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Steinmann Relations and the Two-Loop MHV Amplitude in Eight-Particle Kinematics

John Golden,[1,2] **Andrew J. McLeod**[2,3,4]

[1] *Michigan Center for Theoretical Physics and Randall Laboratory of Physics, Department of Physics, University of Michigan Ann Arbor, MI 48109, USA*

[2] *Kavli Institute for Theoretical Physics, UC Santa Barbara, Santa Barbara, CA 93106, USA*

[3] *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

[4] *Niels Bohr International Academy, Blegdamsvej 17, 2100 Copenhagen, Denmark*

Abstract: We present the full functional form of the two-loop eight-point MHV amplitude in the planar limit of maximally supersymmetric Yang-Mills theory, in terms of cluster polylogarithms. We also compute the two BDS-like ansätze that can be formulated in eight-particle kinematics, and find that ...

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2 Promoting $R_8^{(2)}$ from Symbol to Function

Briefly describe the method outlined in [1] for upgrading the n-point two-loop MHV symbol to a function.

3 The Functions $R_8^{(2)}$, ${}^3\mathcal{E}_8^{(2)}$, and ${}^4\mathcal{E}_8^{(2)}$

When the number of gluons n is not a multiple of 4, the BDS-like ansatz is unique because there exists only a single decomposition

$$A_n^{\text{BDS}} = A_n^{\text{BDS-like}}(\{s_{ij}\}) + Y_n(\{u_i\}), \quad n \neq 4K, \tag{3.1}$$

such that the kinematic dependence of $A_n^{\text{BDS-like}}$ involves only two-particle Mandelstam invariants and Y_n is a function of dual conformal invariant cross ratios [2]. However, when n is a multiple of 4, no decomposition of this type exists, and we are forced to consider multiple BDS-like ansätze in order to expose all Steinmann relations between higher-particle Mandelstam invariants. However, in eight particle kinematics (where this issue first arises), there are two natural normalization choices we might consider. These correspond to letting the BDS-like ansatz depend on either three- or four-particle Mandelstam invariants in addition to two-particle invariants. We therefore consider a pair of Bose-symmetric BDS-like ansätze, respectively satisfying

$$\mathcal{A}_n^{\text{BDS}} = {}^3\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\},\{s_{ijkl}\}) + {}^3Y_8(\{u_i\}), \tag{3.2}$$
$$\mathcal{A}_n^{\text{BDS}} = {}^4\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\},\{s_{ijk}\}) + {}^4Y_8(\{u_i\}). \tag{3.3}$$

In fact, these decompositions each only single out a one-parameter family of Bose-symmetric solutions, so ${}^3A_8^{\text{BDS-like}}$ and ${}^4A_8^{\text{BDS-like}}$ are not uniquely fixed by this choice. [**Do we gauge fix for convenience and then provide the full solution in an appendix?**]

Any choice for the functions ${}^3\mathcal{A}_8^{\text{BDS-like}}$ and ${}^4\mathcal{A}_8^{\text{BDS-like}}$ allow us to define a pair of BDS-like normalized amplitudes that retain Bose symmetry and realize a subset of the Steinmann relations. In particular, defining

$$ {}^X\mathcal{E}_8 \equiv \frac{\mathcal{A}_8^{\text{MHV}}}{{}^X\mathcal{A}_8^{\text{BDS-like}}} = \exp\left[R_8 - {}^XY_8 \frac{\Gamma_{\text{cusp}}}{4}\right] \tag{3.4} $$

for any label X, we expect that ${}^3\mathcal{E}_8$ should satisfy Steinmann relations between all partially overlapping pairs of three-particle invariants, while ${}^4\mathcal{E}_8$ should satisfy Steinmann relations between all partially overlapping pairs of four-particle invariants. That is, ${}^3\mathcal{E}_8$ is expected to satisfy the relations

$$ \text{Disc}_{s_{i+1,i+2,i+3}}\left[\text{Disc}_{s_{i,i+1,i+2}}({}^3\mathcal{E}_8)\right] = 0, \tag{3.5} $$

$$ \text{Disc}_{s_{i+2,i+3,i+4}}\left[\text{Disc}_{s_{i,i+1,i+2}}({}^3\mathcal{E}_8)\right] = 0, \tag{3.6} $$

for all i, while ${}^4\mathcal{E}_8$ is expected to satisfy

$$ \text{Disc}_{s_{i+1,i+2,i+3,i+4}}\left[\text{Disc}_{s_{i,i+1,i+2,i+3}}({}^4\mathcal{E}_8)\right] = 0, \tag{3.7} $$

$$ \text{Disc}_{s_{i+2,i+3,i+4,i+5}}\left[\text{Disc}_{s_{i,i+1,i+2,i+3}}({}^4\mathcal{E}_8)\right] = 0, \tag{3.8} $$

$$ \text{Disc}_{s_{i+3,i+4,i+5,i+6}}\left[\text{Disc}_{s_{i,i+1,i+2,i+3}}({}^4\mathcal{E}_8)\right] = 0. \tag{3.9} $$

However, conditions (3.5) through (3.9) don't exhaust the set of Steinmann relations obeyed by generic eight-particle amplitudes. These additional relations are easiest to expose by breaking Bose symmetry—in particular, it proves possible to define a BDS-like ansatz that depends on all but one of the four-particle invariants (and on no three-particle invariants). That is, we can decompose the BDS ansatz as

$$ \mathcal{A}_n^{\text{BDS}} = {}^{3,q}\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\}, \{s_{ijkl} \neq s_{qrst}\}) + {}^{3,q}Y_8(\{u_i\}), \tag{3.10} $$

and doing so defines a BDS-like normalized amplitude that satisfies the Steinmann relations

$$ \text{Disc}_{s_{i+1,i+2,i+3}}\left[\text{Disc}_{s_{i,i+1,i+2,i+3}}({}^{3,i}\mathcal{E}_8)\right] = 0, \tag{3.11} $$

$$ \text{Disc}_{s_{i+2,i+3,i+4}}\left[\text{Disc}_{s_{i,i+1,i+2,i+3}}({}^{3,i}\mathcal{E}_8)\right] = 0. \tag{3.12} $$

as well as those in eqns. (3.5) and (3.6). It is not possible to make the function ${}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$ appearing in the decomposition (3.10) fully Bose-symmetric, but we can require that it is invariant under the dihedral flip that maps $s_{i\ldots l} \to s_{9-i\ldots 9-l}$. This gives rise to a three-parameter set of solutions to (3.10).

For any choice of ${}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$, momentum conservation implies that ${}^{3,q+4}\mathcal{A}_8^{\text{BDS-like}} = {}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$. This means that every eight-point Steinmann relation is manifestly respected by at least one of the five amplitudes $\{{}^4\mathcal{E}_8, {}^{3,1}\mathcal{E}_8, {}^{3,2}\mathcal{E}_8, {}^{3,3}\mathcal{E}_8, {}^{3,4}\mathcal{E}_8\}$. However, in practice it may be easier to include ${}^3\mathcal{E}_8$ in the set of functions one considers, since it manifests all Steinmann relations between partially overlapping three-particle invariants in a Bose-symmetric way.

Note that by momentum conservation there are only four independent four-particle Mandelstam invariants, so all distinct four-particle invariants are disallowed from appearing in adjacent slots in the symbol.

References

[1] J. Golden and M. Spradlin, "An analytic result for the two-loop seven-point MHV amplitude in $\mathcal{N} = 4$ SYM," *JHEP*, vol. 1408, p. 154, 2014.

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