MAT300 Spring Notes

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Notes

Order relations.

Definition 6 Let R be a relation on A. Then R is called a partial order on A if it is reflexive, antisymmetric, and transitive.

Examples: 1) $\leq$ on $\mathbb{R}$

2) $\subseteq$ on $\mathcal{P}()\mathbb{N})$

3) — on $\mathbb{Z}^+$

A relation R on A is called a total order if it is a partial order and for every $a, b, \in A$ either aRb or bRa

Let R be a partial order on A and let B be a subset of A.

An element $b \in B$ is called minimal if there is no b' in B such that $b' \neq b$ and $b'Rb$.

An element $b \in B$ is called maximal if there is no b' in B such that $b' \neq b$ and bRb'.

An element $b \in B$ is called smallest

Example

a) $Z, \leq$

$B = -1, 0, 1, 2, 17$

minimal: -1

smallest: -1

b) $Z^+, |$

$B = 2, 3, 5, 6, 15$

minimal: $2, 3, 5$

smallest: none