Performance Analyses of Resistojet and Arcjet Engines

John J. Murray[1]

The University of Alabama, Tuscaloosa, AL, 35487

A number of different propulsion systems were analyzed for their thrust and performance characteristics. The specific impulse, thrust, and exhaust temperature of three related engines were determined. An arcjet engine was analyzed for the purposes of determining required design features to meet specified performance requirements. Values for expansion ratio and propellant mass were determined for the arcjet motor.

Nomenclature

c_p = isobaric heat capacity

h = specific enthalpy, kJ/kg

$\boldsymbol{M}$ = molecular mass, g/mol

q = heat transfer, kJ/kg-s

$\boldsymbol{R}$ = universal gas constant, 8.314 J/mol-K

T = temperature, K

Given Quantities

Inlet pressure	1.0 MPa
Mass flow $\dot{m}$	100 mg/s
Resistojet heater power	300 W
Resistojet chamber diameter	2.5 cm

[1] M.S. Candidate, Department of Aerospace Engineering and Mechanics, The University of Alabama.

I. Ammonia Resistojet

Beginning with the basic thermodynamic equation

$$q = c_p(T_H - T_C)\dot{m} \tag{1}$$

the added heat is assumed for this problem to be the heater power with no losses (300 W), and the isobaric heat capacity of gaseous ammonia is found[2] at 300 K to be approximately 2160 J/kg-K (a value that is assumed to remain constant throughout the process for this part; its variation over temperature will be considered in a later section). This results in a temperature change of 1389 K, for a new chamber temperature of **1689 K.**

Using the Chemical Equilibrium with Applications software tool from NASA, the molar mass and specific heat ratio of ammonia at this chamber temperature and 1 MPa of pressure are found to be 8.516 g/mol and 1.327, respectively.

Examining the equation for specific impulse,

$$I_{sp} = \lambda \left\{ \frac{1}{g_c} \sqrt{\frac{2\gamma}{\gamma-1} \frac{\boldsymbol{R} T_C}{\boldsymbol{M}} \left[1 - \left(\frac{P_{exit}}{P_C} \right)^{\frac{\gamma-1}{\gamma}} \right]} + \frac{c^* \varepsilon}{g_c} \left(\frac{(P_{exit} - P_a)}{P_C} \right) \right\} \tag{2}$$

it is apparent that some assumptions must be made to continue with the parameters that are available. The first is that the ratio of exit pressure to chamber pressure is infinitesimal and can be discounted as 0. So too can the environmental pressure (P_a) be set to 0 – the space environment being a near-vacuum. This eliminates a number of the terms and allows the specific impulse to be calculated as **372.9 s**. Assuming an efficiency of 60% for the conversion of thermal energy to kinetic energy, a more realistic value is **233.7 s**.

This specific impulse value informs the thrust calculation

$$F_T = I_{sp} g_c \dot{m} = \mathbf{0.229\ N} \tag{3}$$

which is near (within 15%) of the thrust given by the relation

$$F_T = \frac{2q}{I_{sp} g_C} = \mathbf{0.262\ N} \tag{4}$$

Given an arbitrarily chosen expansion ratio of 10, the following may be solved via an Excel goal seek function for exit Mach number M_e

$$\varepsilon = \frac{1}{M_e} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M_e^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5}$$

which is found to be 3.67. Using this in the isentropic relationship for pressure and setting total temperature to the original 300 K at the inlet, the exit temperature is found to be

$$T_e = T_t \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{-1} = \boldsymbol{93.7\ K} \tag{6}$$

[2] Engineering Toolbox, https://www.engineeringtoolbox.com/ammonia-heat-capacity-specific-temperature-pressure-Cp-Cv-d_2016.html

II. Hydrazine Monopropellant

Solving for the performance characteristics with a liquid hydrazine monopropellant at 298 K and 1.0 MPa, the Chemical Equilibrium with Applications software determines the following parameters:

Combustion (Chamber) Temperature *Tc*	873.5 K
Molar mass $\boldsymbol{M}$	10.73 g/mol
Specific heat ratio γ	1.364

Once more, Equation 2 is employed with these parameters and with the same assumptions as above to determine the specific impulse of the hydrazine monopropellant engine, **229.6 s** (**137.8** s accounting for estimated 60% efficiency). This then feeds into the thrust equation

$$F_T = I_{sp} g_c \dot{m} = \mathbf{0.135\ N}$$

Exit temperature is calculated as in part I, with the same arbitrary expansion ratio of 10, leading to an exit Mach number of 3.79 and a T_e of **83.0 K**.

III. Augmented Hydrazine Thruster

For an augmented thruster such as the Aerojet MR-501B, determining the performance characteristics is a combination of the monopropellant problem and the resistive heater problem. In this section, however, the assumption that the isobaric heat capacity of hydrazine remains constant throughout combustion is discarded in favor of a more realistic approach. Instead of a simple temperature change, the problem is considered in regard to enthalpy change. To begin, a reexamination of Equation 1 is required, starting with the formula for specific enthalpy

$$h = c_p T \tag{7}$$

and relating it back into Equation 1

$$q = \dot{m}\Delta h \tag{8}$$

Knowing the input power and the mass flow, the change in enthalpy is found to be

$$\Delta h = \frac{300\ W}{100\ x\ 10^{-6}\frac{kg}{s}} = 3\ x\ 10^{7}\,\frac{J}{kg} = 3000\,\frac{kJ}{kg}$$

Taking the initial temperature of hydrazine to be 300 K and using results from CEA given in Table 1, highlighted in yellow below, the starting specific enthalpy of the hydrazine propellant is found to be -1901.6 kJ/kg. Taking into account the increase found above, the final enthalpy is determined to be 1098.4 kJ/kg.

Table 1 - Thermodynamic properties of hydrazine (Source: Dr. Richard Branam)

bar	K	kg/m3	kJ/kg	kJ/kg	kJ/kg	kJ/kg-K
p	T	ρ	h	u	G	s
10	200	11.562	-2082.4	-2168.9	-3790.5	8.5402
10	300	7.687	-1901.6	-2031.6	-4682.4	9.2694
10	400	5.4533	-1573	-1756.3	-5650	10.193
10	500	3.5042	-694.78	-980.15	-6757.1	12.125
10	600	2.3632	375.07	-48.086	-8076.1	14.085
10	700	1.8864	972.21	442.09	-9537.3	15.014
10	800	1.6207	1341	724.02	-11065	15.508
10	900	1.4329	1652.6	954.69	-12635	15.875
10	1000	1.2871	1950.9	1174	-14238	16.189
10	1100	1.1691	2247.7	1392.3	-15872	16.472
10	1200	1.0712	2546.3	1612.8	-17532	16.732
10	1300	0.98862	2848	1836.5	-19217	16.973
10	1400	0.9179	3153.3	2063.9	-20926	17.2
10	1500	0.85664	3462.3	2294.9	-22657	17.413

It is necessary to convert this enthalpy value back into a usable temperature to determine the conditions of the heated hydrazine before combustion. This is accomplished by turning to the enthalpy values in Table 1 once more, and locating a corresponding temperature between 700 and 800 K, highlighted in blue. Simple linear interpolation between the two data points approximates this value at 734.2 K. More complex fits generally agree with this value as well.

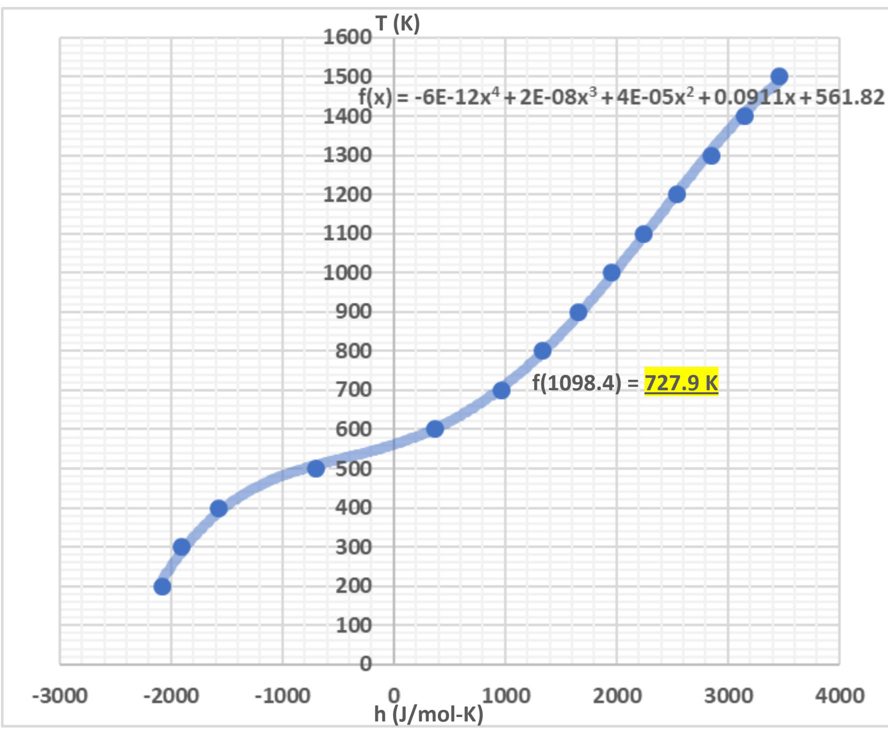

Figure 1 - High-order approximation of CEA results

Taking an approximate value of 730 K as the starting temperature of the hydrazine, and running a CEA combustion analysis at 10 bar once more, the molar mass of the gas is determined to be 10.68 g/mol, the chamber temperature found to be 1628.2 K, and the ratio of specific heats is found to be 1.373. Now Equations 1 and 2 can be solved for I_{sp} and thrust, with the same assumptions as in part I (ratio of pressures at exit and in chamber is negligible, as is ambient pressure). Doing so, the specific impulse of the motor is found to be **311 s** (**186.6 s** at 60% efficiency, which may no longer be as valid an estimate given that there are two types of heating and combustion systems at play). The thrust associated with the reduced value is found via Equation 3 to be **0.183 N**.

Exit temperature is found in the same way as parts I and II – **80.6 K**.

IV. Arcjet Analysis

The analysis of an arcjet takes a different tack. Figure 2 below is an illustration of the basic operation of such a propulsion mechanism, where an electric potential difference between anodic chamber walls and an enveloped cathode creates an electrical arc through the combustion chamber, which is filled with propellant. In the case of this scenario, the propellant is ammonia. The arc heats and energizes the propellant, which is then ejected at high velocity.

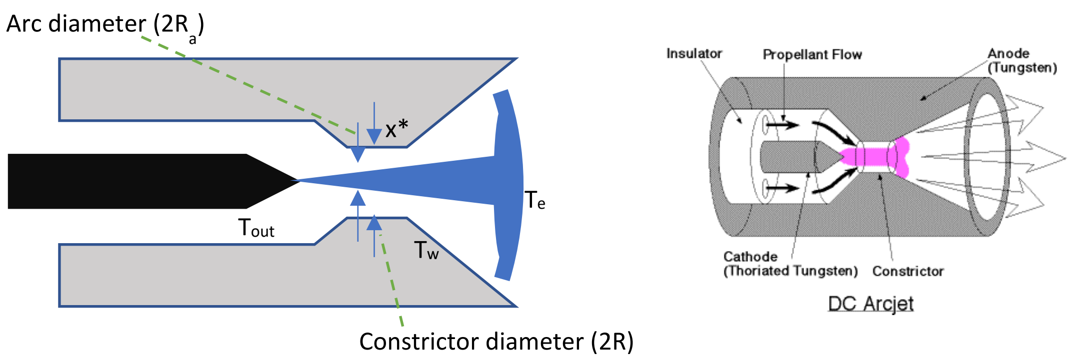

Figure 2 - Key arcjet quantities

Given in the problem are the power of the arcjet (26 kW), the payload mass (180 kg), and desired thrust (1.914 N) and specific impulse (786.8 s). Starting from the equation

$$EI = 4\pi k_c(T_c - T_e) \tag{9}$$

a CEA analysis is performed on the ammonia propellant to determine its at which centerline temperature and corresponding conductivity the realized power of 26 kW is achieved (for simplicity on the left hand side of the equation, the constrictor length x* is assumed to be 1 m). The results are captured in Table 2 and plotted in Figure 2 below. A pressure of 1 MPa is assumed, and T_e is said to be 500 K. It is found to be indeterminate from these results at which temperature the desired power is achieved – there are multiple intersections with 26 kW.

Table 2 - CEA results for conductivity informing calculated power via Equation 9

T, K	K, mW/cm-K	K, kW/m-K	P, kW/m
3500	49.1193	0.000491193	18.51754
4000	83.8802	0.000838802	36.89244
4500	93.2484	0.000932484	46.87176
5000	68.934	0.00068934	38.98126
5500	44.7749	0.000447749	28.1329
6000	34.7897	0.000347897	24.04491
6500	34.129	0.00034129	25.73266
7000	36.7583	0.000367583	30.0247
7500	37.801	0.00037801	33.2515
8000	34.826	0.00034826	32.82273
8500	29.6176	0.000296176	29.77486
9000	25.3316	0.000253316	27.05773
9500	23.0143	0.000230143	26.02856
10000	22.185	0.00022185	26.48457
10500	22.2056	0.000222056	27.90438
11000	22.6718	0.000226718	29.91474

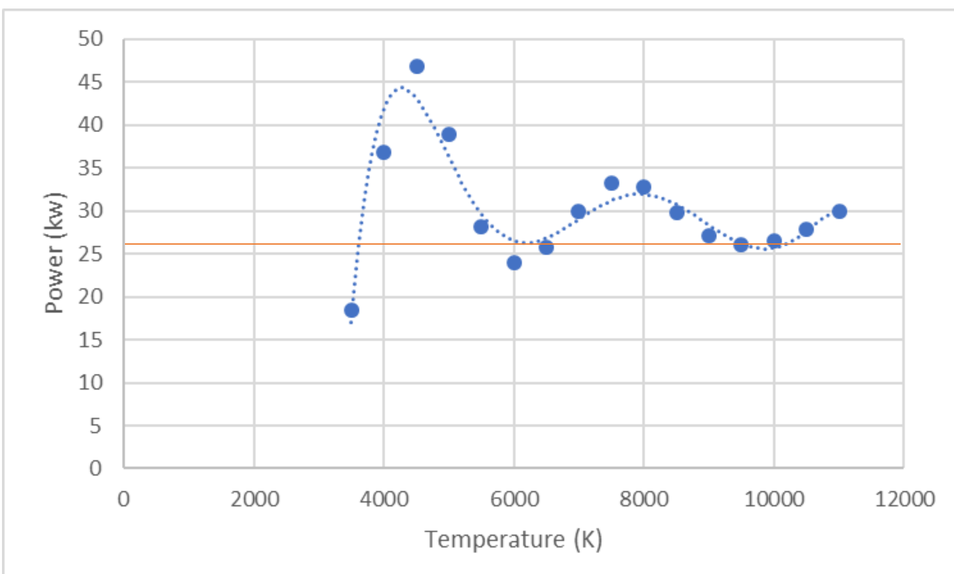

Figure 3 – Plotting power output against varying temperature (and conductivity) leads to inconclusive results, multiple crossings of desired power, best fit by a high (sixth) order polynomial

To achieve more conclusive results, the experimentally determined flow rate (250 mg/s) from the ammonia arcjet system described in *Fife, et al*[3] is used in an enthalpy analysis similar to that employed on the augmented hydrazine thruster above. Multiplying the input power (26 kW) by this mass flow rate results in an enthalpy change of 1.04 x 10^5 kJ/kg. Taking the starting temperature of the propellant to be 500 K and again the starting pressure at 1 MPa, the starting isobaric specific heat capacity of the ammonia is found to be 2.55 kJ/kg-K. Multiplying this by the starting temperature results in a starting enthalpy of 1275 kJ/kg. The target final enthalpy is then 1.053 x 10^5 kJ/kg. The CEA results from Table 2 also include the enthalpy of the ammonia propellant and the various combustion temperatures, captured below.

T,K	**h, kJ/kg**
4000	26135.5
5000	50439.1
6000	65709.1
7000	79551.1
8000	95683.6
9000	107467.6
10000	114884.2
11000	120894.1

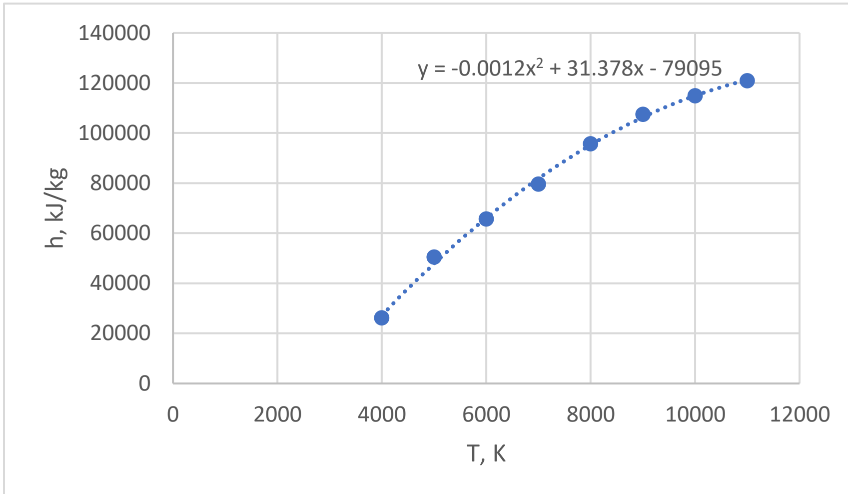

Figure 4 - Specific enthalpy of heated ammonia

The temperature corresponding to the desired enthalpy resides between 8000 and 9000 K. Linear interpolation between the two points finds this temperature to be 8816.1 K. A combustion analysis is then run at this temperature to determine the specific heat ratio for the combusted ammonia – 1.342.

The aim of this process is to determine the expansion ratio ε needed to achieve the given thrust value. Using the thrust coefficient

$$c_F = \frac{F_T}{A_{ce}P_t} = \frac{P_e}{P_t}\varepsilon(1+\gamma M_e^2) \tag{9}$$

making the substitution

$$\frac{P_e}{P_t} = \left[1+\frac{\gamma-1}{2}M_e^2\right]^{-\left(\frac{\gamma}{\gamma-1}\right)} \tag{10}$$

and substituting in Equation 5 leads to

$$\frac{F_T}{A_{ce}P_t} = \left[1+\frac{\gamma-1}{2}M_e^2\right]^{-\left(\frac{\gamma}{\gamma-1}\right)} \frac{1}{M_e}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}(1+\gamma M_e^2) \tag{11}$$

[3] Fife, et al, "Orbital Performance Measurements of Air Force Electric Propulsion Space Experiment Ammonia Arcjet," *Journal of Propulsion and Power*, Vol. 18, No. 4, July-August 2002.

Now the equation is written entirely in terms that are known (assume A_{ce} to be 0.25 m^2 and the stagnation pressure to be 26 psi, or approximately 180 kPa)– save for the exit Mach number. A goal seek function locates the required Mach number at 2.66, giving an expansion ratio of **3.29**.

For a mission requiring a Δv of 5.0 km/s, finding the propellant mass for a required payload mass is a matter of solving the rocket equation

$$\Delta v = v_e \ln\left(\frac{m_f}{m_0}\right) \tag{12}$$

where

$$v_e = g_0 I_{sp} = 9.81 * 786.8 = 7.72\ km/s$$

The solution is as follows

$$m_f = m_0 e^{\frac{\Delta v}{v_e}} = 180 e^{\frac{5}{7.72}} = \boldsymbol{343\ kg}$$

This is slightly less than the performance one would expect from a representative nuclear engine (I_{sp} ~ 800-900 s), where the needed fuel to impart a Δv of 5 km/s on a given payload would require 317-340 kg of propellant under Equation 12.