Homework 3

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Problem 1:

Proof. 1) n^3 is odd $\to$ n is odd

Indirectly. Suppose n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$. Therefore $n^3 = (2k)^3 = 8k^3 = 2(4k^3)$ and $4k^3 \in \mathbb{Z}$. Thus n^3 is even.

2) n is odd $\to$ n^3 is odd

Suppose n is odd. Then $n = 2k+1$ for some $k \in \mathbb{Z}$. Therefore $n^3 = (2k+1)^2 = 8k^3 + 6k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1$ and $4k^3 + 6k^2 + 3k \in \mathbb{Z}$. Thus n^3 is odd. $\square$

Problem 2:

Proof. $\square$

Problem 3:

Proof. $\square$

Problem 4:

Proof. $\square$

Problem 5:

Proof. $\square$

Problem 6:

Proof. $\square$

Problem 7:

Proof. $\square$

Problem 8:

Proof. By contradiction.

Suppose $\sqrt{3}$ is rational. Then $\sqrt{3} = \frac{p}{q}$ where p and q are integers, share no common factors, and $q \neq 0$. Then $3 = \frac{p^2}{q^2}$ then $3q^2 = p^2$. So $3|p^2$ which implies $3|p$ and $3k = p$ where $k \in \mathbb{Z}$ Thus $3q^2 = (3k)^2$ then $q^2 = 3k$ so $3|q^2$ which implies $3|q$. And so $q = 3j$ for some $j \in \mathbb{Z}$. This contradicts our statement that p and q share no common factors. So, there are no integers p and q such that $\sqrt{3}$ is rational by contradiction. Thus, $\sqrt{3}$ must be irrational.

□

Problem 9:

Proof. □

Problem 10:

Proof. □