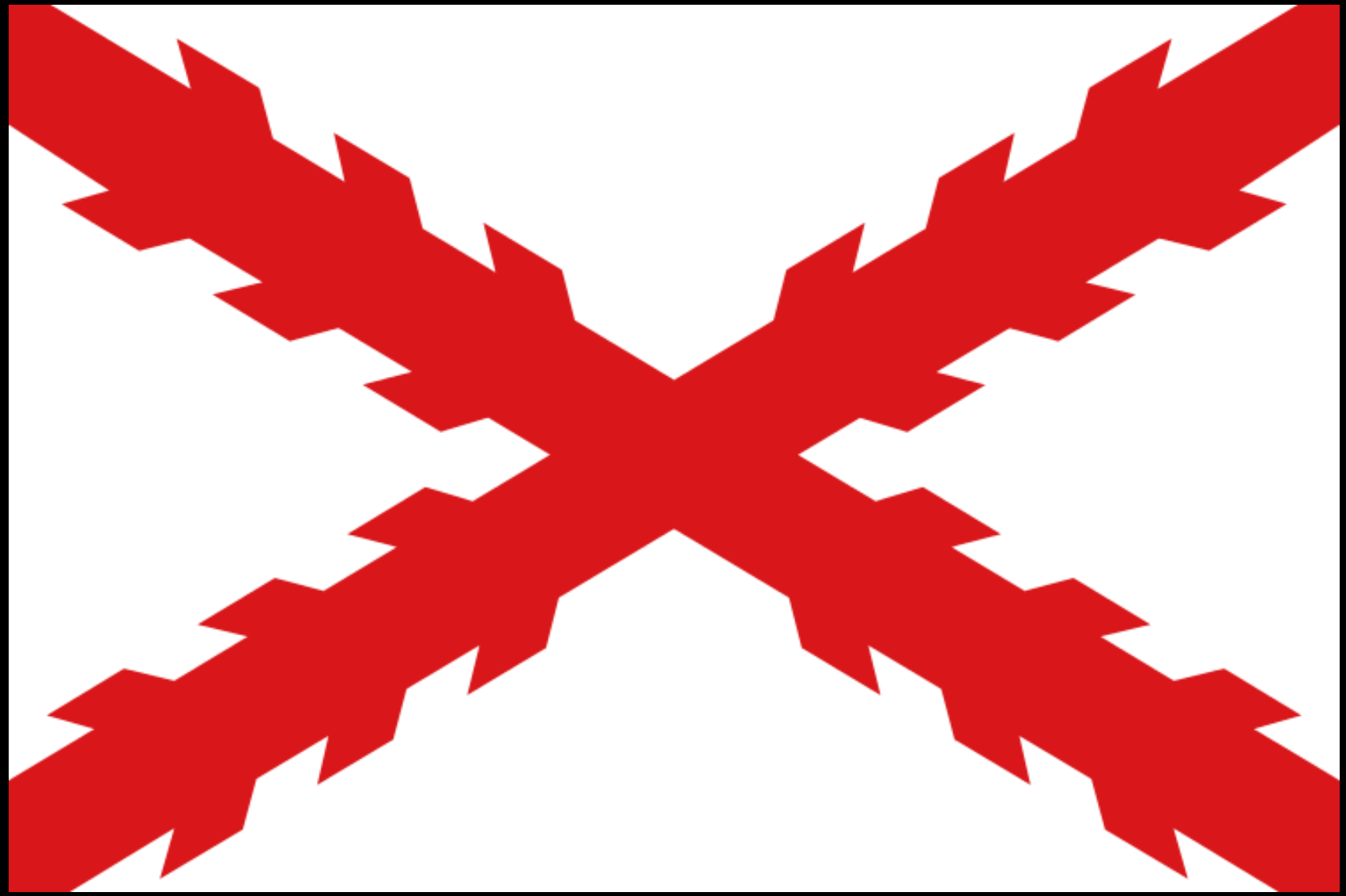
Typos in Tables and One of the Fathers of Computing

Logarithmic Tables, The Difference Engine, and Babbage

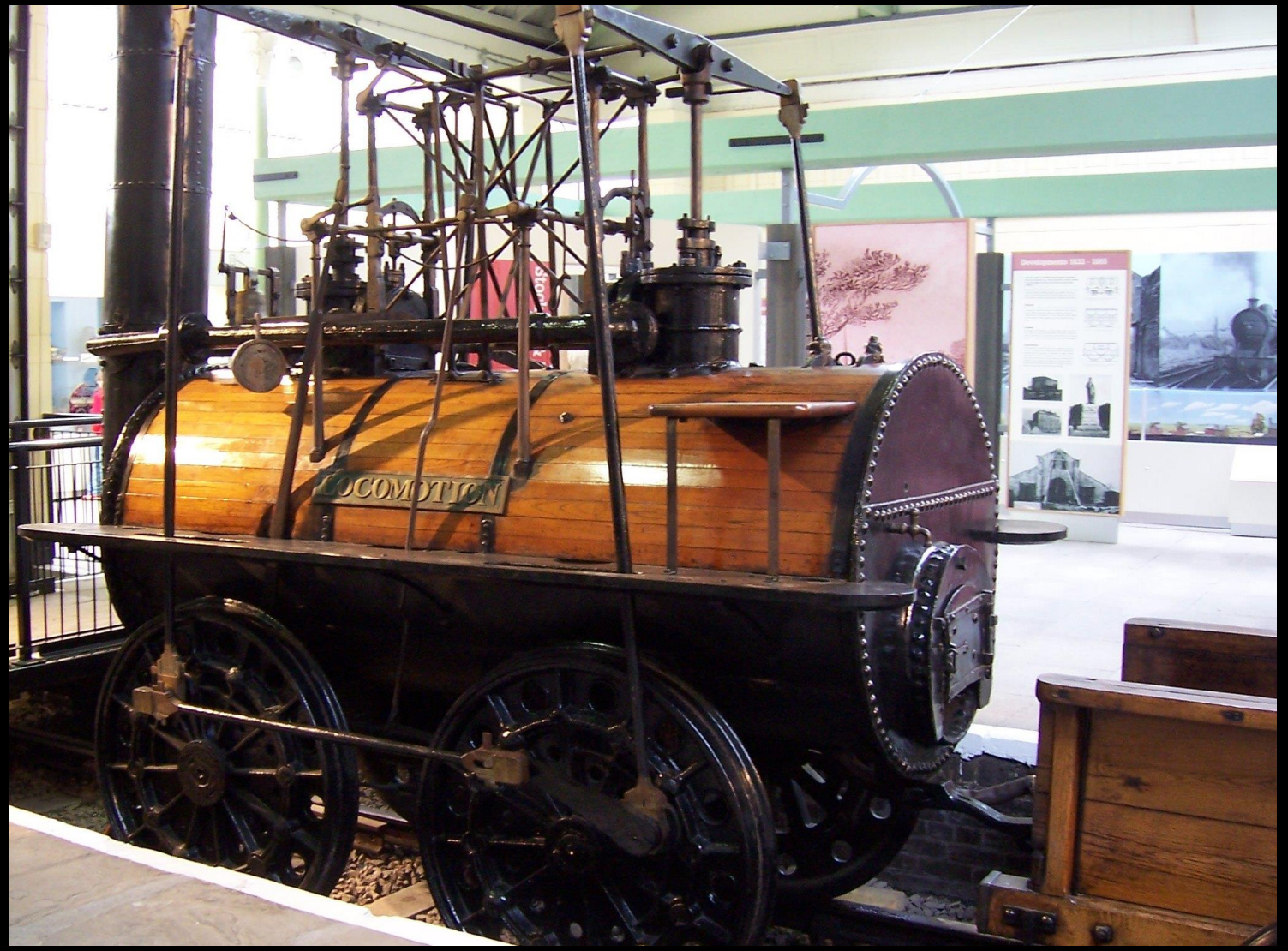
John Haldeman – Hackforge – April/May 2017

Early 1820s

Early 1820s



Early 1820s



Multiply

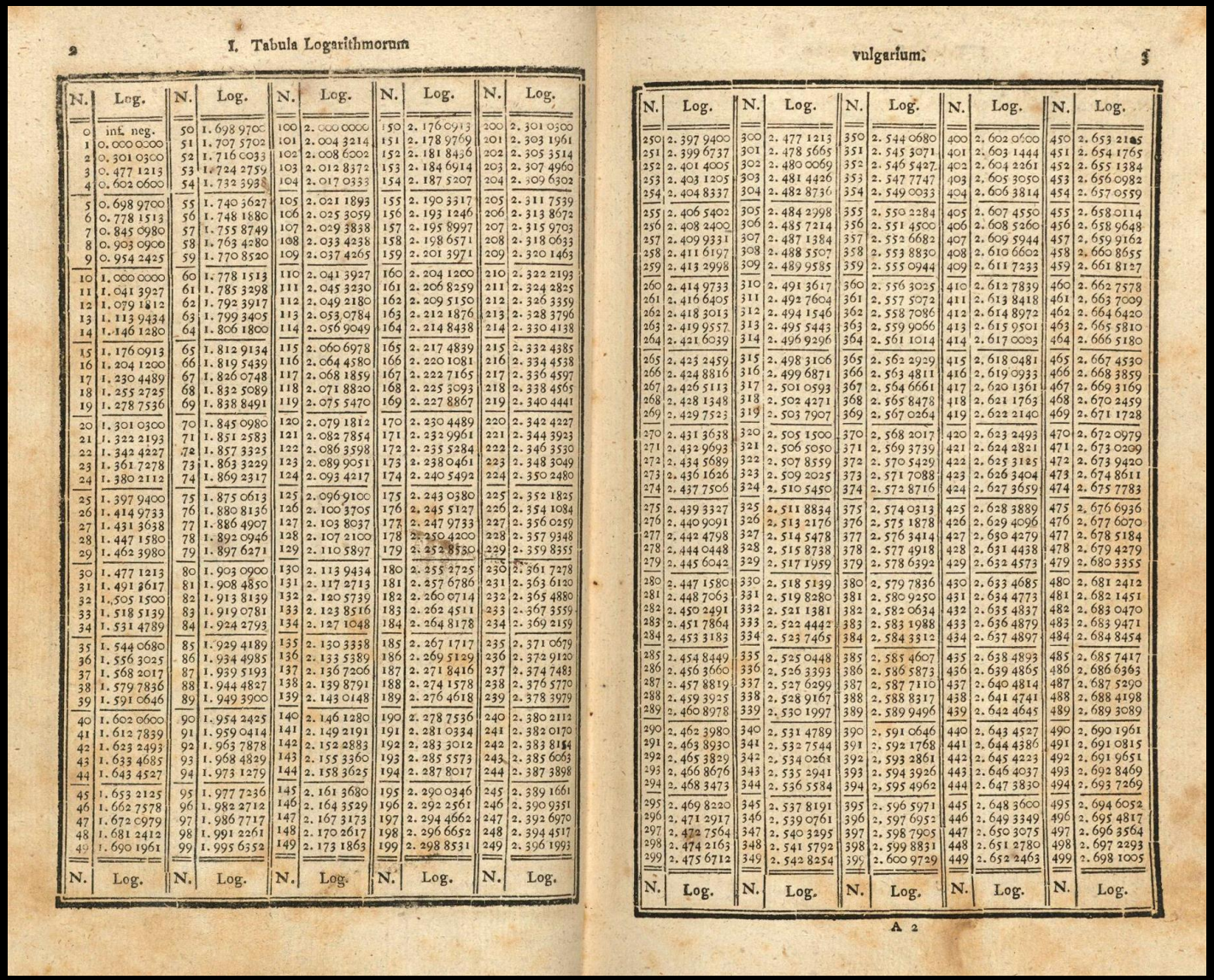
18793.26 and 54778.18

Quickly

Useful Fact

$\log_x(AB) = \log_x(A) + \log_x(B)$

Useful Resource



2 I. Tabula Logarithmorum

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	inf. neg.	50	1. 698 9700	100	2. 000 0000	150	2. 176 0913	200	2. 301 0300
1	0. 000 0000	51	1. 707 5702	101	2. 004 3214	151	2. 178 9769	201	2. 303 1961
2	0. 301 0300	52	1. 716 0033	102	2. 008 6002	152	2. 181 8436	202	2. 305 3514
3	0. 477 1213	53	1. 724 2759	103	2. 012 8372	153	2. 184 6914	203	2. 307 4960
4	0. 602 0600	54	1. 732 3938	104	2. 017 0333	154	2. 187 5207	204	2. 309 6302
5	0. 698 9700	55	1. 740 3627	105	2. 021 1893	155	2. 190 3317	205	2. 311 7539
6	0. 778 1513	56	1. 748 1880	106	2. 025 3059	156	2. 193 1246	206	2. 313 8672
7	0. 845 0980	57	1. 755 8749	107	2. 029 3838	157	2. 195 8997	207	2. 315 9703
8	0. 903 0900	58	1. 763 4280	108	2. 033 4238	158	2. 198 6571	208	2. 318 0633
9	0. 954 2425	59	1. 770 8520	109	2. 037 4265	159	2. 201 3971	209	2. 320 1463
10	1. 000 0000	60	1. 778 1513	110	2. 041 3927	160	2. 204 1200	210	2. 322 2193
11	1. 041 3927	61	1. 785 3298	111	2. 045 3230	161	2. 206 8259	211	2. 324 2825
12	1. 079 1812	62	1. 792 3917	112	2. 049 2180	162	2. 209 5150	212	2. 326 3359
13	1. 113 9434	63	1. 799 3405	113	2. 053 0784	163	2. 212 1876	213	2. 328 3796
14	1. 146 1280	64	1. 806 1800	114	2. 056 9049	164	2. 214 8438	214	2. 330 4138
15	1. 176 0913	65	1. 812 9134	115	2. 060 6978	165	2. 217 4839	215	2. 332 4385
16	1. 204 1200	66	1. 819 5439	116	2. 064 4580	166	2. 220 1081	216	2. 334 4538
17	1. 230 4489	67	1. 826 0748	117	2. 068 1859	167	2. 222 7165	217	2. 336 4597
18	1. 255 2725	68	1. 832 5089	118	2. 071 8820	168	2. 225 3093	218	2. 338 4565
19	1. 278 7536	69	1. 838 8491	119	2. 075 5470	169	2. 227 8867	219	2. 340 4441
20	1. 301 0300	70	1. 845 0980	120	2. 079 1812	170	2. 230 4489	220	2. 342 4227
21	1. 322 2193	71	1. 851 2583	121	2. 082 7854	171	2. 232 9961	221	2. 344 3923
22	1. 342 4227	72	1. 857 3325	122	2. 086 3598	172	2. 235 5284	222	2. 346 3530
23	1. 361 7278	73	1. 863 3229	123	2. 089 9051	173	2. 238 0461	223	2. 348 3049
24	1. 380 2112	74	1. 869 2317	124	2. 093 4217	174	2. 240 5492	224	2. 350 2480
25	1. 397 9400	75	1. 875 0613	125	2. 096 9100	175	2. 243 0380	225	2. 352 1825
26	1. 414 9733	76	1. 880 8136	126	2. 100 3705	176	2. 245 5127	226	2. 354 1084
27	1. 431 3638	77	1. 886 4907	127	2. 103 8037	177	2. 247 9733	227	2. 356 0259
28	1. 447 1580	78	1. 892 0946	128	2. 107 2100	178	2. 250 4200	228	2. 357 9348
29	1. 462 3980	79	1. 897 6271	129	2. 110 5897	179	2. 252 8530	229	2. 359 8355
30	1. 477 1213	80	1. 903 0900	130	2. 113 9434	180	2. 255 2725	230	2. 361 7278
31	1. 491 3617	81	1. 908 4850	131	2. 117 2713	181	2. 257 6786	231	2. 363 6120
32	1. 505 1500	82	1. 913 8139	132	2. 120 5739	182	2. 260 0714	232	2. 365 4880
33	1. 518 5139	83	1. 919 0781	133	2. 123 8516	183	2. 262 4511	233	2. 367 3559
34	1. 531 4789	84	1. 924 2793	134	2. 127 1048	184	2. 264 8178	234	2. 369 2159
35	1. 544 0680	85	1. 929 4189	135	2. 130 3338	185	2. 267 1717	235	2. 371 0679
36	1. 556 3025	86	1. 934 4985	136	2. 133 5389	186	2. 269 5129	236	2. 372 9120
37	1. 568 2017	87	1. 939 5193	137	2. 136 7206	187	2. 271 8416	237	2. 374 7483
38	1. 579 7836	88	1. 944 4827	138	2. 139 8791	188	2. 274 1578	238	2. 376 5770
39	1. 591 0646	89	1. 949 3900	139	2. 143 0148	189	2. 276 4618	239	2. 378 3979
40	1. 602 0600	90	1. 954 2425	140	2. 146 1280	190	2. 278 7536	240	2. 380 2112
41	1. 612 7839	91	1. 959 0414	141	2. 149 2191	191	2. 281 0334	241	2. 382 0170
42	1. 623 2493	92	1. 963 7878	142	2. 152 2883	192	2. 283 3012	242	2. 383 8154
43	1. 633 4685	93	1. 968 4829	143	2. 155 3360	193	2. 285 5573	243	2. 385 6063
44	1. 643 4527	94	1. 973 1279	144	2. 158 3625	194	2. 287 8017	244	2. 387 3898
45	1. 653 2125	95	1. 977 7236	145	2. 161 3680	195	2. 290 0346	245	2. 389 1661
46	1. 662 7578	96	1. 982 2712	146	2. 164 3529	196	2. 292 2561	246	2. 390 9351
47	1. 672 0979	97	1. 986 7717	147	2. 167 3173	197	2. 294 4662	247	2. 392 6970
48	1. 681 2412	98	1. 991 2261	148	2. 170 2617	198	2. 296 6652	248	2. 394 4517
49	1. 690 1961	99	1. 995 6352	149	2. 173 1863	199	2. 298 8531	249	2. 396 1993
N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.

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N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
250	2. 397 9400	300	2. 477 1213	350	2. 544 0680	400	2. 602 0600	450	2. 653 2125
251	2. 399 6737	301	2. 478 5665	351	2. 545 3071	401	2. 603 1444	451	2. 654 1765
252	2. 401 4005	302	2. 480 0069	352	2. 546 5427	402	2. 604 2261	452	2. 655 1384
253	2. 403 1205	303	2. 481 4426	353	2. 547 7747	403	2. 605 3050	453	2. 656 0982
254	2. 404 8337	304	2. 482 8736	354	2. 549 0033	404	2. 606 3814	454	2. 657 0559
255	2. 406 5402	305	2. 484 2998	355	2. 550 2284	405	2. 607 4550	455	2. 658 0114
256	2. 408 2400	306	2. 485 7214	356	2. 551 4500	406	2. 608 5260	456	2. 658 9648
257	2. 409 9331	307	2. 487 1384	357	2. 552 6682	407	2. 609 5944	457	2. 659 9162
258	2. 411 6197	308	2. 488 5507	358	2. 553 8830	408	2. 610 6602	458	2. 660 8655
259	2. 413 2998	309	2. 489 9585	359	2. 555 0944	409	2. 611 7233	459	2. 661 8127
260	2. 414 9733	310	2. 491 3617	360	2. 556 3025	410	2. 612 7839	460	2. 662 7578
261	2. 416 6405	311	2. 492 7604	361	2. 557 5072	411	2. 613 8418	461	2. 663 7009
262	2. 418 3013	312	2. 494 1546	362	2. 558 7086	412	2. 614 8972	462	2. 664 6420
263	2. 419 9557	313	2. 495 5443	363	2. 559 9066	413	2. 615 9501	463	2. 665 5810
264	2. 421 6039	314	2. 496 9296	364	2. 561 1014	414	2. 617 0003	464	2. 666 5180
265	2. 423 2459	315	2. 498 3106	365	2. 562 2929	415	2. 618 0481	465	2. 667 4530
266	2. 424 8816	316	2. 499 6871	366	2. 563 4811	416	2. 619 0933	466	2. 668 3859
267	2. 426 5113	317	2. 501 0593	367	2. 564 6661	417	2. 620 1361	467	2. 669 3169
268	2. 428 1348	318	2. 502 4271	368	2. 565 8478	418	2. 621 1763	468	2. 670 2459
269	2. 429 7523	319	2. 503 7907	369	2. 567 0264	419	2. 622 2140	469	2. 671 1728
270	2. 431 3638	320	2. 505 1500	370	2. 568 2017	420	2. 623 2493	470	2. 672 0979
271	2. 432 9693	321	2. 506 5050	371	2. 569 3739	421	2. 624 2821	471	2. 673 0209
272	2. 434 5689	322	2. 507 8559	372	2. 570 5429	422	2. 625 3125	472	2. 673 9420
273	2. 436 1626	323	2. 509 2025	373	2. 571 7088	423	2. 626 3404	473	2. 674 8611
274	2. 437 7506	324	2. 510 5450	374	2. 572 8716	424	2. 627 3659	474	2. 675 7783
275	2. 439 3327	325	2. 511 8834	375	2. 574 0313	425	2. 628 3889	475	2. 676 6936
276	2. 440 9091	326	2. 513 2176	376	2. 575 1878	426	2. 629 4096	476	2. 677 6070
277	2. 442 4798	327	2. 514 5478	377	2. 576 3414	427	2. 630 4279	477	2. 678 5184
278	2. 444 0448	328	2. 515 8738	378	2. 577 4918	428	2. 631 4438	478	2. 679 4279
279	2. 445 6042	329	2. 517 1959	379	2. 578 6392	429	2. 632 4573	479	2. 680 3355
280	2. 447 1580	330	2. 518 5139	380	2. 579 7836	430	2. 633 4685	480	2. 681 2412
281	2. 448 7063	331	2. 519 8280	381	2. 580 9250	431	2. 634 4773	481	2. 682 1451
282	2. 450 2491	332	2. 521 1381	382	2. 582 0634	432	2. 635 4837	482	2. 683 0470
283	2. 451 7864	333	2. 522 4442	383	2. 583 1988	433	2. 636 4879	483	2. 683 9471
284	2. 453 3183	334	2. 523 7465	384	2. 584 3312	434	2. 637 4897	484	2. 684 8454
285	2. 454 8449	335	2. 525 0448	385	2. 585 4607	435	2. 638 4893	485	2. 685 7417
286	2. 456 3660	336	2. 526 3393	386	2. 586 5873	436	2. 639 4865	486	2. 686 6363
287	2. 457 8819	337	2. 527 6299	387	2. 587 7110	437	2. 640 4814	487	2. 687 5290
288	2. 459 3925	338	2. 528 9167	388	2. 588 8317	438	2. 641 4741	488	2. 688 4198
289	2. 460 8978	339	2. 530 1997	389	2. 589 9496	439	2. 642 4645	489	2. 689 3089
290	2. 462 3980	340	2. 531 4789	390	2. 591 0646	440	2. 643 4527	490	2. 690 1961
291	2. 463 8930	341	2. 532 7544	391	2. 592 1768	441	2. 644 4386	491	2. 691 0815
292	2. 465 3829	342	2. 534 0261	392	2. 593 2861	442	2. 645 4223	492	2. 691 9651
293	2. 466 8676	343	2. 535 2941	393	2. 594 3926	443	2. 646 4037	493	2. 692 8469
294	2. 468 3473	344	2. 536 5584	394	2. 595 4962	444	2. 647 3830	494	2. 693 7269
295	2. 469 8220	345	2. 537 8191	395	2. 596 5971	445	2. 648 3600	495	2. 694 6052
296	2. 471 2917	346	2. 539 0761	396	2. 597 6952	446	2. 649 3349	496	2. 695 4817
297	2. 472 7564	347	2. 540 3295	397	2. 598 7905	447	2. 650 3075	497	2. 696 3564
298	2. 474 2163	348	2. 541 5792	398	2. 599 8831	448	2. 651 2780	498	2. 697 2293
299	2. 475 6712	349	2. 542 8254	399	2. 600 9729	449	2. 652 2463	499	2. 698 1005
N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.

A 2

"Tabula logarithmorum vulgarium" from Logarithmisch-trigonometrische Tafeln nebst andern zum gebrauch der Mathematik eingerichteten Tafeln und Formeln, Leipzig: 1797, by Georg, Freiherr von Vega (1754-1802).

"Math 837.97*, Houghton Library, Harvard University"

Multiplication With Log Table Lookups

$\log_{10}(18793.26) \approx 4.274005$

$\log_{10}(54778.18) \approx 4.738605$

$\log_{10}(18793.26 \times 54778.18) \approx 4.274005 + 4.738605 = 9.01261$

$18793.26 \times 54778.18 \approx 10^{9.01261} = 10^{0.01261} \times 10^{9} =$

1.02946×10^{9}

KEY:

Looked Up in a Table	Useful Fact	Hard Work

Multiplication!

- This method takes 3 lookups and 1 addition – super fast

- Quality Problems?
  - Incorrect Addition
  - Human error in looking up the numbers incorrectly in the table
  - Mistakes in the tables themselves

Babbage May Have Been Preoccupied with Lookup Tables…… (studies in human error in lookups)

40. Specimen of Logarithmic Tables, printed with different coloured inks and on variously-coloured papers, in twenty-one volumes 8vo. London. 1831.

The object of this Work, of which *one single copy only* was printed, is to ascertain by experiment the tints of the paper and colours of the inks least fatiguing to the eye.

One hundred and fifty-one variously-coloured papers were chosen, and the same two pages of my stereotype Table of Logarithms were printed upon them in inks of the following

green, dark green, olive, yellow, lig

Each of these twenty volumes

numbered in the same order, and

each kind of ink.

The twenty-first volume contains

in gold, silver, and copper, upon vell

For the same purpose, about thirt

logarithms were printed on thick dr

An account of this work may be

(*Brewster's,*) 1832. Vol. vi. p. 144

38 ——— Specimen of Logarithmic Tables, printed with different coloured inks on variously coloured papers, in 21 vols., 8vo, *half-bound*, UNIQUE, this being the only copy printed. 1831

["The object of the experiment was to ascertain the colour of the inks and the tints of papers least fatiguing to the eye. For that purpose about 140 differently coloured papers were chosen, and ten different colours of ink were employed." One volume is printed in metallic inks.]

Errors in the Tables Themselves:

“I wish to God these calculations had been executed by steam”

- Charles Babbage (1821) - Pictured

"An undetected error in a logarithmic table is like a sunken rock at sea yet undiscovered, upon which it is impossible to say what wrecks may have taken place.“

- Sir John Herschel, 1842

Useful Fact

$$\ln(1 - x) = \quad - (x + x^2/2 + x^3/3 + x^4/4 + x^5/5 + \ldots)$$

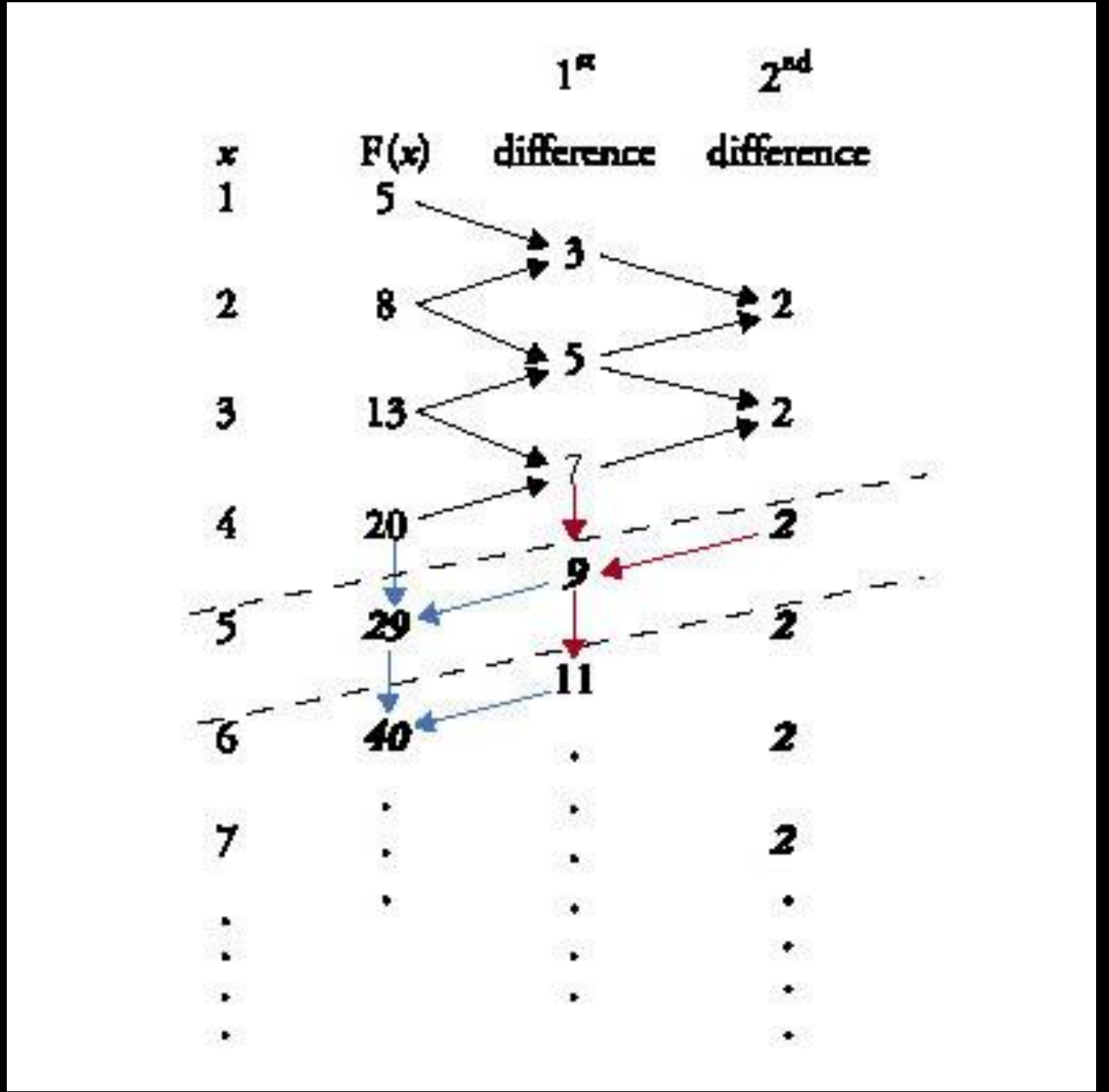
Important thing – if you can calculate a polynomial you can approximate a logarithm

Similar stuff exists for other common items in lookup tables (trig functions for example)

The Difference Method

For an arbitrary polynomial, you can calculate the values in the series using only subtraction/addition

Pictured: $f(x) = x^2 + 4$

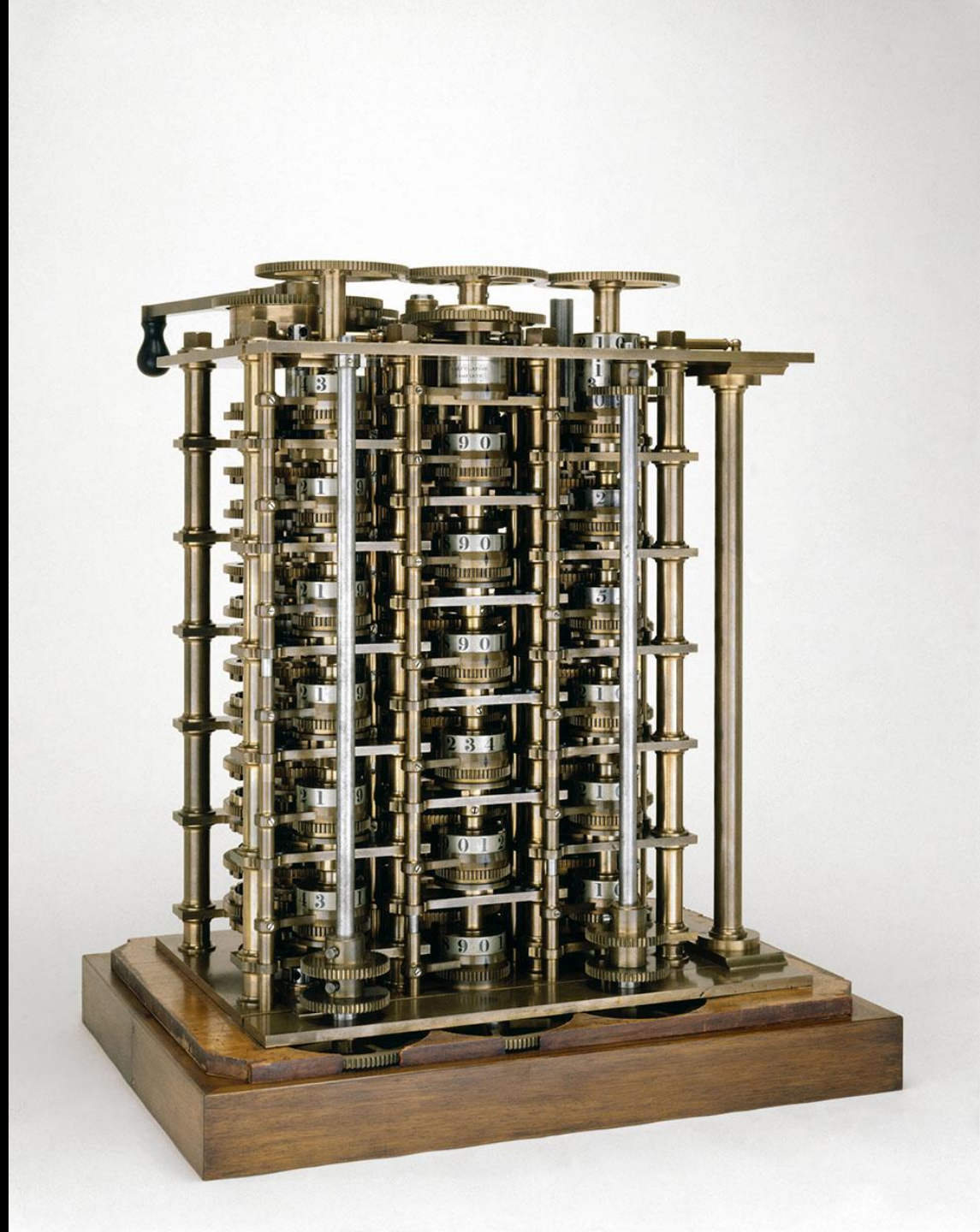


The Difference Engine

A Machine to Calculate Polynomials Using the Difference Method and Print the Results

8 Storage Registers and 7 Adders

Shown: The Sample built in 1820



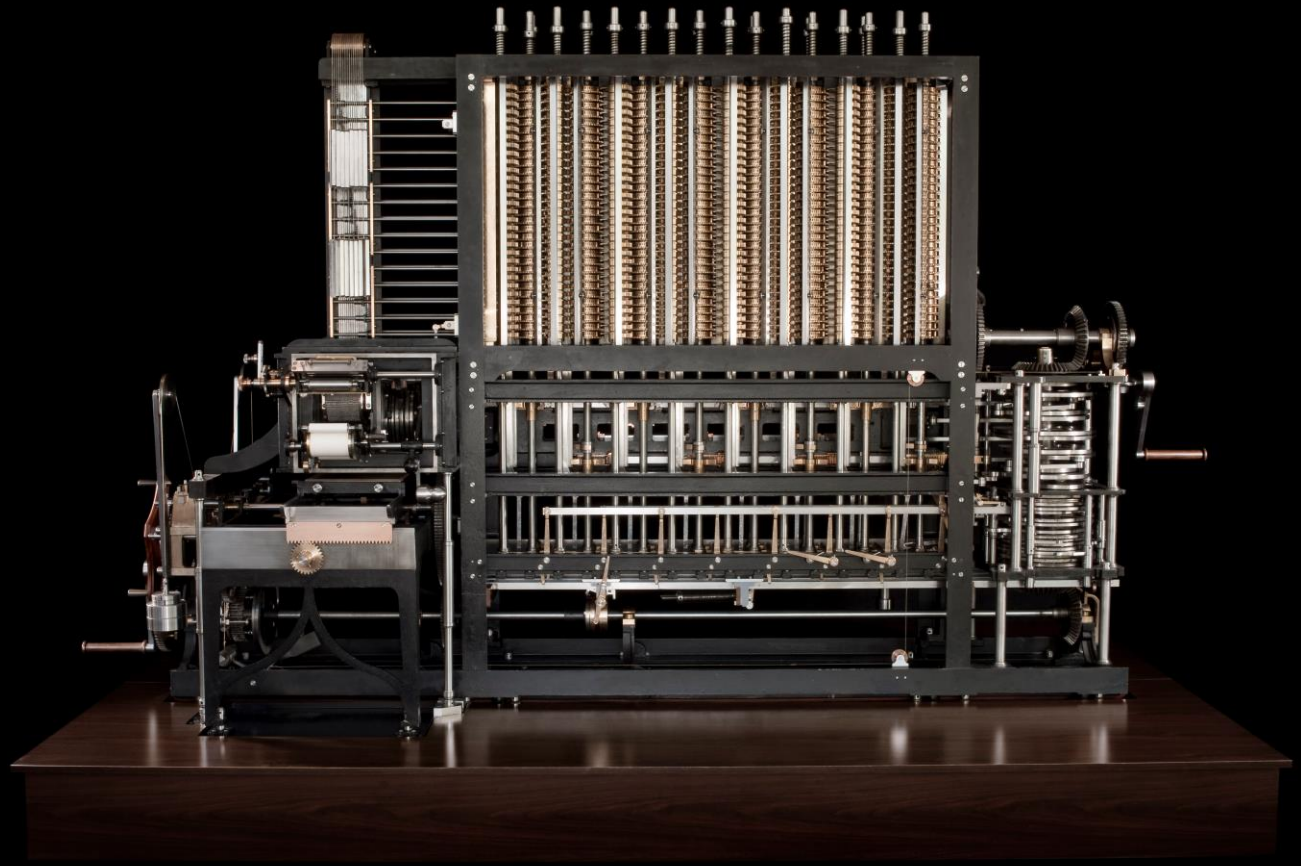
$x^7 + 4x$: Initialization

x	f(x)	1	2	3	4	5	6	7
1	5	131	1932	10206	25200	31920	20160	5040
2	136	2063	12138	35406	57120	52080	25200	5040
3	2199	14201	47544	92526	109200	77280	30240	5040
4	16400	61745	140070	201726	186480	107520	35280	
5	78145	201815	341796	388206	294000	142800		
6	279960	543611	730002	682206	436800			
7	823571	1273613	1412208	1119006				
8	2097184	2685821	2531214					
9	4783005	5217035						
10	10000040							

$x^7 + 4x$: 7 Register Continuous Addition (continuously add adjacent columns)

x	f(x)	1	2	3	4	5	6	7
1	5	131	1932	10206	25200	31920	20160	5040
2	136	2063	12138	35406	57120	52080	25200	5040
3	2199	14201	47544	92526	109200	77280	30240	5040
4	16400	61745	140070	201726	186480	107520	35280	5040
5	78145	201815	341796	388206	294000	142800	40320	5040
6	279960	543611	730002	682206	436800	183120	45360	5040
7	823571	1273613	1412208	1119006	619920	228480	50400	5040
8	2097184	2685821	2531214	1738926	848400	278880	55440	5040
9	4783005	5217035	4270140	2587326	1127280	334320	60480	5040
10	10000040	9487175	6857466	3714606	1461600	394800	65520	5040
11	19487215	16344641	10572072	5176206	1856400	460320	70560	5040
12	35831856	26916713	15748278	7032606	2316720	530880	75600	5040
13	62748569	42664991	22780884	9349326	2847600	606480	80640	5040
14	105413560	65445875	32130210	12196926	3454080	687120	85680	5040
15	170859435	97576085	44327136	15651006	4141200	772800	90720	5040
16	268435520	141903221	59978142	19792206	4914000	863520	95760	5040
17	410338741	201881363	79770348	24706206	5777520	959280	100800	5040

The Difference Engine – Completed Machine (2002)



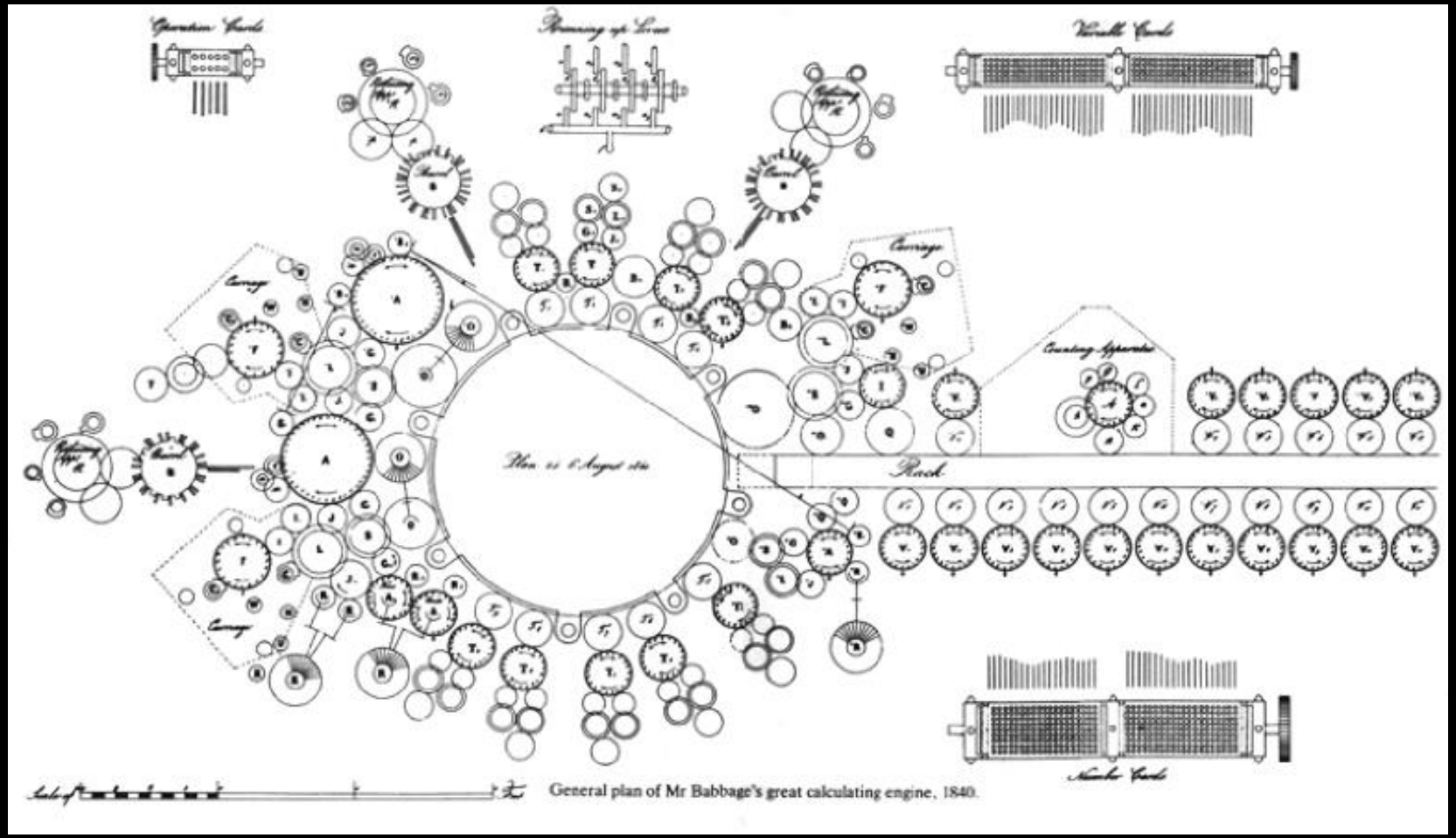
https://youtu.be/qctHEGKr9Zs?t=53

The Difference Engine's Legacy

Babbage never completed his machine due to:

- Economics
- Politics
- Personalities (Babbage, Joseph Clement, Robert Peel)

Moved on to the Analytical Engine: a general-purpose programmable computing machine



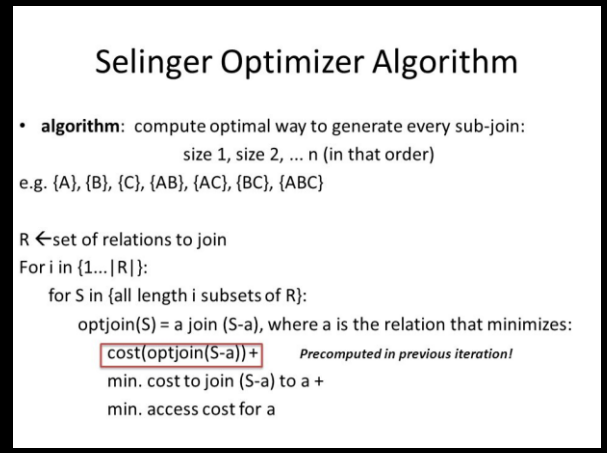
General plan of Mr Babbage's great calculating engine, 1840.

Lessons for Today

- Precomputation Saves Time/Resources
  - A reason caching is popular

- Technology and ingenuity is an important for success, but so is politics/personalities/economics

- The difference method and Babbage's memory efficient computation of it has interesting lessons algorithmically



Lessons for Today

Caching

Selinger Optimizer Algorithm

- **algorithm**: compute optimal way to generate every sub-join:
  size 1, size 2, ... n (in that order)

e.g. {A}, {B}, {C}, {AB}, {AC}, {BC}, {ABC}

R ←set of relations to join
For i in {1...|R|}:
  for S in {all length i subsets of R}:
    optjoin(S) = a join (S-a), where a is the relation that minimizes:
      cost(optjoin(S-a))+ *Precomputed in previous iteration!*
      min. cost to join (S-a) to a +
      min. access cost for a

Memory Efficient Dynamic Programming Algorithms

Sir Robert Peel

Computers are Old

- Well…. Computation is old
- A great deal of the things you are working on have long histories
- I think every modern developer can relate to Babbage in several ways
- Today:
  - Saying the industry or profession is “immature” is not really true
  - Keeping Up:
    - Obsolescence anxiety might be a little bit of an illusion
    - Problem anxiety is essential

Great Resources

CHARLES BABBAGE and the Difference Engine - Aditi Kar
http://people.maths.ox.ac.uk/kar/Babbage.html

A Practical Use For Logarithms, Part 2 - Santo D'Agostino
https://qedinsight.wordpress.com/2011/04/22/a-practical-use-for-logarithms-part-2-how-we-multiplied-large-numbers-40-years-ago-and-how-integral-transforms-use-the-same-basic-idea/

The Babbage Engine – Computer History Museum
http://www.computerhistory.org/babbage/

Babbage's Difference Engine No. 2, Part 1: The method of finite differences
https://www.youtube.com/watch?v=PFMBU17eo_4