18.01 Single Variable Calculus
Fall 2006

18.01 Practice Exam 4

Problem 1. (15) Evaluate $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$ by making a trigonometric substitution; remember to change the limits also.

Problem 2. (15) Find the volume of the solid obtained by rotating about the y-axis (that's the y-axis) the area under the graph of $y = e^x$ and over the interval $0 \le x \le 1$.

(Suggestion: use cylindrical shells.)

Problem 3. (20) Evaluate $\displaystyle\int \frac{4x}{(x^2-1)(x-1)}\,dx.$

(Begin by factoring the denominator completely; don't forget the final integration.)

Problem 4. (15: 8, 7)

a) Write down the definite integral of the form $\displaystyle\int_a^b f(x)\,dx$ which represents the arclength of the curve $y = \sin^2 x$ for $0 \le x \le \pi/4$.

b) Show that the arclength is less than 1.2 by estimating the integral.
(Indicate how you are estimating it. It helps to write the integral in the simplest-looking form.)

Problem 5. (15)

A circle in the xy-plane has radius a and center at the point $(b, 0)$ on the x-axis; assume $a < b$.

a) Write its equation in rectangular coordinates, and change the equation to polar coordinates.

Problem 5b can be skipped; no question similar to 5b will appear on the 2006 exam.

b) Check your answer by deriving it directly in polar coordinates, using a geometric formula.

I forgot how professor derived it in the lec 32

Problem 6. (20)

Once every century the Klingon moon Bosok completely eclipses the moon Yan-ki. As seen from the city Mitek, the two moons appear as circular discs, with radii respectively $\sqrt{2}$ and 1 (units: dorks).

When the center of Bosok is exactly on the circumference of Yan-ki, the problem is to find the area (in square dorks) of the region of the Yan-ki disc that is not yet covered.

a) Draw a diagram in polar coordinates, placing the center of Bosok at the origin, and the other center on the positive x-axis. Set up a definite integral representing the area of the desired region. Indicate how the significant limit on the integral was determined.

very interesting question it is

b) Evaluate the integral.

(If you have time, check your answer by elementary geometry, but no credit for this, just a warm glow of satisfaction and a smug feeling of superiority.)