Hall Effect Thruster Analyses

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A Hall effect thruster is analyzed to determine its component and total efficiencies as a function of discharge voltage. Additionally, the specific impulse of the engine is calculated at the tested mass flows given.

Nomenclature

e	=	elementary charge
η	=	efficiency
I_b	=	beam current
I_{sp}	=	specific impulse
M	=	molar mass
T	=	thrust force
$\bar{V}_b$	=	effective beam voltage

I. Data Given & Assumptions

The following data are the entirety of what is known about the engine:

Design Mass flow = 3E-5 kg/s (Krypton) *Discharge Power = 8000 W*
Keeper Power = 50 W *Magnet Power = 150 W*
L = 0.0742 m *Ri = 0.015 m*
Ro = 0.03 m *B field = 0.02 T*

The following data are obtained via test

Table 1 – Test data

V Discharge (V)	I beam (A)	Thrust measured (N)
100	62.4	0.681
200	31.2	0.489
300	21.1	0.410
400	15.7	0.356
500	12.6	0.323
600	10.6	0.299
700	9.4	0.286
800	8.5	0.278
900	7.6	0.265
1000	7.0	0.254

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Additionally, the following assumptions are made:

1. Adjust anode mass flow for optimal operation
2. Assume cathode flow is 10% of the anode flow
3. The total mass utilization is relatively constant (0.713)
4. Beam divergence seems to be 23 degrees (half angle)
5. Assume 10% of the beam current is doubly ionized krypton

II. Hall Effect Thruster Analysis

A. Efficiencies & Total Efficiency of Thruster

An efficiency analysis of a Hall effect thruster requires the assessment of a number of efficiencies along the way, related to the various components of such a thruster. The following analysis closely follows the one given in section 7.3.1 of *Goebel & Katz* and takes into account all the data given and assumptions made above.

The cathode efficiency—accounting for cathode gas flow that does not reach the ionization region—is given by

$$\eta_c = \frac{\dot{m}_a}{\dot{m}_a + \dot{m}_c} = \frac{\dot{m}_a}{1.1\dot{m}_a} = 0.909$$

and is said to be constant so long as the cathode flow remains 10% of the anode mass flow.

The electrical utilization efficiency—accounting for discharge power as a ratio to total power consumed to operate the thruster (cathode keeper power and requisite power to maintain the magnetic field)—is calculated by

$$\eta_o = \frac{P_d}{P_d + P_k + P_{mag}} = \frac{8000}{8000+50+150} = 0.976$$

and is said to be constant so long as these power values remain so.

While it is now possible to calculate the total efficiency of the Hall effect thruster, more efficiency terms will be analyzed to allow for a more complete analysis of the system. Of the discharge voltage and current, for example, only a portion is employed in the beam. These ratios give the current and voltage efficiencies

$$\eta_b = \frac{I_b}{I_d}\,;\eta_v = \frac{V_b}{V_d}$$

Discharge voltage and beam current are given by the test data in Table 1. To find discharge current, the discharge power (8000 W) is divided by the discharge voltage in each test case.

Table 2 - Finding current efficiency

V Discharge (V)	I Discharge (A)	I beam (A)	Current Efficiency
100	80.0	62.4	**0.780**
200	40.0	31.2	**0.780**
300	26.7	21.1	**0.790**
400	20.0	15.7	**0.785**
500	16.0	12.6	**0.788**
600	13.3	10.6	**0.797**
700	11.4	9.4	**0.825**
800	10.0	8.5	**0.850**
900	8.89	7.6	**0.855**
1000	8.00	7.0	**0.875**

Finding the beam voltage is more involved. Given that the thrust of a Hall effect engine can be found by

$$\mathrm{T} = \gamma\sqrt{\frac{2\mathrm{M}}{\mathrm{e}}}I_b\sqrt{V_b} \tag{1}$$

to find beam voltage the only variable that needs determining is γ, an adjustment term that accounts for double ionization of particles within the beam and also for beam divergence. The underlying relationships are as follows:

$$\gamma = \alpha F_t$$

$$\alpha = \frac{I^+ + \frac{1}{\sqrt{2}}I^{++}}{I^+ + I^{++}}\ ; F_t = cos\theta\ ; \theta \equiv half\ angle\ of\ exhaust\ plume$$

The assumptions made and known data allow for calculation of γ in the following manner

$$F_t = \cos(23°) = 0.921$$

$$\alpha = \frac{\left(1+\frac{r}{\sqrt{2}}\right)}{1+r} = \frac{\left(1+\frac{0.1}{\sqrt{2}}\right)}{1+0.1} = 0.973$$

where α is rewritten to account for 10% of the krypton atoms being doubly ionized. Now that γ is found to be 0.896, the thrust equation can be used to determine beam voltage, knowing the molar mass of krypton (0.0838 kg/mol)

Table 3 - Finding beam voltage and voltage efficiency

I beam (A)	**Thrust measured (N)**	**M (kg)**	**e (C)**	**gamma**	**V beam (V)**	**V Discharge (V)**	**Voltage efficiency**
62.4	0.681	1.36E-25	1.60E-19	0.896	**87**	100	**0.874**
31.2	0.489	1.36E-25	1.60E-19	0.896	**180**	200	**0.901**
21.1	0.41	1.36E-25	1.60E-19	0.896	**277**	300	**0.923**
15.7	0.356	1.36E-25	1.60E-19	0.896	**377**	400	**0.943**
12.6	0.323	1.36E-25	1.60E-19	0.896	**482**	500	**0.964**
10.6	0.299	1.36E-25	1.60E-19	0.896	**584**	600	**0.973**
9.4	0.286	1.36E-25	1.60E-19	0.896	**679**	700	**0.970**
8.5	0.278	1.36E-25	1.60E-19	0.896	**785**	800	**0.981**
7.6	0.265	1.36E-25	1.60E-19	0.896	**892**	900	**0.991**
7	0.254	1.36E-25	1.60E-19	0.896	**966**	1000	**0.966**

Lastly, mass utilization efficiency is simply given as a constant

$$\eta_m = 0.713$$

The total efficiency of the Hall thruster can now be found as

$$\eta_t = \gamma^2 \eta_b \eta_v \eta_m \eta_0$$

and, furthermore, as the anode efficiency

$$\eta_a = \frac{\eta_T}{\eta_0 \eta_c}$$

is often used as a metric, it is calculated as well.

Table 4 - Calculation of total and anode efficiencies

V Discharge (V)	**I beam (A)**	**Thrust measured (N)**	$\boldsymbol{\eta_v}$	$\boldsymbol{\eta_b}$	$\boldsymbol{\eta_0}$	$\boldsymbol{\eta_m}$	$\boldsymbol{\eta_T}$	$\boldsymbol{\eta_a}$
100	62.4	0.681	0.874	0.780	0.976	0.713	**0.381**	**0.429**
200	31.2	0.489	0.901	0.780	0.976	0.713	**0.393**	**0.443**
300	21.1	0.410	0.923	0.790	0.976	0.713	**0.408**	**0.459**
400	15.7	0.356	0.943	0.785	0.976	0.713	**0.414**	**0.466**
500	12.6	0.323	0.964	0.788	0.976	0.713	**0.424**	**0.478**
600	10.6	0.299	0.973	0.797	0.976	0.713	**0.433**	**0.488**
700	9.4	0.286	0.970	0.825	0.976	0.713	**0.447**	**0.504**
800	8.5	0.278	0.981	0.850	0.976	0.713	**0.466**	**0.525**
900	7.6	0.265	0.991	0.855	0.976	0.713	**0.473**	**0.534**
1000	7.0	0.254	0.966	0.875	0.976	0.713	**0.472**	**0.532**

Results are presented below in the same manner as Figure 7-9 from *Goebel & Katz*, with the addition of total efficiency to the plot.

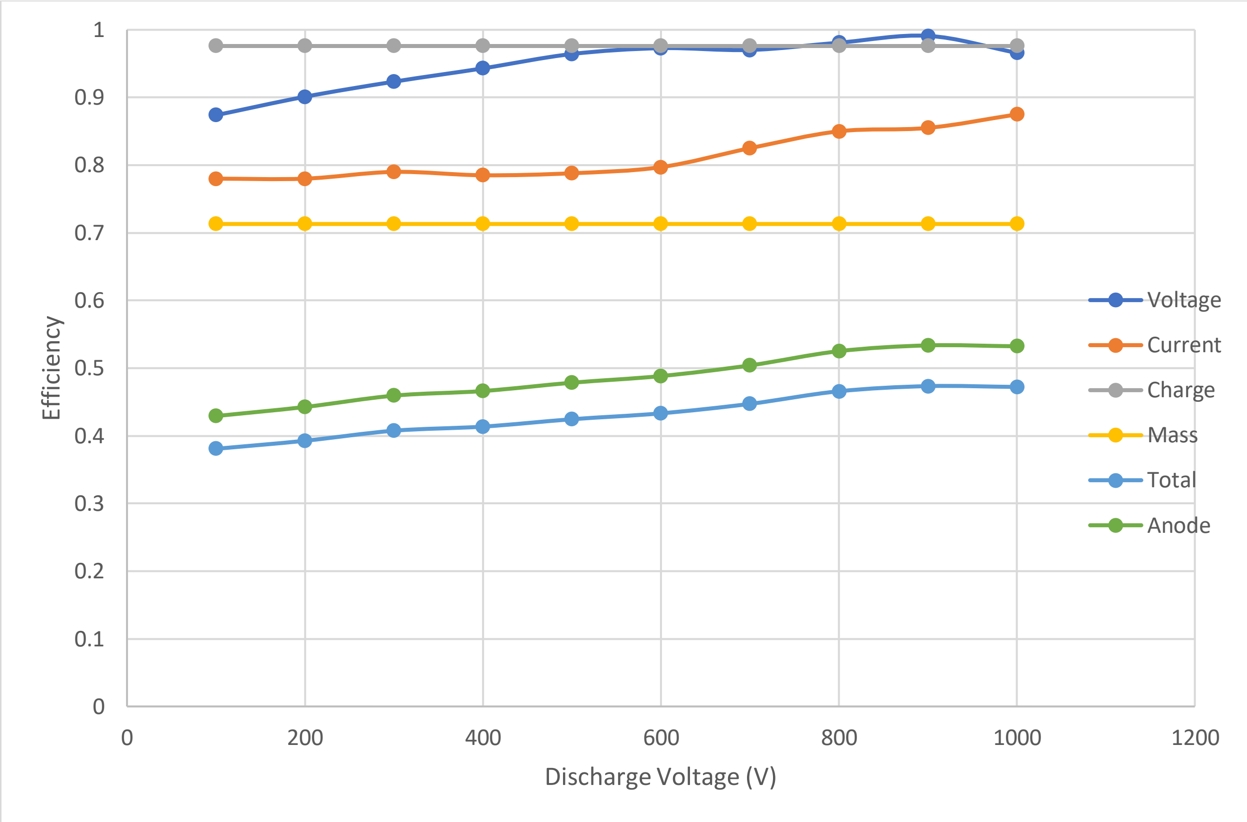

Figure 1 - Component and total efficiency

B. Specific Impulse Calculation

Given the relationships

$$I_{sp} = \frac{v_i}{g_0}\,; T = \dot{m}_i v_i$$

(where g_0 is the constant acceleration due to gravity, 9.81 m/s^2) and the ion mass flow of the beam found by

$$\frac{M}{e} I_d \eta_b = \dot{m}_i$$

the specific impulse of the engine in each test case may be found

I Discharge (A)	$\boldsymbol{\eta_b}$	$\boldsymbol{\dot{m}_i}$ **(kg/s)**	**Thrust measured (N)**	$\boldsymbol{I_{sp}}$ **(s)**
80.0	0.780	5.30E-05	0.681	**1310**
40.0	0.780	2.65E-05	0.489	**1882**
26.7	0.790	1.79E-05	0.41	**2334**
20.0	0.785	1.33E-05	0.356	**2723**
16.0	0.788	1.07E-05	0.323	**3076**
13.3	0.797	9.00E-06	0.299	**3387**
11.4	0.825	7.98E-06	0.286	**3651**
10.0	0.850	7.22E-06	0.278	**3927**
8.89	0.855	6.45E-06	0.265	**4186**
8.00	0.875	5.94E-06	0.254	**4357**