# Trajectory Design for Encountering Planet Nine

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**Planet Nine is an as-yet hypothetical body presumed to exist at a distance of several hundred astronomical units from the Sun. The following paper puts forth a number of scenarios by which an encounter with Planet Nine may be achieved by a spacecraft sent from Earth, and presents analysis on these trajectories from an orbital mechanics standpoint to assess their feasibility. The scenarios range from the simplistic estimation to the more detailed considerations of a multi-stage trajectory. Time of flight and total $\Delta v$ are taken into consideration when optimizing trajectories, time of flight being fixed, when possible, to 50 years, and $\Delta v$ being minimized to the greatest extent possible given a number of stated assumptions. The resulting $\Delta v$ values are found to be consistently in excess of the current capabilities of existing spacecraft.**

## Nomenclature

| | | |
|---|---|---|
| $a$ | = | semi-major axis |
| $e$ | = | eccentricity |
| $i$ | = | inclination |
| $p$ | = | semi-latus rectum |
| $\Omega$ | = | right ascension of the ascending node |
| $\omega$ | = | argument of periapsis |
| $r_a$ | = | distance to apoapsis |
| $r_p$ | = | distance to periapsis |
| $\theta^*$ | = | true anomaly |
| $TOF$ | = | time of flight |

*Subscripts*

| | | |
|---|---|---|
| a | = | arrival (on transfer orbit) |
| d | = | departure (on transfer orbit) |
| E | = | Earth |
| 9 | = | Planet Nine |
| S | = | Sun |
| T | = | transfer orbit |

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## I. Introduction

PLANET Nine is an as-yet hypothetical body orbiting the Sun at the fringe of our Solar System whose existence is inferred by observed perturbations in the orbits of known bodies in the same region. Its existence is the subject of much speculation in the astronomical and planetary science communities, with mounting support for the affirmative in recent years [1,2]. Two of its greatest proponents, Drs. Konstantin Batygin and Michael Brown, put forth as evidence the clustering in argument of perihelion and other orbital elements of numerous Kuiper Belt objects, (KBOs) [1]. Such a clustering, they claim, has a small probability of occurring due to chance: a more statistically reasonable explanation is that there exists, at a great distance from the Sun, a massive perturbing body (christened Planet Nine) which encounters these objects with some regularity. Should Planet Nine exist, it would be of great interest to the scientific community to send a probe to encounter it and conduct studies from nearby.

## II. Background

When considering a space mission, a key parameter to keep in mind is $\Delta v$. This quantity is the total change in velocity of a body—in this case, of a spacecraft that may be sent to distant regions of space, or even just to Earth orbit. The $\Delta$v of a maneuver is a measure of how much fuel will be needed, given an engine of known thrust capabilities, to perform said maneuver. Given that fuel cost and weight are major constraints on all space missions, it is in the best interests of the mission stakeholders to accurately calculate—and, beyond this, to minimize—the required $\Delta$v for accomplishing the desired orbit, or trajectory, or maneuver. Launch, orbital insertion, on-orbit maneuvering, escape from Earth's gravitational influence, orbit transfers—all of these have an associated $\Delta$v. Rocket engines are often described in terms of the $\Delta$v they can provide to a payload, a metric sums up their effectiveness in a succinct way.

A second key constraint in the design of a mission is *time of flight*, i.e., how long it will take for a spacecraft to reach its destination. When the intended target of a mission is very distant, it may take years or even decades for a spacecraft to reach it. This poses a number of issues: for one, it guarantees that by the time the target is reached, any technology aboard the spacecraft will be considered obsolete, having remained stagnant while technology on Earth continues to progress during its flight time. Secondly, the timeline of such missions may be longer than the lifespan of their architects and engineers, putting immense pressure on proper knowledge transfer between generations to mitigate the risk of mission failure. There is also the emotional benefit of being able to see a mission to completion.

It is with these two parameters in mind that the efforts to optimize trajectories to Planet Nine were conducted. The time of flight of the mission was set as a static constraint: 50 years. The models were then iterated through a number of starting positions for the Earth and the spacecraft such that a minimum Δv requirement was achieved.

## III. Orbital Mechanics – Relevant Equations and Concepts

In the following section, a number of concepts and equations are introduced that are of relevance to the following analyses. They are presented with the intent that a reader with minimal knowledge of the field of orbital mechanics may gain a basic understanding of key terms and definitions and thus follow along with the paper.

### A. Keplerian Elements

Fundamental to understanding any analyses in the field of orbital mechanics is a knowledge of the orbital elements used to define an object's motion around a central body. These five elements $(a, e, i, \Omega, \omega)$, referred to as "Keplerian elements," exactly define the shape and orientation of an orbit, and when a sixth element (true anomaly, $\theta^*$) is added, an object's exact position in that orbit is known. How orbit definition is accomplished using these elements is illustrated in Figure 1 below.

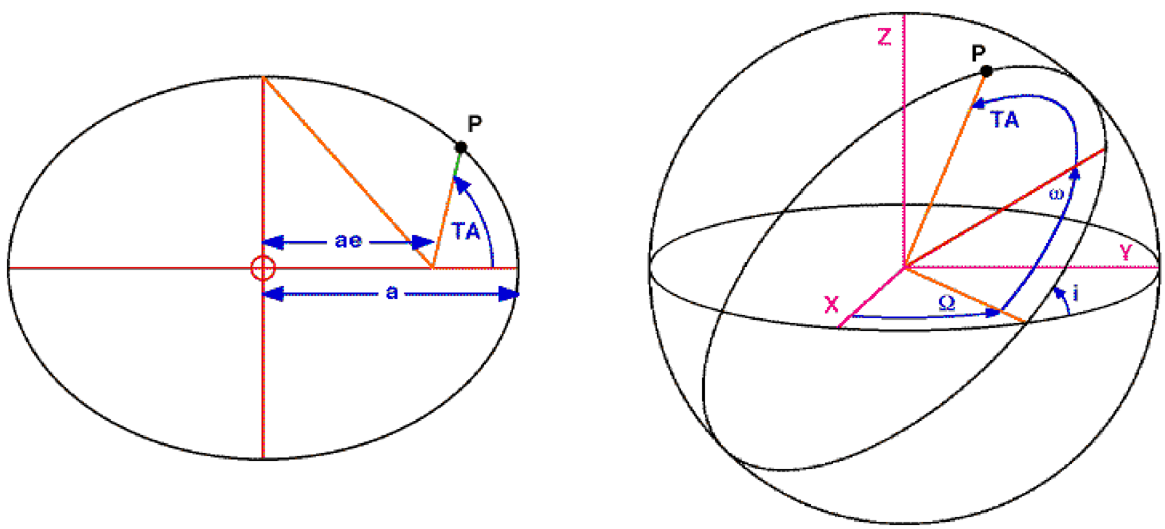


**Figure 1 - Keplerian elements illustrated for clarity**
*[Source: AstroGrav Astronomy Software Documentation, accessed via http://www.astrograv.co.uk/Documents/AstroGrav-Help/Data-Elements/Orbital-Elements.html]*

To define the elements briefly: $a$ is the semi-major axis of the elliptical path, and $e$ is the eccentricity of the same. $\Omega$ is the angle between the inertial-frame $\hat{x}$ direction (also referred to as the vernal equinox direction) and the ascending node, in the ecliptic plane of the central body. $\omega$ is the angle between the ascending node and the periapsis (closest approach to the central body) along the orbit. $\theta^*$ (denoted "TA" in Figure 1), is the angle between periapsis and the current position of the orbiting body along the orbital path. Once the Keplerian elements are defined, a number of important quantities may be derived from them.

### B. Reference Frames and Direction Cosine Matrices

An object's position along its orbit can be redefined to locate it within greater 3-dimensional space—or, more aptly, within an inertial reference frame exterior to the central body-orbiting body system. Visible in Figure 1 is the dextral orthogonal triad that defines this frame, composed of the direction unit vectors $\hat{x}, \hat{y}, \hat{z}$. This frame, fixed in space, is useful in defining the position and movement of objects relative to one another. The conversion of an object's determined position along its orbit to its position in the inertial reference frame is accomplished through the use of the direction cosine matrix

$$^{I}C^{R} = \begin{bmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\Omega C_i C_\theta & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{bmatrix} \tag{1}$$

where *I* represents the inertial reference frame, *R* represents the rotating reference frame, *C* and *S* are abbreviations of the *cos* and *sin* functions, and $\theta$ is the sum of the argument of periapsis $\omega$ and the true anomaly $\theta^*$, representing the angle between the orbiting body's location and the right ascension of the ascending node $\Omega$ along the orbit path. When a vector written in the inertial reference frame is multiplied by this matrix, it will be rewritten in the rotational reference frame, defined by the $\hat{r}$ (radial), $\hat{\theta}$ (tangential), $\hat{h}$ (normal) direction unit vector triad. For conversion of a vector from the rotational reference frame to the inertial, it will be multiplied by the transpose of this matrix. This particular matrix represents a rotation of a vector between frames using the Body 3-1-3 rotation sequence.

### C. Position, Velocity, and Energy of an Orbiting Body

The radial distance of an orbiting body from the central body is defined as

$$r = \frac{p}{1+e\cos(\theta^*)} \tag{2}$$

where $p$ is the semi-latus rectum of the orbit, defined by

$$p = a(1 - e^2) \tag{3}$$

or, for hyperbolic orbits,

$$p = |a|(e^2 - 1) \tag{4}$$

The velocity of the orbiting body is always tangent to the arc of the orbit in the orbital plane, and the angle between the position vector and the velocity vector of the orbiting body is $\gamma$, the flight path angle.

The energy of an object in orbit is constant, and defined by

$$\varepsilon = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \tag{5}$$

where $\mu$ is the gravitational constant of the central body. In an elliptic orbit, $a$ is positive and $\varepsilon$ is negative; in a hyperbolic orbit, the opposite is true. By knowing the orbiting body's position and the semi-major axis of the orbit, it is possible to determine the body's speed through Equation 5, and furthermore its direction of motion using the flight path angle.

When initial position and velocity vectors $\bar{r}_0, \bar{v}_0$ are known for an orbiting body, the position and velocity at any other true anomaly can be determined using the following functions

$$\bar{r} = \left\{a - \frac{r}{p}[1 - \cos(\theta^* - \theta_0^*)]\right\}\bar{r}_0 + \frac{r r_0}{\sqrt{\mu p}}\sin(\theta^* - \theta_0^*)\,\bar{v}_0 \tag{6}$$

$$\bar{v} = \left\{\frac{\bar{r}_0 \cdot \bar{v}_0}{p r_0}[1 - \cos(\theta^* - \theta_0^*)] - \frac{1}{r_0}\sqrt{\frac{\mu}{p}}\sin(\theta^* - \theta_0^*)\right\}\bar{r}_0 + \left\{1 - \frac{r_0}{p}[1 - \cos(\theta^* - \theta_0^*)]\right\}\bar{v}_0 \tag{7}$$

**D. Orbit Transfers**

An orbit transfer is self-explanatory: it is a maneuver to bring a spacecraft from one orbit into another. There are two distinctive approaches to orbit transfers employed in this analysis, and the first one, the *Hohmann transfer*, is very simple in design and explanation. In a Hohmann transfer, the initial and final orbits are both circular and coplanar, distinguished only by differing radii (in a circular orbit, the semi-major axis and the radial distance to the orbiting body are always equivalent). Δv is applied tangentially to the spacecraft on the initial orbit path (which is to say, velocity is applied along the direction of initial velocity). This puts the spacecraft on a new, elliptical orbit, with a periapsis distance equal to the initial orbit radius (or the final orbit radius, if the transfer is from a higher to a lower orbit) and an apoapsis distance equal to the final orbit radius. Once the spacecraft reaches the final transfer point (at the opposite apsis of the transfer orbit), a second tangential burn is conducted to circularize the orbit at the new radial distance. The simplicity of this transfer arises from the assumption that both orbits are circular, so any position at a certain radial distance within the orbit plane will suffice as a starting point, and that velocity is applied tangentially in all cases. The Hohmann provides a good initial basis of estimate for the Δv required to reach a target orbit.

The second means of orbit transfer zeroes in on a particular starting point along the initial orbit and a particular arrival point along the final orbit. Given that an infinite number of curves may intersect two points in space, some constraint must be applied to precisely define a transfer orbit between them. This is the basis of Lambert arcs and the algorithm that produces them. Lambert's algorithm for determining a transfer path between two specific points on different orbits is summarized as follows:

1. Determine the transfer angle (TA) between the initial and final points ($r_1$ , $r_2$)
2. Calculate the time of flight (TOF) of a parabolic arc between them, using

$$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu}}\left[s^{\frac{3}{2}} - (s \pm c)^{\frac{3}{2}}\right] \tag{8}$$

3. Obtain $TOF_{min}$ using $a_{min}$
   a. for elliptic transfers

$$a_{min} = \frac{s}{2}\ ; s = \frac{1}{2}(r1 + r2 + c) \tag{9}$$

   where c is the third side of the space triangle enclosed by r1, r2, c, and including the angle between r1 and r2, or the TA. *s* is the semi-parameter of this space triangle.

   b. for hyperbolic transfers, $a_{min} = 0$
4. Calculate the Lambert angles $\alpha_0, \beta_0$ via

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a}}\ ;\ \beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a}} \tag{10}$$

or, for hyperbolic paths,

$$\alpha_0{}' = 2\sinh^{-1}\sqrt{\frac{s}{2|a|}}\ ;\ \beta_0{}' = 2\sinh^{-1}\sqrt{\frac{s-c}{2|a|}} \tag{11}$$

5. Iterate on *a* to achieve the desired TOF via

$$TOF = \sqrt{\frac{a^3}{\mu}}[(\alpha - \sin\alpha) - (\beta - \sin\beta)] \tag{12}$$

or, for hyperbolic orbits,

$$TOF = \sqrt{\frac{|a|^3}{\mu}}[(\alpha' - \sin\alpha') - (\beta' - \sin\beta')] \tag{13}$$

where $\alpha$ and $\beta$ are defined by the principal values $\alpha_0$ and $\beta_0$ and the geometry of the orbit path.

Once $a$ is determined, the other Keplerian elements of the transfer path follow. If the desired time of flight is less than the parabolic time of flight, the transfer path will be a hyperbolic one. If greater, the path will be elliptic.

**E. Transfer Maneuvers and Escape/Arrival**

When transferring orbits, as mentioned above, a $\Delta v$ is required. This is calculated by simple vector addition as follows:

$$\bar{v}_f = \bar{v}_i + \Delta\bar{v} \tag{14}$$

where $\bar{v}_i$and $\bar{v}_f$ represent the initial and final intended velocities, respectively. The magnitude $\Delta v$ is sufficient to judge the fuel costs associated with a maneuver.

When transferring orbits from one central body to another in the solar system, as in this analysis, it is required that the orbiting spacecraft escape from the gravitational influence of its origin central body, and then, once the transfer is complete and it has arrived at the destination body, insert itself into a new orbit around the body using a capture maneuver. For escape, the required Δv to depart a circular orbit around a body is found by

$$\Delta v_i = \sqrt{v_\infty^2 + 2\left(\frac{\mu}{r}\right)} - v_c \tag{15}$$

and, for a capture into a circular orbit, is written as

$$\Delta v_p = \sqrt{v_\infty^2 + 2\left(\frac{\mu}{r_p}\right)} - v_c \tag{16}$$

where

$$\bar{v}_\infty = \bar{v}^+ - \bar{v}_b \tag{17}$$

($\bar{v}^+$ is the spacecraft velocity with regard to the Sun, $\bar{v}_b$ is the planetary body's velocity with regard to the Sun).

## IV. Scenario Modeling

The orbital elements of each planetary body as used in this analysis are defined in Table 1. Planet Nine's orbital elements are defined as ranges in [1] and [2]. Precise values were then chosen from these ranges for the purposes of these analyses.

**Table 1 - Orbital elements as used in scenarios**

| | **Earth [3]** | **Planet Nine [2]** |
|---|---|---|
| ***a*, km (AU)** | $1.496 \times 10^{11}$ (1) | $1.047 \times 10^{14}$ (700) |
| ***e*** | 0.017 | 0.6 |
| ***i*, deg** | 0.0005 | 30 |
| ***Ω*, deg** | -11.26 | 90 |
| ***ω*, deg** | 114.21 | 150 |

A number of trajectories from Earth to Planet Nine are considered below, spanning from the quick estimation to the more detailed considerations of multi-stage flight paths. A number of assumptions are considered in all cases. For one, the target of each trajectory is taken to be the perihelion of Planet Nine's orbit as defined by the elements specified above. This is to set the closest possible encounter location as our target—when dealing with a highly elliptic orbit 700 AU away, the difference between targeting periapsis and targeting apoapsis is vast. The location of the perihelion in the inertial reference frame is defined as

$$\bar{r}_{p9} = (-1.814\,\hat{x} - 3.628\,\hat{y} + 1.047\hat{z}) \times 10^{13}\ m$$

In the cases that include optimizing the departure point along Earth's orbit, no consideration is made for the movement of Planet Nine during this time. It is assumed that no matter what time departure takes place, the mission target is Planet Nine's perihelion point.

All thrust maneuvers are taken to be instantaneous in duration. Except where specified (the last case), the local gravity fields of Earth, Planet Nine, and other bodies were not taken into account, and the spacecraft operates solely under the gravitational influence of the Sun.

**A. Hohmann Transfer – Earth Periapsis to Planet Nine Periapsis**

For the purposes of quick estimation of the Δv requirements for such a mission, a Hohmann transfer is considered. Ignoring the local gravity fields of both Earth and Planet Nine, the model only considers the Δv required to transfer from Earth's orbit around the Sun to Planet Nine's. Both bodies are said to be in circular orbits, with radii equivalent to their respective perihelion distances.†

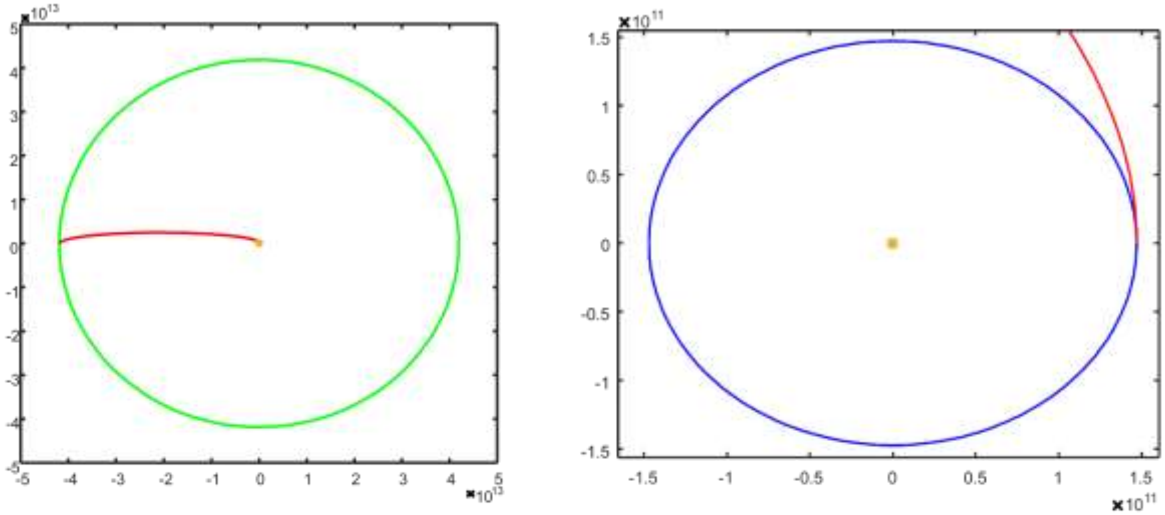


**Figure 2 – Hohmann transfer path between circularized initial and final orbits. Earth departure at right.**

The time of flight of this maneuver comes out to 2.628 x $10^{10}$ s, or 833.5 years. The Δv budget for this transfer is a reasonable 13.996 km/s.

**B. Lambert Arc – Earth Perihelion to Planet Nine Perihelion**

To more accurately assess the actual time of flight and Δv requirements of this mission, a more complex and detailed model is created. This transfer takes into consideration all Keplerian elements of the initial and final orbits. Lambert's algorithm is employed to find a transfer path between Earth perihelion and Planet Nine perihelion with a more attainable time of flight of 50 years.

Earth perihelion has been found, using the Keplerian elements of Table 1, to be located at

$$\bar{r}_{pE} = -3.296 \times 10^{10}\,\hat{x} + 1.433 \times 10^{11}\hat{y} + 1.170 \times 10^{6}\hat{z}\ m$$

† Earth perihelion distance (nearly equivalent to Earth semi-major axis) is chosen as the circular radius for consistency with the use of Planet Nine's perihelion.

The time of flight associated with the minimum semi-major axis of an elliptical transfer orbit is 832.9 years, so for the desired TOF of 50 years a hyperbolic orbit is needed. Using this knowledge and the desired TOF, Lambert's algorithm produces a transfer arc with the characteristics listed in Table 2 below.

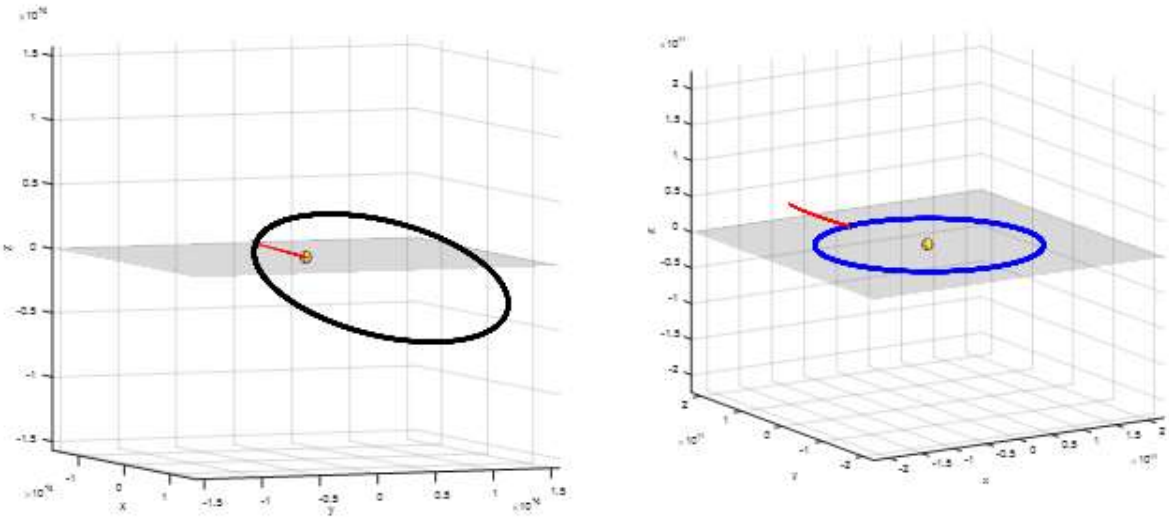

**Figure 3 - Lambert arc from Earth perihelion to Planet Nine perihelion. Earth departure at right.**

**Table 2 – Lambert transfer arc parameters (Earth perihelion to Planet Nine perihelion)**

| | |
|---|---|
| $\bar{r}_d$ | $-3.296 \times 10^{10}\hat{x} + 1.433 \times 10^{11}\hat{y} + 1.17 \times 10^{6}\hat{z}\ m$ |
| $\bar{r}_a$ | $(-1.814\ \hat{x} - 3.628\ \hat{y} + 1.047\hat{z}) \times 10^{13}\ m$ |
| $a$ | $-1.960 \times 10^{11} m$ |
| $e$ | 1.737 |
| $i$ | 22.09° |
| $\Omega$ | 103° |
| $\omega$ | 13.56° |
| $\theta_d^*$ | −13.56° |
| $\theta_a^*$ | 124.77° |
| $TOF$ | 50 years |
| $\Delta\bar{v}_d$ | 25.18 $km/s$ |
| $\Delta\bar{v}_a$ | 26.22 $km/s$ |

### C. Lambert Arc – Optimized Earth Orbit Departure Point to Planet Nine Perihelion

To minimize the $\Delta v$ necessary to transfer from Earth orbit to Planet Nine, this analysis was repeated for various positions along Earth's orbit, as perihelion may not be the most ideal location to initiate the transfer. Iterating $\theta_E$ from 0° to 360° in 1-degree increments, one finds the following results:

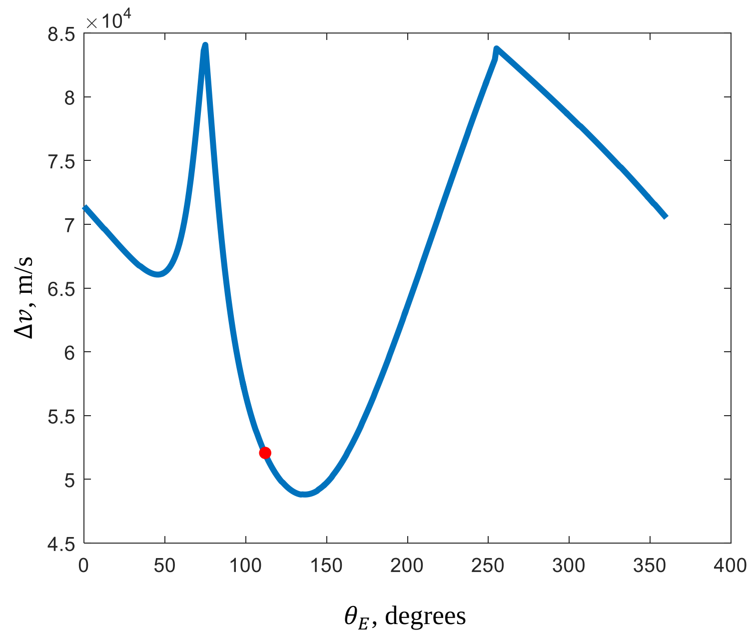


**Figure 4 - Optimizing departure time using $\theta_E$. Earth perihelion marked for reference (red dot).**

$\Delta v$ is found to be a minimum at $\theta_E = 134°$, just beyond Earth perihelion along its orbit. Using this value and stepping through Lambert's algorithm again, we find the following associated transfer arc, with parameters specified in Table 3.

**Table 3 – Lambert transfer arc parameters ($\boldsymbol{\theta_E = 134°}$ to Planet Nine perihelion)**

| | |
|---|---|
| $\bar{r}_d$ | $-7.961 \times 10^{10}\hat{x} + 1.238 \times 10^{11}\hat{y} + 9.241 \times 10^{5}\hat{z}\ m$ |
| $\bar{r}_a$ | $(-1.814\ \hat{x} - 3.628\ \hat{y} + 1.047\hat{z}) \times 10^{13}\ m$ |
| $a$ | $-1.961 \times 10^{11} m$ |
| $e$ | 1.749 |
| $i$ | 16.71° |
| $\Omega$ | 123° |
| $\omega$ | 4.87° |
| $\theta_d^*$ | 4.87° |
| $\theta_a^*$ | 124.49° |
| *TOF* | 50 years |
| $\Delta\bar{v}_d$ | $22.61\ km/s$ |
| $\Delta\bar{v}_a$ | $26.20\ km/s$ |

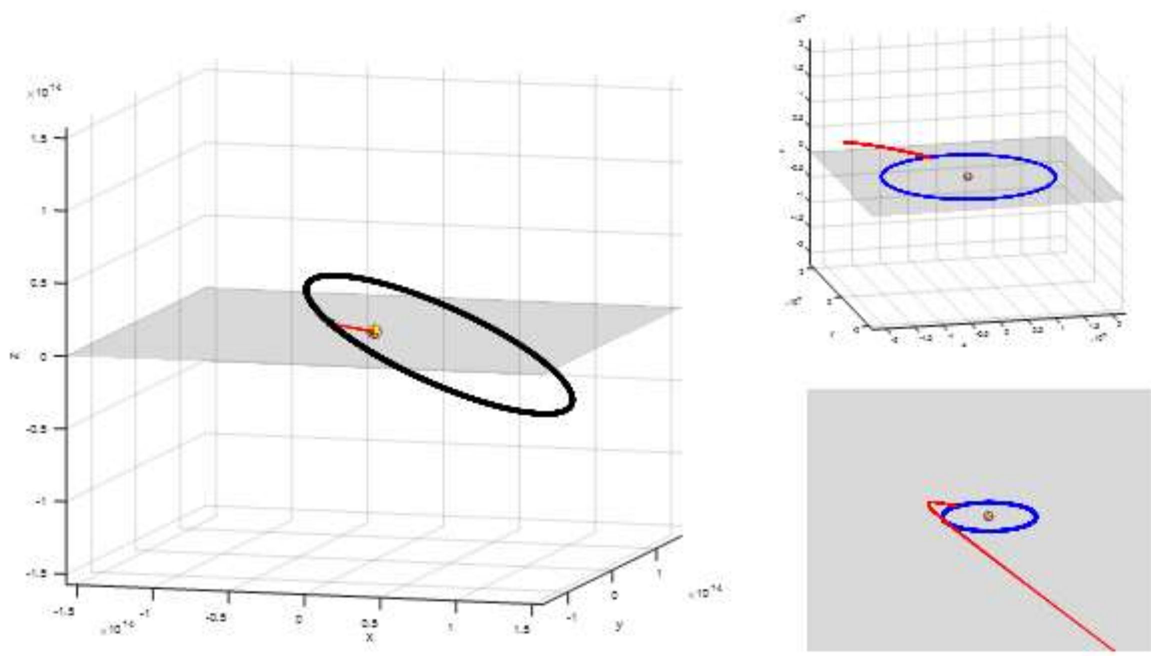

**Figure 1 - Lambert arc from $\boldsymbol{\theta_E = 134°}$ to Planet Nine perihelion. Earth departure detail shown at right. $\boldsymbol{\Delta v = 48.81\ km/s}$**

## D. Lambert Arc – Earth to Planet Nine Periapsis with Intermediate Sun-Proximity Transfer Orbit

Finally, it was explored whether a near-Sun approach would provide any advantage in saving $\Delta v$. This final trajectory plan involves a transfer from Earth orbit to an intermediate, coplanar, highly elliptical orbit with a perihelion of 3 solar radii, a distance suggested by Adams and Richardson [4] as a survivable proximity to the Sun by current spacecraft design standards. The departure point on Earth orbit is regarded as the aphelion of this intermediate orbit. In this case, both the position of the Earth and the departure position from this elliptical orbit were iterated in 15-degree steps to find the most advantageous combination from a $\Delta v$ perspective. The results of this iteration are presented below in Figure 5, and the combination that results in the lowest $\Delta v$ spent is a transfer from Earth orbit onto the intermediate arc at $\theta_E = 255°$, a propagation along the intermediate orbit until $\theta_T = 225°$, and finally a departure onto the Lambert transfer arc to Planet Nine.

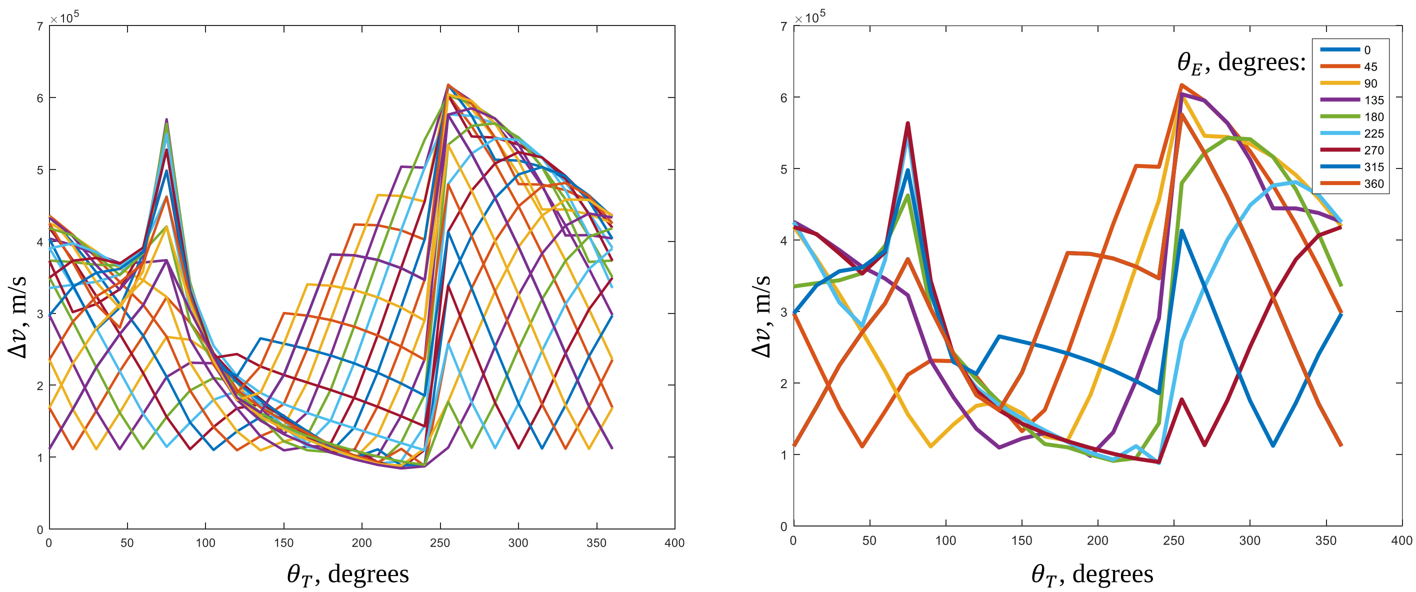


**Figure 5 – $\Delta v$ vs. $\theta_T$ for various $\theta_E$. Optimized combination: $\theta_E = 255°, \theta_T = 225°$**

Using these values and stepping through Lambert's algorithm we find a transfer orbit with the following attributes, collected in Table 4.

**Table 4 - Lambert transfer arc parameters ($\theta_E = 255°, \theta_T = 225°$ to Planet Nine perihelion)**

| | |
|---|---|
| $\bar{r}_d$ | $-2.174 \times 10^{10}\hat{x} - 1.452 \times 10^{10}\hat{y} - 1.613 \times 10^{5}\hat{z}\ m$ |
| $\bar{r}_a$ | $(-1.814\ \hat{x} - 3.628\ \hat{y} + 1.047\hat{z}) \times 10^{13}\ m$ |
| $a$ | $-1.972 \times 10^{11} m$ |
| $e$ | 1.017 |
| $i$ | $27.53°$ |
| $\Omega$ | $214°$ |
| $\omega$ | $-136.69°$ |
| $\theta_d^*$ | $-136.69°$ |
| $\theta_a^*$ | $169.43°$ |
| $TOF$ | 50 years |
| $\Delta\bar{v}_d$ | $19.99\ km/s$ |
| $\Delta\bar{v}_a$ | $26.14\ km/s$ |

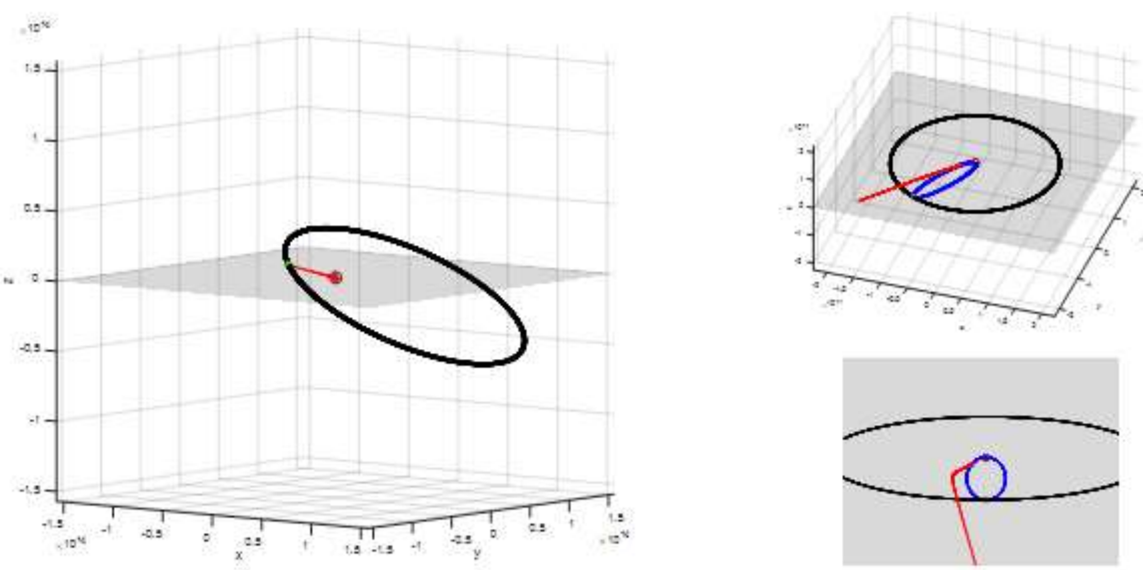

**Figure 1 - Lambert arc from intermediate transfer orbit to Planet Nine perihelion. Intermediate transfer orbit detail shown at right. $\theta_E = 255°, \theta_T = 225°,\ \Delta v = 84.40\ km/s$**

In this case as well, to obtain the greatest detail possible, the local gravity fields of both Earth and Planet Nine are taken into consideration, and with them the associated $\Delta v$ for Earth escape and for Planet Nine capture. It is assumed that Planet Nine has a radius of twice that of Earth, and that the spacecraft is escaping from an orbit 200 km above Earth and entering into an orbit 200 km above Planet Nine. From Equations 15 and 16, we find the associated $\Delta v$ with these maneuvers to be

$$\Delta v_{escape} = 19.77\frac{km}{s};\ \Delta v_{capture} = 18.51\frac{km}{s}$$

Summing these together with the required $\Delta v$ for entering and exiting the Lambert arc, the resultant $\Delta v$ is 84.40 km/s. Without these escape/capture considerations, for comparison to the previous scenario, the resultant $\Delta v$ is 46.13 km/s.

## V. Conclusions

Given the time-of-flight constraint of 50 years, and the target destination as the perihelion of Planet Nine, the $\Delta v$ requirements for the trajectories contained herein are far beyond the current capabilities of aerospace vehicle designs. This is not to definitively conclude that the target is unattainable within a greater, but still reasonable, timeframe. A more complex trajectory design, taking into account gravity assist maneuvers from Jupiter or other planets, or more appropriately taking advantage of the Sun's gravitational influence, may lower the $\Delta v$ required to be provided by fuel. For perspective, the hypothetical periapsis of Planet Nine is still nearly ten times the distance as of yet covered by the Voyager probes, launched three decades ago.

As with all space missions, this becomes more feasible if one looks forward to newer technologies and advances in rocketry and space vehicle design. Planet Nine, if it indeed exists, may be a worthy first target for ion engines, which provide slow acceleration but low fuel cost, if time of flight is relaxed as a constraint. Targeting Planet Nine's aphelion, too, may be a worthy next step, as this is statistically where it is most likely to be located along its orbit at any given time, and its reduced speed at this location may allow for more forgiveness in fine targeting once the spacecraft nears capture.

## VI. References


[1] Batygin, K., and Brown, M., "Evidence for a Distant Giant Planet in the Solar System," *The Astronomical Journal*, Vol. 151, No. 22, February 2016, doi:10.3847/0004-6256/151/2/22

[2] Batygin, K., and Brown, M., "Observational Constraints on the Orbit and Location of Planet Nine in the Outer Solar System," *The Astronomical Journal Letters*, Vol. 824, No. 2, 17 June 2016, doi:10.3847/2014-8205/824/2/L23

[3] NASA Goddard Spaceflight Center, "Earth Fact Sheet", accessed via https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html

[4] Adams, R.B., and Richardson, G.A., "Using the Two-Burn Escape Maneuver for Fast Transfers in the Solar System and Beyond," *NASA Technical Reports Server,* accessed 23 April 2018


All analysis and plotting conducted in MATLAB.