

*Goal: closed-form representation of  $n$ -particle, two-loop MHV amplitude in  $N=4$  SYM.*

*Fall-back goal: 8-particle, two-loop MHV amplitude.*

### Individual steps

- Generate subalgebras of finite cluster algebras:  $A_2, A_3, A_4, D_4, A_5, D_5, E_6$ .
- Apply  $f_{A_2}$  and  $f_{A_3}$  across subalgebras, cataloguing identities etc.
- Find larger subalgebras with “unique” functions by applying analytic and/or algebraic criteria such as:
  - collinear limits
  - branch cuts (first-entry, Steinmann)
  - Poisson bracket structure
- Related, find “completions” (products of lower weight functions) for  $f_{A_2}$  and  $f_{A_3}$  by considering similar criteria.
- Build 7- and 8-pt amplitudes out of new larger subalgebra functions, hopefully in a fully unique way. (Can build 8-pt amplitude out of pre-existing functions, so this is the fall-back paper.)
- Use 7- and 8-pt amplitude structure as guide for constructing closed-form representation of  $n$ -particle, two-loop MHV amplitude.

### Other Opportunities

- Higher-weight cluster polylogarithms (test Goncharov hypothesis, try to re-write 6-pt 3-loop in terms of cluster functions, etc).
- Re-write known non-MHV results in cluster algebra language.
- New functional relations for  $\text{Li}_4$  (and higher?) of cluster algebraic origin.
- Understand larger connections between Feynman diagram integrals and cluster algebra polylogarithms.