Transmission Lines

TRANSMISSION LINES

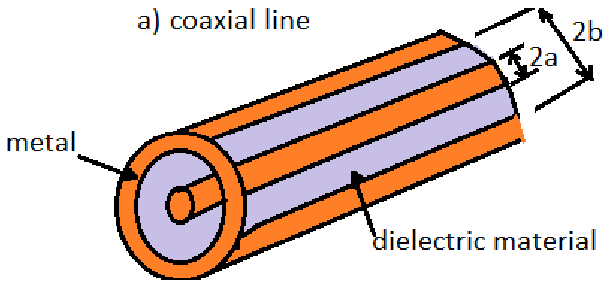
Transmission lines are structures or media that serves to transfer energy or information between two points.

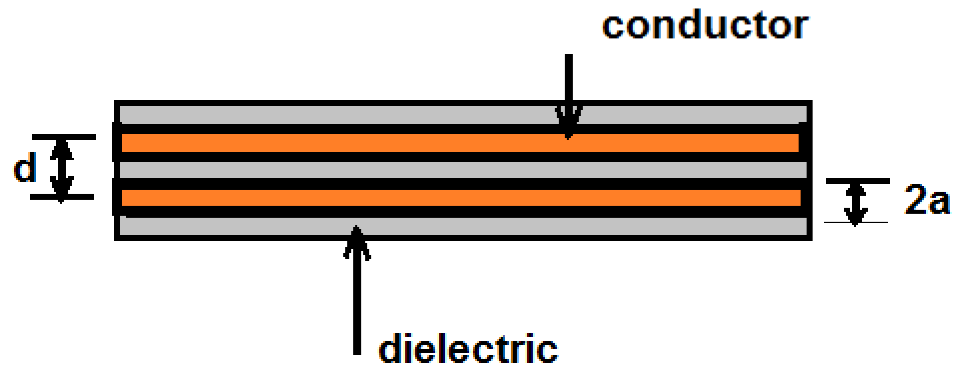
Examples:

- Nerve Fibres in human bodies,
- Fluid Media that acts as channels for acoustic waves
- Solid Media that acts as channels for mechanical pressure waves
- Two Conductors that guide transverse electromagnetic (TEM) waves;
  - Coaxial cables,
  - Two-wire lines,
  - Parallel plate lines strip-lines, and
  - Microstrip lines
- Single Conductor that guide transverse magnetic and transverse electric waves include
  - Rectangular waveguides,
  - cylindrical waveguides
- Optical Fibres constitute the high order transmission lines.

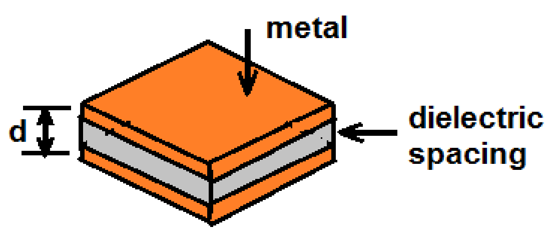
Examples of transmission lines

a) coaxial line





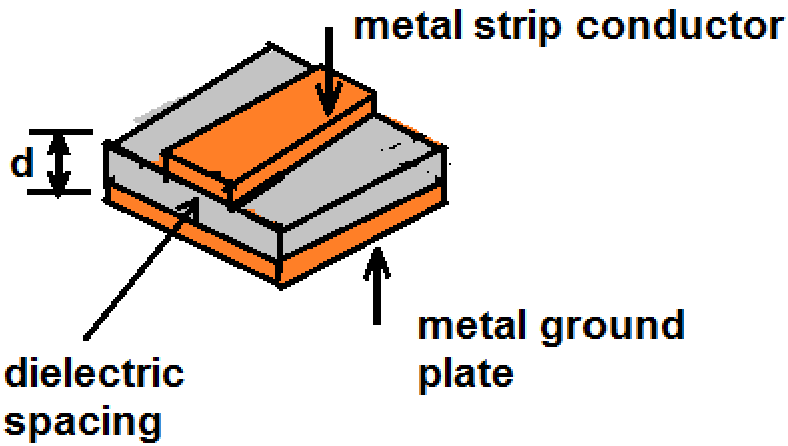
b) Two-wire line



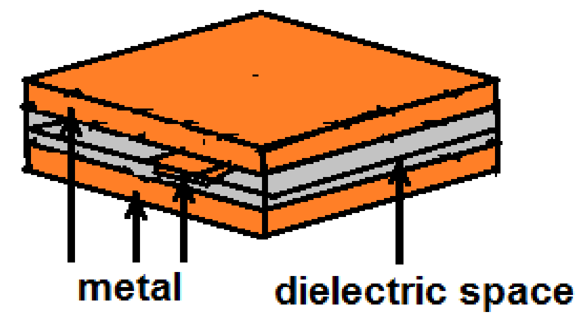
c) Parallel plate line

TEM TRANSMISSION LINES

Examples of transmission lines



d) Microstrip line



e) Strip line

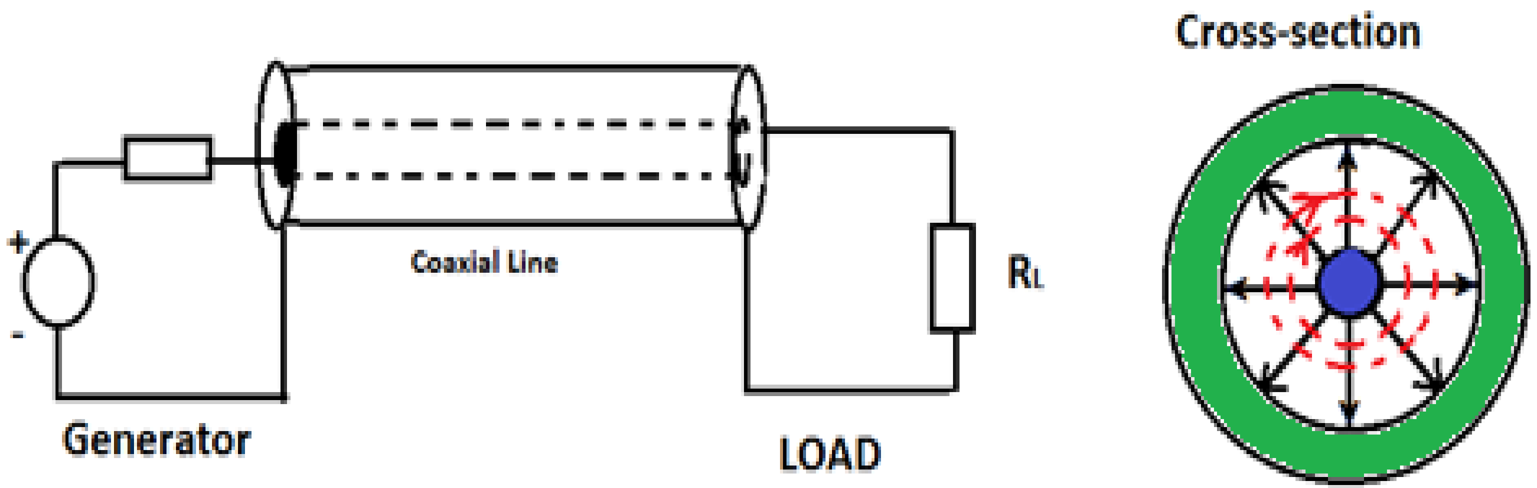
TEM TRANSMISSION LINES

Propagation modes

Transverse electromagnetic (TEM) transmission lines:

- Wave propagating in such lines are characterized by electric and magnetic fields that are entirely transverse to the direction of propagation.
- There is no component along the direction of propagation
- TEM lines consist of two parallel transmission lines.
- Such waves are referred to as the TEM mode

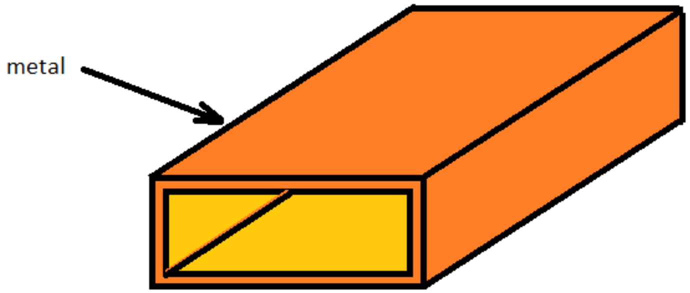
Example:



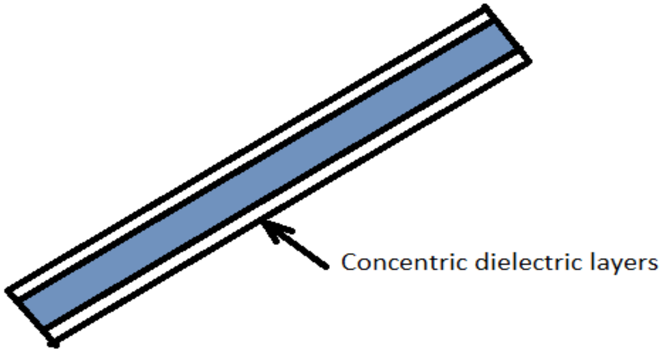
→ Electric Field---->radial

----- Magnetic field ----> Circumferential

Examples of transmission lines

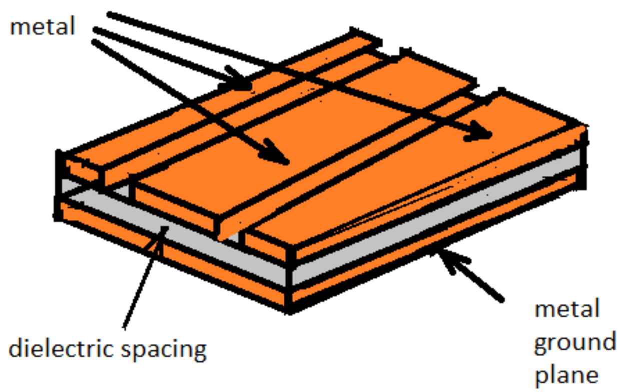


f) Rectangular waveguide



g) Optic fibre

Higher order transmission lines



h) Coplanar waveguide

High order transmission lines

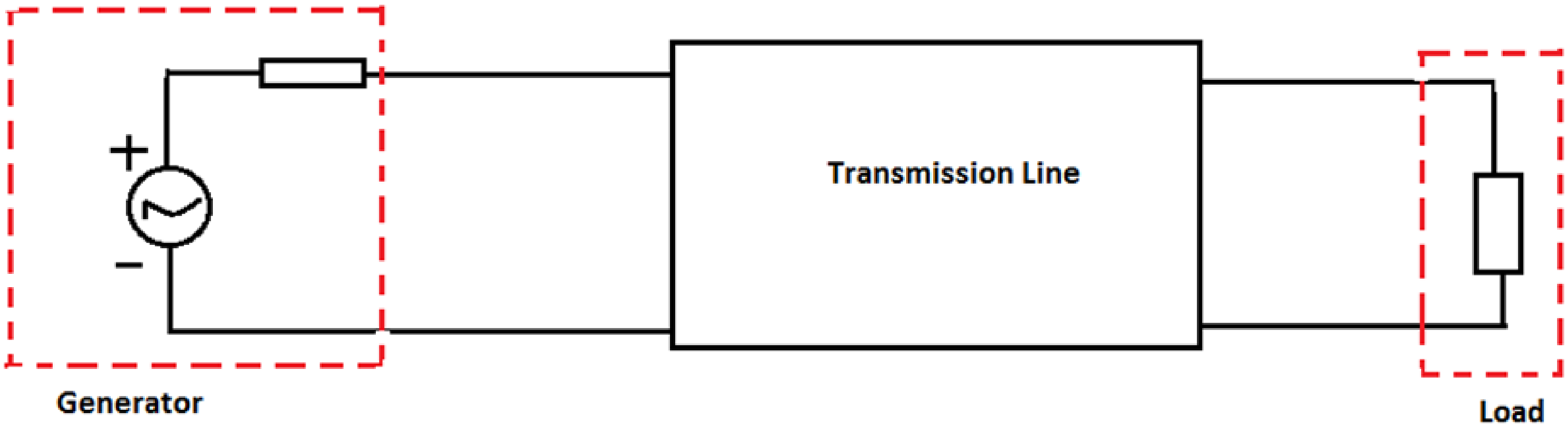
- They are characterized by a single conductor transmission line
- Such lines propagate transverse electric or transverse magnetic fields
- They have at least one significant field component in the direction of propagation.

Example:

Waveguides

Optical fibres also are in this category

Basic elements of a Communication System



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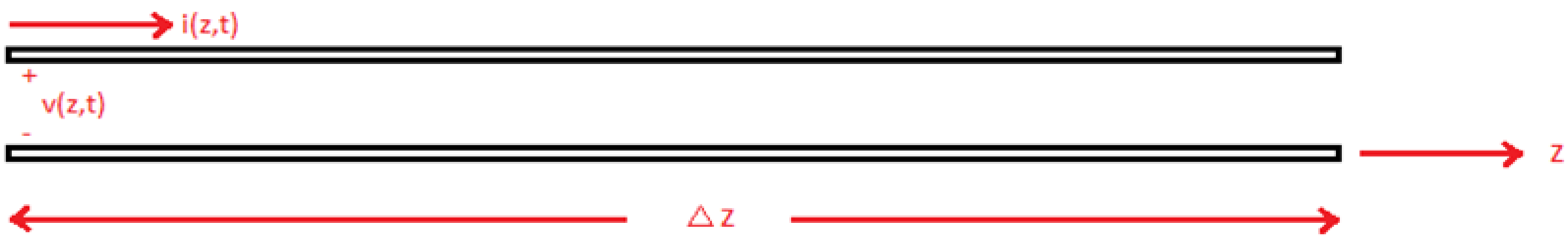
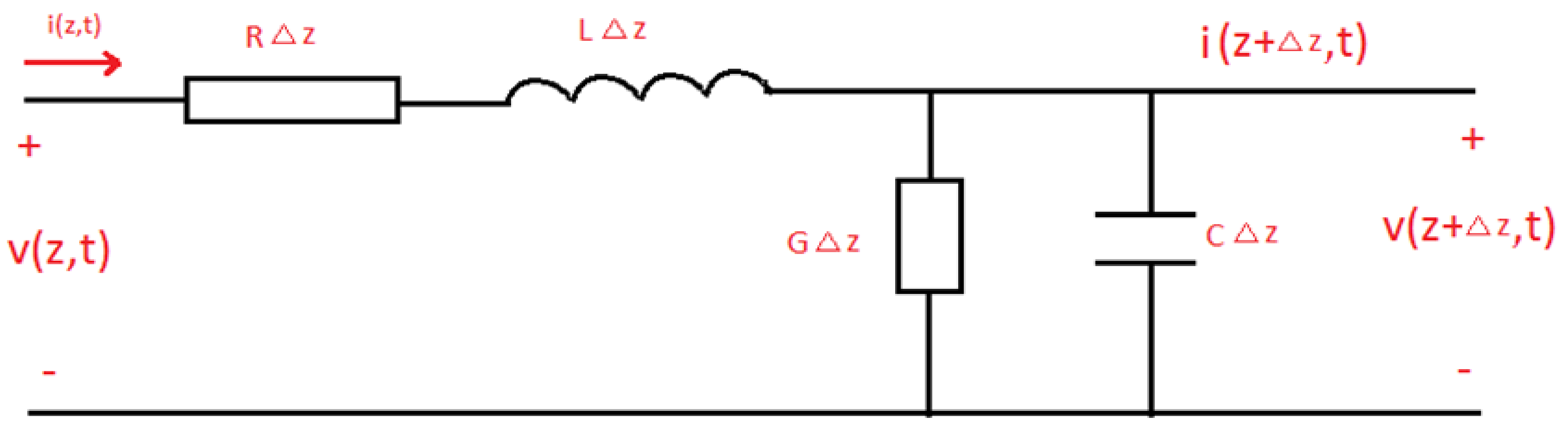
- Transmission lines are used to connect antennas to transmitters and receivers, for impedance matching in mixers and amplifiers and as resonant elements in oscillators and filters.

Lumped element model



Lumped element Model:



Lumped element model

R= series resistance/unit length for both conductors in Ω/m

L= Series inductance/ unit length in H/m

G= shunt conductance /unit length in S/m

C=Shunt capacitance/unit length in F/m

- Applying Kirchhoff's voltage law

$$\boldsymbol{v(z,t) - i(z,t)R\Delta z - L\Delta z\frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0} \quad (1)$$

- Applying Kirchhoff's current law

$$\boldsymbol{i(z,t) - v(z+\Delta z,t)G\Delta z - C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0} \quad (2)$$

Dividing (1) by Δz and taking the limit

$$\lim_{\Delta z\to 0}\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\boldsymbol{\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}} \quad (3)$$

- Dividing (2) by Δz and taking the limit

$$\lim_{\Delta z \to 0} \frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t} \qquad (4)$$

Equations (3) and (4) and are telegrapher or transmission line equations. These are time domain equations.

With sinusoidal steady-state condition, with cosine based phasors

$$(3) \xrightarrow{yields}$$

$$\boldsymbol{\frac{dV(z)}{dz} = -(R + j\omega L)I(z)} \qquad (5)$$

$$(4) \xrightarrow{yields}$$

$$\boldsymbol{\frac{dI(z)}{dz} = -(G + j\omega c)V(z)} \qquad (6)$$

Wave propagation on a transmission line

Differentiating (5) with respect to z

$$\frac{d^2V(z)}{dz^2} = -(R + j\omega L)\frac{dI(z)}{dz} \qquad (7)$$

Substituting (6) into (7) we get

$$\boldsymbol{\frac{d^2V(z)}{dz^2} = +(R + j\omega L)(G + j\omega C)V(z)} \qquad (8)$$

Similarly differentiating (6) with respect to z

$$\frac{d^2I(z)}{dz^2} = -(G + j\omega C)\frac{dV(z)}{dz} \qquad (9)$$

Substituting (5) into (9) we get

$$\boldsymbol{\frac{d^2I(z)}{dz^2} = +(G + j\omega C)(R + j\omega L)I(z)} \qquad (10)$$

Substituting

$$\boldsymbol{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$(8) \xrightarrow{yields}$$

$$\boldsymbol{\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0} \qquad (11)$$

$$(10) \xrightarrow{yields}$$

$$\boldsymbol{\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0} \qquad (12)$$

where

$\boldsymbol{\gamma}$ is the complex propagation constant,

$\boldsymbol{\beta}$ is the phase constant, and

$\boldsymbol{\alpha}$ is the attenuation constant.

Solution to (11) and (12)

$$\boldsymbol{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}} \qquad (13)$$
$$\mathbf{I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}} \qquad (14)$$

where the factors
$e^{-\gamma z}$represents wave propagating in the +z direction, and
$e^{+\gamma z}$ represents wave propagating in the –z direction.

Differentiating (13) with respect to z

$$\frac{dV(z)}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -\gamma[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}]$$

Substituting (5)

$$-(R + j\omega L)I(z) = -\gamma[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}]$$

$$\boldsymbol{I(z) = \frac{\gamma}{R+j\omega L}[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}]} \qquad (15)$$

Compare with (14)

$$\boldsymbol{I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}}$$

Comparing (14) and (15) we get

$$I_0^+ = \frac{\gamma}{R + j\omega L}[V_0^+]$$

$$\frac{V_0^+}{I_0^+} = z_0 = \frac{R + j\omega L}{\gamma} = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\boldsymbol{z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} \quad (16)$$

Reproducing (13)

$$\boldsymbol{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}} \quad (13)$$

Substituting $z_0 = \frac{R+jwL}{\gamma}$ in (15)

$$\boldsymbol{I(z) = \frac{V_0^+}{z_0} e^{-\gamma z} - \frac{V_0^-}{z_0} e^{+\gamma z}} \quad (17)$$

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- Reproducing 13

$$\boldsymbol{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}}$$

- We note
- $V^+ = |V^+| \measuredangle \emptyset^+$ and $V^- = |V^-| \measuredangle \emptyset^-$
  - $\gamma = \alpha + j\beta$
  - $V(z) = |V^+| e^{-az} e^{-j\beta z} e^{\emptyset^+} + |V^-| e^{az} e^{j\beta z} e^{\emptyset^-}$
  - $\boldsymbol{V(z) = |V^+| e^{-az} e^{(-j\beta z + \emptyset^+)} + |V^-| e^{az} e^{(j\beta z + \emptyset^-)}}$
  - To change phasor into time domain
  - $v(z,t) = Re\left[V(z) e^{j\omega t}\right]$

Converting phasor voltages of (13) to the time-domain gives

$$\boldsymbol{v(z,t) = |V_0^+|e^{-\alpha z}\, cos(\omega t - \beta z + \emptyset^+) \; + |V_0^-|e^{\alpha z}\, cos(\omega t + \beta z + \emptyset^-)} \qquad (18)$$

The wavelength of the travelling waves is defined as the distance between two successive points of equal phase on the wave at a fixed instant of time

Let the wavelength be λ

$\beta\lambda = 2\pi$, $\quad\quad \boldsymbol{\therefore \lambda = {}^{2\pi}/_{\beta}}$ (19)

The phase velocity of the wave is defined as the speed at which a constant phase point travels down the line.

$$\beta z - \boldsymbol{\omega} t = const$$

$$z = \frac{const + \boldsymbol{\omega} t}{\beta}$$

$$\boldsymbol{v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \lambda f} \qquad (20)$$

Lossless transmission lines

For a lossless transmission line R=G=0

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$$

$$\therefore \boldsymbol{\alpha = 0,\ \beta = \omega\sqrt{LC}} \qquad (21)$$

$$\boldsymbol{z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}} \qquad (22)$$

The general solutions for the diff equations becomes:

$$\boldsymbol{V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}} \qquad (23(a))$$

$$\boldsymbol{I(z) = \frac{V_0^+}{z_0} e^{-j\beta z} - \frac{V_0^-}{z_0} e^{+j\beta z}} \qquad (23(b))$$

The wavelength on the line

$$\boldsymbol{\lambda = {}^{2\pi}/_{\beta} = \frac{2\pi}{\omega\sqrt{LC}}} \qquad (24)$$

The phase velocity

$$\boldsymbol{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}} \qquad (25)$$

Lossy transmission lines

- In practice all transmission lines have losses due to finite conductivity and/or lossy dielectrics.
- In many cases these losses are small and may be neglected.
- However, there are cases when the losses are of interest.
- In most microwave transmission lines the losses are small.
- In such cases approximations can be made that simplify the general expressions for γ and Z_0.

Lossy transmission lines

- General expressions for propagation constant is
- $\gamma = \sqrt{(R + j\omega L)(G + j\omega C}$
- which can be re-arranged to
- $\gamma = \sqrt{(j\omega L)(j\omega C)(1 + \frac{R}{j\omega L})(1 + \frac{G}{j\omega c})}$
- $= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$ (26)
- If the line is low loss we can assume that
- $R \ll \omega L$ and $G \ll \omega C$ and (26) reduces to

Lossy transmission lines

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} \quad (27)$$

Using Taylor series expansion ($\sqrt{1+x} \cong 1 + {}^{x}/_{2} + \cdots$)

$$\gamma = j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right] \quad (28)$$

which gives

$$\boldsymbol{\alpha} \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \quad (29)$$

$$\boldsymbol{\beta} \cong \boldsymbol{\omega}\sqrt{\boldsymbol{LC}} \quad (30)$$

Similarly the characteristic impedance can be approximated to be

$$\boldsymbol{z_0} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \cong \sqrt{\frac{L}{C}} \quad (31)$$

The distortionless transmission line

- From the exact equation of the transmission line the propagation constant of a lossy line is given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C}$$

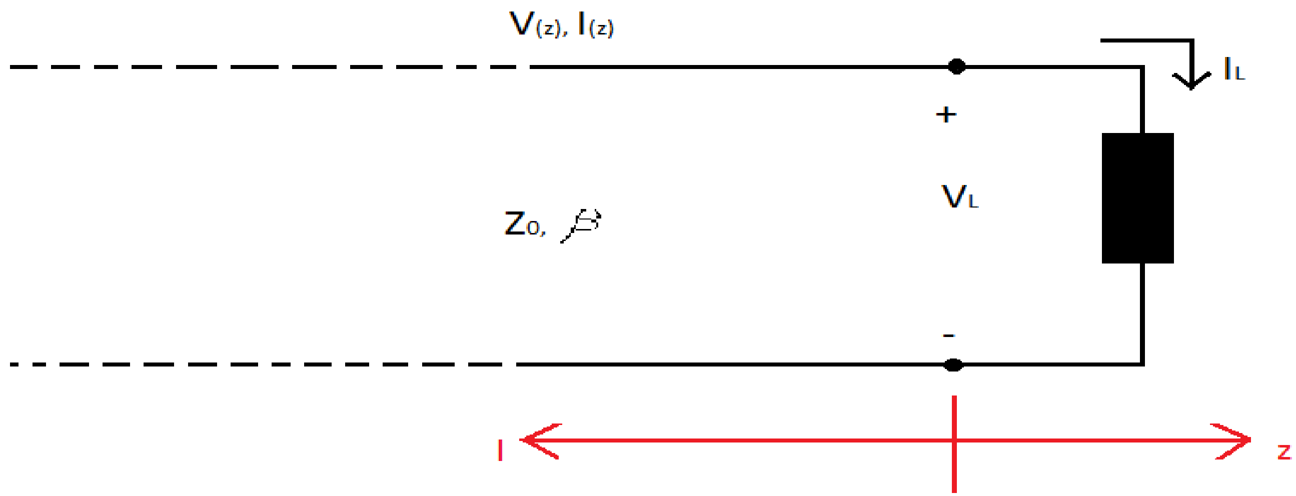
$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

- It can be seen that the phase constant β is not a linear function of frequency.
- The phase velocity $v_p = \frac{\omega}{\beta}$ is therefore different for different frequencies.
- The various frequency components of a wideband signal will travel at different velocities and will arrive at the receiver end at slightly different times.
- This will cause distortion of the resultant signal.
- This form of distortion is referred to as **dispersion.**

The distortionless transmission line

- There is a special case where a lossy line has a linear phase factor as a function of frequency.
- Such a line is referred to as a **distortionless** line.
- It is characterized by the line parameters that satisfy $\frac{R}{L} = \frac{G}{C}$
- Equation (26) reproduced here
- $\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$
- reduces to
- $\alpha + j\beta = j\omega\sqrt{LC}\sqrt{1 - j2\left(\frac{R}{\omega L}\right) - \frac{R^2}{\omega^2 L^2}} = j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right)$
- $\beta = \omega\sqrt{LC}$ which is a linear phase frequency relation

Terminated transmission lines



For a lossless transmission line we have shown

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \qquad (23(a))$$

$$I(z) = \frac{V_0^+}{z_0} e^{-j\beta z} - \frac{V_0^-}{z_0} e^{+j\beta z} \qquad (23(b))$$

At z=0

$$\frac{V(0)}{I(0)} = z_L = \frac{V_0^+ + V_0^-}{\dfrac{V_0^+}{z_0} - \dfrac{V_0^-}{z_0}}$$

Solving for V_0^- gives

$$V_0^- = \frac{z_L - z_0}{z_L + z_0} V_0^+$$

But the voltage reflection coefficient at the load point is given by

$$\boldsymbol{\Gamma_L \triangleq \frac{V_0^-}{V_0^+} = \frac{z_L - z_0}{z_L + z_0}} \qquad (32)$$

Equation (23(a)) can be developed further

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ \left[e^{-j\beta z} + \frac{V_0^-}{V_0^+} e^{+j\beta z} \right]$$

$$= V_0^+ [e^{-j\beta z} + \Gamma_L e^{+j\beta z}]$$

Also equation (23(b)) can be developed as

$$I(z) = \frac{V_0^+}{z_0} e^{-j\beta z} - \frac{V_0^-}{z_0} e^{+j\beta z}$$

$$= \frac{V_0^+}{z_0} \left[e^{-j\beta z} - \frac{V_0^-}{V_0^+} e^{+j\beta z} \right] = \frac{V_0^+}{z_0} [e^{-j\beta z} - \Gamma_L e^{+j\beta z}]$$

Thus

$$V(z) = V_0^+ [e^{-j\beta z} + \Gamma_L e^{+j\beta z}] \qquad (33(a))$$

$$I(z) = \frac{V_0^+}{z_0} [e^{-j\beta z} - \Gamma_L e^{+j\beta z}] \qquad (33(b))$$

The voltage on the line consists of the superposition of an incident and reflected wave. Such a wave is referred to as a **Standing Wave**.

When $\boldsymbol{\Gamma_L = 0}$ there is no reflected wave and $\boldsymbol{z_L = z_0}$.

When the load is mismatched, not all available power from the generator is delivered to the load.

The loss is called the return loss (RL) and is defined as

$$\boldsymbol{RL = -20\ log|\Gamma|\ dB} \tag{34}$$

For matched load $\Gamma_L = 0$, RL$=\infty$

This implies that there is no reflected power.

For $\Gamma_L = 1$, RL$=0$

This implies that all incident power is reflected.

When the load is matched to the line, $\Gamma_L = 0$ and $|v(z)| = |V_0^+|$ which is constant. The line is said to be flat.

When the line is not matched to the load the presence of reflected waves leads to standing waves.

$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right] \qquad \text{(from (33(a))}$$

$$|V(z)| = |V_0^+|\left[1 + \Gamma_L e^{+j2\beta z}\right]\left|e^{-j\beta z}\right|$$

$$= |V_0^+|\left|[1 + |\Gamma_L| e^{+j(\theta+2\beta z)}]\right|$$

But $l = -z$

$$\boldsymbol{|V(-l)| = |V_0^+|\left|[1 + |\Gamma_L| e^{+j(\theta-2\beta l)}]\right|} \qquad (34)$$

Maximum value of $|V(-l)|$ occurs when $e^{+j(\theta-2\beta l)} = 1$ resulting

$$V_{max} = |V_0^+|(1 + |\Gamma_L|)$$

Minimum value of $|V(-l)|$ occurs when

$$e^{+j(\theta-2\beta l)} = -1$$

$$V_{min} = |V_0^+|(1 - |\Gamma_L|)$$

We define the standing wave ratio (SWR) as

$$\boldsymbol{SWR = \frac{V_{max}}{V_{min}} = \frac{(1+|\Gamma_L|)}{(1-|\Gamma_L|)}} \qquad (35)$$

This quantity is also known as the voltage standing wave ratio **(VSWR)**

VSWR is a real number such that

$$\mathbf{1 \leq \boldsymbol{VSWR} \leq \infty}$$

When $\boldsymbol{VSWR = 1}$

the load impedance is matched to the transmission line.

The load impedance is equal to the characteristic impedance we say that the load is matched to the line.

The time average power flow

$$P_{avg} = \frac{1}{2}\mathrm{Re}[V(z)I^*(z)]$$ using equations (33) we get

$$= \frac{1}{2}\frac{|V_0|^2}{z_0} Re\left[1 - \Gamma_L^* e^{-j2\beta z} + \Gamma_L e^{+j2\beta z} - |\Gamma_L|^2\right]$$

$$P_{avg} = \frac{1}{2}\frac{|V_0|^2}{z_0}[1 - |\Gamma_L|^2]$$

$\therefore$ **Incindent Power** $= \frac{1}{2}\frac{|V_0|^2}{z_0}$; 36(a)

Reflected Power $= \Gamma_L^2 \frac{1}{2}\frac{|V_0|^2}{z_0}$ 36(b)

Maximum power is delivered to the load when $\Gamma_L = 0$.

No power is delivered to the load when $\Gamma_L = 1$.

Distance between 2 successive voltage maxima

From equation (34) reproduced here

$$|V(z)| = |V_0^+|\left|[1 + |\Gamma_L|e^{+j(\theta-2\beta l)}]\right|$$

It can be seen that

the first peak, Vmax, occurs when $\theta - 2\beta l_1 = 0$,

the second peak, Vmax occurs when $\theta - 2\beta l_2 = 2\pi$

Therefore the distance between 2 successive voltages maximum

$$= 2\beta(l_2 - l_1) = 2\pi, \qquad \Delta l = \frac{2\pi}{2\beta} = {}^{\pi}/_{\beta}$$

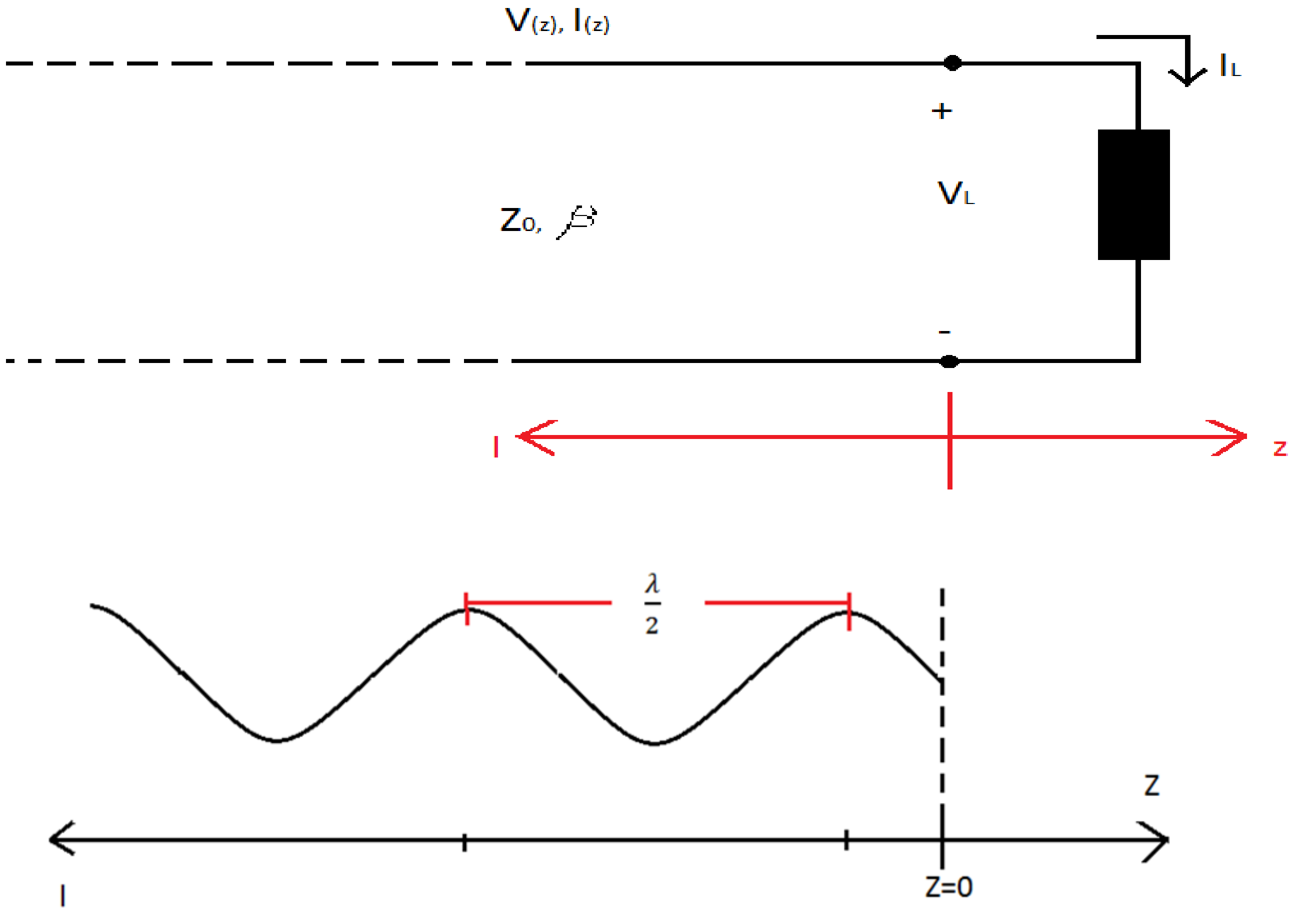
But from equation (24)

$\lambda = {}^{2\pi}/_{\beta}$ *or* $\beta = {}^{2\pi}/_{\lambda}$ $\qquad \therefore \boldsymbol{\Delta l = \frac{\pi}{2\pi/\lambda} = \frac{\lambda}{2}}$ **(37)**

Distance between 2 successive voltage maxima (or minima) $= \frac{\lambda}{2}$

Distance between a maxima and minima $= \frac{\lambda}{4}$

Distance between 2 successive voltage maxima



Reflection Coefficient at any point on the line

Recall equation (23(a))

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

The reflection coefficient at any point along the line is defined as the reflected voltage divided by the incident voltage;

$$\Gamma(z) = \frac{V_0^- e^{+j\beta z}}{V_0^+ e^{-j\beta z}},$$

With z=-l we have

$$\boldsymbol{\Gamma(-l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{+j\beta l}}}$$

$$\boldsymbol{= \Gamma(0) e^{-j2\beta l}} \qquad (38)$$

With $\Gamma(0)$ is the reflection coefficient at z=0

Input impedance

At a distance $z = -l$ from the load the input impedance seen looking toward the load is (from equation 33)

$$z_{in} = \frac{V(-l)}{I(-l)} = \frac{V_0^+[e^{j\beta l} + \Gamma_L e^{-j\beta l}]}{\frac{V_0^+}{z_0}[e^{j\beta l} - \Gamma_L e^{-j\beta l}]}$$

$$= z_0 \frac{[e^{j\beta l} + \Gamma_L e^{-j\beta l}]}{[e^{j\beta l} - \Gamma_L e^{-j\beta l}]}$$

Substituting $\Gamma_L = \frac{z_L - z_0}{z_L + z_0}$ from (32) we get

$$z_{in} = z_0 \frac{[e^{j\beta l} + \frac{z_L - z_0}{z_L + z_0} e^{-j\beta l}]}{[e^{j\beta l} - \frac{z_L - z_0}{z_L + z_0} e^{-j\beta l}]}$$

$$= z_0 \frac{(z_L + z_0)e^{j\beta l} + (z_L - z_0)e^{-j\beta l}}{(z_L + z_0)e^{j\beta l} - (z_L - z_0)e^{-j\beta l}}$$

$$= z_0 \frac{z_l\left(e^{j\beta l} + e^{-j\beta l}\right) + z_0\left(e^{j\beta l} - e^{-j\beta l}\right)}{z_0(e^{j\beta l} + e^{-j\beta l}) + z_l(e^{j\beta l} - e^{-j\beta l})}$$

$$= z_0 \frac{z_l \cos \beta l + j z_0 \sin \beta l}{z_0 \cos \beta l + j z_l \sin \beta l}$$

- Dividing by $\cos \beta l$ gives

$$\boldsymbol{z_{in} = z_0 \frac{z_l + j z_0 \tan \beta l}{z_0 + j z_l \tan \beta l}} \qquad (39)$$

- This is the transmission line impedance equation.

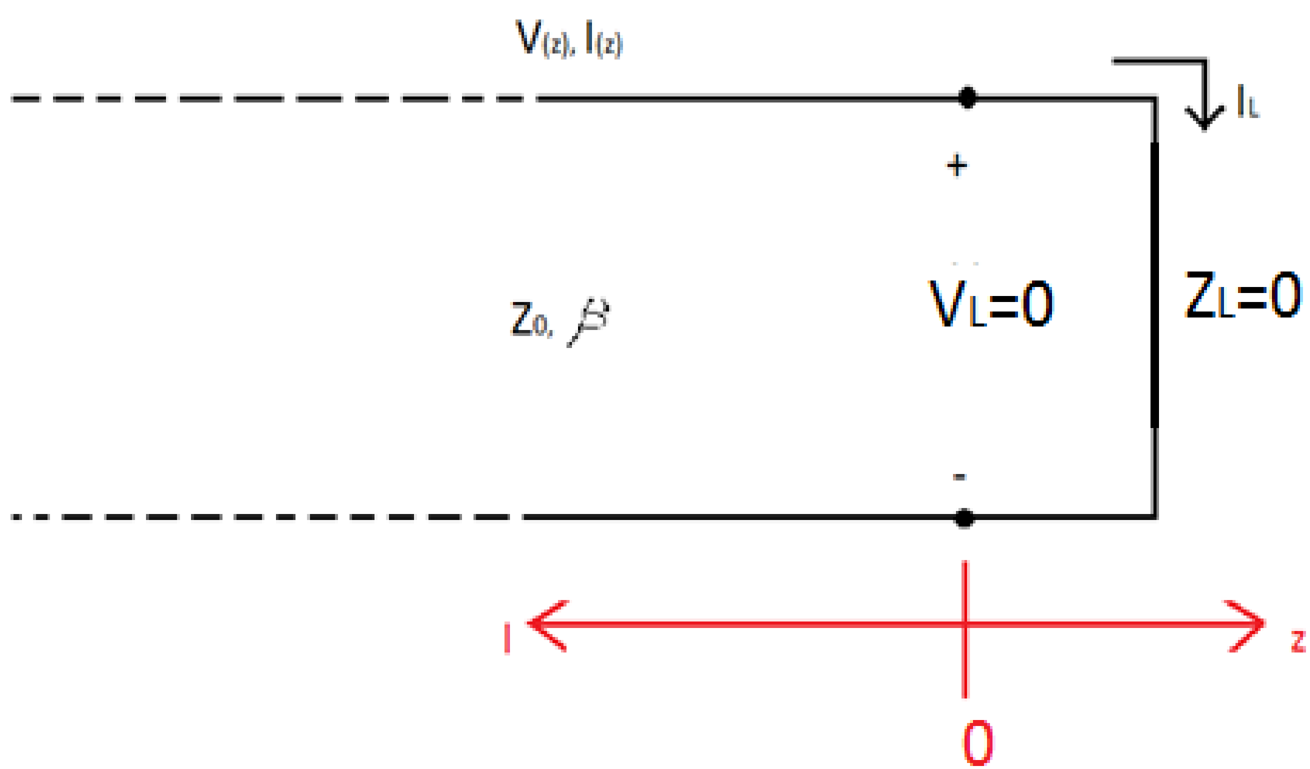
Example

- A load impedance of $130+j90\ \Omega$ terminates a $50\ \Omega$ transmission line that is $0.3\ \lambda$ long. Find the following:
- (i) Γ_L (the reflection coefficient at the load),
- (ii) the SWR on the line,
- (iii) RL (the return loss), and
- (iv) Z_{in} (the input impedance).

- (i) $0.598\angle 21.8^o$ (ii) 3.98 (iii) 4.47 dB
- (iv) $12.75+j6.8\ \Omega$

Special case of termination

Case 1: Short circuit termination



When $Z_L = 0$

$$\Gamma_L = \frac{z_L - z_0}{z_L + z_0} = -1$$

and therefore

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \infty$$

The standing wave voltage on the line is

$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right] = V_0^+\left[e^{-j\beta z} - e^{+j\beta z}\right]$$

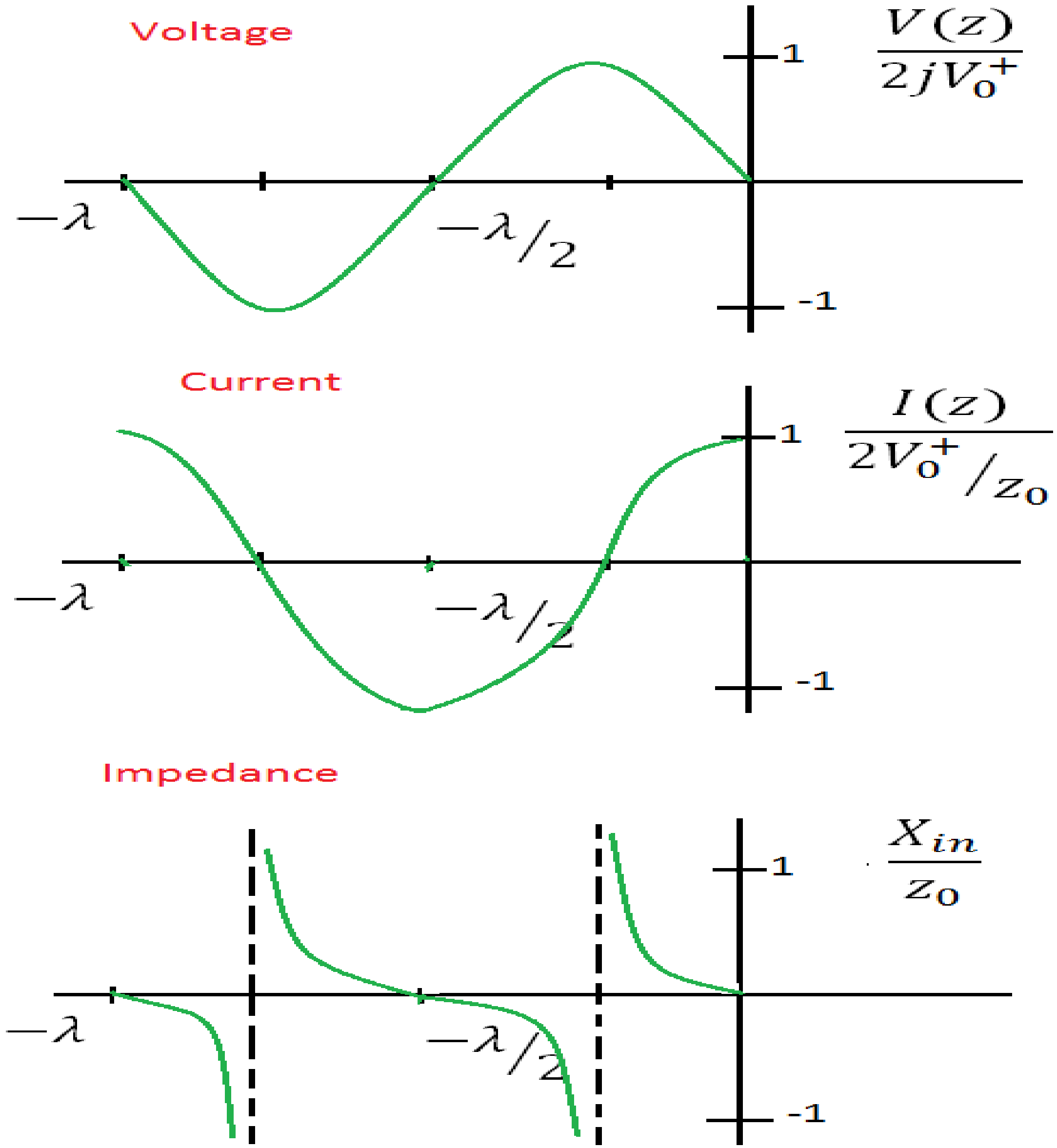
$$V(z) = -j2V_0^+ \sin \beta z$$

and the current is given by

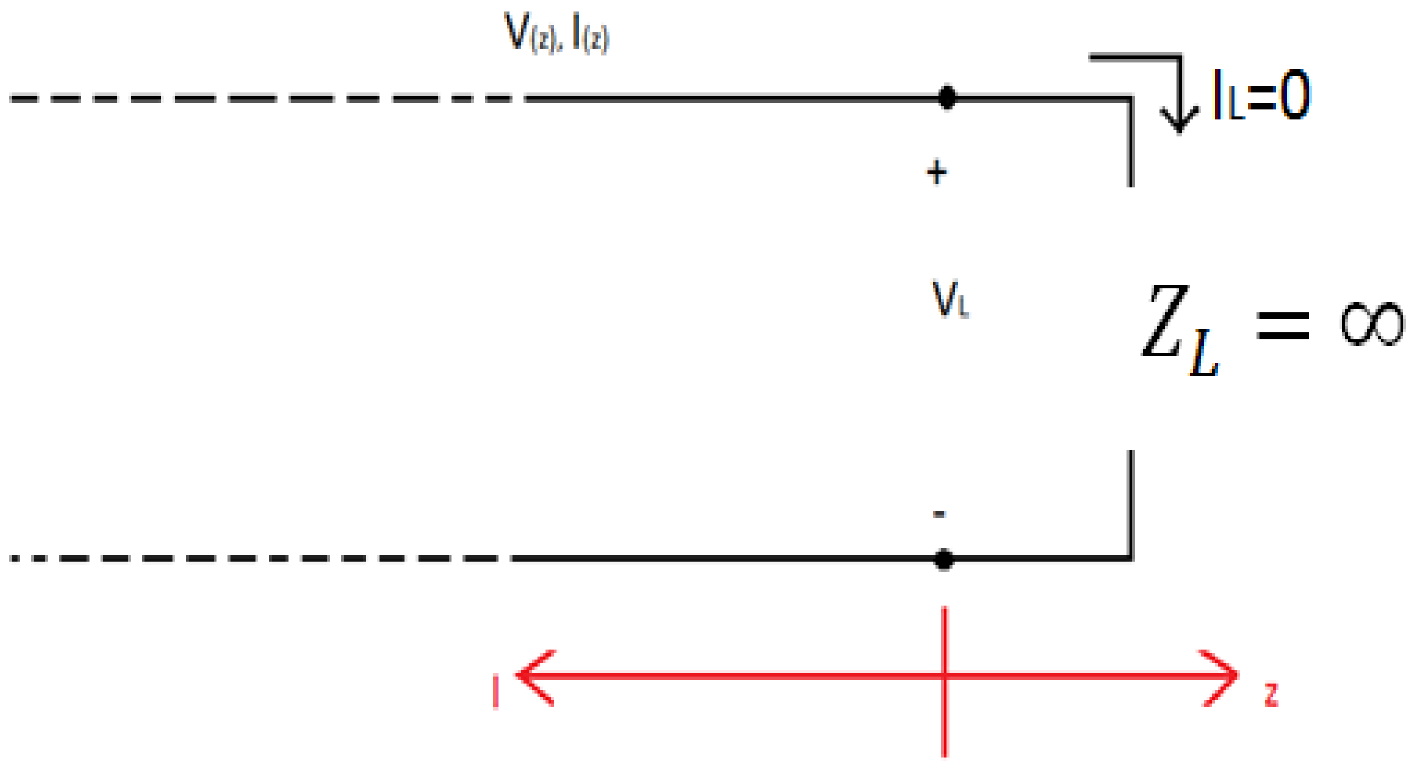
$$I(z) = \frac{V_0^+}{z_0}\left[e^{-j\beta z} - \Gamma_L e^{+j\beta z}\right] = \frac{V_0^+}{z_0}\left[e^{-j\beta z} + e^{+j\beta z}\right]$$

$$I(z) = 2\frac{V_0^+}{z_0}\cos \beta z$$

$$\boldsymbol{z_{in} = \frac{V(z)}{I(z)} = \frac{V(-l)}{I(-l)} = jz_0\, tan\, \beta l}$$


Voltage
$\frac{V(z)}{2jV_0^+}$
1
-1
$-\lambda$
$-\lambda/2$
Current
$\frac{I(z)}{2V_0^+/Z_0}$
1
-1
$-\lambda$
$-\lambda/2$
Impedance
$\frac{X_{in}}{Z_0}$
1
-1
$-\lambda$
$-\lambda/2$

Case 2: Open circuit termination



When $z_l = \infty$

$$\Gamma = \frac{z_L - z_0}{z_L + z_0} = \frac{1 - {z_0}/{z_L}}{1 - {z_0}/{z_L}} = 1 \ ,$$

and therefore

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

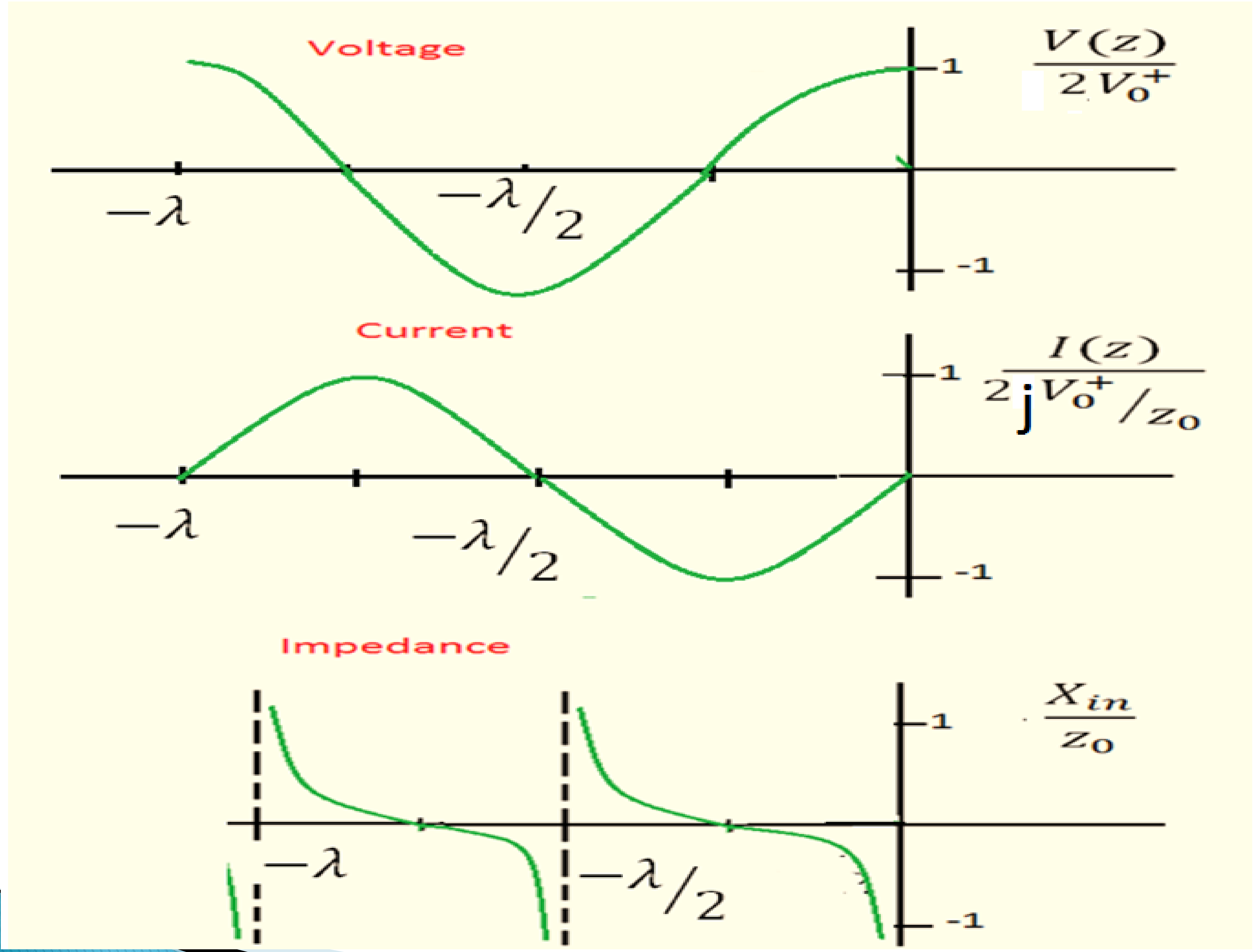
$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right] = V_0^+\left[e^{-j\beta z} + e^{+j\beta z}\right]$$

$$\boldsymbol{V(z) = 2V_0^+ \cos\beta z}$$

$$I(z) = \frac{V_0^+}{z_0}\left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right] = \frac{V_0^+}{z_0}\left[e^{-j\beta z} - e^{+j\beta z}\right]$$

$$\boldsymbol{I(z) = -j2\frac{V_0^+}{z_0}\sin\beta z}$$

$$\boldsymbol{z_{in} = \frac{V(z)}{I(z)} = \frac{V(-l)}{I(-l)} = jz_0 \cot\beta l}$$



Voltage
$\frac{V(z)}{2V_0^+}$
1
-1
$-\lambda$
$-\lambda/2$
Current
$\frac{I(z)}{2jV_0^+/z_0}$
1
-1
$-\lambda$
$-\lambda/2$
Impedance
$\frac{X_{in}}{z_0}$
1
-1
$-\lambda$
$-\lambda/2$

Case 3: terminated transmission line with line length $l = {}^{\lambda}/_{2}$

General input impedance

$$z_{in} = z_0 \frac{z_l + jz_0 \tan \beta l}{z_0 + jz_l \tan \beta l},$$

When $l = {}^{\lambda}/_{2}$

$$\beta l = {}^{2\pi}/_{\lambda} . {}^{\lambda}/_{2} = \pi,$$

$$z_{in} = z_0 \frac{z_l + jz_0 \tan \pi}{z_0 + jz_l \tan \pi}$$

$$z_{in} = Z_L$$

A half-wavelength line (or any multiple of ${}^{\lambda}/_{2}$) does not alter or transform the load impedance regardless of its characteristic impedance.

Case 4: terminated transmission line with line length $l = \lambda/_4$

General input impedance

$$z_{in} = z_0 \frac{z_l + jz_0 \tan \beta l}{z_0 + jz_l \tan \beta l}$$

- When $l = \lambda/_4$
- $\beta l = {}^{2\pi}/_{\lambda} . {}^{\lambda}/_{4} = {}^{\pi}/_{2}$

$$z_{in} = \lim_{\beta l \to \pi/_2} z_0 \frac{{}^{z_l}/_{\tan \beta l} + jz_0}{{}^{z_0}/_{\tan \beta l} + jz_l} = \frac{Z_0^2}{Z_L};$$

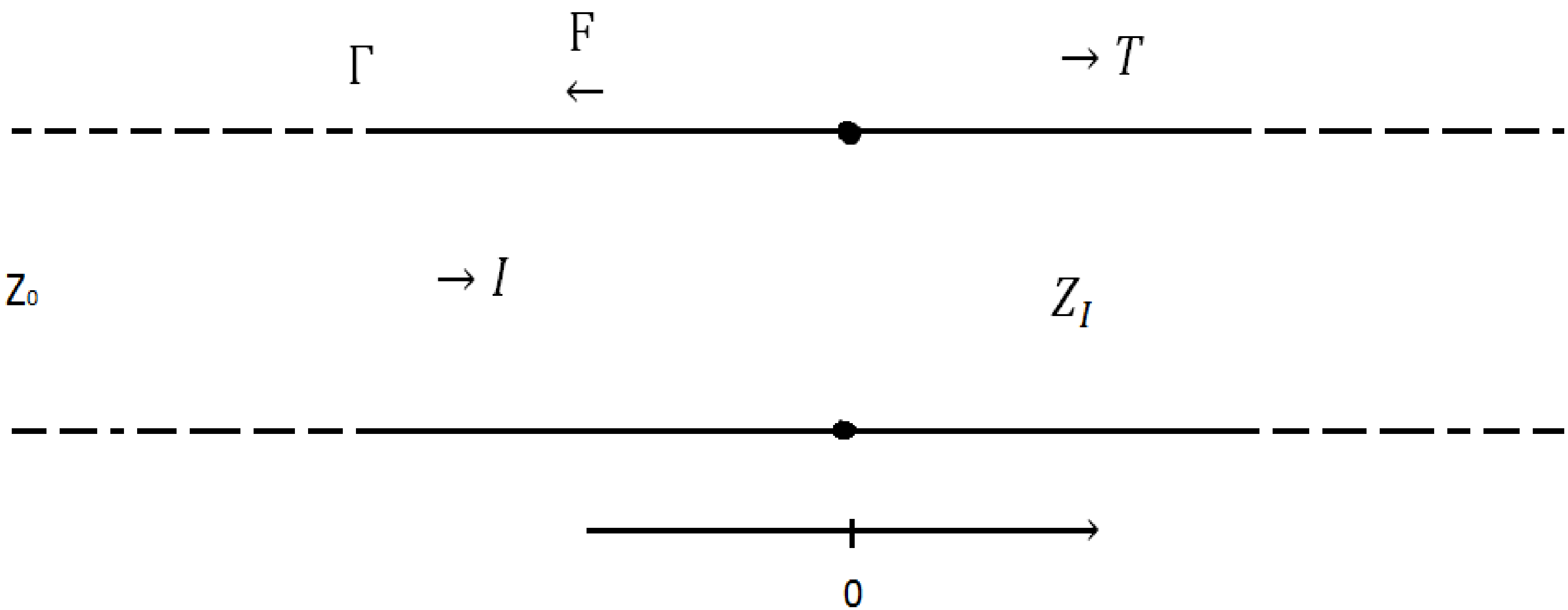
$$\frac{z_{in}}{z_0} = \frac{1}{{}^{Z_L}/_{z_0}}$$

$$\Rightarrow \mathfrak{z}_{in} = {}^{1}/_{\mathfrak{z}_L}$$

where $\mathfrak{z}$ is a normalized impedance

A ${}^{\lambda}/_{4}$ line (or any $l = {}^{\lambda}/_{4} + n\,{}^{\lambda}/_{2}$ for n= 1, 2, 3,…) has the effect of transforming the normalized load impedance in an inverse manner.

Case 5: Transmission line with characteristic impedance z_0 feeding a line of a different characteristic impedance z_I



If the load line is infinitely long, or if it is terminated by its own characteristic impedance then

$$\Gamma = \frac{z_I - z_0}{z_I + z_0}$$

Some of the incident waves will be transmitted.

Let T be the transmission coefficient

for $Z \leq 0$, $\qquad V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)$

$for\ Z > 0;$ $\qquad V(z) = V_0^+ T e^{-j\beta z}$ (*no reflections*)

$at\ Z = 0;$ $\qquad V(0) = V_0^+(1 + \Gamma) = V_0^+ T$

$$T = 1 + \Gamma = 1 + \frac{z_I - z_0}{z_I + z_0} = \frac{2z_I}{z_I + z_0}$$