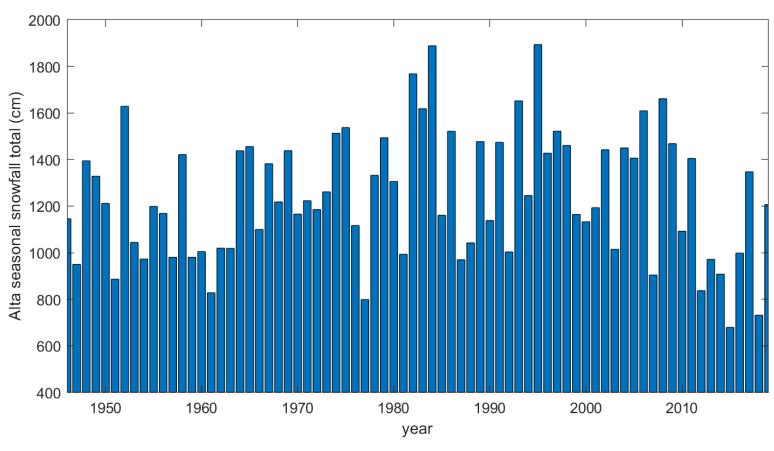
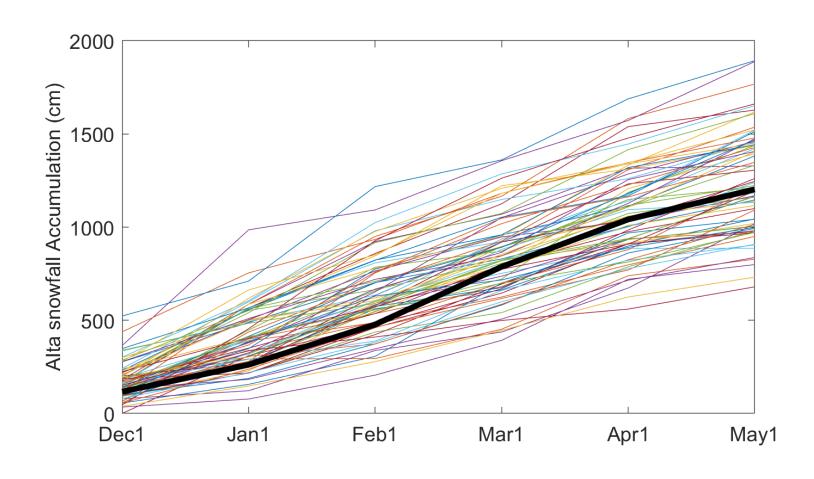
### Alta Snowfall Seasonal Totals



https://utahavalanchecenter.org/alta-monthly-snowfall

### Atla Snowfall Accumulation Each Winter

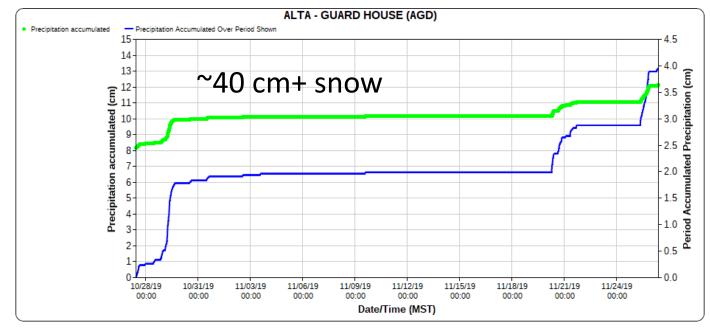


Predict May 1 Snowfall from Dec 1 Snowfall Below< 121 cm; Above> 193 cm

So far: ~40 cm at Alta Guard

Case 1. Predictor: Dec1 total	I snowfall (	cm)
-------------------------------	--------------	-----

<b>Predictand:</b>		Below	Near	Above (E <sub>3</sub> )	M Marginal
May 1 Total		(E <sub>1</sub> )	(E <sub>2</sub> )		Totals
snowfall at					
Alta (cm)	Below (M <sub>1</sub> )	14	10	1	25
	Near (M <sub>2</sub> )	7	7	10	24
	Above (M <sub>3</sub> )	4	7	14	25
	E Marginal Totals	25	24	25	74

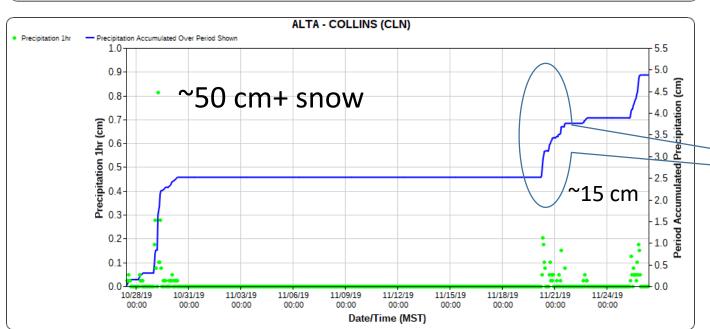


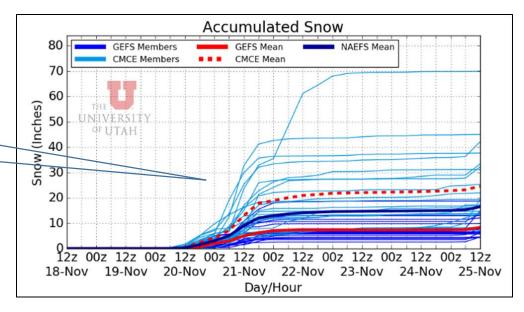
# Alta Guard & Alta Collins Precipitation

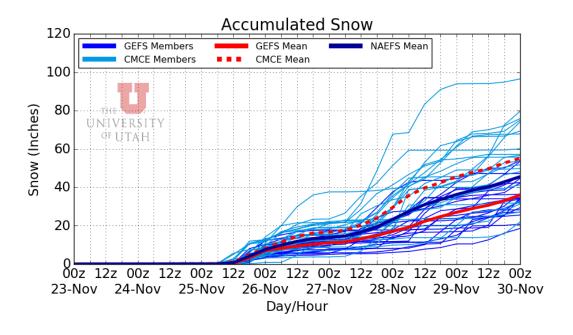
Ensemble Forecasts of Snow (5-70 inch)

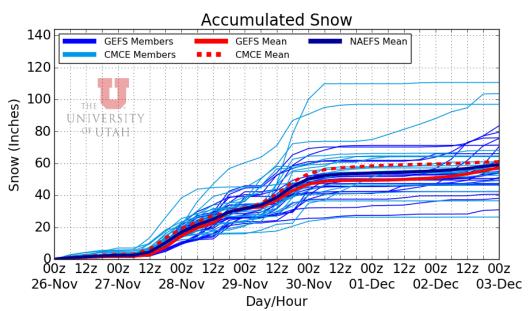
Canadian mean: 20+ inch (~50 cm)

GFS mean: 5 inch (~11 cm)









# Ensemble Forecasts of Snow at Alta-Collins

Saturday forecast

Ensemble Forecasts of Snow (10-100inch)

Canadian mean: 55 inch (~135 cm)

GFS mean: 40 inch (~90 cm)

Tuesday forecast ending Dec 1

Ensemble Forecasts of Snow (20-110inch)

Canadian mean: 60 inch (~150 cm)

GFS mean: 50 inch (~125 cm)

http://weather.utah.edu/index.php?runcode=2019112600&t=naefs&d=PL&r=CLN

## Central Limit Theorem

sum (or mean) of a sample (6 dice) will have a Gaussian distribution even if the original distribution (one die) does not have a Gaussian distribution, especially as the sample size increases.

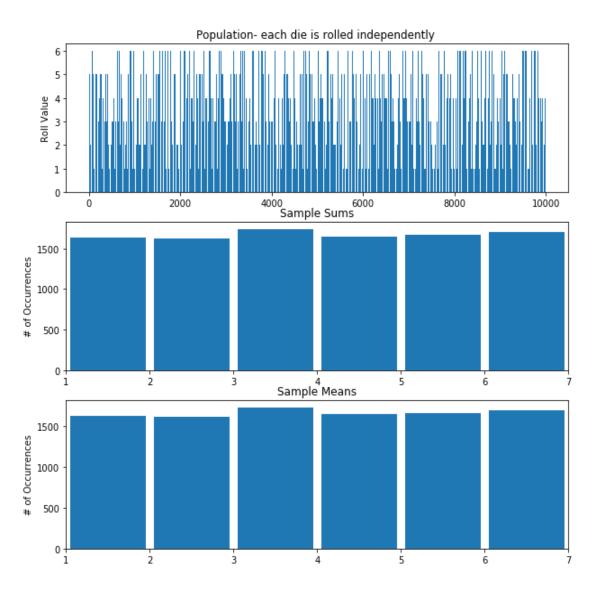
$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

 $\sigma_{\bar{x}}$  standard deviation of the sample means  $\sigma$  standard deviation of the original population n sample size

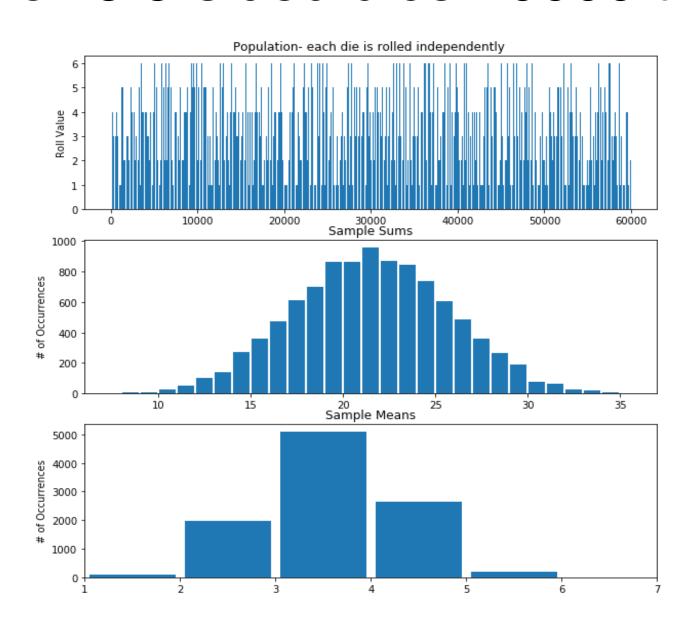
$$s_x = \sqrt{\frac{n-1}{n}}\sigma$$
  $\sigma_{\bar{x}} = s_x / \sqrt{n-1}$ 

degrees of freedom: n-1, since sample can be described by the mean (1 value) plus n-1 others

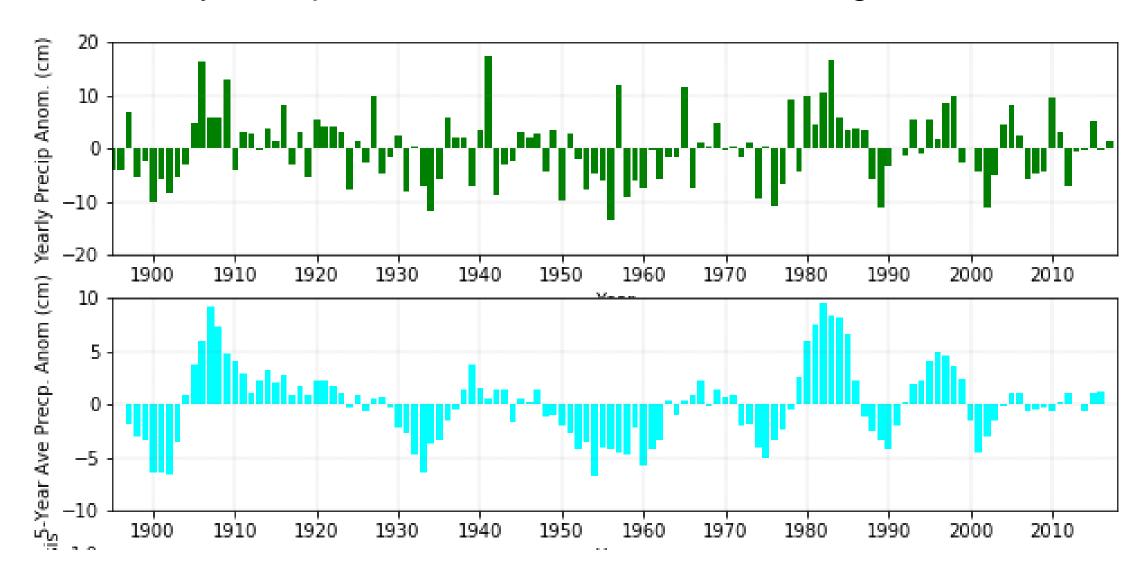
## Roll 1 6 sided die 10000 times



## Roll 6 6-sided dice 10000 times



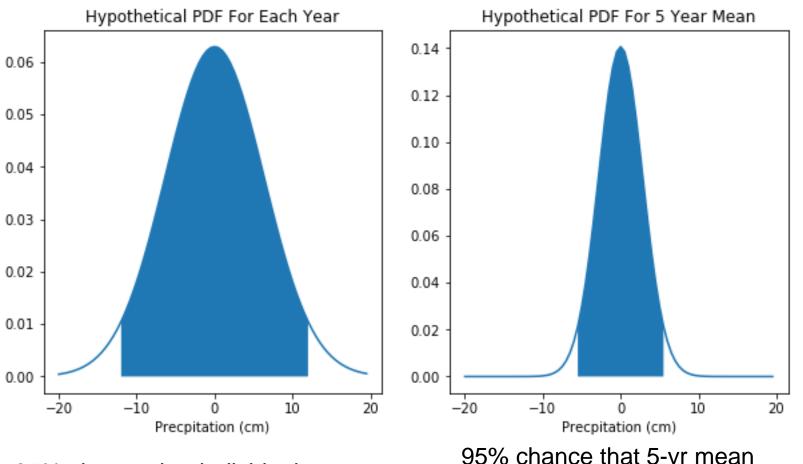
### Which 5-yr samples would be considered a drought for Utah?



# Steps of Hypothesis Testing

- Identify a test statistic that is appropriate to the data and question at hand
  - Computed from sample data values. 5 yr sample means
- Define a null hypothesis, H<sub>0</sub> to be rejected
  - 5 yr sample mean 0
- Define an alternative hypothesis, H<sub>A</sub>
  - 5 yr sample mean < 0</li>
- Estimate the null distribution
  - Sampling distribution of the test statistic IF the null hypothesis were true
  - Making assumptions about which parametric distribution to use (Gaussian, Weibull, etc.)
  - Use sample mean of 0 and 124 yr sd of 6.3 cm
- Compare the observed test statistics (5-yr means) to the null distribution. Either
  - Null hypothesis is rejected as too unlikely to have been true IF the test statistic fall in an improbable region of the null distribution
    - Possibility that the test statistics has that particular value in the null distribution is small
    - OR
    - The null hypothesis is not rejected since the test statistic falls within the values that are relatively common to the null distribution

# Each Year individually: sample standard deviation = 6.3 cm 5 Year sample: sample standard deviation = 2.7cm



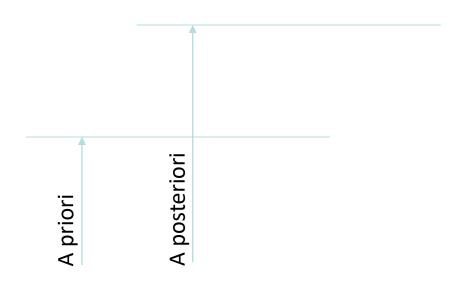
95% chance that individual year within 12.4 cm

Conclusion:
Less likely to have
a 5-yr drought
(really large 5-yr
mean) than to have
a single really dry
year

95% chance that 5-yr mean anomaly within 5.4 cm

# Setting the statistical bar

- a priori ("from the earlier") vs.
   a posteriori ("from the later")
  - a priori- can be verified independently: any conclusions drawn do not depend on a prior analysis of empirical evidence
  - a posteriori- involves some level of subjective exposure to the data from which a conclusion will be drawn



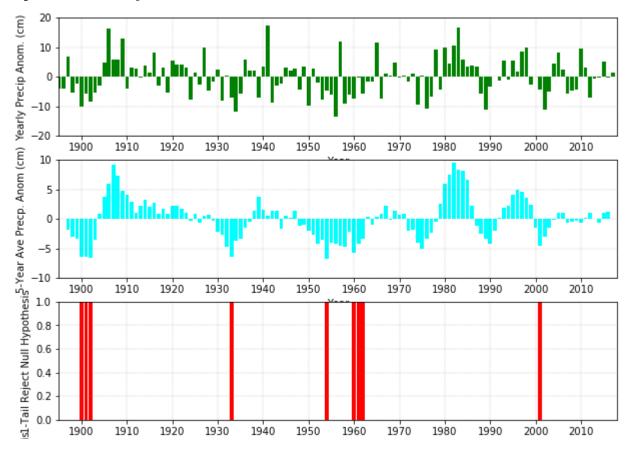
## Students' t test

- $\bullet \quad \sigma_{\overline{X}} = \frac{s_x}{\sqrt{n-1}}$
- Estimate of population variance from sample
- T value:
- Numerator: signal

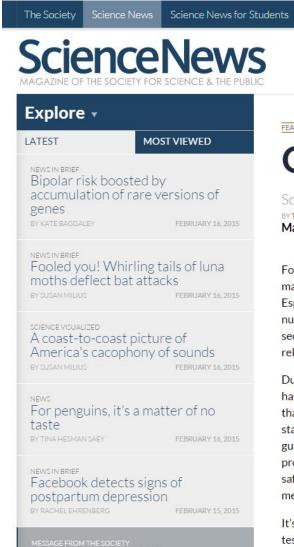
$$t = (\overline{x} - \mu)\sqrt{n-1}/s_x$$

- Denominator: noise
- At t gets larger, confidence in rejecting the null hypothesis (sample mean differs from population mean) gets higher
- T large IF:
  - Spread between sample and population means large
  - Degrees of freedom is large
  - Variability in sample is small

### Which 5-yr samples would be considered a drought?



# https://www.sciencenews.org/article/e/odds-are-its-wrong



FEATURE HUMANS & SOCIETY, NUMBERS

Student Science

### Odds Are, It's Wrong

Science fails to face the shortcomings of statistics

BY TOM SIEGFRIED 2:40PM, MARCH 12, 2010

Magazine issue: Vol. 177 #7, March 27, 2010

For better or for worse, science has long been married to mathematics. Generally it has been for the better.

Especially since the days of Galileo and Newton, math has nurtured science. Rigorous mathematical methods have secured science's fidelity to fact and conferred a timeless reliability to its findings.

During the past century, though, a mutant form of math has deflected science's heart from the modes of calculation that had long served so faithfully. Science was seduced by statistics, the math rooted in the same principles that guarantee profits for Las Vegas casinos. Supposedly, the proper use of statistics makes relying on scientific results a safe bet. But in practice, widespread misuse of statistical methods makes science more like a crapshoot.

It's science's dirtiest secret: The "scientific method" of testing hypotheses by statistical analysis stands on a In github!



Search Science News...

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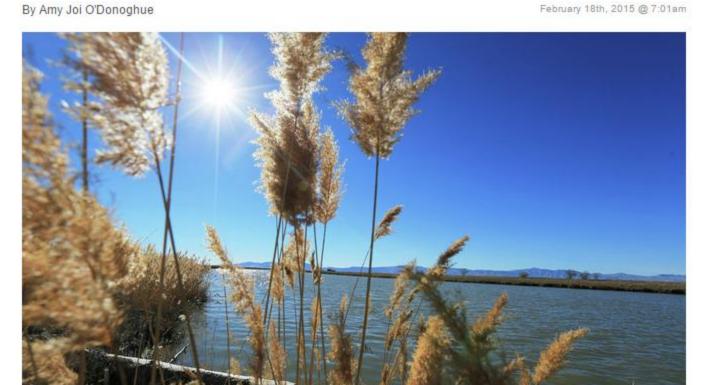


# Tired about complaints about poor statistical analyses??

Here's a good one

rte 1 200 year history of Poor Dive

Study charts 1,200-year history of Bear River



## In canvas

#### ARTICLE IN PRESS

Journal of Hydrology xxx (2015) xxx-xxx



Contents lists available at ScienceDirect

#### Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol



#### A millennium-length reconstruction of Bear River stream flow, Utah

R.J. DeRose <sup>a,\*</sup>, M.F. Bekker <sup>b</sup>, S.-Y. Wang <sup>c</sup>, B.M. Buckley <sup>d</sup>, R.K. Kjelgren <sup>c</sup>, T. Bardsley <sup>e</sup>, T.M. Rittenour <sup>f</sup>, E.B. Allen <sup>g</sup>

<sup>&</sup>lt;sup>a</sup> USDA, Forest Service, Forest Inventory and Analysis, Rocky Mountain Research Station, 507 25th Street, Ogden, UT 84401, United States

<sup>&</sup>lt;sup>b</sup> Department of Geography, 690 SWKT, Brigham Young University, Provo, UT 84602, United States

<sup>&</sup>lt;sup>c</sup> Plant, Soil, and Climate Department, 4820 Old Main Hill, Utah State University, Logan, UT 84322-4820, United States

<sup>&</sup>lt;sup>d</sup> Tree Ring Lab, Room 108, Lamont-Doherty Earth Observatory, Columbia University, 61 Route 9W, Palisades, NY 10964, United States

<sup>&</sup>lt;sup>e</sup>Western Water Assessment, 2242 West North Temple, Salt Lake City, UT 84116, United States

<sup>&</sup>lt;sup>f</sup>Department of Geology, 4505 Old Main Hill, Utah State University, Logan, UT 84322-4505, United States

g United States Geological Survey, 4200 New Haven Road, Columbia, MO 65201, United States

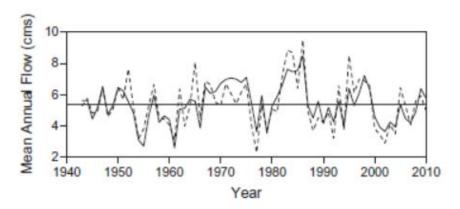


Fig. 2. Observed (dashed line) versus predicted (solid line) Bear River stream flow for the instrumental period (1943–2010). Horizontal line indicates instrumental mean water year flow (5.412 cms). Linear regression model explained 67% of the variation in instrumental Bear River flow.

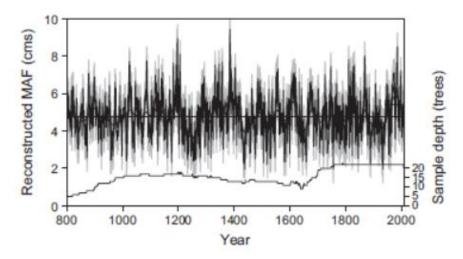


Fig. 3. Reconstructed Bear River stream flow from 800 to 2010 AD (thin black line), dark bold solid line cubic smoothing spline with 50% frequency cut-off at wavelength 20 years, light bold solid line cubic smoothing spline with 50% frequency at wavelength 60 years. Gray bands indicate 80% confidence interval calculated from the Bear River reconstruction model RMSE. Solid horizontal line is reconstructed MAF (4.796 cms). Dashed horizontal line is instrumental MAF (5.412 cms). Sample depth (number trees) for SFC indicated on the right.

Table 1

Model skill statistics and calibration-verification results for the Bear River reconstruction.

	r	R <sup>2</sup>	Adj. R <sup>2</sup>	RE	CE	Sign test (hit/miss)	RMSE (cms)
Calibrate (1943-1976)	0.72	0.52	0.50	0.66	0.39		
Calibrate (1977-2010)	0.90	0.81	0.80	0.23	0.13		
Full model	0.82	0.68	0.67			54/12 <sup>a</sup>	0.8156

<sup>(</sup>r) – Pearson's correlation coefficient, (R<sup>2</sup>) – coefficient of determination, (adj. R<sup>2</sup>) coefficient of determination adjusted for degrees of freedom, RE – reduction of error statistic, CE – coefficient of efficiency statistic, RMSE – root mean-squared error. Full model: 1.9414 + 2.9048 \* SFC.

a Sign test significant at the alpha <0.01 level (Fritts, 1976).

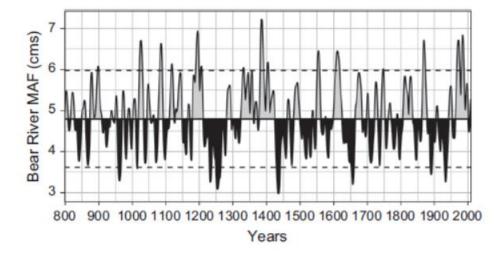
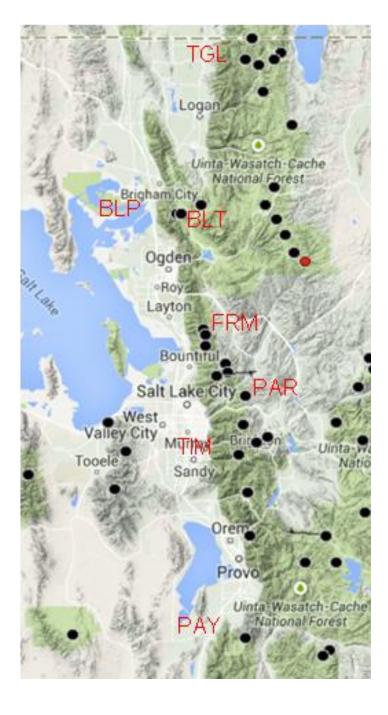


Fig. 5. Reconstructed Bear River decadal-scale drought (black) and pluvial (gray) periods from cubic smoothing spline with frequency response of 25% at wavelength 10 years. Dashed lines indicate 1 SD from reconstruction mean. See Table 3 for ranked dry and wet periods.

# SNOTEL Sites



## Accumulated Annual Precipitation

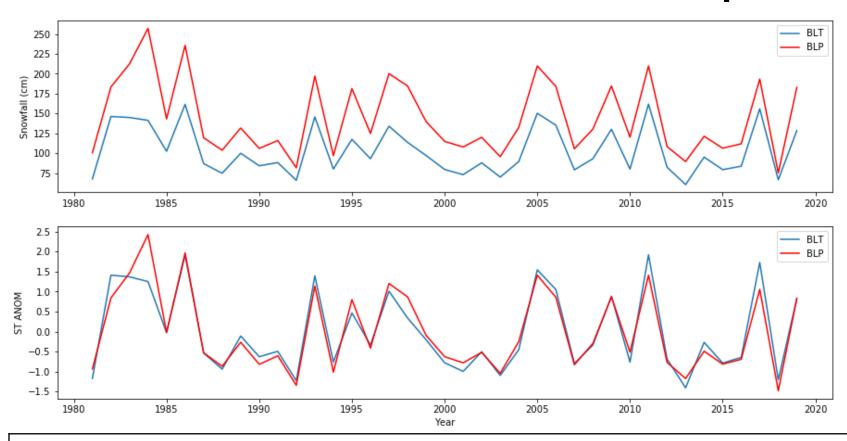


Figure 4.2. Time series (cm) of seasonal total precipitation at Ben Lomond Peak and Trail (top panel) and standardized anomalies (nondimensional) of the time series in the lower panel.

## Scatter plots

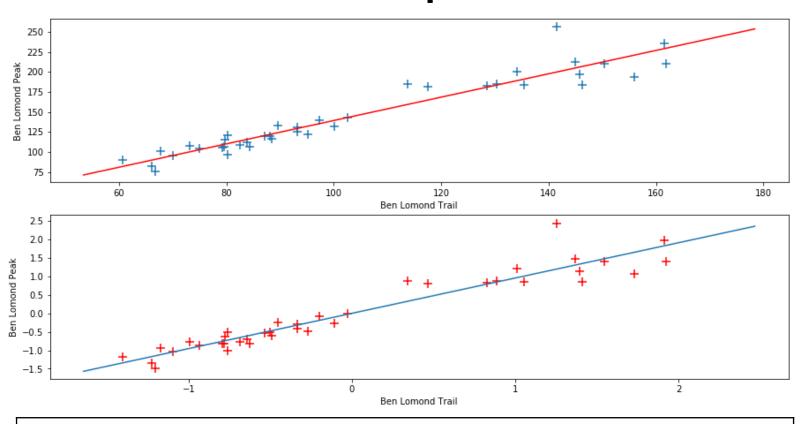


Figure 4.3. Scatter plot of total precipitation (cm) at Ben Lomond Peak vs. Trail (top panel) and of their standardized anomalies (bottom panel).

# Estimating Values of One Variable From Another

- X- Ben Lomond Trail
- Y- Ben Lomond Peak
- Want to estimate Peak from Trail
- Use pairs of observations from sample
- Need to determine coefficient b or r
- b- slope of linear estimate
- r- linear correlation

$$\hat{y}_i = \overline{y} + b(\hat{x}_i - \overline{x})$$

$$\hat{y}^*_i = r\hat{x}^*_i$$

## **Definitions**

Estimate

$$\hat{y}_i = \bar{y} + b(\hat{x}_i - \bar{x})$$

Error of estimate

$$e_i = y_i' - \hat{y}_i'$$

- Want  $\sum_{i=1}^{n} e_i^2$  to be a minimum
- Need to find the value of b that minimizes that sum

$$\frac{\partial}{\partial b} \sum_{i=1}^{n} e_i^2 = 0$$

$$b = \overline{x_i' y_i'} / \overline{(x_i')^2} = \overline{x_i' y_i'} / s_x^2$$

The value of b that minimizes the total error in the sample

## Covariance

- Relates how departures of x and y from respective means are related
- Units are the product of the units of the two variables x and y
- Large and positive if sample tendency for:
  - large + anomalies of x occurring when large + anomalies of y
     AND
  - large anomalies of x occurring when large anomalies of y
- Large and negative if sample tendency for:
  - large + anomalies of x occurring when large anomalies of y
     AND
  - large anomalies of x occurring when large + anomalies of y
- Near zero when tendency for cancellation
  - large + anomalies of x occurring when both large and + anomalies of y AND
  - large anomalies of x occurring when both large and + anomalies of y

## **Linear Correlation**

$$r^{2} = b^{2} s_{x}^{2} / s_{y}^{2} = (\overline{x'_{i} y'_{i}})^{2} / (s_{x}^{2} s_{y}^{2}) \qquad r = (\overline{x'_{i} y'_{i}}) / \sqrt{\overline{x'_{i}^{2}} \overline{y'_{i}^{2}}}$$

$$x_{i}^{*} = x'_{i} / s_{x}, y_{i}^{*} = y'_{i} / s_{y}, r = (\overline{x'_{i} y'_{i}})$$

$$1 = r^2 + \frac{\overline{e_i^2}}{s_y^2}$$
 y's total sample variance = fraction of variance estimated by x + fraction of variance NOT explained by x

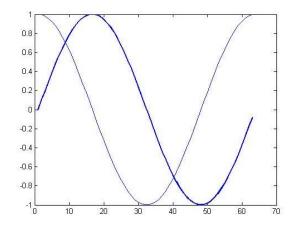
- Dimensionless number relates how departures of x and y from respective means are related taking into account variance of x and y
- r = 1. Linear fits estimates ALL of the variability of the y anomalies and x and y vary identically
- r = -1 perfect linear estimation but when x is positive, y is negative and vice versa
- r = 0. linear fit explains none of the variability of the y anomalies in the sample. Best estimate of y is the mean value

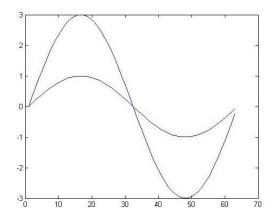
# Linear Algebra is your friend

$$\vec{X}' = \begin{bmatrix} x_1' \\ x_2' \\ \dots \\ x_n' \end{bmatrix} \vec{Y}' = \begin{bmatrix} y_1' \\ y_2' \\ \dots \\ y_n' \end{bmatrix} \qquad \overline{x_i' y_i'} = \vec{X}'^T \vec{Y}' / n$$

#### Stop and think before blindly computing correlations

- tendency to use correlation coefficients of 0.5 -0.6 to indicate "useful" association.
  - 75%-64% of the total variance is NOT explained by a linear relationship if the correlation is in that range
- linear correlations can be made large by leaving in signals that may be irrelevant to the analysis. Annual and diurnal cyles may need to be removed
- large linear correlations may occur simply at random, especially if we try to correlate one variate with many, many others
- relationships in the data that are inherently nonlinear will not be handled well
- when two time series are in quadrature with one another then the linear correlation is 0
- Linear correlation provides no information on the relative amplitudes of two time series



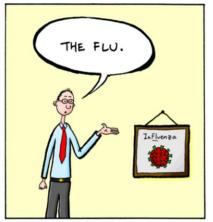


# Getting things mucked up

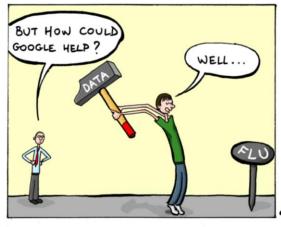
- Incorrectly assuming causation from correlation
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  - Something else is the underlying cause: decreased arctic ice will cause temperatures in Utah to increase (global warming?)

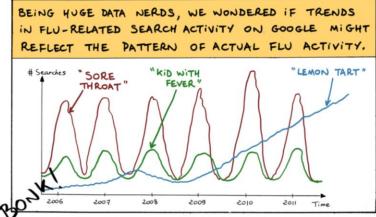
## Google correlate











## Resources for humor...

Google Correlate will shut down on December 15th 2019 as a result of low usage.



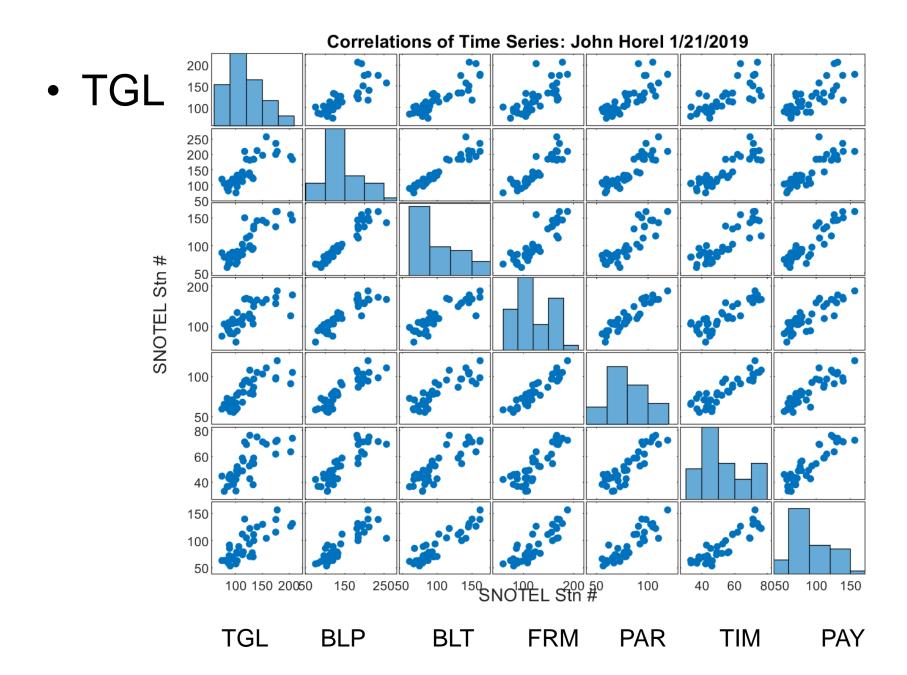
https://www.google.com/trends/correlate/search?e=war&e
 =air+pollution&t=monthly&p=us#default,30

http://tylervigen.com/spurious-correlations

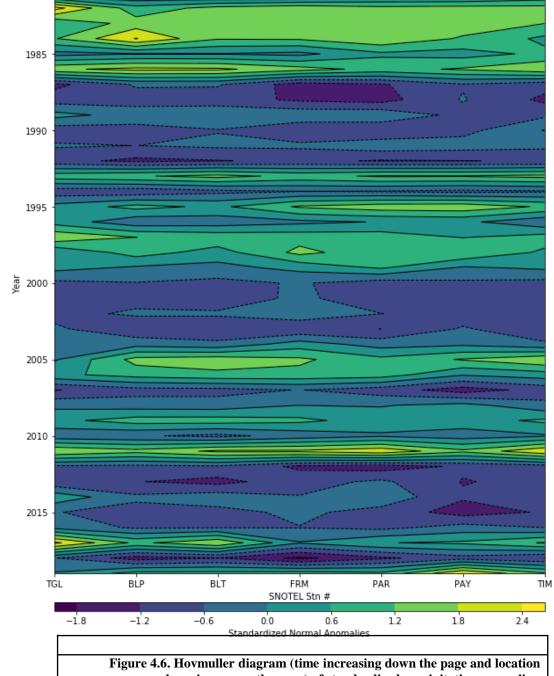
## Multivariate Linear Correlations

$$\vec{X}^* = \begin{bmatrix} x^*_{11} & x^*_{12} & \dots & x^*_{17} \\ x^*_{21} & x^*_{22} & \dots & x^*_{27} \\ \dots & \dots & \dots \\ x^*_{n1} & x^*_{n2} & \dots & x^*_{n7} \end{bmatrix}$$

- 7 stations and n=39 years
- Standardized anomalies

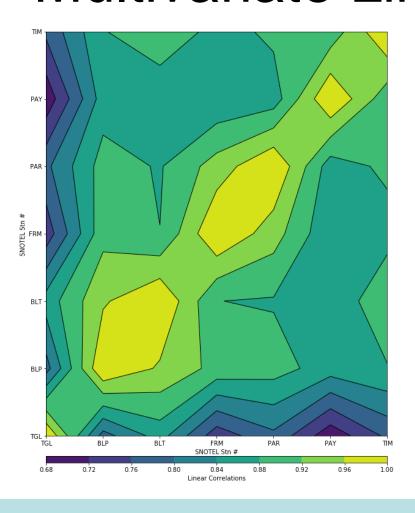


# Hovmuller Diagram



advancing across the page) of standardized precipitation anomalies.

## Multivariate Linear Correlations

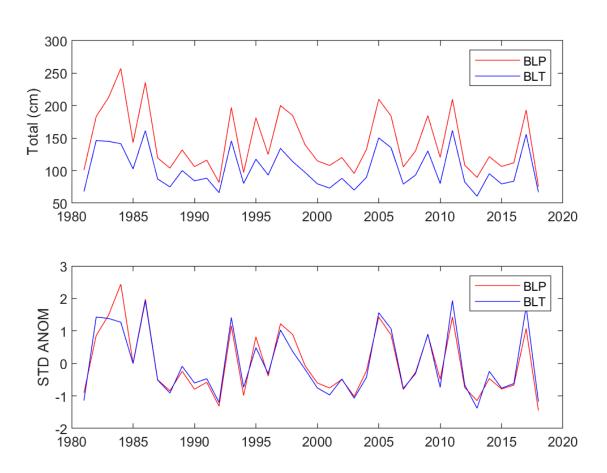


$$\vec{X}^* = \begin{bmatrix} x *_{11} & x *_{12} & \dots & x *_{17} \\ x *_{21} & x *_{22} & \dots & x *_{27} \\ \dots & \dots & \dots \\ x *_{n1} & x *_{n2} & \dots & x *_{n7} \end{bmatrix}$$

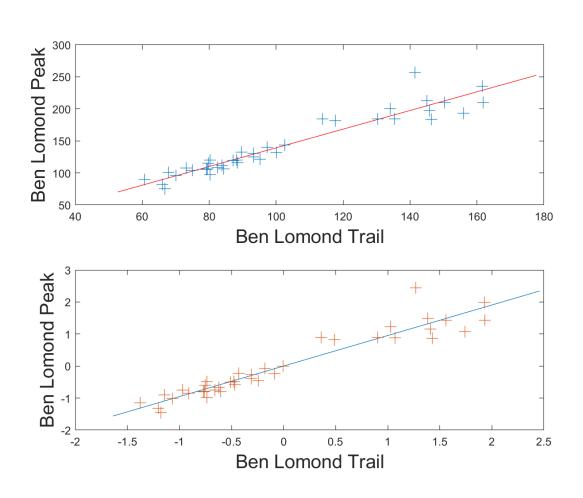
$$\vec{R} = \vec{X} *^T \vec{X} * / n$$

Figure 4.7. Correlation between all pairs of precipitation time series computed over the 39 year sample.

# Accum Precip



## Scatter plots



# Estimating Values of One Variable From Another

- X- Ben Lomond Trail
- Y- Ben Lomond Peak
- Want to estimate Peak from Trail
- Use pairs of observations from sample
- Need to determine coefficient b or r
- b- slope of linear estimate
- r- linear correlation

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#### **Definitions**

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- Need to find the value of b that minimizes that sum

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The value of b that minimizes the total error in the sample

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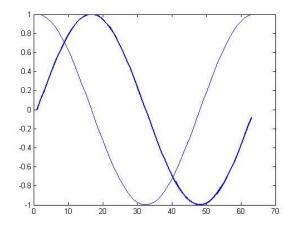
$$x_{i}^{*} = x'_{i} / s_{x}, y_{i}^{*} = y'_{i} / s_{y}, r = (\overline{x'_{i} y'_{i}})$$

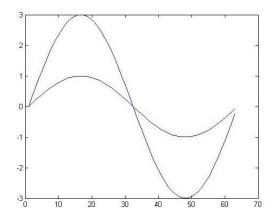
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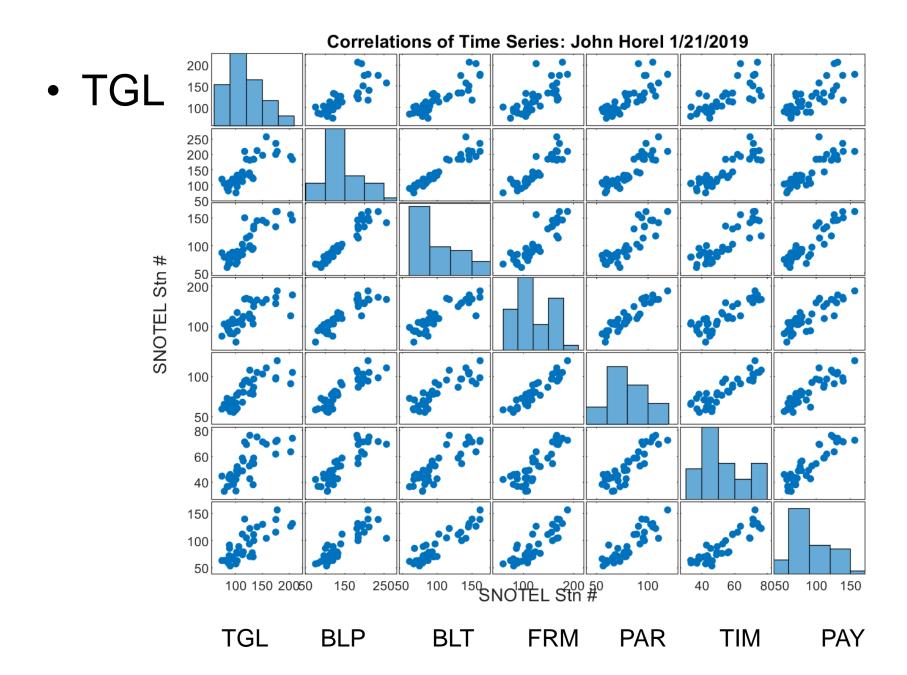
## Getting things mucked up

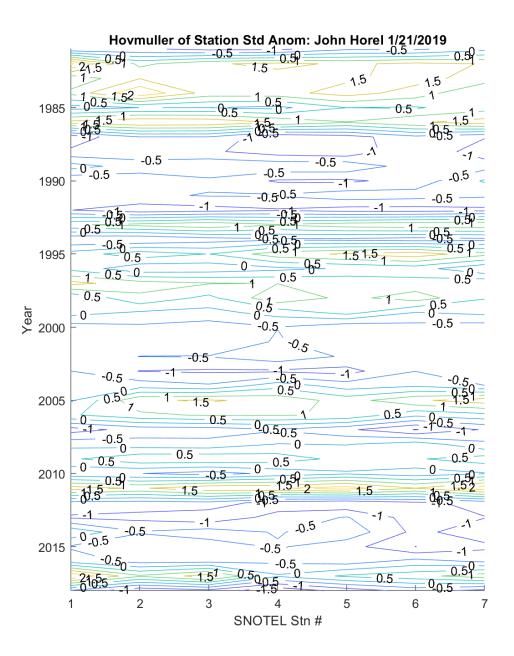
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#### Multivariate Linear Correlations

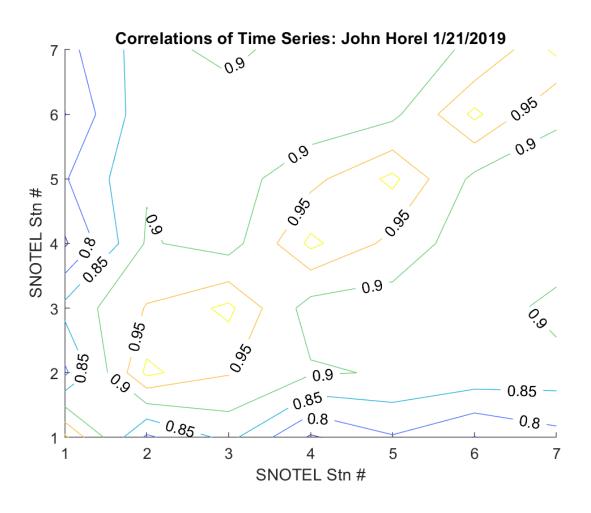
$$\vec{X}^* = \begin{bmatrix} x^*_{11} & x^*_{12} & \dots & x^*_{17} \\ x^*_{21} & x^*_{22} & \dots & x^*_{27} \\ \dots & \dots & \dots \\ x^*_{n1} & x^*_{n2} & \dots & x^*_{n7} \end{bmatrix}$$

- 7 stations and n=38 years
- Standardized anomalies





#### Multivariate Linear Correlations



$$\vec{X}^* = \begin{bmatrix} x^*_{11} & x^*_{12} & \dots & x^*_{17} \\ x^*_{21} & x^*_{22} & \dots & x^*_{27} \\ \dots & \dots & \dots \\ x^*_{n1} & x^*_{n2} & \dots & x^*_{n7} \end{bmatrix}$$

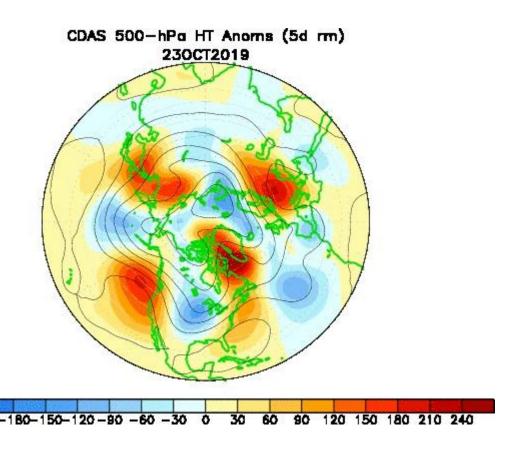
$$\vec{R} = \vec{X} *^T \vec{X} * / n$$

## Assignments: wrapping things up

- Read Chapter 4 notes
- Online final will be posted after Thanksgiving
- Two remaining classes, don't miss!

### Daily Anomaly Maps

http://www.cpc.ncep.noaa.gov/products/intraseasonal/z500\_nh\_anim.shtml



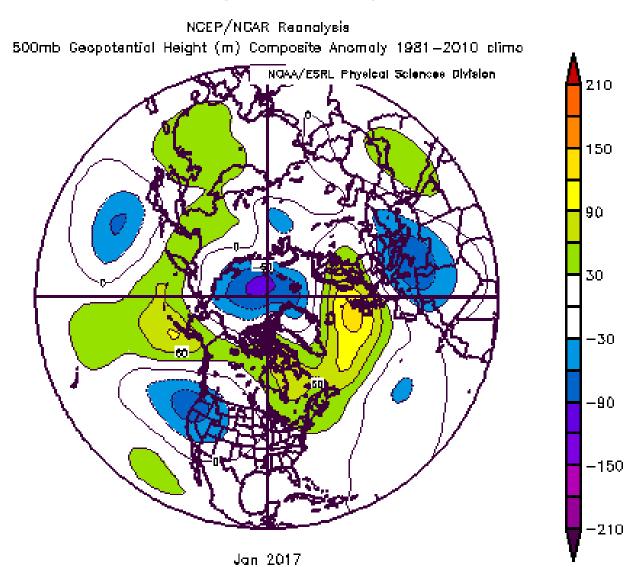
#### Teleconnection

a causal connection or correlation between meteorological or other environmental phenomena that occur a long distance apart

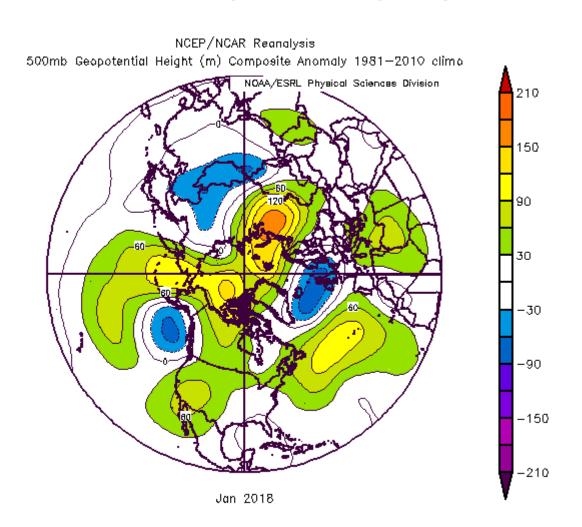
http://www.cpc.ncep.noaa.gov/data/teledoc/teleintro.shtml

Teleconnections: "the study of blobs"

## 500 hPa Height Anomalies Jan 2017

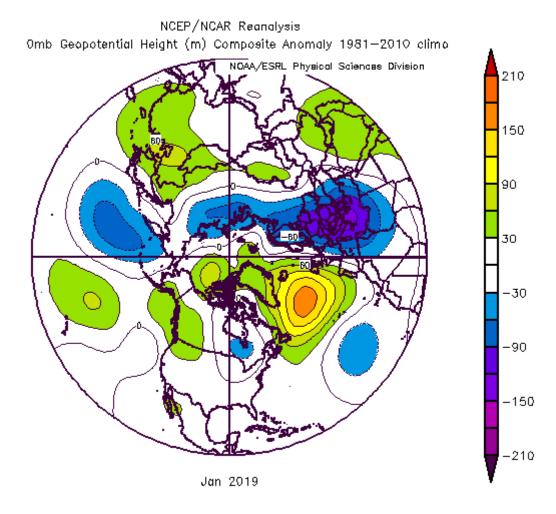


## 500 hPa Height Anomalies Jan 2018

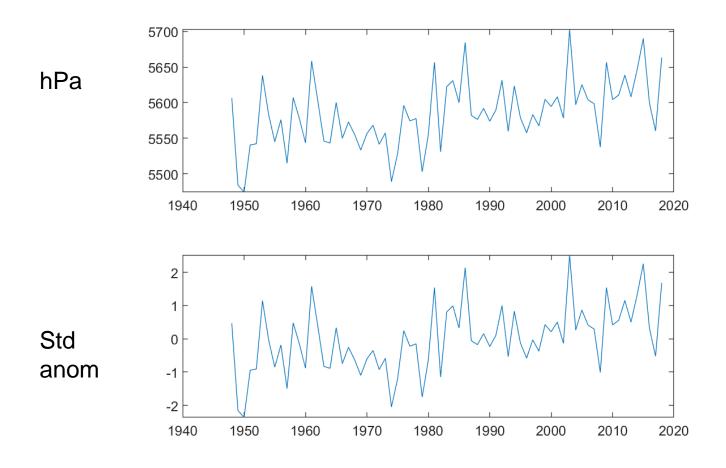


## 500 hPa Height Anomalies Jan 2019

https://www.esrl.noaa.go v/psd/cgibin/data/composites/print page.pl



## January SLC 500 hPa time series

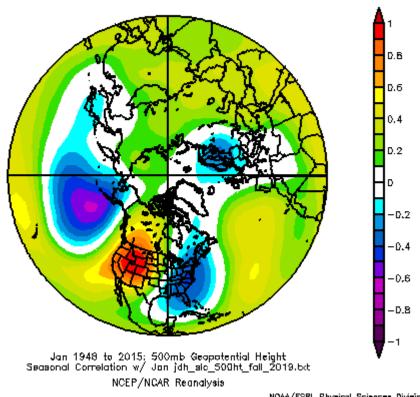


#### Correlate 500 hPa heights over SLC with every other point in NH: a teleconnection map

 Create a time series for **SLC:** location 42.5 40.0 247.5 250. 500 mb

https://www.esrl.noaa.gov/p sd/data/correlation/ /Public/incoming/timeseries/ jdh\_slc\_500ht\_fall2019.txt

Create plot



NOAA/ESRL Physical Sciences Division

## Correlation between SLC time series and precipitation in the Northern Hemisphere

