A_3-constructible function counts

In this section we extend the subalgebra-constructibility results to the more intricate case of $A_2 \subset A_3$-constructible functions.

For A_3 there are two possible functions, as described in a previous section: $f_{A_3}^{+-}$ and $f_{A_3}^{--}$. We will work throught the case of $f_{A_3}^{--}$ as it is known to be connected with $R_7^{(2)}$ [], however it remains an interesting open question to find applications for $f_{A_3}^{+-}$-constructible functions.

The algorithm described in the previous section applies, changing only $A_2 \to A_3$:

- begin with an ansatz of all $f_{A_3}^{+-}$ applied across all A_3 subalgebras of a given larger algebra,
- impose that the overall function is invariant under automorphisms up to overall sign choices,
- count the number of solutions to these constraints.

To reiterate, the functions that satisfy these constraints are decomposable in to both A_2 and A_3 building blocks, thus making contact with multiple layes of the intricate cluster algebraic structure.

There are no $A_3^{--} \subset A_4$ or $A_3^{--} \subset D_4$ functions. For A_5 we have

	$\sigma^+\tau^+$	$\sigma^+\tau^-$	$\sigma^-\tau^+$	$\sigma^-\tau^-$
$A_3^{--} \subset A_5$	1	1	0	3

(1)

For D_5 we have

$\sigma^+\tau^+$		$\sigma^+\tau^-$		$\sigma^-\tau^+$		$\sigma^-\tau^-$	
$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$
2	0	3	0	0	1	0	5

(2)

And finally E_6 gives

$\sigma^+\tau^+$		$\sigma^+\tau^-$		$\sigma^-\tau^+$		$\sigma^-\tau^-$	
$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$	$\mathbb{Z}_2^+$	$\mathbb{Z}_2^-$
-	-	-	-	-	-	-	-

(3)