18.01 Single Variable Calculus
Fall 2006

18.01 Practice Questions for Exam 1

Solutions will be posted on the 18.01 website

Problem 1. Evaluate each of the following:

a) $\dfrac{d}{dx}\left.\dfrac{\sqrt{x}}{1+2x}\right|_{x=1}$

b) $\dfrac{d}{du}(u \ln 2u)$ (simplify your answer)

Problem 2. a) Evaluate $\dfrac{d}{dt}\sqrt{1-k\cos^2 t}$, where k is constant.

b) Check your answer to part (a) by showing that if $k = 1$, your answer agrees with the derivative calculated by a simpler method.

Problem 3. Derive the formula for $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)$ directly from the definition of derivative.

(You will need to transform the difference quotient algebraically before taking the limit.)

Problem 4. Derive the formula for $\dfrac{d}{dx}\sin^{-1} x$ by solving $y = \sin^{-1} x$ for x and using implicit differentiation.

(You may use the known values of $D\sin x$ and $D\cos x$ in your derivation. Your answer must be expressed in terms of x.)

Problem 5. Find all values of the constants a and b for which the function defined by

$$f(x) = \begin{cases} ax + b, & x > 1 \\ x^2 - 3x + 2, & x \le 1 \end{cases}$$

will be differentiable. differentiable and continuous diff

Problem 6. Evaluate the following, with enough indications to show you are not just guessing:

a) $\lim\limits_{u\to 0}\dfrac{\tan 2u}{u}$

b) $\lim\limits_{h\to 0}\dfrac{e^h - 1}{h}$ (relate it to the value of a derivative)

Problem 7. A hawk is pursuing a mouse. We choose a coordinate system so the mouse runs along the x-axis in the negative direction, and the hawk is flying over the x-axis, swooping down along the exponential curve $y = e^{kx}$, for some positive constant k. The hawk in flight is always aimed directly at the mouse. It is noon at the equator, and the sun is directly overhead.

When the hawk's shadow on the ground is at the point x_0, where is the mouse?