MAT300 Spring 2021 Final Review

John J Li

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Problem 1

Prove that for every nonnegative integer n, $1 + 6n \leq 7^n$.

Proof. Base case: Let $n = 0$. Then $1 + 6(0) = 1 \leq 7^0 = 1$.

Inductive step: Suppose $P(n)$, we will show $P(n+1)$. $\square$

Problem 2

Prove that every nonnegative integer n, $101|(10^2n - (-1)^n)$.

Problem 3

Let R be an equivalence relation on A and let $a, b \in A$.

Show that if aRb then $|a| \subseteq |b|$.

Show that $R \circ R \subseteq R$.

Problem 4

Let R be the following relation on $\mathbb{Z}$: xRy if $2|(x + y^2)$. Check if R is reflexive, symmetric, antisymmetric, and transitive (give a proof or a counterexample).

Problem 5

Let R be the following relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$, $(x, y)R(z, w)$, if $x|z$ and $y|w$.

Show that R is a partial order but not a total order.

Final all minimal elements (or state that none exists) of $(\mathbb{Z}^+ \times \mathbb{Z}^+) \setminus \{(1, 1)\}$.

Problem 6

Prove that if $A \subseteq B$ and $C \subseteq B$, then $A \times \mathcal{P}(C) \subseteq B \times \mathcal{P}(B)$.

Problem 7

Let R be a relation on A. Show that R is symmetric if and only if $R^{-1} = R$.

Problem 8

Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be given by $f((x, y)) = (3y - x, x - y)$. Check if f is injective and surjective.

Problem 9

Consider the following relation R on $\mathbb{Z} \times \mathbb{Z}$, $(x, y)R(z, w)$ if $2|(x - z)$ and $2|(y - w)$. Prove that R is an equivalence relation and find $\mathbb{Z} \times \mathbb{Z} \setminus R$.

Problem 10

Let E be the set of positive even integers. Show that $E \times \{1, 2\}$ is denumerable.