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18.01 Single Variable Calculus

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Problem 1. (10 pts.) Find the tangent line to $y = \frac{1}{3}x^2$ at $x = 1$

Problem 2. Find the derivative of the following functions:

a. (7 pts.) $\frac{x}{\sqrt{1-x}}$

b. (8 pts.) $\frac{\cos(2x)}{x}$

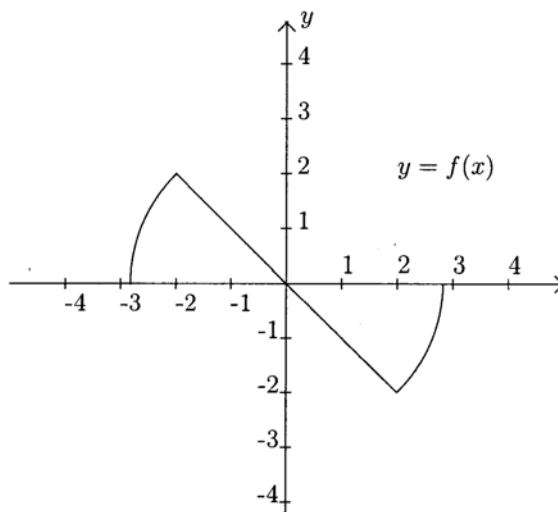
c. (5 pts.) $e^{2f(x)} = g(x)$

d. (5 pts.) $\ln(\sin x)$

Problem 3. (15 pts.) Find $\frac{dy}{dx}$ for the function for the function y defined implicitly by

$$y^4 + xy = 4 \text{ at } x=3, y=1$$

Problem 4. (15 pts.) Draw the graph of the derivative of the function (qualitatively accurate) directly under the graph of the function.



Problem 5. (15 pts) Let

$$f(x) = \begin{cases} ax + b & x < 1 \\ x^4 + x + 1 & x \geq 1 \end{cases}$$

Find all a and b such that the function $f(x)$ is differentiable.

Problem 6. Evaluate these limits by relating them to a derivative.

a. (5 pts.) Evaluate $\lim_{x \rightarrow 0} \frac{(1+2x)^{10} - 1}{x}$

b. (5 pts.) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x}$

Problem 7. (10 pts.) Derive the formula $\frac{d}{dx} a^x = M(a)a^x$ directly from the definition of the derivative, and identify $M(a)$ as a limit. ?