A complete basis of weight n Steinmann-satisfying HPLs (in the cross ratios u_i, and modulo MZVs) is given by all functions that can be written in one of the following four ways:

- $f^p_{\vec{w}}(1-u_1, u_3u_4u_7^2/u_2^2u_5u_6)\log^{n-p-|\vec{w}|}(u_1u_4u_5/u_3u_6)$
- $f^{n-|\vec{w}|}_{\vec{w}}(1-u_1u_4, u_2^2u_1u_5^2/u_3^2u_4u_7^2)$
- $\log^m(u_1u_4u_5/u_3u_6)\log^{n-m}(u_1u_4u_7/u_2u_6)$
- $\log^n(u_1u_4u_5/u_3u_6)$

plus all cyclic permutations $u_i \to u_{i+1}$, where

$$\begin{aligned} f^p_{\vec{w}}(a,b) = &\ \big(H_{\vec{w}01}(a) + H_{\vec{w}11}(a)\big)\log^{p-2}(b) \\ &+ (p-2)(p-3)\big(H_{\vec{w}0111}(a) + H_{\vec{w}1111}(a)\big)\log^{p-4}(b) \\ &+ (p-2)(p-3)(p-4)(p-5)\big(H_{\vec{w}011111}(a) + H_{\vec{w}111111}(a)\big)\log^{p-6}(b) \\ &+ \dots \end{aligned}$$

for any integer $p \geq 2$ and any vector of zeros and ones $\vec{w}$ (the sum in the definition of $f^p_{\vec{w}}(a,b)$ terminates when the power of $\log b$ becomes negative). $|\vec{w}|$ denotes the length of $\vec{w}$, which is allowed to be zero. This gives rise to $7 \times (3 \times 2^n - 4)/2$ functions at weight n. Since this basis is written modulo MZVs it only satisfies the Steinmann condition at symbol level, although it should be straightforward to promote to a basis that satisfies the Steinmann condition at function-level (whenever I get around to it...).

Note that the functions involving $f^p_{\vec{w}}(a,b)$ are particularly easy to project out of an integrability ansatz when $|\vec{w}| > 0$, because each of these functions has a unique $\{n-1,1\}$ coproduct term associated with it—for instance, one can project $f^4_{010}(a,b)\log^m(c)$ out of one's set of weight n integrability equations by setting the coefficient multiplying $f^4_{10}(a,b)\log^m(c) \otimes \log(a)$ to zero, and similarly project out $f^4_{110}(a,b)\log^m(c)$ by setting the coefficient multiplying $f^4_{10}(a,b)\log^m(c) \otimes \log(1-a)$ to zero.