Chapter 2 Notes

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1 Quantifiers

$\forall x\, P(x)$ reads: "For all x, $P(x)$." This symbol is called the *universal quantifier* because the statement states that P(x) is universally true. This statement is saying that the truth set of $P(x)$ is equal to U.

$\exists x\, P(x)$ reads: "There exists an x such that $P(x)$." This symbol is called the *existential quantifier*. $P(x)$ is true for at least one value of x.

The quantifiers *binds* a variable. So x is a bound variable in the statements $\forall x\, P(x)$ and $\exists x\, P(x)$.

1.1 Example

What do the following formulas mean? Are they true or false?

1. $\forall x\, (x^2 \geq 0) \in \mathbb{R}$
2. $\exists x\, (x^2 - 2x + 3 = 0) \in \mathbb{R}$
3. $\exists x\, (M(x) \wedge B(x))$ where the universe is the set of all people. $M(x)$ stands for: "x is a man," and $B(x)$ stands for: "x has brown hair."
4. $\forall x\, (M(x) \rightarrow B(x))$ with same universe and definition as 3.
5. $\forall x\, L(x, y)$ where the universe is the set of all people, and $L(x, y)$ means "x likes y."

Solutions

1. true
2. true
3. true
4. false
5. not enough information

1.2 Example

Analyze the logical forms of the following statements.

1. Someone didn't do the homework
2. Everything in that store is either overpriced or poorly made
3. Nobody's perfect
4. Susan likes everyone who dislikes Joe
5. $A \subseteq B$
6. $A \cap B \subseteq B \setminus C$

Solutions

1. $\exists x$ (x didn't do the hw)
2. $\forall x$ ((x in the store) $\rightarrow$ ((x is overpriced) $\vee$ (x is poorly made))
3. $\neg\exists x$ (x is perfect)
4.
5. not enough information