Homework 3

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Problem 1:

Proof. Suppose $0 < a < b$. Then multiplying the inequality $a < b$ by the positive number a can conclude that $a^2 < ab$; similarly, multiplying the inequality $a < b$ by the positive number b can conclude that $ab < b^2$. Thus $a^2 < ab < b^2$. Then by multiplying the new inequality $a^2 < ab < b^2$ by the positive number a, we can conclude $a^3 < a^2b < ab^2$; furthermore, by multiplying the new inequality $a^2 < ab < b^2$ by the positive number b, we can conclude $a^2b < ab^2 < b^3$. So $a^3 < a^2b < ab^2 < b^3$, thus if $0 < a < b$ then $a^3 < b^3$. □

Problem 2:

Proof. We will prove the contrapositive. Suppose if $x = 9$ then $x^3 - \sqrt{x} \neq 3x^2 + sin(x)$. Then $(9)^3 - \sqrt{9} \neq 3(9)^2 + sin(9)$ and $726 \neq 243.412$. Therefore if $x^3 - \sqrt{x} = 3x^2 + sin(x)$ then $x \neq 9$. □

Problem 3:

Proof. We will prove the contrapositive. Suppose n is not even. Then $n = 2k+1$ where $k \in \mathbb{Z}$. Then $3(2k+1)^3 + 6$ and $24k^3 + 36k^2 + 18k^2 + 9$ which can be written as $2(12k^3 + 13k^2 + 12k + 4) + 1$ which is not even. Therefore if $3(n)^3 + 6$ then n is not even. □

Problem 4:

Proof. Let x be arbitary. Suppose A and $B \setminus C$ are disjoint. This means $\forall_x (x \in A \wedge \neg(x \in B \wedge x \notin C))$ and so $\forall_x (x \in A \wedge x \in B \rightarrow x \in C)$ and this is the logic form of $A \cap B \subseteq C$ since x is arbitary. Therefore if A and $B \setminus C$ are disjoint then $A \cap B \subseteq C$. □