18.01 Single Variable Calculus
Fall 2006

EXAM 3 PRACTICE : SOLUTIONS : FALL 2006

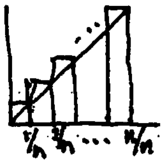
$\boxed{1}$ a) $\int_0^1 \frac{x\,dx}{\sqrt{1+3x^2}} = \frac13(1+3x^2)^{1/2}\Big]_0^1 = \frac13(2-1) = 1/3$

b) $\int_{\pi/3}^{\pi/2} \cos^3 x \sin 2x\,dx = \int_{\pi/3}^{\pi/2} 2\cos^4 x \sin x\,dx$

$= -\frac25 \cos^5 x\Big]_{\pi/3}^{\pi/2} = -\frac25\left[0-\left(\frac12\right)^5\right]$

$= \frac{1}{5\cdot 16}$

$\boxed{2}$



Upper sum is sum of circums. rectangles

$= \sum_{i=1}^{n} \left(\frac{i}{n}\right)\frac1n$
height width

$= \frac{1}{n^2}\sum_1^n i = \frac{n^2+n}{2n^2} = \frac12 + \frac{1}{2n}$

$\lim_{n\to\infty} = \int_0^1 x\,dx = \frac12$

$\boxed{3}$ $\underset{0}{\vdash}\overset{t_1\ t_2\ \cdots}{\rule{5em}{0.4pt}}\underset{1}{\dashv}$ Divide t-interval into n equal subintervals, length Δt.

⊛ $k\Delta t$ deposited in the i^{th} time interval grows by year-end to $k\Delta t e^{r(1-t_i)}$

?

Total at year-end $\approx \sum^n k e^{r(1-t_i)}\Delta t$

As $n\to\infty$, this $\to \int_0^1 k e^{r(1-t)}dt$

[or can make argument using dt instead of Δt : replace Δt by dt in ⊛, then pass directly to the last line]

$\boxed{4}$ a) On $[0,1]$, $\sin t < 1$

So $\int_0^1 \sqrt{3+\sin t}\,dt < \int_0^1 \sqrt{4}\,dt = 2.$

b) $F'(x) = \sqrt{3+\sin x}$ (2^{nd} F.T.)

$F''(x) = \frac{\cos x}{2\sqrt{3+\sin x}} > 0$ on $[0,1]$

$\therefore$ convex (concave up).

c) Set $u = 2t$, so $du = 2dt$

$\int_1^2 \sqrt{3+\sin 2t}\,dt = \int_2^4 \sqrt{3+\sin u}\cdot\frac12 du$

$= \frac12 F(u)\Big]_2^4 = \frac12[F(4)-F(2)]$

$\boxed{5}$ $\int_0^0 f(t)\,dt = 0 = e^{2\cdot 0}\cos 0 + c$

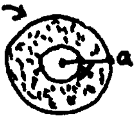
$\therefore c = -1$

By 2^{nd} F.T. :

$f(x) = D(e^{2x}\cos x - 1)$

$= 2e^{2x}\cos x - e^{2x}\sin x.$

$\boxed{6}$

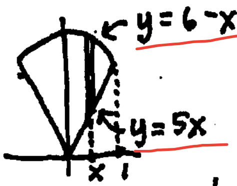


$\int_0^{\sqrt a} (\pi a^2 - \pi x^2)\,dy = \pi\int_0^{\sqrt a}(a^2-y^4)\,dy$

$= \pi\left(a^2 y - \frac{y^5}{5}\right)\Big]_0^{\sqrt a} = \pi\left[a^{5/2} - \frac15 a^{5/2}\right]$

$= \pi\cdot\frac45 a^{5/2}$

$\boxed{7}$



The strip rotated has volume $2\pi x(6-x^2-5x)\,dx$

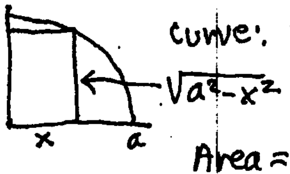
volume $= 2\pi\int_0^1 x(6-x^2-5x)\,dx$

$= 2\pi\left[3x^2 - \frac14 x^4 - \frac53 x^3\right]_0^1$

$= 2\pi\left[3 - 1/4 - 5/3\right] = 2\pi\cdot\frac{13}{12}$

$\boxed{8}$

?



curve: $y = \sqrt{a^2-x^2}$

Area $= x\sqrt{a^2-x^2}$

Average area $= \frac1a\int_0^a x\sqrt{a^2-x^2}\,dx$

$= \frac1a(a^2-x^2)^{3/2}\left(-\frac13\right)\Big]_0^a = \frac13(a^2)^{3/2} = \frac{a^2}{3}$

$\boxed{9}$

x=	0	π/4	π/2
sin x	0	$1/\sqrt2$	1
sin⁶x	0	1/8	1

$\Delta x = \pi/4$

TRAPEZOIDAL

$\approx \frac{\pi}{4}\left(\frac12\cdot 0 + \frac18 + \frac12\cdot 1\right)$

$= \frac{\pi}{4}\cdot\frac58 = \frac{5\pi}{32}$

SIMPSON

$\int_0^{\pi/2} \sin^6 x\,dx$

$\approx \frac{\pi/4}{3}\left(0 + 4\cdot\frac18 + 1\right)$

$= \frac{\pi}{12}\left(\frac32\right) = \frac{\pi}{8}$