18.01 Single Variable Calculus
Fall 2006

18.01 Problem Set 4

Due Friday 10/20/06, 1:55 pm

This is all of Problem Set 4 (not split into 4A and 4B). Although it is not due until after Exam 2, you should do all the Part I exercises through Lecture 16 and all the Part II problems through Problem 4 before the exam, in order to prepare for it. Practice exam problems and an actual past exam will be posted on line as usual.

Part I (20 points)

Lecture 14. Fri. Oct. 6 Mean-value theorem. Inequalities.
Read: 2.6 to middle p. 77, Notes MVT Work: 2G-1b, 2b, 5, 6

(Columbus Day Holiday. No classes Mon and Tues, Oct 9 and 10)

Lecture 15. Thurs. Oct. 12 Differentials and antiderivatives.
Read: 5.2, 5.3 Work: 3A-1de, 2acegik, -3aceg

Lecture 16. Fri. Oct. 13 Differential equations; separating variables.
Read: 5.4, 8.5 Work: 3F-1cd, 2ae, 4bcd, 8b

Lecture 17. Tues. Oct 17 **Exam 2** Covers Lectures 8–16.

Lecture 18. Thurs. Oct. 19 Definite integral; summation notation.
Read: 6.3 though formula (4); skip proofs; 6.4, 6.5
Work: 3B-2ab, 3b, 4a, 5 4J-1 (set up integral; do not evaluate)

Lecture 19. Fri. Oct. 20 First fundamental theorem. Properties of integrals.
Read: 6.6, 6.7 to top p. 215 (Skip the proof pp. 207-8, which will be discussed in Lec 20.)
Work: assigned on PS 5

Part II (36 points + 10 extra credit)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

0. (not until due date; 3 pts) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". (See full explanation on PS1).

1. (Lec 14, 10pts: 2 + 2 + 2 + 2 + 2))

a) Use the mean value property to show that if $f(0) = 0$ and $f'(x) \geq 0$, then $f(x) \geq 0$ for all $x \geq 0$.

b) Deduce from part (a) that $\ln(1 + x) \leq x$ for $x \geq 0$. Hint: Use $f(x) = x - \ln(1 + x)$.

c) Use the same method as in (b) to show $\ln(1+x) \geq x - x^2/2$ and $\ln(1+x) \leq x - x^2/2 + x^3/3$ for $x \geq 0$.

d) Find the pattern in (b) and (c) and make a general conjecture.

e) Show that $\ln(1+x) \le x$ for $-1 < x \le 0$. (Use the change of variable $u = -x$.)

2. (Lec 15, 4 pts: 2 + 2)

a) Do 5.3/68

b) Show that both of the following integrals are correct, and explain.

$$\int \tan x \sec^2 x dx = (1/2)\tan^2 x; \quad \int \tan x \sec^2 x dx = (1/2)\sec^2 x$$

3. (Lec 16, 6 pts: 3 + 3)

a) Do 8.6/5 (answer in back of text)

b) Do 8.6/6 (optional?)

4. (Lec 16, 7 pts: 2 + 3 + 2) Do 3F-5abc

STOP HERE. DO THE REST AFTER EXAM 2.

5. (Lec 18, 6 pts) Calculate $\int_0^1 e^x dx$ using lower Riemann sums. (You will need to sum a geometric series to get a usable formula for the Riemann sum. To take the limit of Riemann sums, you will need to evaluate $\lim_{n\to\infty} n(e^{1/n} - 1)$, which can be done using the standard linear approximation to the exponential function.)

6. (Lec 16; extra credit: 10 pts: 2 + 2 + 3 + 3) **More about the hypocycloid.** We use differential equations to find the curve with the property that the portion of its tangent line in the first quadrant has fixed length.

a) Suppose that a line through the point (x_0, y_0) has slope m_0 and that the point is in the first quadrant. Let L denote the length of the portion of the line in the first quadrant. Calculate L^2 in terms of x_0, y_0 and m_0. (Do not expand or simplify.)

b) Suppose that $y = f(x)$ is a graph on $0 \le x \le L$ satisfying $f(0) = L$ and $f(L) = 0$ and such that the portion of each tangent line to the graph in the first quadrant has the same length L. Find the differential equation that f satisfies. Express it in terms of L, x, y and $y' = dy/dx$. (Hints: This requires only thought, not computation. Note that $y = f(x)$, $y' = f'(x)$. Don't take square roots, the expression using L^2 is much easier to use. Don't expand or simplify; that would make things harder in the next step.)

c) Differentiate the equation in part (b) with respect to x. Simplify and write in the form

$$(\text{something})(xy' - y)y'' = 0$$

(This starts out looking horrendous, but simplifies considerably.)

d) Show that one solution to the equation in part (c) is $x^{2/3} + y^{2/3} = L^{2/3}$. What about two other possibilities, namely, those solving $y'' = 0$ and $xy' - y = 0$?