18.01 Single Variable Calculus
Fall 2006

18.01 Practice Exam 3 Sol'ns Fall 2006

slice, dx or dy

1 a) $\cos 2x = \cos^2 x - \sin^2 x$
$= (1-\sin^2 x) - \sin^2 x$
$= 1 - 2\sin^2 x$

$$\int \sin^2 x\,dx = \int \frac{(1-\cos 2x)}{2}\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

b) $D(x\ln x) = \ln x + x\cdot\frac{1}{x} = \ln x + 1$

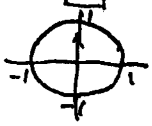
$\therefore$ by the fundamental theorem,

$$x\ln x\Big]_1^e = \int_1^e \ln x\,dx + \int_1^e 1\,dx$$

$$e\cdot 1 - 0 = \int_1^e \ln x\,dx + e - 1$$

$$\therefore \int_1^e \ln x\,dx = 1.$$

2 By horizontal slices, (calculate vol. of top half + double it)



$$= \int_0^1 \pi x^2\,dy = \pi\int_0^1 (1-y^4)\,dy$$

$$= \pi\left(y - \frac{y^5}{5}\right)\Big|_0^1 = \pi\cdot\frac{4}{5}$$

By cylindrical shells: $y = (1-x^2)^{1/4}$

$$= \int_0^1 2\pi x\cdot(1-x^2)^{1/4}$$

$$= -\frac{4\pi}{5}(1-x^2)^{5/4}\Big]_0^1 = \frac{4\pi}{5}$$

$\therefore$ Volume is $\frac{8\pi}{5} \approx \frac{8\cdot(3.14)}{5} > \frac{25}{5}$

5 cubic feet is not enough.

3 a) $F(x) = \int_0^x t^2 e^{-t^2}\,dt$; $F'(x) = x^2 e^{-x^2}$ (second fund thm)

b) $F' = 0$ when $x = 0$; otherwise $F'(x) > 0$. Thus F is increasing, so $x = 0$ is a point of horiz. inflection (not a max or min)

c) $u = t^2$; $du = 2t\,dt$ $\quad \int_0^9 \sqrt{u}\,e^{-u}\,du = \int_0^3 t\,e^{-t^2}\,2t\,dt = 2\cdot F(3)$

d) $e^{-t^2} \le 1$

$$\therefore \int_0^x t^2 e^{-t^2}\,dt \le \int_0^x t^2\,dt = \frac{x^3}{3}.$$

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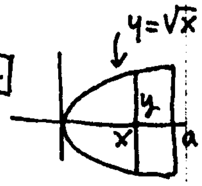

Area of slice at x is $\pi y^2 = \pi x$

Average area of slices $= \frac{1}{a}\int_0^a \pi x\,d$

$$= \frac{1}{a}\pi\frac{x^2}{2}\Big]_0^a$$

Therefore average area $= \frac{\pi a}{2}$

which is the area of the slice at $x_0 = a/2$ (halfway)

$$\pi\cdot(\sqrt{x_0})^2 = \frac{\pi a}{2} \Rightarrow x_0 = a/2$$

5

1	8	15	22	29
3	2	0	1	3

a) by trapezoidal rule:

Total # hits $\approx \left(\frac{3}{2} + 2 + 0 + 1 + \frac{3}{2}\right)7 = 6\cdot 7 = \boxed{42}$

b) by Simpson's rule:

Total # hits $\approx \frac{(3 + 4\cdot 2 + 2\cdot 0 + 4\cdot 1 + 3)}{6}\cdot 14 = \frac{18}{6}\cdot 14 = \boxed{42}$

6 In an infinitesimal time interval dt at time t,

$C = 2 - \frac{1}{10}t$

concentration over time slope $= -\frac{1}{10}$

flow rate of water at time t into pool (cc/hour) $= t^2(10-t)^2\cdot 10^4$

pool surface is $(100\text{ cm})^2$

$\therefore$ amt entering from time t to $t+dt$

(*) $= t^2(10-t)^2\cdot 10^4\cdot\left(2 - \frac{t}{10}\right)\cdot d$

Total amt $= 10^4\int_0^{10} t^2(10-t)^2\cdot\left(2-\frac{t}{10}\right)dt$ (**) nanograms

For Δt calculation:
replace dt by Δt in (*)
write (**) as $\sum_1 10^4 t_i^2(10-t_i)^2(2 - t_i/10)\Delta$
and pass to limit as $n \to \infty$ $\quad$ integral given.

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