Conjectures – John 3-15-21

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1 Conjectures for m=14:

- Things relating to residues 3 apart:
- $N(1, 14, n) \geq N(4, 14, n)$, $\forall n$.
- $N(2, 14, n) \geq N(5, 14, n)$, $\forall n$.
- $N(3, 14, n) \geq N(6, 14, n)$, $\forall n$.
- Things relating to the middle:
- $N(3, 14, n) \geq N(7, 14, n)$, $\forall n$.
- $N(2, 14, n) \geq N(7, 14, n)$, $\forall n$.
- $N(1, 14, n) \geq N(7, 14, n)$, $\forall n$.
- Things relating to residues 2 apart:
- $N(0, 14, n) \geq N(2, 14, n)$, $\forall n$.
- $N(1, 14, n) \geq N(3, 14, n)$, $\forall n$.
- $N(2, 14, n) \geq N(4, 14, n)$, $\forall n$.
- $N(3, 14, n) \geq N(5, 14, n)$, $\forall n$.
- $N(4, 14, n) \geq N(6, 14, n)$, $\forall n$.
- NOTE that it is NOT true that $N(5, 14, n) \geq N(7, 14, n)$, $\forall n$!

2 Conjectures for m=16:

3 Conjectures for m=18:

4 Conjectures for m=20:

5 Conjectures for m=22:

- Things dealing with residues 2 apart:
- $N(0, 22, n) \geq N(2, 22, n)$, $\forall n$.
- $N(1, 22, n) \geq N(3, 22, n)$, $\forall n$.
- $N(2, 22, n) \geq N(4, 22, n)$, $\forall n$.
- $N(3, 22, n) \geq N(5, 22, n)$, $\forall n$.
- $N(4, 22, n) \geq N(6, 22, n)$, $\forall n$.
- $N(5, 22, n) \geq N(7, 22, n)$, $\forall n$.
- $N(6, 22, n) \geq N(8, 22, n)$, $\forall n$.
- $N(7, 22, n) \geq N(9, 22, n)$, $\forall n$.
- $N(8, 22, n) \geq N(10, 22, n)$, $\forall n$.
- Note it is NOT true that $N(9, 22, n) \geq N(11, 22, n)$, $\forall n$.
- Things dealing with residues 3 apart:
- $N(0, 22, n) \geq N(3, 22, n)$, $\forall n$.
- $N(1, 22, n) \geq N(4, 22, n)$, $\forall n$.
- $N(2, 22, n) \geq N(5, 22, n)$, $\forall n$.
- $N(3, 22, n) \geq N(6, 22, n)$, $\forall n$.
- $N(4, 22, n) \geq N(7, 22, n)$, $\forall n$.
- $N(5, 22, n) \geq N(8, 22, n)$, $\forall n$.
- $N(6, 22, n) \geq N(9, 22, n)$, $\forall n$.
- $N(7, 22, n) \geq N(10, 22, n)$, $\forall n$.
- It is NOT true that $N(8, 22, n) \geq N(11, 22, n)$, $\forall n$.
- Things dealing with residues 5 apart:
- $N(0, 22, n) \geq N(5, 22, n)$, $\forall n$.

- $N(1, 22, n) \geq N(6, 22, n)$, $\forall n$.
- $N(2, 22, n) \geq N(7, 22, n)$, $\forall n$.
- $N(3, 22, n) \geq N(8, 22, n)$, $\forall n$.
- $N(4, 22, n) \geq N(9, 22, n)$, $\forall n$.
- $N(5, 22, n) \geq N(10, 22, n)$, $\forall n$.
- $N(6, 22, n) \geq N(11, 22, n)$, $\forall n$.
- $N(7, 22, n) \geq N(12, 22, n)$, $\forall n$.
- Note that it is NOT true that $N(8, 22, n) \geq N(13, 22, n)$, $\forall n$.
- Things dealing with residues 7 apart:
- $N(0, 22, n) \geq N(7, 22, n)$, $\forall n$.
- $N(1, 22, n) \geq N(8, 22, n)$, $\forall n$.
- $N(2, 22, n) \geq N(9, 22, n)$, $\forall n$.
- $N(3, 22, n) \geq N(10, 22, n)$, $\forall n$.
- $N(4, 22, n) \geq N(11, 22, n)$, $\forall n$.
- $N(5, 22, n) \geq N(12, 22, n)$, $\forall n$.
- $N(6, 22, n) \geq N(13, 22, n)$, $\forall n$.
- Note that it is NOT true that $N(14, 22, n) \geq N(7, 22, n)$, $\forall n$.
- Things dealing with residues 11 apart:
- ??? yeah
- Things relating to $r = \frac{m}{2}$:
- If i is any natural number in $[0, 7]$, $N(i, 22, n) \geq N(11, 22, n)$, $\forall n$.
- Also, note that by the above inequalities, residues that differ by multiples of the numbers mentioned above will also possess this inequality, so long as the residues involved are less than $r = m/2$.

6 Now, conjectures for all even m:

General ideas:

- For any residues that are a distance of >1 apart (NOT adjacent), if $r+d < m/2$, then $N(r,m,n) \geq N(r+d,m,n)$, $\forall n$, where d is the distance between the residue classes.
- When two residues ARE adjacent, then for large enough n, $N(r,m,n) \geq N(r+1,m,n)$.
- For n smaller than this "large enough" n, if n is even, then N of the ODD residues will be greater than the residues that immediately come before them. Likewise if n is even, then N of the even residues will be greater than the one that came immediately before it as well.
- For r which are
- For large enough n, r=0 will have the most partitions of ANY residue class, much like in the odd case. For smaller n, it is subject to the constraints mentioned above.
- Basically, greater than some n, our distribution will have the nice, monotone "V" shape that we're so familiar with from the evens. Before that, though, it will be jagged.
- For $r = \frac{m}{2}$, the first nonzero value for N occurs at $n = \frac{m}{2} + 1$.
- and of course, the whole sequence thing...