In this note we analyze "the" D_5 function, f_{D_5}, which we generate by:

- start with an ansatz of all distinct f_{A_3}'s in D_5,
- impose antisymmetry under all of the D_5 automorphisms $\{\sigma, \tau, \mathbb{Z}_2\}$,
- take the fully symmetric sum of this f_{D_5} over E_6 and fit to $R_7^{(2)}$.

The resulting function has 1 free parameter, which represents an internal degree of freedom in f_{D_5} that cancels in the symmetric sum over E_6. Later in the note we will explore tuning these free parameters to make some (tbd) nice property manifest.

Reader's Digest: Deriving f_{D_5}

There are 65 distinct A_3's in D_5. Of these, only 42 are linearly independent. Imposing full antisymmetry leaves only 5 degrees of freedom. Requiring that the fully symmetric sum of f_{D_5} gives $R_7^{(2)}$ fixes 4 of these parameters, leaving us with only 1.

Describing f_{D_5}

I can express f_{D_5} as a sum over 13 A_3's – maybe less? What is the fewest number of $B_2 \wedge B_2$ terms I can express it in terms of?

Representing $R_7^{(2)}$ in terms of f_{D_5}

$$R_7^{(2)} = \sum_{i=1}^{14} \pm f_{D_5}^{(i)} \tag{1}$$

where i indexes the 14 D_5's in E_6, and the $\pm$ is to get the symmetries to work out right – half of the f_{D_5}'s have $+$ and half have $-$. This needs sharper restatement.

Constraining the remaining parameter

Can this last parameter be tuned to make f_{D_5} have some other nice amplitudes property? For example, can f_{D_5} be well-defined in the collinear limit? What are other properties it can have?