We wish to define a function $f_{A_2}(x,y)$. It is a function of two variables, and is smooth and real for $x,y>0$.

$$\sigma_{A_2} = \mathcal{X}_i \to \mathcal{X}_{i+1}, \qquad \tau_{A_2} = \mathcal{X}_i \to \mathcal{X}_{6-i} \tag{1}$$

$$\begin{aligned}\sum_{\text{skew-dihedral}} &\mathrm{Li}_{(2,2)}(-1/x_i,-1/x_{i+2}) - \mathrm{Li}_{(1,3)}(-1/x_i,-1/x_{i+2}) - 6\,\mathrm{Li}_3(-x_i)\log(x_{i+2}) \\ &+ \mathrm{Li}_2(-x_i)\log(x_{i+2})\left(3\log(x_i)+\log(x_{i+2})-\log(x_{i+1})\right) + \frac{1}{2}\log(x_i)^2\log(x_{i+1})\log(x_{i-2})\end{aligned} \tag{2}$$

$$\sum_{\text{skew-dihedral}}^{A_2} f = \sum_{k=1}^{5}\Big((\sigma_{A_2})^k - (\sigma_{A_2})^k \circ \tau_{A_2}\Big) f \tag{3}$$

$$\sum_{\text{skew-dihedral}}^{A_2} f(x,y) = \sum_{i=1}^{5} f(\mathcal{X}_i, 1/\mathcal{X}_{i+1}) - f(\mathcal{X}_{6-i}, 1/\mathcal{X}_{5-i}) \tag{4}$$

where

$$\mathcal{X}_1 = x, \quad \mathcal{X}_2 = 1/y, \quad \mathcal{X}_i = (1+\mathcal{X}_{i-1})/\mathcal{X}_{i-2} \tag{5}$$