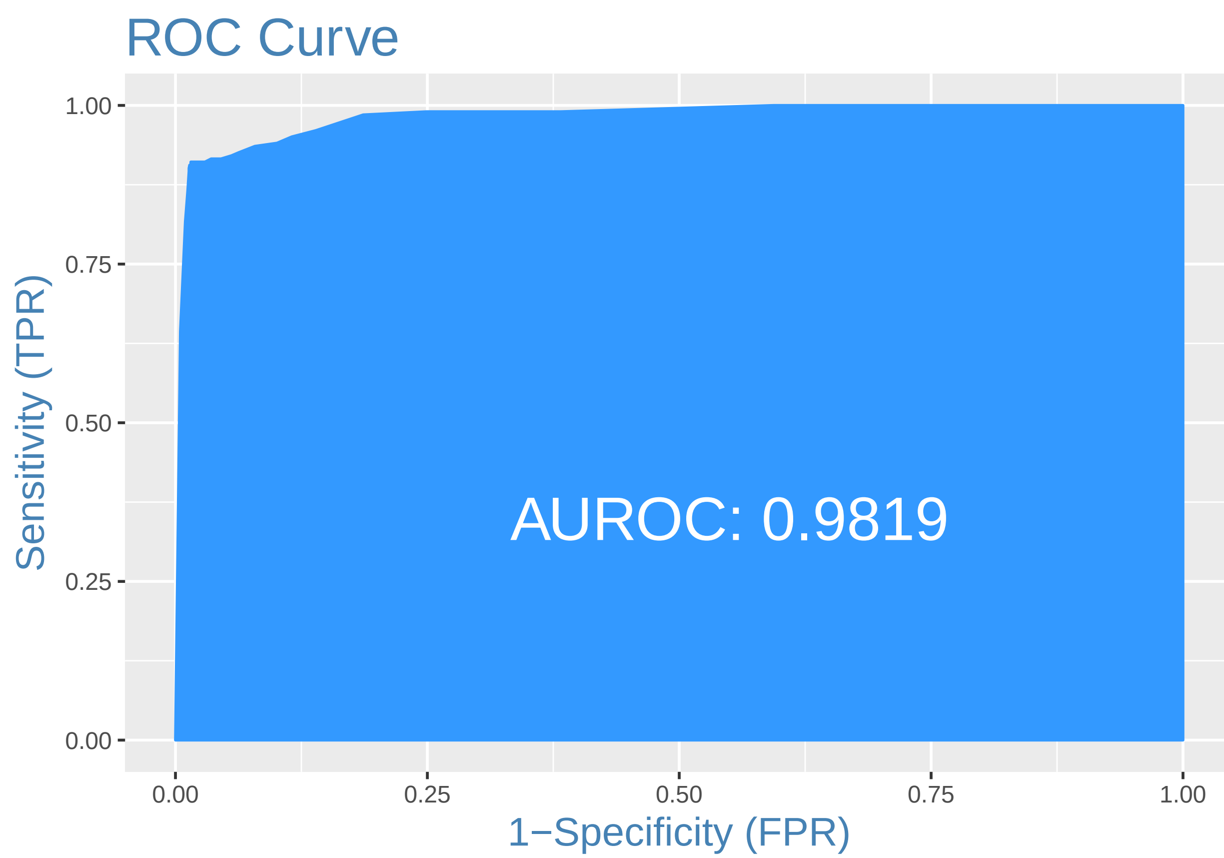
```
predicted1 <- plogis(predict(logit1, testData1))
misClassError(testData1$index_now, predicted1)
```

```
[1] 0.0234
```

ROC

Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 98.2%.

```
plotROC(testData1$index_now, predicted1)
```



Concordance

Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 98.2% is a good quality model.

```
Concordance(testData1$index_now, predicted1)
```

```
$Concordance
[1] 0.9827534

$Discordance
[1] 0.01724659

$Tied
[1] 4.163336e-17

$Pairs
[1] 330674
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 90.5%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 98.6%.

```
sensitivity(testData1$index_now, predicted1, threshold = optCutOff1)
```

```
[1] 0.9059406
```

```
specificity(testData1$index_now, predicted1, threshold = optCutOff1)
```

```
[1] 0.9859499
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData1$index_now, predicted1, threshold = optCutOff1)
```

```
     0   1
0 1614  19
1   23 183
```

4.2 Model 2: Total Rebalancing (November & May) Model

Since the index is rebalanced twice a year (once in November and once in May), it makes sense to look at a model for both of these months. Thus, a subset of the data was taken for May and November, by combining the data sets from Model 2 and Model 3.

```
# Subset data for dates from May and November
november_final <- filter(monthly_data, date == "2011-11-30" | date == "2012-11-30"
# Remove NA values from set
november_final <- subset(november_final, !is.na(index_before))

both_final <- rbind(may_final, november_final)
```

4.2.1 Data Cleaning - Checking for Class Bias

Ideally, the proportion of stocks in and out of the Min Vol index should approximately be the same. However, after checking this, it is clear that this is not the case, as, just around 25% of the data is from stocks that are currently in the index, meaning there is a class bias. As a result, observations must be sampled in approximately equal proportions to get a better model.

```
table(both_final$index_now)
```

```
   0    1
4182 1385
```

4.2.2 Create Training and Test Samples

One way to address the problem of class bias is to draw the 0's and 1's for the trainingData (development sample) in equal proportions. In doing so, we will put rest of the inputData not included for training into testData (validation sample). As a result, the size of development sample will be smaller that validation, which is okay, because, there are large number of observations.

```
# Create Training Data
input_ones2 <- both_final[which(both_final$index_now == 1), ]
input_zeros2 <- both_final[which(both_final$index_now == 0), ]
set.seed(100)  # for repeatability of samples
input_ones_training_rows2 <- sample(1:nrow(input_ones2), 0.7*nrow(input_ones2))
input_zeros_training_rows2 <- sample(1:nrow(input_zeros2), 0.7*nrow(input_ones2))
training_ones2 <- input_ones2[input_ones_training_rows2, ]
```

```r
training_zeros2 <- input_zeros2[input_zeros_training_rows2, ]
trainingData2 <- rbind(training_ones2, training_zeros2)
# Create Test Data
test_ones2 <- input_ones2[-input_ones_training_rows2, ]
test_zeros2 <- input_zeros2[-input_zeros_training_rows2, ]
testData2 <- rbind(test_ones2, test_zeros2)
```

There is no more class bias, as the sample is now evenly weighted now, with each outcome being represented by 969 observations.

```r
table(trainingData2$index_now)
```

```
  0   1
969 969
```

4.2.3 Logistic Regression Model

```r
# Model 2
logit2 <- glm(index_now ~  volatility + beta + price_to_book + index_before, data=traini

# Summary of Model 2
summary(logit2)
```

```
Call:
glm(formula = index_now ~ volatility + beta + price_to_book +
    index_before, family = binomial(link = "logit"), data = trainingData2)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.4421  -0.3205   0.0156   0.2131   3.3425

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)    1.858e+00  3.623e-01   5.128 2.93e-07 ***
volatility    -3.976e-02  4.072e-02  -0.977    0.329
beta          -4.193e+00  3.933e-01 -10.661  < 2e-16 ***
price_to_book  5.059e-05  1.775e-02   0.003    0.998
index_before1  5.432e+00  2.382e-01  22.802  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2686.64  on 1937  degrees of freedom
Residual deviance:  745.23  on 1933  degrees of freedom
AIC: 755.23

Number of Fisher Scoring iterations: 6
```

```
# Coefficient Interpretation
## Log Odds
exp(coef(logit2))
```

```
(Intercept)    volatility          beta price_to_book index_before1
 6.41055627    0.96101526    0.01510306    1.00005059  228.61385918
```

```
## Probability
(exp(coef(logit2))) / (1+(exp(coef(logit2))))
```

```
(Intercept)    volatility          beta price_to_book index_before1
 0.86505736    0.49006006    0.01487835    0.50001265    0.99564486
```

Looking at this model will be helpful for someone looking to predict index rebalancing, generally, for both months.

4.2.4 Interpretation of Model

The model can be interpreted as:
$\ln[\frac{p}{1-p}]$ = -1.84 + 0.003 x vol - 0.31 x beta - 0.0019 x price_to_book + 5.89 x index_before
$\frac{p}{1-p}$ = exp(-1.84 + 0.003 x vol - 0.31 x beta - 0.0019 x price_to_book + 5.89 x index_before)
The coefficients can be interpreted as:
- Volatility: The odds ratio of being added to the index is 1.0029 times greater, given a one unit increase in volatility. This response variable is not statistically significant.
- Beta: The odds ratio of being added to the index is 0.73 times smaller, given a one unit increase in beta. This response variable is statistically significant.
- Price to Book: The odds ratio of being added to the index is 0.99 times smaller, given a one unit increase in price to book ratio. This response variable is not statistically significant.
- Index before: The odds ratio of being added to the index is 360.28 times greater if the stock was in the index 6 months ago. This response variable is statistically significant.

4.2.5 Sanity Check

Will do later if useful.

4.2.6 Model Quality

To test the quality of the model, several tests were done:
Predictive Power
The default cutoff prediction probability score is 0.5 or the ratio of 1's and 0's in the training data. But sometimes, tuning the probability cutoff can improve the accuracy in both the development and validation samples. The InformationValue::optimalCutoff function provides ways to find the optimal cutoff to improve the prediction of 1's, 0's, both 1's and 0's and to reduce the misclassification error. Here, the optimal cut off is 0.86.

```
library(InformationValue)
predicted2 <- plogis(predict(logit2, testData2))
optCutOff2 <- optimalCutoff(testData2$index_now, predicted2)[1]
```

*VIF**_*
Like in case of linear regression, we should check for multicollinearity in the model. As seen below, all X variables in the model have VIF well below 4.

```
library(car)
vif(logit2)
```

```
 volatility           beta price_to_book  index_before
   1.018738       1.053972      1.020580      1.065880
```

Misclassification Error
Misclassification error is the percentage mismatch of predicted vs. actuals, irrespective of 1's or 0's. The lower the misclassification error, the better the model. Here it is 4.0%, which is quite low, and good.

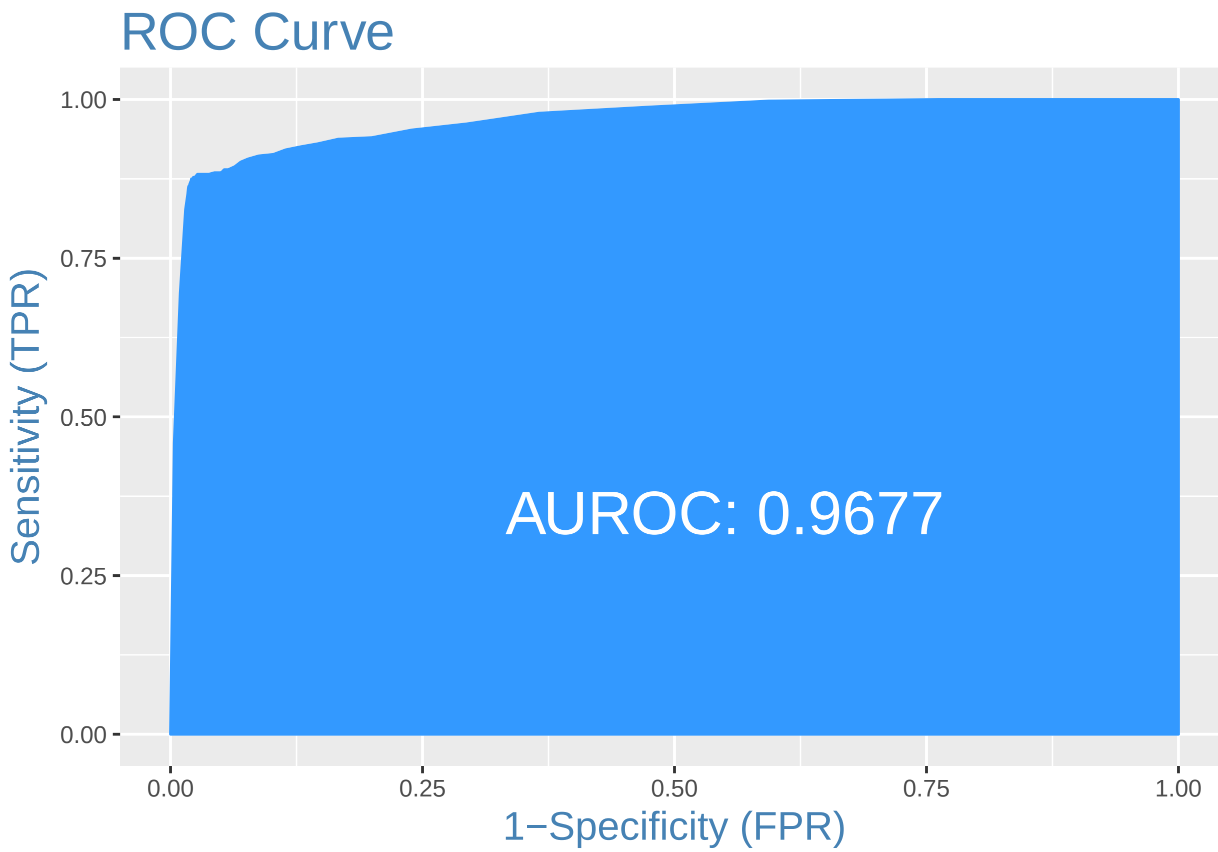
```
predicted2 <- plogis(predict(logit2, testData2))
misClassError(testData2$index_now, predicted2)
```

```
[1] 0.0402
```

ROC
Receiver Operating Characteristics Curve traces the percentage of true positives accurately predicted by a given logit model as the prediction probability cutoff is

lowered from 1 to 0. For a good model, as the cutoff is lowered, it should mark more of actual 1's as positives and lesser of actual 0's as 1's. So for a good model, the curve should rise steeply, indicating that the TPR (Y-Axis) increases faster than the FPR (X-Axis) as the cutoff score decreases. Greater the area under the ROC curve, better the predictive ability of the model. Here, it is 96.8%.

```
plotROC(testData2$index_now, predicted2)
```



Concordance
Ideally, the model-calculated-probability-scores of all actual Positive's, (aka Ones) should be greater than the model-calculated-probability-scores of ALL the Negatives (aka Zeroes). Such a model is said to be perfectly concordant and a highly reliable one. This phenomenon can be measured by Concordance and Discordance.

In simpler words, of all combinations of 1-0 pairs (actuals), Concordance is the percentage of pairs, whose scores of actual positive's are greater than the scores of actual negative's. For a perfect model, this will be 100%. So, the higher the concordance, the better is the quality of model. This model with a concordance of 96.8% is a good quality model.

```
Concordance(testData2$index_now, predicted2)
```

```
$Concordance
[1] 0.968147
```

```
$Discordance
[1] 0.03185302

$Tied
[1] 3.469447e-17

$Pairs
[1] 1336608
```

Specificity and Sensitivity
- Sensitivity (or True Positive Rate) is the percentage of 1's (actuals) correctly predicted by the model, while, specificity is the percentage of 0's (actuals) correctly predicted. In this model, it was found to be 86.3%.
- Specificity can also be calculated as 1 - False Positive Rate. In this model, it was found to be 98.2%.

```
sensitivity(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
[1] 0.8629808
```

```
specificity(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
[1] 0.9819483
```

Confusion Matrix
In the confusion matrix, the columns are actuals, while rows are predicteds

```
confusionMatrix(testData2$index_now, predicted2, threshold = optCutOff2)
```

```
     0   1
0 3155  57
1   58 359
```

Chapter 5

Conclusion

All in all, the models were comprable in terms of statistical resiliency and predictive power. Each model may have a different usage, based of each model's strengths and weaknesses, and what goals the investor has in mind for the model. For example, an investor looking to capture an arbitrage opportunity in November, might be best suited in looking at the November specific model. Someone looking for abritrage opportunities throughout the year during both rebalances might look at the combined May and November model.

5.1 Side by Side Model Comparison

```
Warning: package 'tidyr' was built under R version 3.4.2

Warning: 'rbind_list' is deprecated.
Use 'bind_rows()' instead.
See help("Deprecated")

`mutate_each()` is deprecated.
Use `mutate_all()`, `mutate_at()` or `mutate_if()` instead.
To map `funs` over a selection of variables, use `mutate_at()`

# A tibble: 10 x 4
            term       key   `1`   `2`
 *         <chr>     <chr> <dbl> <dbl>
 1   (Intercept)  estimate  1.39  1.86
 2   (Intercept) std.error  0.59  0.36
 3          beta  estimate -4.38 -4.19
 4          beta std.error  0.68  0.39
 5 index_before1  estimate  6.53  5.43
 6 index_before1 std.error  0.43  0.24
 7 price_to_book  estimate  0.00  0.00
 8 price_to_book std.error  0.03  0.02
 9    volatility  estimate -0.06 -0.04
10    volatility std.error  0.10  0.04
```

As seen, each model gave out pretty similar coefficient values for the various response variables. Beta ranged between -0.31 and -0.64, index_before ranged from 5.08 to 7.01, price to book ranged from 0.00 to -0.01, and volatility ranged between -0.04 and 0.06.

Chapter 6

Discussion

After comparing all of the models, it makes sense to discuss the applications of these various models to the real world, and to finance.

6.1 Understanding of Relationships

Through these models, we can get a better understanding of the relationships between the predictor variables, and whether or not the stock is in the Min Vol index. In general, each model suggested an increase in beta will reduce the likelihood of a stock being in the min vol index, with all else held constant. This makes sense, as beta is one measure of risk and volatility. Moreover, it is a widely used metric in finance, so it is not surpising that it is a statistically significant variable. Moreover, the most significant variable was whether or not the stock was in the index before. This makes a lot of sense, as a stock currently in the index presumably has many min vol characteristics from before, that must be signficantly altered if it were to be removed. Moreover, stocks that were in the index previously were many times more likely to be in the index currently, than stocks that had previously not been in the index. This variable was also statistically significant. Surpisingly, volatility was not statistically signficant, though the index itself is called the "Minimum Volatility" Index. Moreover, price to book was also an insignficant variable, which does make sense. Each model was able to quantify these relationships, and help us better understand what

6.2 Arbitrage

Each model was able to take various attributes of a stock, and calculate a probability for it currently being in the index. Using the optimal cutoffs, we were able to get a sense of the probability value that would be significant in determining when a stock would be in or out of the index. For example, at a cutoff of 0.9, this would tell us that we could reasonably expect stocks with a probability of over 90% to be in the index, and stocks with less than a 90% probability to not be in the index. With this information, there are many different arbitrage opportunities. One could long stocks currently not in the index that have a probability greater than the optimal cutoff for that model. This would represent the stocks with the greatest chance of being added to the index, that are currently not in the index. If correct, prior studies would suggest that the stock price would consequently increase from this happening. Moreover, one could short stocks that are currently in the index, that have a probability value less than the cutoff. This could lead to an arbitrage opportunity if the stock is removed from the index, as one is short it.

Appendix A

The First Appendix

This first appendix includes all of the R chunks of code that were hidden throughout the document (using the `include = FALSE` chunk tag) to help with readibility and/or setup.

In the main Rmd file

In Chapter ??:

```
# This chunk ensures that the thesisdown package is
# installed and loaded. This thesisdown package includes
# the template files for the thesis and also two functions
# used for labeling and referencing
if(!require(devtools))
  install.packages("devtools", repos = "http://cran.rstudio.com")
if(!require(dplyr))
    install.packages("dplyr", repos = "http://cran.rstudio.com")
if(!require(ggplot2))
    install.packages("ggplot2", repos = "http://cran.rstudio.com")
if(!require(ggplot2))
    install.packages("bookdown", repos = "http://cran.rstudio.com")
if(!require(thesisdown)){
  library(devtools)
  devtools::install_github("ismayc/thesisdown")
  }
library(thesisdown)
flights <- read.csv("data/flights.csv")
```

Appendix B

The Second Appendix, for Fun

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