

# Homework 2

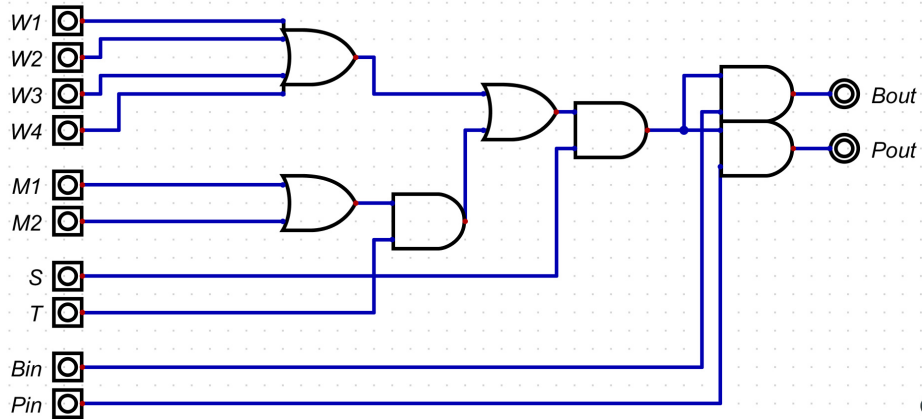
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**Problem 1:**

- (1)  $(abc') + (bd) + (a'cd') + (b'd) + (a'c'd')$
- (2) Associative  $(bd + b'd) + (a'cd' + a'c'd') + (abc')$
- (3) Distributive  $d(b + b') + a'd'(c + c') + (abc')$
- (4) Inverse element  $d(\cancel{b+b'}) + a'd'(\cancel{c+c'}) + (abc')$
- (5) Identity  $d + a'd' + abc'$
- (6) Distributive  $(d + a')(d + d') + abc'$
- (7) Inverse element  $(d + a')(\cancel{d+d'}) + abc'$
- (8) Identity  $(d + a') + abc'$
- (9) Distributive  $d + (a' + a)(a' + b)(a' + c')$
- (10) Inverse element  $d + (\cancel{a'+a})(a' + b)(a' + c')$
- (11) Identity  $d + (a' + b)(a' + c')$
- (12) Distributive  $d + a' + (bc')$

**Problem 2:**



**Problem 3:**

(a)

#	x	y	z	G(x,y,z)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

(b)

cononical SOP

$$G(x, y, z) = x'y'z' + x'y'z + x'yz + xy'z' + xyz$$

(c)

$$\begin{aligned} G(x, y, z) &= x'y'z' + x'y'z + x'yz + xy'z' + xyz & (1) \\ G(x, y, z) &= x'y'z' + x'y'z + x'yz + xyz + xy'z' & (2) \text{ Associative} \\ G(x, y, z) &= x'y'(z' + z) + (x' + x)yz + xy'z' & (3) \text{ Distributive} \\ G(x, y, z) &= x'y'(\cancel{z' + z}) + (\cancel{x' + x})yz + xy'z' & (4) \text{ Inverse element} \\ G(x, y, z) &= x'y' + yz + xy'z' & (5) \text{ Identity} \\ G(x, y, z) &= x'y' + xy'z' + yz & (6) \text{ Associative} \\ G(x, y, z) &= y'(x' + xz') + yz & (7) \text{ Distributive} \end{aligned}$$

(d)

cononical POS

$$G(x, y, z) = (x + y' + z) \cdot (x' + y + z') \cdot (x' + y' + z)$$

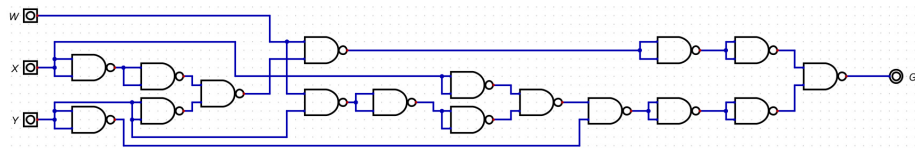
(e)

$$G(x, y, z) = \pi M(2, 5, 6)$$

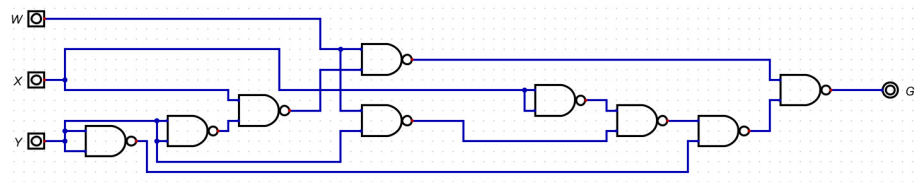
(f)

$$\begin{aligned} G(x, y, z) &= (x + y' + z) \cdot (x' + y + z') \cdot (x' + y' + z) & (1) \\ G(x, y, z) &= (x' + y + z') \cdot (x + y' + z) \cdot (x' + y' + z) & (2) \text{ Associative} \\ G(x, y, z) &= (x' + y + z') \cdot (y' + z) \cdot (x + x') & (3) \text{ Distributive} \\ G(x, y, z) &= (x' + y + z') \cdot (y' + z) \cdot (\cancel{x + x'}) & (4) \text{ Inverse element} \\ G(x, y, z) &= (x' + y + z') \cdot (y' + z) & (5) \text{ Identity} \end{aligned}$$

**Problem 4:**



Simplified:



### Problem 5:

A	B	C	D	$A'B' + ABD'$	$AC'D' + BD'$	$AB'CD$	F
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	1
0	0	1	0	1	0	0	1
0	0	1	1	1	0	0	1
0	1	0	0	0	1	0	1
0	1	0	1	0	0	0	0
0	1	1	0	0	1	0	1
0	1	1	1	0	0	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0
1	0	1	1	0	0	1	1
1	1	0	0	1	1	0	1
1	1	0	1	0	0	0	0
1	1	1	0	1	1	0	1
1	1	1	1	0	0	0	0

AB	CD			
	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	0	1	0

0/1/3/2 row: 0000, 0001, 0011, 0010  $\rightarrow$  C and D changes

$$= A'B'$$

0/4/12/8 column: 0000, 0100, 1100, 1000  $\rightarrow$  A and B changes

$$= C'D'$$

3/11 pair: 0011, 1011  $\rightarrow$  A changes

$$= B'CD$$

4/12/6/14 square: 0100, 0110, 1100, 1110  $\rightarrow$  A and C changes

$$= BD'$$

$$\boxed{F = A'B' + C'D' + B'CD + BD'}$$

**Problem 6:**

(A)

WX	YZ	00	01	11	10
00		1	1	0	1
01		1	1	0	1
11		0	0	0	0
10		1	1	0	0

0/1/4/5/square: 0000, 0001, 0100, 0101  $\rightarrow$  X and Z changes

$$= W'Y'$$

0/1/8/9 square: 0000, 0001, 1000, 1001  $\rightarrow$  W and Z changes

$$= X'Y'$$

0/4/2/6 square: 0000, 0100, 0010, 0110  $\rightarrow$  X and Y changes

$$W'Z'$$

$$\boxed{A = W'Y' + X'Y' + W'Z'}$$

(B)

WX	YZ				
		00	01	11	10
	00	1	1	1	1
	01	1	1	1	1
	11	0	0	0	0
	10	1	1	1	1

0/1/3/2/4/5/7/6 rectangle: 0000, 0001, 0011, 0010, 0100, 0101, 0111, 0110 → X, Y, and Z changes

$$= W'$$

0/1/3/2/8/9/11/10 rectangle: 0000, 0001, 0011, 0010, 1000, 1001, 1011, 1010 → W, Y, and Z changes

$$= X'$$

$$B = W' + X'$$

**Problem 7:**

w	x	y	z	w'y'z	w'xz	yz+xy'z	G
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0
0	0	1	1	0	0	1	1
0	1	0	0	0	0	0	0
0	1	0	1	1	1	1	1
0	1	1	0	0	0	0	0
0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0
1	0	1	0	0	0	0	0
1	0	1	1	0	0	1	1
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	0	0
1	1	1	1	0	0	1	1

wx	yz				
		00	01	11	10
	00	0	1	1	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	0	1	0

POS

0/4/12/8/2/6/14/10 rectangle: 0000, 0100, 1100, 1000, 0010, 0110, 1110, 1010  
 $\rightarrow$  w,x and y changes

$$= z$$

8/9 pair: 1000, 1001  $\rightarrow$  z changes

$$= w' + x + y$$

$$\boxed{G(w, x, y, z) = z(w' + x + y)}$$

SOP

3/7/15/11 column: 0011, 0111, 1111, 1011  $\rightarrow$  w and x changes

$$= yz$$

5/7/13/15 square: 0101, 0111, 1101, 1111  $\rightarrow$  w and y changes

$$= xz$$

1/3/5/7 square: 0001, 0011, 0101, 0111  $\rightarrow$  x and y changes

$$= w'z$$

$$G(w, x, y, z) = yz + xz + w'z$$

**Problem 8:**

$$F(a, b, c, d) = \sum m(0, 1, 2, 3, 6, 7, 8, 10, 12, 13)$$

ab	cd				
		00	01	11	10
00		1	1	1	1
01		0	0	1	1
11		1	1	0	0
10		1	0	0	1

0/2/8/10 square: 0000, 0010, 1000, 1010  $\rightarrow$  a and c changed

$$= b'd'$$

0/1/3/2 row: 0000, 0001, 0011, 0010  $\rightarrow$  c and d changed

$$= a'b'$$

3/2/7/6 square: 0011, 0010, 0111, 0110  $\rightarrow$  b and d changed

$$= a'c$$

12/13 pair: 1100, 1101  $\rightarrow$  d changed

$$= abc'$$

$$F(a, b, c, d) = b'd' + a'b' + a'c + abc'$$

$$G(a, b, c, d) = \sum m(0, 1, 2, 3, 8, 9, 10, 13)$$

ab	cd				
		00	01	11	10
00		1	1	1	1
01		0	0	0	0
11		0	1	0	0
10		1	1	0	1

0/2/8/10 square: 0000, 0010, 1000, 1010  $\rightarrow$  a and c changed

$$= b'd'$$

0/1/3/2 row: 0000, 0001, 0011, 0010  $\rightarrow$  c and d changed

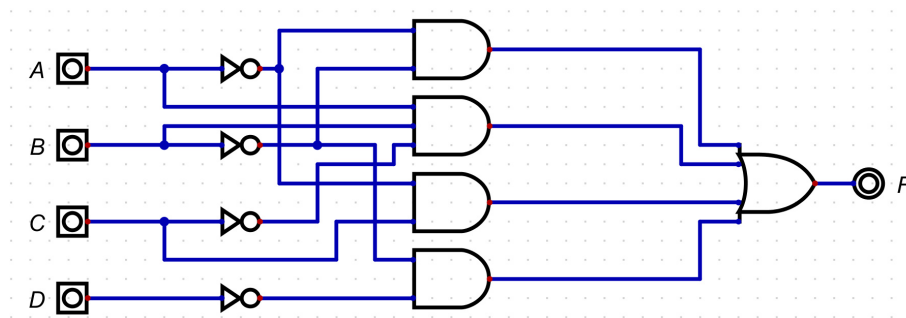
$$= a'b'$$

9/13 pair: 1011, 1101  $\rightarrow$  b changed

$$= ac'd$$

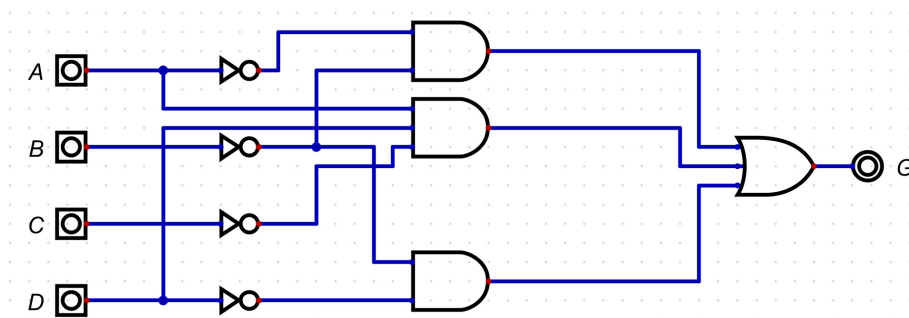
$$G(a, b, c, d) = b'd' + a'b' + ac'd$$

F Circuit

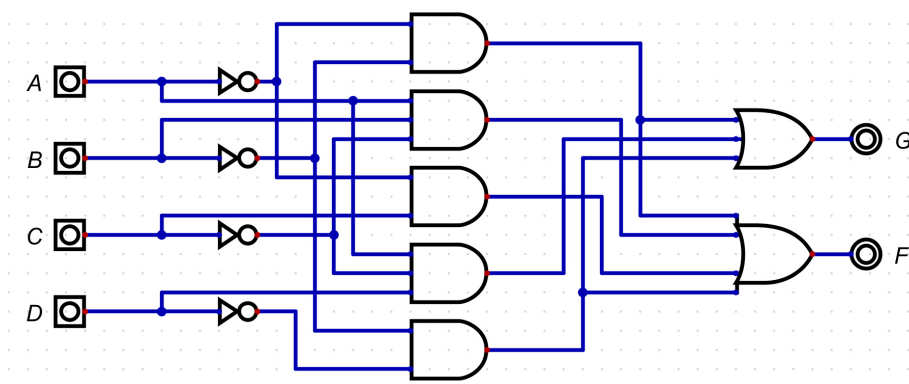




G Circuit



Shared Circuit



**Problem 9:**

$$F(a, b, c, d) = \sum m(0, 3, 4, 6, 9, 10, 11, 13, 14) + \sum d(1, 2, 7, 15)$$

ab	cd				
		00	01	11	10
00		1	x	1	x
01		1	0	x	1
11		0	1	x	1
10		0	1	1	1

3/2/7/6/15/14/10/11 rectangle: 0011, 0010, 0111, 0110, 1111, 1110, 1011, 1010  
 → a, b, and d changes

$$= c$$

13/15/9/11 square: 1101, 1111, 1001, 1011  $\rightarrow$  b and c changes

$$= ad$$

0/4/2/6 square: 0000, 0100, 0010, 0110  $\rightarrow$  b and c changes

$$= a'd'$$

$$\boxed{c + ad + a'd'}$$

**Problem 10:**

w	x	y	z	G
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

wx	yz				
		00	01	11	10
00		1	0	0	1
01		1	0	0	1
11		1	1	1	1
10		1	1	1	1

POS

1/3/5/7 square: 0001, 0011, 0101, 0111  $\rightarrow$  x and y changes

$$= w + z'$$

$$\boxed{G(w, x, y, z) = w + z'}$$

SOP

1/4/12/8/2/6/14/10 rectangle: 0000, 0100, 1100, 1000, 0010, 0110, 1110, 1010  
 $\rightarrow$  w, x, and y changes

$$= z'$$

12/13/15/14/8/9/11/10 rectangle: 1100, 1101, 1111, 1110, 1000, 1001, 1011,  
 1010  $\rightarrow$  x, y, and z changes

$$= w$$

$$\boxed{G(w, x, y, z) = z' + w}$$