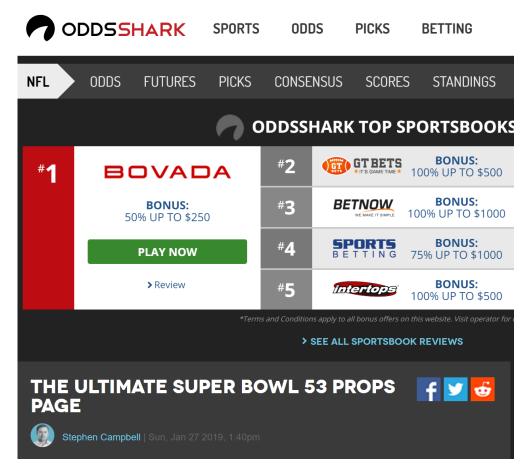
### Assignments

- Assignment 5: extra credit. Due Thursday
  - <a href="https://www.meted.ucar.edu/training">https://www.meted.ucar.edu/training</a> module.php?id=500#.VMpjz2jF-Sp
- Assignment 7: Super Bowl. Due Friday
- Assignment 8: Due at beginning of class next Tuesday
- Read Chapter 3 notes
- Read text: Text Chapter 3.1-3.3

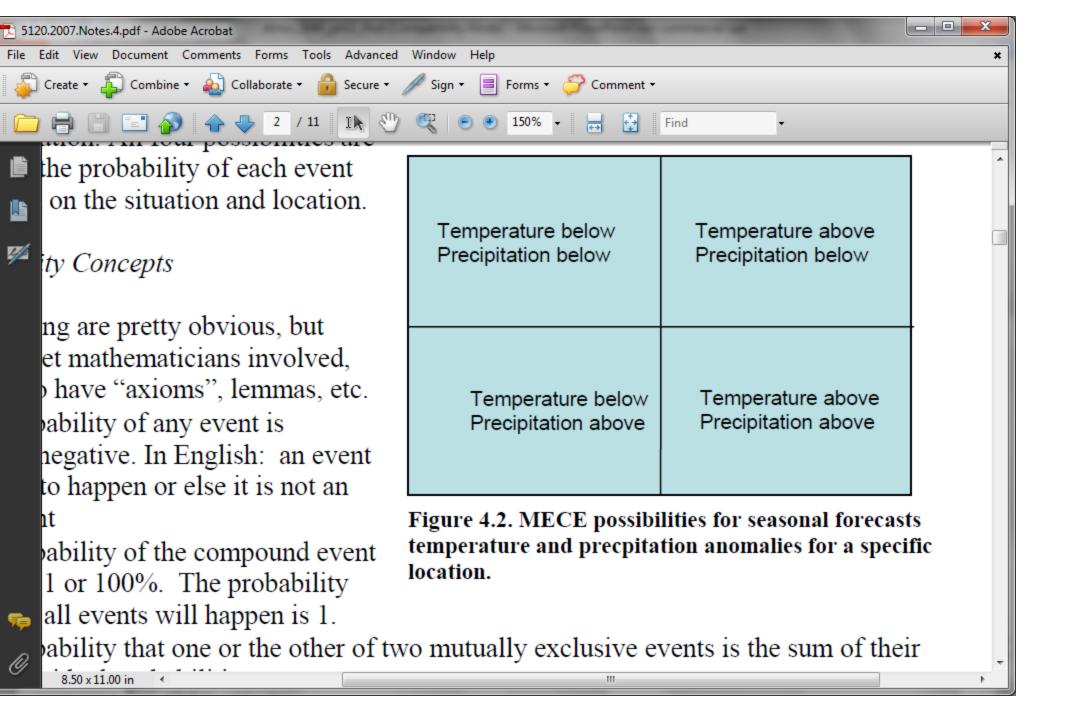
# SuperBowl!! <a href="https://www.oddsshark.com/super-bowl/props">https://www.oddsshark.com/super-bowl/props</a>

- bet 115 on NE to win get 100 (go home with 225 if NE wins; lose 115 if NE loses)
- Patriots favored by 1: bet 101 on Patriots to win by at least 1 point (go home with 201 if Patriots win by 1; go home with 101 if Patriots win by 1- push; lose 101 if Patriots tie or lose)



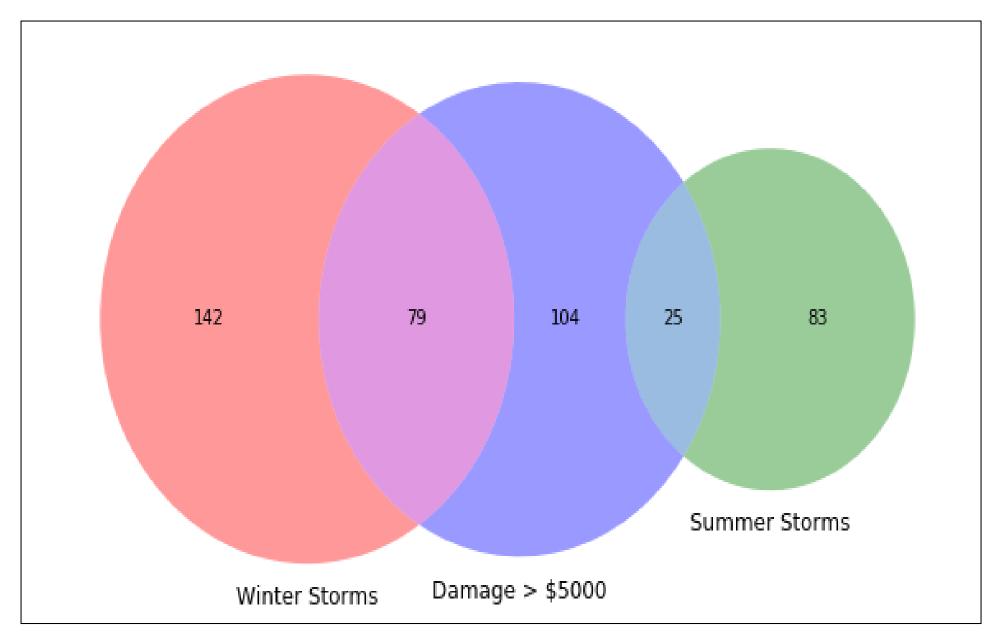
# Probability Definitions

- Event- possible uncertain outcomes
- Null event- can't happen
- Elementary event- can't be decomposed into other events
- Compound event- decomposable into 2 or more elementary events
- S- sample or event space- all possible elementary events
- Mutually exclusive- two events that can't occur at same time
- MECE- Mutually exclusive and collectively exhaustive- no more than 1 event can occur and at least one event will occur



#### More Definitions

- E- Event
- Pr{E}- probability of Event E; 0≤ Pr{E} ≤ 1
- Pr{E} = 0, event does not occur
- Pr{E} = 1 event occurs



Number of Opportunities: 2340 (180 days \* 13 years)

# Two Statistical Frameworks: Frequency vs. Bayesian

- Frequency- probability of an event is its relative frequency after many trials
- a- number of occurrences of E
- n- number of opportunities for E to take place
- a/n- relative frequency of E occurring
- $Pr\{E\} \rightarrow a/n \text{ as } n \rightarrow \infty$

# Two Statistical Frameworks: Frequency vs. Bayesian

- Bayesian- probability represents the degree of belief of an individual about an outcome of an uncertain event
- Some events occur so rarely that there is no long-term relevant probability
- Two individuals can have different probabilities for same outcome
- Bookies are Bayesian
  - Super Bowl odds 2019: Heavy Patriots money has sportsbooks rooting for Rams

## More concepts

- {E}<sup>c</sup>- complement of {E}, event does not occur
- $Pr\{E\}^c = 1-\{E\}$
- $Pr\{E_1 \cap E_2\}$  joint probability that  $E_1 \& E_2$  occur
- $Pr\{E_1 \cap E_2\} = 0$  if  $E_1 \& E_2$  are mutually exclusive
- Pr{E<sub>1</sub> U E<sub>2</sub>}- probability that E<sub>1</sub> OR E<sub>2</sub> occur
- $Pr\{E_1 \cup E_2\} = Pr\{E_1\} + Pr\{E_2\} Pr\{E_1 \cap E_2\}$

# Conditional Probability

- Conditional probability: probability of {E<sub>2</sub>} given that {E<sub>1</sub>} has occurred
- $Pr\{E_2 \mid E_1\} = Pr\{E_1 \cap E_2\} / Pr\{E_1\}$
- E<sub>1</sub> is the conditioning event
- If  $E_1$  and  $E_2$  are independent of each other, then  $Pr\{E_2 \mid E_1\} = Pr\{E_2\}$  and  $Pr\{E_1 \mid E_2\} = Pr\{E_1\}$
- Fair coin- Pr{heads} = 0.5
  - chance of getting heads on second toss is independent of the first Pr{heads | heads} = 0.5
     Pr{heads} twice = 0.5 \* 0.5 = .25

#### Deck of Cards

 If don't look at cards, then the odds of getting any specific card when dealt cards is the same as if you only were dealt one card: they are independent events

Pr{ace}	Pr{10-K}	Pr{2-9}	Pr{21}
7.7	30.8	61.5	4.8

	Tally of	Total	Tally of	Total # of	Observed	Expected	Expected
	occurrences	number of	opportunities	opportunities	probability	Probability	number of
		occurrences	(2 x number	(n)	a/n *100	from Part 1	outcomes if n
		(a)	of hands)		(%)	(%)	opportunities
ace				20		7.7	1.5
10-K				20		30.8	6
2-9				20		61.5	12
21				10		4.8	.5

# Bayes Theorem

- $Pr\{E_2 \mid E_1\} = Pr\{E_1 \cap E_2\} / Pr\{E_1\}$
- E<sub>1</sub> is the conditioning event
- What is the advantage? Probability of conditioning event E<sub>1</sub> only computed once
- $Pr\{E_1 \mid E_2\} = Pr\{E_2 \mid E_1\} * Pr\{E_1\} / Pr\{E_2\}$
- $Pr\{E_1 \cap E_2\} = Pr\{E_2 \mid E_1\} * Pr\{E_1\}$
- $Pr\{E_1 \cap E_2\} = Pr\{E_1 \mid E_2\} * Pr\{E_2\}$  then

Bayesian Application: how you should respond to "evidence"

	Pos Test	Neg Test	TOTAL
DRUG USER	0.495%	0.005%	0.5%
NOT DRUG USER	.995%	98.505%	99.5%
TOTAL	1.49%	98.51%	1

How many drug users test positive? 99%
Out of a ten thousand people, how many drug users do they catch? .00495\*10000=~50

What are odds of falsely accusing non drug user?

E₁ – not drug user

E<sub>2</sub> - positive test

 $Pr\{E_1\} - 99.5\%$ 

 $Pr\{E_2\} - 1.49\%$ 

 $Pr\{E_2 \mid E_1\} - .995\%$ 

 $Pr\{E_1 \mid E_2\} = Pr\{E_2 \mid E_1\} * Pr\{E_1\} / Pr\{E_2\} = 0.995 * 99.5 / 1.49 = 68\%$ 

- If you are a non drug user, you have a 68% chance of getting a "false positive"
- How many non-drug users get falsely accused?.00995\*10000=~100

Conclusion: always always ask for second opinion if clean and test positive

# Application of Bayes theorem: how to be rational responding to probabilities

	Pos Test	Neg Test	TOTAL
DRUG USER	0.495%	0.005%	0.5%
NOT DRUG USER	.995%	98.505%	99.5%
TOTAL	1.49%	98.51%	1

What are odds of a drug user skating?

E₄ – drug user

E<sub>3</sub> - negative test

 $Pr{E_3} - 98.51\%$ 

 $Pr\{E_4\} - 0.5\%$ 

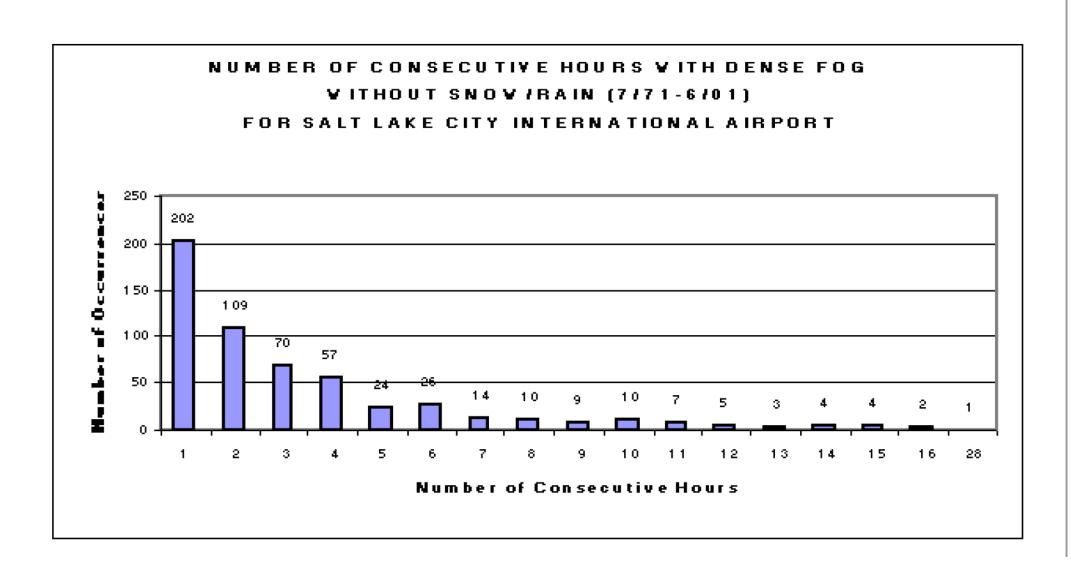
 $Pr\{E_3 \cap E_4\} - .005\%$ 

 $Pr{E_4 | E_3} = Pr{E_4 \cap E_3} / Pr{E_3} = 0.005 / 98.51 = .0051\%$ 

Out of 10000 people, maybe 1 drug user will test negative Conclusion: people who give drug tests are more interested in making sure drug users are caught than worrying about innocent people being falsely accussed

### Olympic Fog Climatology

When phenomena are persistent, then "odds" are higher once an event is underway



### **Forecast Verification**

- What is your reason for doing it?
- (Brier and Allen 1951; Compendium of Meteorology)
  - Administrative: who's blowing the forecasts?
  - Scientific: why do errors happen?
  - Economic: what's the impact of forecast errors?

# Measures oriented: "give me a number!"

Distill set of forecasts and observations into small # of metrics

		Observed	Observed	Forecast marginal totals
		Yes	No	
Forecast	Yes	а	b	a+b
Forecast	No	С	d	c+d
	Observed marginal totals	a+c	b+d	n=a+b+c+d sample size

• PC = percent correct = 
$$\frac{a+d}{n}$$

• FAR = false alarm ratio = 
$$\frac{b}{a+b}$$

• TS = CSI = 
$$\frac{a}{a+b+c}$$

• POD = HR = 
$$\frac{a}{a+c}$$

# What if it just happened by chance?

		Observed	Observed	Forecast marginal totals
		Yes	No	
Forecast	Yes	а	b	a+b
Forecast	No	С	d	c+d
	Observed marginal totals	a+c	b+d	n=a+b+c+d sample size

- Random correct yes forecast by chance =  $\frac{(a+b)}{n} \frac{(a+c)}{n}$
- Random correct no forecast by chance =  $\frac{(b+d)}{n} \frac{(c+d)}{n}$
- $SS = \frac{(correct forecasts random correct forecasts)}{(total forecasts random correct forecasts)}$

$$\bullet \quad HSS = \frac{2(ad-bc)}{(a+c)(b+d)+(a+b)(b+d)}$$