Homework 4

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Problem 5:

Prove that for every real number x, if $x \neq 5$, then there is an unique real number y such that $5y = x(y-2)$.

Proof. Let x be an arbitrary real number, an suppose $x \neq 5$. Let $y = \frac{2x}{x-5}$, which is defined since $x \neq 5$. By solving for x, it is seen that $x = \frac{5y}{y-2}$ then

$$\frac{5y}{y-2} = \frac{\frac{10x}{x-5}}{\frac{2x}{x-5} - 2} = \frac{\frac{10x}{x-5}}{\frac{10}{x-5}} = \frac{10x}{10} = x$$

To see that this solution is unique, suppose $\frac{5z}{z-2} = x$. Then $5z = x(z-2)$, so $z(5-x) = -2x$ which is also $z = \frac{2x}{x-5} = y$. $\square$

Problem 6:

Let $\mathcal{F}$ and $\mathcal{G}$ be families of sets. Show that if $\mathcal{F} \cap \mathcal{G} \neq \emptyset$, then $\mathcal{P}(\bigcap \mathcal{F}) \subseteq \mathcal{P}(\bigcup \mathcal{G})$.

Proof. $\square$

Problem 7:

Prove that $\lim_{x \to 2} \frac{3x^2-12}{x-2} = 12$

Proof. Let $\epsilon > 0$. Consider $\delta = \frac{\epsilon}{3}$. Then for every x, if $0 < |x-2| < \delta$ then $|f(x) - 12| = |\frac{3x^2-12}{x-2} - 12| = |3x + 6 - 12| = 3|x-2| < 3\delta = 3 \cdot \frac{\epsilon}{3} = \epsilon$. $\square$