

In momentum twistor language we have the  $n$  momentum twistors  $Z_i$ , which together form the  $4 \times n$  matrix

$$K = \begin{pmatrix} z_{11} & \dots & z_{n1} \\ z_{12} & \dots & z_{n2} \\ z_{13} & \dots & z_{n3} \\ z_{14} & \dots & z_{n4} \end{pmatrix}. \quad (1)$$

As long as the first 4 columns are non-singular, we can row reduce  $K$  in to the form

$$K' = \begin{pmatrix} 1 & 0 & 0 & 0 & y_{51} & \dots & y_{n1} \\ 0 & 1 & 0 & 0 & y_{52} & \dots & y_{n2} \\ 0 & 0 & 1 & 0 & y_{53} & \dots & y_{n3} \\ 0 & 0 & 0 & 1 & y_{54} & \dots & y_{n4} \end{pmatrix}. \quad (2)$$

The columns of  $K'$  define a new set of momentum twistors  $Z'_i$ , where for example  $Z'_1 = \{1, 0, 0, 0\}$  and  $Z'_5 = \{y_{51}, y_{52}, y_{53}, y_{54}\}$ . It is easy to check that

$$y_{ij} = (-1)^j \langle \{1, 2, 3, 4\} \setminus \{j\}, i \rangle / \langle 1234 \rangle, \quad (3)$$

$$\langle abcd \rangle' = \det(Z'_a Z'_b Z'_c Z'_d) = \langle abcd \rangle / \langle 1234 \rangle. \quad (4)$$

You can then define the Sklyanin bracket as an operation on these  $y_{ij}$  by

$$\{y_{ij}, y_{ab}\} = (\text{sgn}(a - i) - \text{sgn}(b - j)) y_{ib} y_{aj}. \quad (5)$$

Which then extends to a bracket on functions of the  $y_{ij}$  via

$$\{f(y), g(y)\} = \sum_{i,a=1}^n \sum_{j,b=1}^4 \frac{\partial f}{\partial y_{ij}} \frac{\partial g}{\partial y_{ab}} \{y_{ij}, y_{ab}\}. \quad (6)$$

Now if we want to evaluate the Poisson bracket between two  $\mathcal{X}$ -coordinates, we can instead treat them as functions of the  $y_{ij}$  and use eq. (6), dividing by an overall factor of 2.<sup>1</sup> To be precise, for each four-bracket  $\langle abcd \rangle$  in the  $\mathcal{X}$ -coordinates, replace them with  $\langle abcd \rangle'$  expanded out in terms of  $y_{ij}$  (e.g.  $\langle 1256 \rangle' = y_{53}y_{64} - y_{54}y_{63}$ ). Then you can calculate eq. (6) directly in terms of the  $y_{ij}$ , though note that it will be a VERY long expression.

To actually extract a real value for the Poisson bracket, you need to numerically evaluate the remaining expression. I think you can choose arbitrary numerical values of the  $y_{ij}$ , but to be sure I choose some point  $K$  in the positive grassmannian and then use eq. (3) to get values for the  $y_{ij}$ . After plugging in the numbers, if you get some nasty fraction dependent on

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<sup>1</sup>Of course also remember that the Poisson bracket on  $\mathcal{X}$ -coordinates is actually defined on the Log's of the  $\mathcal{X}$ -coordinates, so in practice not only do you have to divide by 2 but you also have to divide by the product of the two  $\mathcal{X}$ -coordinates in order to get what we commonly refer to as “the Poisson bracket”.

the particular kinematic choice, then the two  $\mathcal{X}$ -coordinates have a “bad” Poisson bracket. However if you get some nice number, such as  $\pm 1$  or  $0$  (it’s also possible to get values such as  $\pm 2$ , but in practical cases that doesn’t occur), then you can say that the  $\mathcal{X}$ -coordinates have nice Poisson bracket.

As for why this whole procedure actually works, I have no idea, but apparently it’s explained in <http://arxiv.org/abs/math/0208033>.