

Steinmann Relations and the Two-Loop MHV Amplitude in Eight-Particle Kinematics

John Golden,^{1,2} **Andrew J. McLeod**^{2,3,4}

¹ *Michigan Center for Theoretical Physics and Randall Laboratory of Physics, Department of Physics, University of Michigan Ann Arbor, MI 48109, USA*

² *Kavli Institute for Theoretical Physics, UC Santa Barbara, Santa Barbara, CA 93106, USA*

³ *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA*

⁴ *Niels Bohr International Academy, Blegdamsvej 17, 2100 Copenhagen, Denmark*

ABSTRACT: We present the full functional form of the two-loop eight-point MHV amplitude in the planar limit of maximally supersymmetric Yang-Mills theory, in terms of cluster polylogarithms. We also compute the two BDS-like ansätze that can be formulated in eight-particle kinematics, and find that ...

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1 Introduction

2 Promoting $R_8^{(2)}$ from Symbol to Function

Briefly describe the method outlined in [1] for upgrading the n -point two-loop MHV symbol to a function.

3 The Steinmann Relations for Eight Particles

When the number of gluons n is not a multiple of 4, the BDS-like ansatz is unique because there exists only a single decomposition

$$A_n^{\text{BDS}} = A_n^{\text{BDS-like}}(\{s_{ij}\}) + Y_n(\{u_i\}), \quad n \neq 4K, \quad (3.1)$$

such that the kinematic dependence of $A_n^{\text{BDS-like}}$ involves only two-particle Mandelstam invariants and Y_n is a function of dual conformal invariant cross ratios [2]. However, when n is a multiple of 4, no decomposition of this type exists, and we are forced to consider multiple BDS-like ansätze in order to expose all Steinmann relations between higher-particle Mandelstam invariants. However, in eight particle kinematics (where this issue first arises), there are two natural normalization choices we might consider. These correspond to letting the BDS-like ansatz depend on either three- or four-particle Mandelstam invariants in addition to two-particle invariants. We therefore consider a pair of Bose-symmetric BDS-like ansätze, respectively satisfying

$$\mathcal{A}_n^{\text{BDS}} = {}^3\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\}, \{s_{ijkl}\}) + {}^3Y_8(\{u_i\}), \quad (3.2)$$

$$\mathcal{A}_n^{\text{BDS}} = {}^4\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\}, \{s_{ijk}\}) + {}^4Y_8(\{u_i\}). \quad (3.3)$$

In fact, these decompositions only single out a one-parameter family of Bose-symmetric solutions for ${}^3\mathcal{A}_8^{\text{BDS-like}}$ and ${}^4\mathcal{A}_8^{\text{BDS-like}}$, so these functions are not uniquely fixed by this choice. **[Do we gauge fix for convenience and then provide the full solution in an appendix?]**

Any choice for the functions ${}^3\mathcal{A}_8^{\text{BDS-like}}$ and ${}^4\mathcal{A}_8^{\text{BDS-like}}$ allow us to define a pair of BDS-like normalized amplitudes that retain Bose symmetry and realize a subset of the Steinmann relations. In particular, defining

$${}^X\mathcal{E}_8 \equiv \frac{\mathcal{A}_8^{\text{MHV}}}{{}^X\mathcal{A}_8^{\text{BDS-like}}} = \exp \left[R_8 - {}^XY_8 \frac{\Gamma_{\text{cusp}}}{4} \right] \quad (3.4)$$

for any label X , we expect that ${}^3\mathcal{E}_8$ should satisfy Steinmann relations between all partially overlapping pairs of three-particle invariants, while ${}^4\mathcal{E}_8$ should satisfy Steinmann relations between all partially overlapping pairs of four-particle invariants. That is, ${}^3\mathcal{E}_8$ is expected to satisfy the relations

$$\text{Disc}_{s_{i+1,i+2,i+3}} [\text{Disc}_{s_{i,i+1,i+2}} ({}^3\mathcal{E}_8)] = 0, \quad (3.5)$$

$$\text{Disc}_{s_{i+2,i+3,i+4}} [\text{Disc}_{s_{i,i+1,i+2}} ({}^3\mathcal{E}_8)] = 0, \quad (3.6)$$

for all i , while ${}^4\mathcal{E}_8$ is expected to satisfy

$$\text{Disc}_{s_{i+1,i+2,i+3,i+4}} [\text{Disc}_{s_{i,i+1,i+2,i+3}} ({}^4\mathcal{E}_8)] = 0, \quad (3.7)$$

$$\text{Disc}_{s_{i+2,i+3,i+4,i+5}} [\text{Disc}_{s_{i,i+1,i+2,i+3}} ({}^4\mathcal{E}_8)] = 0, \quad (3.8)$$

$$\text{Disc}_{s_{i+3,i+4,i+5,i+6}} [\text{Disc}_{s_{i,i+1,i+2,i+3}} ({}^4\mathcal{E}_8)] = 0. \quad (3.9)$$

However, conditions (3.5) through (3.9) don't exhaust the set of Steinmann relations obeyed by generic eight-particle amplitudes—there are also Steinmann relations between partially overlapping three- and four-particle invariants. In order to make use of these additional relations, we use the fact that it proves possible to define a BDS-like ansatz that depends on all but one of the four-particle invariants (and on no three-particle invariants). That is, we decompose the BDS ansatz as

$$\mathcal{A}_n^{\text{BDS}} = {}^{3,q}\mathcal{A}_8^{\text{BDS-like}}(\{s_{ij}\}, \{s_{ijkl} \neq s_{qrst}\}) + {}^{3,q}Y_8(\{u_i\}), \quad (3.10)$$

and in so doing define a BDS-like normalized amplitude that satisfies the Steinmann relations

$$\text{Disc}_{s_{i+1,i+2,i+3}} [\text{Disc}_{s_{i,i+1,i+2,i+3}} ({}^{3,i}\mathcal{E}_8)] = 0, \quad (3.11)$$

$$\text{Disc}_{s_{i+2,i+3,i+4}} [\text{Disc}_{s_{i,i+1,i+2,i+3}} ({}^{3,i}\mathcal{E}_8)] = 0, \quad (3.12)$$

$$\text{Disc}_{s_{i+1,i+2,i+3}} [\text{Disc}_{s_{i,i+1,i+2}} ({}^{3,q}\mathcal{E}_8)] = 0, \quad (3.13)$$

$$\text{Disc}_{s_{i+2,i+3,i+4}} [\text{Disc}_{s_{i,i+1,i+2}} ({}^{3,q}\mathcal{E}_8)] = 0, \quad (3.14)$$

where we note that (3.13) and (3.14) were also satisfied by ${}^3\mathcal{E}_8$. It is not possible to make the function ${}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$ appearing in the decomposition (3.10) fully Bose-symmetric, but we can require that it is invariant under the dihedral flip that maps $s_{i\dots l} \rightarrow s_{9-i\dots 9-l}$. This gives rise to a three-parameter set of solutions to (3.10).

For any choice of ${}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$, momentum conservation implies that ${}^{3,q+4}\mathcal{A}_8^{\text{BDS-like}} = {}^{3,q}\mathcal{A}_8^{\text{BDS-like}}$. This means that every eight-point Steinmann relation is manifestly respected by

at least one of the five amplitudes $\{^4\mathcal{E}_8, ^{3,1}\mathcal{E}_8, ^{3,2}\mathcal{E}_8, ^{3,3}\mathcal{E}_8, ^{3,4}\mathcal{E}_8\}$. However, in practice it may be easier to include $^3\mathcal{E}_8$ in the set of functions one considers, since it manifests all Steinmann relations between partially overlapping three-particle invariants in a Bose-symmetric way.

...how the Steinmann relations manifest at symbol level...

Note that by momentum conservation there are only four independent four-particle Mandelstam invariants, so all distinct four-particle invariants are disallowed from appearing in adjacent slots in the symbol.

References

- [1] J. Golden and M. Spradlin, “An analytic result for the two-loop seven-point MHV amplitude in $\mathcal{N} = 4$ SYM,” *JHEP*, vol. 1408, p. 154, 2014.
- [2] G. Yang, “Scattering amplitudes at strong coupling for 4K gluons,” *JHEP*, vol. 12, p. 082, 2010.