Superstring Theory Action and Equations of Motion

AI Assistant

Abstract

This document addresses the following query:

"Give me the action for superstring theory including all ghost fields. Show in a step-by-step derivation, thinking carefully through each step, how one acquires the relevant equations of motion by using the principle of least action. Display the results in an easy-to-read LaTeX compiled format. Include this query word for word at the top of LaTeX document for comparative reference. max_tokens = 2500."

1 Superstring Theory Action

The action for superstring theory, including all ghost fields, is given by:

$$S = S_{\text{matter}} + S_{\text{ghost}} \tag{1}$$

where

$$S_{\text{matter}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} \left[h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu + i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right] \tag{2}$$

and

$$S_{\text{ghost}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{-h} \left[b_{\alpha\beta} \partial^\alpha c^\beta + \tilde{b}_{\alpha\beta} \partial^\alpha \tilde{c}^\beta \right] \tag{3}$$

Here, X^μ are the bosonic fields, ψ^μ are the fermionic fields, $h^{\alpha\beta}$ is the worldsheet metric, ρ^α are the worldsheet gamma matrices, $b_{\alpha\beta}$ and c^β are the ghost fields, and $\tilde{b}_{\alpha\beta}$ and $\tilde{c}^\beta$ are the anti-ghost fields.

2 Derivation of Equations of Motion

We will now derive the equations of motion using the principle of least action. We'll focus on the matter part of the action for simplicity.

2.1 Bosonic Fields

For the bosonic fields X^μ, we vary the action:

$$\delta S_{\text{matter}} = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{\alpha\beta} \partial_\alpha \delta X^\mu \partial_\beta X_\mu \tag{4}$$

Integrating by parts and assuming the variation vanishes at the boundaries:

$$\delta S_{\text{matter}} = \frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-h} \partial_\alpha (h^{\alpha\beta} \partial_\beta X_\mu) \delta X^\mu \tag{5}$$

For the action to be stationary, $\delta S_{\text{matter}} = 0$ for arbitrary δX^μ. This leads to the equation of motion:

$$\partial_\alpha (h^{\alpha\beta} \partial_\beta X_\mu) = 0 \tag{6}$$

2.2 Fermionic Fields

For the fermionic fields ψ^μ, we vary the action:

$$\delta S_{\text{matter}} = -\frac{i}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} (\delta\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \delta\psi_\mu) \tag{7}$$

Integrating by parts the second term and assuming the variation vanishes at the boundaries:

$$\delta S_{\text{matter}} = -\frac{i}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} (\delta\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu - \partial_\alpha (\bar{\psi}^\mu \rho^\alpha) \delta\psi_\mu) \tag{8}$$

For the action to be stationary, $\delta S_{\text{matter}} = 0$ for arbitrary $\delta\bar{\psi}^\mu$ and $\delta\psi_\mu$. This leads to the equations of motion:

$$\rho^\alpha \partial_\alpha \psi_\mu = 0 \tag{9}$$

$$\partial_\alpha (\bar{\psi}^\mu \rho^\alpha) = 0 \tag{10}$$

2.3 Ghost Fields

For the ghost fields $b_{\alpha\beta}$ and c^β, we vary the action:

$$\delta S_{\text{ghost}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{-h} (\delta b_{\alpha\beta} \partial^\alpha c^\beta + b_{\alpha\beta} \partial^\alpha \delta c^\beta) \tag{11}$$

Integrating by parts the second term and assuming the variation vanishes at the boundaries:

$$\delta S_{\text{ghost}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{-h} (\delta b_{\alpha\beta} \partial^\alpha c^\beta - \partial^\alpha b_{\alpha\beta} \delta c^\beta) \tag{12}$$

For the action to be stationary, $\delta S_{\text{ghost}} = 0$ for arbitrary $\delta b_{\alpha\beta}$ and δc^β. This leads to the equations of motion:

$$\partial^\alpha c^\beta = 0 \tag{13}$$

$$\partial^\alpha b_{\alpha\beta} = 0 \tag{14}$$

Similar equations can be derived for the anti-ghost fields $\tilde{b}_{\alpha\beta}$ and $\tilde{c}^\beta$.

3 Conclusion

We have derived the equations of motion for the superstring theory action, including ghost fields, using the principle of least action. The key equations are:

For bosonic fields:

$$\partial_\alpha(h^{\alpha\beta}\partial_\beta X_\mu) = 0 \tag{15}$$

For fermionic fields:

$$\rho^\alpha \partial_\alpha \psi_\mu = 0, \quad \partial_\alpha(\bar{\psi}^\mu \rho^\alpha) = 0 \tag{16}$$

For ghost fields:

$$\partial^\alpha c^\beta = 0, \quad \partial^\alpha b_{\alpha\beta} = 0 \tag{17}$$

These equations describe the dynamics of the string worldsheet in superstring theory.