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18.02 Multivariable Calculus  
Fall 2007

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## 18.02 Practice Exam 1 A – Solutions

### Problem 1.

a)  $\overrightarrow{OQ} = \hat{i} + \hat{j} + \hat{k}$ ;  $\overrightarrow{OR} = \frac{1}{2}\hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ .

b)  $\cos \theta = \frac{\overrightarrow{OQ} \cdot \overrightarrow{OR}}{|\overrightarrow{OQ}| |\overrightarrow{OR}|} = \frac{\langle 1, 1, 1 \rangle \cdot \langle \frac{1}{2}, 1, \frac{1}{2} \rangle}{\sqrt{3} \sqrt{\frac{3}{2}}} = \frac{2\sqrt{2}}{3}$ .

### Problem 2.

Velocity:  $\vec{V} = \frac{d\vec{R}}{dt} = \langle -3 \sin t, 3 \cos t, 1 \rangle$ . Speed:  $|\vec{V}| = \sqrt{9 \sin^2 t + 9 \cos^2 t + 1} = \sqrt{10}$ .

### Problem 3.

a) Minors:  $\begin{bmatrix} 1 & 1 & 2 \\ -2 & -2 & -2 \\ -3 & -5 & -6 \end{bmatrix}$ . Cofactors:  $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 2 \\ -3 & 5 & -6 \end{bmatrix}$ . Inverse:  $\frac{1}{2} \begin{bmatrix} 1 & \boxed{2} & \boxed{-3} \\ -1 & -2 & 5 \\ 2 & 2 & -6 \end{bmatrix}$ .

b)  $X = A^{-1}B = \begin{bmatrix} -3 \\ 4 \\ -4 \end{bmatrix}$ .

### Problem 4.

Q = top of the ladder:  $\overrightarrow{OQ} = \langle 0, L \sin \theta \rangle$ ; R = bottom of the ladder:  $\overrightarrow{OR} = \langle -L \cos \theta, 0 \rangle$ .

Midpoint:  $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OQ} + \overrightarrow{OR}) = \langle -\frac{L}{2} \cos \theta, \frac{L}{2} \sin \theta \rangle$ .

Parametric equations:  $x = -\frac{L}{2} \cos \theta$ ,  $y = \frac{L}{2} \sin \theta$ .

### Problem 5.

a)  $\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 0 & -2 & 1 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$ . Area =  $\frac{1}{2} |\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}| = \frac{1}{2} \sqrt{6}$ .

b) Normal vector:  $\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \hat{i} + \hat{j} + 2\hat{k}$ . Equation:  $x + y + 2z = 3$ .

c) Parametric equations for the line:  $x = -1 + t$ ,  $y = t$ ,  $z = t$ .

Substituting:  $-1 + 4t = 3$ ,  $t = 1$ , intersection point  $(0, 1, 1)$ .

### Problem 6. work like a mathematician

a)  $\frac{d}{dt}(\vec{R} \cdot \vec{R}) = \vec{V} \cdot \vec{R} + \vec{R} \cdot \vec{V} = \underline{2\vec{R} \cdot \vec{V}}$ .

ⓑ Assume  $|\vec{R}|$  is constant: then  $\frac{d}{dt}(\vec{R} \cdot \vec{R}) = 2\vec{R} \cdot \vec{V} = 0$ , i.e.  $\vec{R} \perp \vec{V}$ .

ⓒ  $\vec{R} \cdot \vec{V} = 0$ , so  $\frac{d}{dt}(\vec{R} \cdot \vec{V}) = \vec{V} \cdot \vec{V} + \vec{R} \cdot \vec{A} = 0$ . Therefore  $\vec{R} \cdot \vec{A} = -|\vec{V}|^2$ .