The basic idea for generating a well-defined Poisson bracket on $\mathcal{A}$-coordinatesis to start with some seed (in terms of $\mathcal{A}$-coordinates) with standard adjacency matrix B written as a $n \times k$ matrix (this the case when you have n total nodes with k mutable). Then find some skew-symmetric $n \times n$ matrix Ω such that

$$\Omega B = \begin{pmatrix} \mathbb{1}_{k\times k} \\ 0_{(n-k)\times k} \end{pmatrix} \tag{1}$$

Then the claim is that the entries ω_{ij} form a Poisson bracket on $\mathcal{A}$-coordinates, i.e.

$$\{\mathcal{A}_i, \mathcal{A}_j\} = \omega_{ij}\mathcal{A}_i\mathcal{A}_j \Rightarrow \{\bar{\mathcal{A}}_i, \bar{\mathcal{A}}_j\} = \bar{\omega_{ij}}\bar{\mathcal{A}}_i\bar{\mathcal{A}}_j \tag{2}$$

where $^-$ indicates mutation. The interesting feature here is that imposing eq. (1) does not entirely constrain Ω. For example, let's take a seed from $\mathrm{Gr}(2,5)$:

$$\begin{tikzcd}
\boxed{\langle 12\rangle} \arrow[dr] & & & \\
& \langle 13\rangle \arrow[r] \arrow[d] & \langle 14\rangle \arrow[r] \arrow[d] & \boxed{\langle 15\rangle} \\
& \boxed{\langle 23\rangle} & \boxed{\langle 34\rangle} \arrow[ul] & \boxed{\langle 45\rangle} \arrow[ul]
\end{tikzcd} \tag{3}$$

Associating $\langle 13\rangle$ and $\langle 14\rangle$ with nodes 1 and 2, resp., and then $\langle 12\rangle, \dots, \langle 15\rangle$ with nodes $3, \dots, 7$ we have

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}$$

in which case Ω takes the form

$$\Omega = \left(\begin{array}{ccccccc}
0 & c_{13}-c_{14}+c_{15}-1 & c_{13} & c_{14} & c_{15} & c_{16} & -c_{15} \\
-c_{13}+c_{14}-c_{15}+1 & 0 & c_{23} & c_{24} & c_{24}-c_{23} & c_{26} & -c_{13}+c_{14}-c_{15}+c_{23}-c_{24} \\
-c_{13} & -c_{23} & 0 & c_{34} & c_{34}-c_{23} & c_{36} & -c_{13}+c_{23}-c_{34} \\
-c_{14} & -c_{24} & -c_{34} & 0 & c_{34}-c_{24} & c_{46} & -c_{14}+c_{24}-c_{34} \\
-c_{15} & c_{23}-c_{24} & c_{23}-c_{34} & c_{24}-c_{34} & 0 & c_{26}-c_{36}+c_{46} & -c_{15} \\
-c_{16} & -c_{26} & -c_{36} & -c_{46} & -c_{26}+c_{36}-c_{46} & 0 & -c_{16}+c_{26}-c_{36}+c_{46} \\
c_{15} & c_{13}-c_{14}+c_{15}-c_{23}+c_{24} & c_{13}-c_{23}+c_{34} & c_{14}-c_{24}+c_{34} & c_{15} & c_{16}-c_{26}+c_{36}-c_{46} & 0
\end{array}\right)$$

The c_{ij} are arbitrary, and I guess we really only care about the 2×2 entries in the upper left corner but I present the full matrix for completion's sake.

Some considerable confusion remains, at least on my part. In particular, what mutation rule does one use to generate mutated Ω's? One can just follow the standard B-matrix mutation rules, although it becomes tricky because the B-matrix rules require checking the sign of elements, which is difficult if you want to keep the c_{ij} generic. Furthermore, how does this relate to the Sklyanin bracket calculations done on $\mathcal{A}$-coordinates? If one chooses to mutate Ω via the B-matrix rules, then the Sklyanin bracket (as we know it) is not compatible with any choice of c_{ij}.

The best case scenario here is to find some Poisson bracket on the $\mathcal{A}$'s which makes manifest some particularly nice adjacency structure in the (Steinmann) remainder function. Perhaps this is equivalent to finding some representation of the symbol in terms of $\mathcal{X}$-coordinates where again the Poisson bracket between adjacent pairs follows some nice pattern.