Differential Equations

Calendar

The calendar below provides information on the course's lecture (L) and recitation (R) sessions. There is also a list of skills and concepts and where they are first introduced. Problem Set (PS) distribution and due dates are also provided.

I. First-order differential equations

		Modeling: exponential growth with harvesting Growth rate Separating variables Solutions, general and particular	
R1	Natural growth, separable equations	Amalgamating constants of integration Use of ln|y|, and its elimination Reintroduction of lost solutions Initial conditions - satisfying them by choice of integration constant	
L1	Direction fields, existence and uniqueness of solutions	Direction fields Integral curve Isoclines Funnels Implicit solutions Failure of solutions to continue: infinite derivative	PS 1 out
R2	Direction fields, integral curves, isoclines, separatrices, funnels	Separatrix Extrema of solutions	
L2	Numerical methods	Euler's method First order linear equation System/signal perspective	
L3	Linear equations, models	Bank account model RC circuit Solution by separation if forcing term is constant	
R3	Euler's method; linear models	Mixing problems Homogeneous equation, null signal	
L4	Solution of linear equations, integrating factors	Integrating factors Transients Diffusion example; coupling constant	
R4	First order linear ODEs; integrating factors	Sinusoidal input signal	

L5	Complex numbers, roots of unity	Complex numbers Roots of unity	PS 1 due; PS 2 out
L6	Complex exponentials; sinusoidal functions	Complex exponential Sinusoidal functions: Amplitude, Circular frequency, Phase lag	
L7	Linear system response to exponential and sinusoidal input; gain, phase lag	First order linear response to exponential or sinusoidal signal Complex-valued equation associated to sinusoidal input PS: half life	
R5	Complex numbers; complex exponentials		
L8	Autonomous equations; the phase line, stability	Autonomous equation Phase line Stability e^{k(t-t_0)} vs ce^{kt}	PS 2 due; PS 3 out
L9	Linear vs. nonlinear	Non-continuation of solutions	
R6	Review for exam I		
	Exam I		Hour exam I
II. Second-order linear equations			
R7	Solutions to second order ODEs	Harmonic oscillator Initial conditions Superposition in homogeneous case	
L11	Modes and the characteristic polynomial	Spring/mass/dashpot system General second order linear equation Characteristic polynomial Solution in real root case	
L12	Good vibrations, damping conditions	Complex roots Under, over, critical damping Complex replacement, extraction of real solutions Transience Root diagram	
R8	Homogeneous 2nd order linear constant coefficient equations	General sinusoidal response Normalized solutions	
L13	Exponential response formula, spring drive	Driven systems Superposition Exponential response formula Complex replacement Sinusoidal response to sinusoidal signal	
R9	Exponential and sinusoidal input signals		

L14	Complex gain, dashpot drive	Gain, phase lag Complex gain	PS 3 due; PS 4 out
L15	Operators, undetermined coefficients, resonance	Operators Resonance Undetermined coefficients	
R10	Gain and phase lag; resonance; undetermined coefficients		
L16	Frequency response	Frequency response	
R11	Frequency response	First order frequency response	
L17	LTI systems, superposition, RLC circuits.	RLC circuits Time invariance	PS4 due; PS 5 out
L18	Engineering applications	Damping ratio	
R12	Review for exam II		
L19	Exam II		Hour Exam II

III. Fourier series

R13	Fourier series: introduction	Periodic functions Fourier series	
L20	Fourier series	Orthogonality Fourier integral	
L21	Operations on fourier series	Squarewave Piecewise continuity Tricks: trig id, linear combination, shift	
R14	Fourier series	Different periods Differentiating and integrating fourier series	
L22	Periodic solutions; resonance	Harmonic response Amplitude and phase expression for Fourier series	
R15	Fourier series: harmonic response		
L23	Step functions and delta functions	Step function Delta function Regular and singularity functions Generalized function Generalized derivative	PS 5 due; PS 6 out
L24	Step response, impulse response	Unit and step responses Rest initial conditions First and second order unit step or unit impulse response	
R16	Step and delta functions, and step and delta responses		

L25	Convolution	Post-initial conditions of unit impulse response Time invariance: Commutation with D Time invariance: Commutation with t-shift Convolution product Solution with initial conditions as w * q	
R17	Convolution	Delta function as unit for convolution	
L26	Laplace transform: basic properties	Laplace transform Region of convergence L[t^n] s-shift rule L[sin(at)] and L(cos(at)] t-domain vs s-domain	PS 6 due; PS 7 out
L27	Application to ODEs	L[delta(t)] t-derivative rule Inverse transform Partial fractions; coverup Non-rest initial conditions for first order equations	
R18	Laplace transform	Unit step response using Laplace transform.	
L28	Second order equations; completing the squares	s-derivative rule Second order equations	
R19	Laplace transform II		
L29	The pole diagram	Weight and transfer function L[weight function] = transfer function t-shift rule Poles Pole diagram of LT and long term behavior	PS 7 due; PS 8 out
L30	The transfer function and frequency response	Stability Transfer and gain	
R20	Review for exam III		
	Exam III		Hour Exam III
IV. First order systems			
L32	Linear systems and matrices	First order linear systems Elimination Matrices Anti-elimination: Companion matrix	
R21	First order linear systems		

L33	Eigenvalues, eigenvectors	Determinant Eigenvalue Eigenvector Initial values	
R22	Eigenvalues and eigenvectors	Solutions vs trajectories Eigenvalues vs coefficients	
L34	Complex or repeated eigenvalues	Complex eigenvalues Repeated eigenvalues Defective, complete	PS 8 due; PS 9 out
L35	Qualitative behavior of linear systems; phase plane	Trace-determinant plane Stability	
R23	Linear phase portraits	Morphing of linear phase portraits	
L36	Normal modes and the matrix exponential	Matrix exponential Uncoupled systems Exponential law	
R24	Matrix exponentials	Inhomogeneous linear systems (constant input signal)	
L37	Nonlinear systems	Nonlinear autonomous systems Vector fields Phase portrait Equilibria Linearization around equilibrium Jacobian matrices	PS 9 due
L38	Linearization near equilibria; the nonlinear pendulum	Nonlinear pendulum Phugoid oscillation Tacoma Narrows Bridge	
R25	Autonomous systems	Predator-prey systems	
L39	Limitations of the linear: limit cycles and chaos	Structural stability Limit cycles Strange attractors	
R26	Reviews Final exam