MAT300 Spring Notes

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Notes

The domain of R is:

$$Dom(R) = \{a \in A | (a, b) \in R\}$$

Definition 5 Let R be a relation from A to B and let S be a relation from B to C. Then the composition of S and R is the relation $S\dot{R}$ from A to C.

$$S\dot{R} = \{(a, c) \in A \times C | \exists_{b \in B}(a, b) \in R \wedge (b, c) \in S\}$$

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Properties of relations

Let R be a relation on A. $(R \in A \times A)$

R is called reflexive if for every $a \in A$.

$$aRa$$

$$\forall_{a \in A}(a, a) \in R$$

R is called irreflexive if for every $a \in A$,

$$\neg aRa$$

$$\forall_{a \in A}(a, a) \notin R$$

R is called symmetric if for every $a, b \in A$

$$aRb \to bRa$$

$$\forall_a \forall_b (a, b) \in R \to (b, a) \in R$$

R is called asymmetric if for every $a, b, \in A$,

$$aRb \to \neg bRa$$

$$\forall_a \forall_b (a, b) \in R \to (b, a) \notin R$$

R is antisymmetric if for every $a, b \in A$,

$$(aRb \wedge bRa) \to (a = b)$$

$$\forall a \in A \forall_{b \in A} ((a, b) \in R \wedge (b, a))$$

R is called transitive if for every $a, b, c \in A$,

$$(aRb \wedge bRc) \to aRc$$

Examples

The relation of less than or equal to, $\leq$, on R.

1. reflexive: $\forall_{a \in \mathbb{R}} a = a$
2. symmetric: $\forall_{a,b \in \mathbb{R}} a = b \to b = a$
3. antisymmetric: $\forall_{a,b \in \mathbb{R}} (a = b \wedge b = a) \to a = b$
4. transitive

R on $z \times z$ $(a, b)R(c, d)$ if $a - c$ is even and $3|(b - d)$.

1. reflexive: Yes, let $(a, b) \in Z \times Z$. Then $a - a = 0 = 0 \cdot 2$ is even and $b - b = 0 = 0 \cdot 3; 3|(b - b)$.