18.01 Single Variable Calculus
Fall 2006

18.01 Practice Exam 2 Fall 2006

Problem 1. $f(x) = 2x^3 + 3x^2 - 12x + 1$

$f'(x) = 6x^2 + 6x - 12$ $\quad f'(x) = 0 \quad x = 1, -2$

$f''(x) = 12x + 6$ $\quad f''(x) = 0 \quad x = -1/2$

$f''(1) = 18 > 0$ so $(1, -6)$ is a loc. min

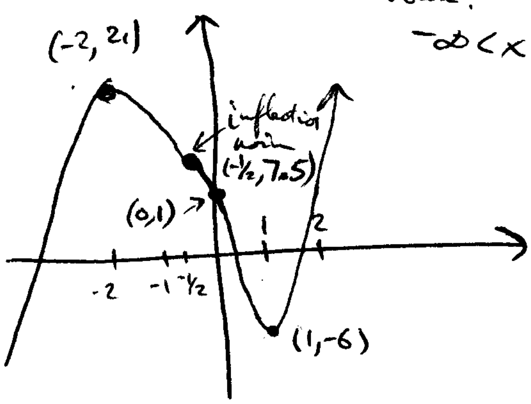
$f''(-2) = -18 < 0$ so $(-2, 21)$ is a loc. max

$x \to \infty \quad f(x) \to \infty$

$x \to -\infty \quad f(x) \to -\infty$

Concave up: $-1/2 < x < \infty$

Concave down: $-\infty < x < -1/2$



Problem 2. $V = \pi r^2 h = 64\pi$

$r^2 h = 64 \quad h = 64/r^2$

$A = 2\pi r h + \pi r^2$

$= \dfrac{128\pi}{r} + \pi r^2$

$0 < r < \infty$

$\dfrac{dA}{dr} = \dfrac{-128\pi}{r^2} + 2\pi r \qquad \dfrac{dA}{dr} = 0,\ r = 4$ (so $h = 4$)

$A(r=4) = 48\pi$

$\dfrac{d^2A}{dr^2} = \dfrac{256\pi}{r^3} + 2\pi > 0$ at $r = 4$

so $r=4, h=4, A = 48\pi$ is the minimum, since as $r \to 0$ or as $r \to \infty$, $A \to \infty$.

Problem 3. a) $\int e^{-3x}\,dx = -\frac{1}{3} e^{-3x} + C$

b) $\int \cos^2 x \sin x\,dx = -\int u^2\,du$

$u = \cos x$

$du = -\sin x\,dx$

$= -\frac{1}{3} u^3 + C$

$= -\frac{1}{3} \cos^3 x + C$

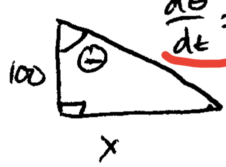
c) $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{-1}{2} \cdot 2u^{1/2} + C$

$u = 1 - x^2$

$du = -2x\,dx$

$= -\sqrt{1-x^2} + C$

Problem 4.



$\dfrac{d\theta}{dt} = \dfrac{\pi}{4}$ rad/min

$\tan\theta = x/100 \qquad 100 \sec^2\theta \dfrac{d\theta}{dt} = \dfrac{dx}{dt}$

$\left.\dfrac{dx}{dt}\right|_{\theta = \pi/3} = 100 \cdot 4 \cdot \dfrac{\pi}{4} = \boxed{100\pi} \approx 314$ m/min $= 18$ km/h

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Problem 5. a) $e^{-x}\sqrt{1+cx} \approx (1-x)(1+\frac{1}{2}cx) = 1 + (\frac{c}{2} - 1)x - \frac{1}{2}cx^2$, so f is const to first order if $c = 2$.

b) $\dfrac{dx}{\sqrt{1-x^2}} = 2t\,dt$

$\arcsin x = t^2 + c$

$x = \sin(t^2 + c)$

$1 = x(0) = \sin c$

$c = \pi/2$

$x = \sin(t^2 + \frac{\pi}{2})$

Problem 6. ⓐ $\dfrac{\ln(1+x) - \ln(1+0)}{x - 0} = \left.\dfrac{d}{dx}(\ln(1+x))\right|_{x=c}$

$\ln(1+x) = \dfrac{1}{1+c} x < x$, if $x > 0$. $\quad 0 < c < x$

ⓑ Suppose $f(a) = f(b) = 0$, $a \neq b$. So there is $a < d < b$ s.t. $f'(d) = 0$. $f'(d) = 3d^2 + 1 > 0$ for any d. So f can't have 2 zeros.