In this note we explore different attempts at expressing the $B_2 \wedge B_2$ component of $R_8^{(2)}$ with cluster polylogarithms.

First let us define "good" $\mathcal{X}$-coordinates: an $\mathcal{X}$-coordinate R is good if R and $1+R$ can be expressed as ratios of $\mathcal{A}$-coordinates of the form

$$\begin{aligned} \langle i\ i{+}1\ jk\rangle, &\quad \langle i(i{-}1\ i{+}1)(j\ j{+}1)(k\ k{+}1)\rangle, \\ \langle i\ i{+}1\ \bar{j}\cap\bar{k}\rangle, &\quad \langle i(i{-}2\ i{-}1)(i{+}1\ i{+}2)(j\ j{+}1)\rangle. \end{aligned} \tag{1}$$

An algebra is good if it contains only good $\mathcal{X}$-coordinates.

Fitting with A_3's

There are 1600 good A_3's in $\mathrm{Gr}(4,8)$, with only 513 actual degrees of freedom in $B_2 \wedge B_2$. An ansatz of these functions can be fit to $R_8^{(2)}$. A zero-thought choice of free parameters leaves us with an expression involving 223 A_3's.

Fitting with D_5's

There are NO good D_5's in $\mathrm{Gr}(4,8)$ (this is a deeply surprising fact to me). Furthermore, NO D_5 contains enough good A_3's to allow us to tune the free parameters in f_{D_5} in order to have only good A_3's appear in f_{D_5}. In other words, any representation of $R_8^{(2)}$ in terms of f_{D_5} will necessarily involve "bad" ratios inside the f_{D_5}'s which must cancel in the full sum. This is an aesthetically upsetting fact to me!

Fitting with E_6's

Are there E_6's in $\mathrm{Gr}(4,8)$ which have $\{v,z\}$ squares? What if I try the 42-term $\{v,z\}$ expression for $R_7^{(2)} \sim f_{E_6}$ and see if that evaluates to "good" things on (at least some of) the E_6's I've found in $\mathrm{Gr}(4,8)$.