18.01 Single Variable Calculus
Fall 2006

18.01 Practice Exam 4 Solutions

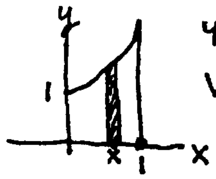
1 $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/6} \frac{\sin^2 u}{\cancel{\cos u}} \cdot \cancel{\cos u}\,du$

Put $x = \sin u$ (or $x = \cos u$) $\frac{1}{2} = \sin \pi/6$

$$= \int_0^{\pi/6} \frac{1-\cos 2u}{2}\,du$$

$$= \frac{u}{2} - \frac{\sin 2u}{4}\Big]_0^{\pi/6} = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$

2



$y = e^x$

$$\text{Volume} = \int_0^1 2\pi x e^x\,dx$$

$$\int x e^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x$$

$$\therefore \text{volume} = 2\pi(xe^x - e^x)\Big|_0^1 = 2\pi(0-(-1)) = 2\pi$$

3 I forgot

$$\frac{4x}{(x^2-1)(x-1)} = \frac{4x}{(x-1)^2(x+1)}$$

$$= \frac{2}{(x-1)^2} + \frac{B}{x-1} + \frac{-1}{x+1}$$

by coverup   by coverup

Put $x=0$: $0 = 2 - B - 1$; $B = 1$

Integrating: silly me

$$\int \frac{4x\,dx}{(x^2-1)(x+1)} = \frac{-2}{x-1} + \ln(x-1) - \ln(x+1) + c$$

$$= \frac{-2}{x-1} + \ln\left(\frac{x-1}{x+1}\right) + c$$

4 $\displaystyle\int_a^b \sqrt{1+y'^2}\,dx$ $y = \sin^2 x$, $y' = 2\sin x\cos x = \sin 2x$

Arclength

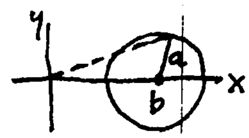
$$= \int_0^{\pi/4} \sqrt{1+\sin^2 2x}\,dx$$

Since $0 \le \sin^2 2x \le 1$ on the interval,

$$\int_0^{\pi/4} \sqrt{1+\sin^2 2x}\,dx \le \frac{\pi}{4} \cdot \sqrt{2} < \frac{3.2}{4} \cdot \frac{3}{2} = 1.2$$

length of interval a mess

5

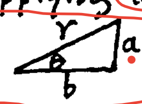


a) $(x-b)^2 + y^2 = a^2$ (by translation of $x^2+y^2=a^2$ to $(b,0)$ center)

$x^2 + y^2 - 2bx + b^2 = a^2$

$$\boxed{r^2 - 2br\cos\theta = a^2 - b^2}$$

b) Applying law of cosines to

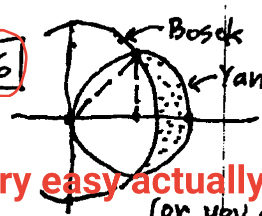


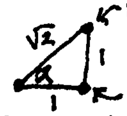
we get

$a^2 = r^2 + b^2 - 2br\cos\theta$, same as part (a).

DON'T HAVE TO DO!

6





shows this is a right Δ, $\alpha = \pi/4$

very easy actually

(or you can solve $\begin{cases} r = 2\cos\theta \\ r = \sqrt{2} \end{cases}$ simultaneously, for θ)

Using symmetry:

$$\text{Area} = 2 \cdot \frac{1}{2}\int_0^{\pi/4} (2\cos\theta)^2 - (\sqrt{2})^2\,d\theta$$

b) $$= 2\int_0^{\pi/4} (\underbrace{2\cos^2\theta - 1}_{\cos 2x})\,d\theta = \frac{2\sin 2x}{2}\Big]_0^{\pi/4} = 1$$

[By elem. geometry:

$= \frac{1}{2} + \frac{\pi}{4}$

$= \frac{\pi(\sqrt{2})^2}{8} + A$ $\therefore A = \frac{1}{2}$, $2A = 1$]