Proof. Let $t, u \in \mathbb{R}$ where $t = xy$ and $u = zw$. So,

$$\begin{aligned}
4xyzw &= 2 \cdot 2tu \\
&\leq 2 \cdot (t^2 + u^2) \\
&= 2 \cdot ((xy)^2 + (zw)^2) && \text{(substituting variables)} \\
&= 2 \cdot (x^2y^2 + z^2w^2) \\
&= 2x^2y^2 + 2z^2w^2 \\
&\leq ((x^2)^2 + (y^2)^2) + ((z^2)^2) + (w^2)^2) \\
&= x^4 + y^4 + z^4 + w^4
\end{aligned}$$

□