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18.02 Multivariable Calculus  
Fall 2007

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## 18.02 Practice Exam 1B Solutions

### Problem 1.

a)  $P = (1, 0, 0)$ ,  $Q = (0, 2, 0)$  and  $R = (0, 0, 3)$ . Therefore  $\overrightarrow{QP} = \hat{i} - 2\hat{j}$  and  $\overrightarrow{QR} = -2\hat{j} + 3\hat{k}$ .

$$\text{b) } \cos \theta = \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{|\overrightarrow{QP}| |\overrightarrow{QR}|} = \frac{\langle 1, -2, 0 \rangle \cdot \langle 0, -2, 3 \rangle}{\sqrt{1^2 + 2^2} \sqrt{2^2 + 3^2}} = \frac{4}{\sqrt{65}}$$

### Problem 2.

a)  $\overrightarrow{PQ} = \langle -1, 2, 0 \rangle$ ,  $\overrightarrow{PR} = \langle -1, 0, 3 \rangle$ .

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\hat{i} + 3\hat{j} + 2\hat{k}.$$

$$\text{Then } \text{area}(\Delta) = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2} \sqrt{6^2 + 3^2 + 2^2} = \frac{1}{2} \sqrt{49} = \frac{7}{2}.$$

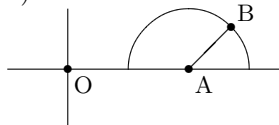
b) A normal to the plane is given by  $\vec{N} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 6, 3, 2 \rangle$ . Hence the equation has the form  $6x + 3y + 2z = d$ . Since  $P$  is on the plane  $d = 6 \cdot 1 + 3 \cdot 1 + 2 \cdot 1 = 11$ . In conclusion the equation of the plane is

$$6x + 3y + 2z = 11.$$

c) The line is parallel to  $\langle 2 - 1, 2 - 2, 0 - 3 \rangle = \langle 1, 0, -3 \rangle$ . Since  $\vec{N} \cdot \langle 1, 0, -3 \rangle = 6 - 6 = 0$ , the line is parallel to the plane.

### Problem 3.

a)



$\overrightarrow{OA} = \langle 10t, 0 \rangle$  and  $\overrightarrow{AB} = \langle \cos t, \sin t \rangle$ , hence

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \langle 10t + \cos t, \sin t \rangle.$$

The rear bumper is reached at time  $t = \pi$  and the position of  $B$  is  $(10\pi - 1, 0)$ .

b)  $\vec{V} = \langle 10 - \sin t, \cos t \rangle$ , thus

$$|\vec{V}|^2 = (10 - \sin t)^2 + \cos^2 t = 100 - 20 \sin t + \sin^2 t + \cos^2 t = 101 - 20 \sin t.$$

The speed is then given by  $\sqrt{101 - 20 \sin t}$ . The speed is smallest when  $\sin t$  is largest i.e.  $\sin t = 1$ . It occurs when  $t = \pi/2$ . At this time, the position of the bug is  $(5\pi, 1)$ . The speed is largest when  $\sin t$  is smallest; that happens at the times  $t = 0$  or  $\pi$  for which the position is then  $(0, 0)$  and  $(10\pi - 1, 0)$ .

### Problem 4.

a)  $|M| = -12$ .

b)  $a = -5$ ,  $b = 7$ .

$$\text{c) } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{12} \begin{bmatrix} 1 & 1 & 4 \\ -5 & 7 & -8 \\ 7 & -5 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ t \\ 3 \end{bmatrix} = \begin{bmatrix} t/12 + 1 \\ 7t/12 - 2 \\ -5t/12 + 1 \end{bmatrix}$$

$$\text{d) } \frac{d\vec{r}}{dt} = \left\langle \frac{1}{12}, \frac{7}{12}, -\frac{5}{12} \right\rangle.$$

### Problem 5.

a)  $\vec{N} \cdot \vec{r}(t) = 6$ , where  $\vec{N} = \langle 4, -3, -2 \rangle$ .

b) We differentiate  $\vec{N} \cdot \vec{r}(t) = 6$ :

$$0 = \frac{d}{dt} (\vec{N} \cdot \vec{r}(t)) = \frac{d}{dt} \vec{N} \cdot \vec{r}(t) + \vec{N} \cdot \frac{d}{dt} \vec{r}(t) = \vec{0} \cdot \vec{r}(t) + \vec{N} \cdot \frac{d}{dt} \vec{r}(t) \quad \text{and hence } \vec{N} \perp \frac{d}{dt} \vec{r}(t).$$