# Design of a 220 kN Upper-Stage Methane Rocket Engine

John Murray[1]
*The University of Alabama, Tuscaloosa, AL, 35487*

**Space and Missile Center (Los Angeles AFB, CA) is looking for a reusable engine concept capable of being used for both an expendable upper stage (orbit transfer vehicle) and for future development for a space plane with rapid response capability. Hydrogen is often the preferred fuel for this task but supportability is an issue. The commander has heard methane will provide almost as good of a performance as hydrogen and is much easier to tank and pump. You are tasked to design and evaluate the ability of using methane for this application. (Requirements: maximize T/W, Specific Impulse Isp ≥375 sec, Thrust 220 kN). An initial mass budget is needed to justify T/W predictions. This rocket engine will replace the RL-10 cryogenic hydrogen/oxygen engine as the upper stage of the Atlas V 541 (provide the same mission ΔV). Values for the power provided by the pump, and the entrance temperature to the turbine are critical design factors for the engine. The propellant requirement is methane and LOX. The requirements suggest the design can be met with a simpler cycle design (expander cycle).**

**Table 1: Final Design Parameters**

| | |
|---|---|
| **Expansion Ratio** | **300** |
| **Throat Diameter** | 11 cm |
| **O/F** | 3.0 |
| **Pc** | 15 MPa |

**Table 2: Final Design Outputs**

| | |
|---|---|
| **Propellant Mass (kg)** | **13673** |
| **Engine Cycle** | Expander, single turbine |
| **Cooling** | single cooling circuit |
| **Turbine Pressure Ration (TPR)** | 1.5 |
| **Turbine Inlet Temperature (Tt,i)** | 1000 K |

## Nomenclature

| | | |
|---|---|---|
| $\varepsilon$ | = | Expansion Ratio |
| $M_e$ | = | Exit Mach Number |
| $\gamma$ | = | Specific Heat Ratio |
| $P_c$ | = | Combustion Pressure |
| $P_e$ | = | Exit Pressure |
| $c_f$ | = | Coefficient of Thrust |
| $A^*$ | = | Throat Area |
| $A_e$ | = | Exit Area |
| F | = | Thrust |
| $c^*$ | = | Characteristic Velocity |
| R | = | Gas Constant |
| $T_c$ | = | Combustion Temperature |
| $M$ | = | Molar Mass |
| $\dot{m}$ | = | Mass Flow Rate of Propellant |
| $I_{sp}$ | = | Specific Impulse |
| $g_0$ | = | Gravity Constant |
| $D_E$ | = | Exit Diameter |
| $P_{P,o}$ | = | Pressure at Pump Outlet |
| $\Delta P_P$ | = | Pressure Change across Pump |
| $\Delta P_{inj}$ | = | Pressure Loss across Injector |
| $\Delta P_{turb}$ | = | Pressure Change across Turbine |
| $\Delta P_{cool}$ | = | Pressure Change across Cooling Jacket |

[1] M.S. Candidate, Department of Aerospace Engineering and Mechanics, The University of Alabama

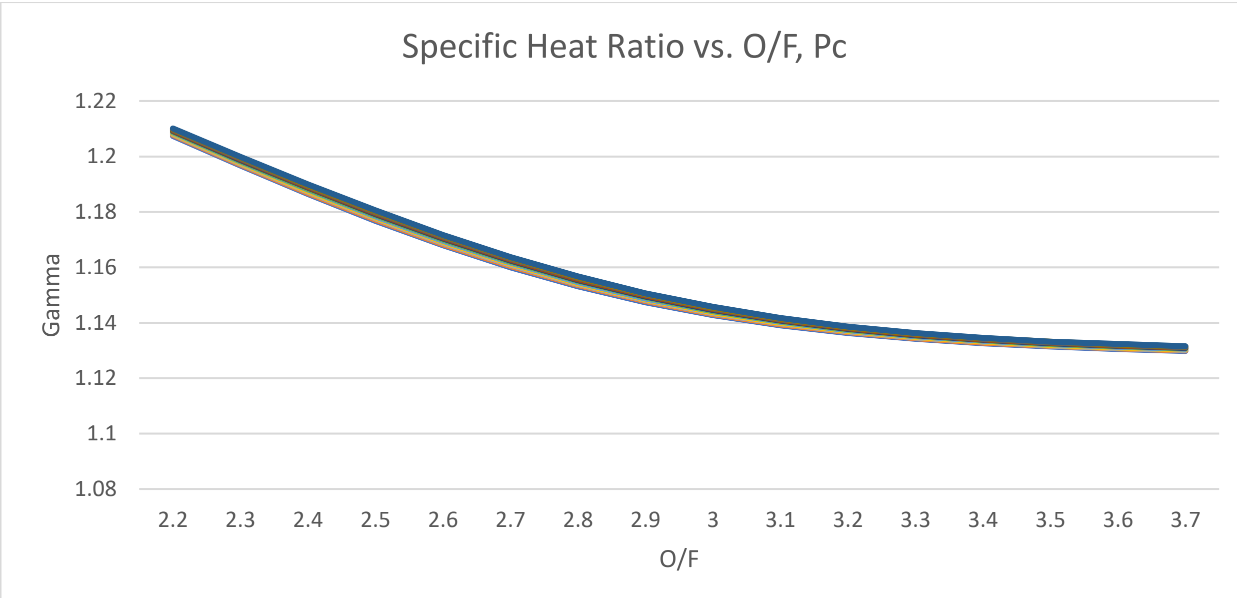
| | | | | | |
|---|---|---|---|---|---|
| $\Delta P_{feed}$ | = | Pressure Loss across Feed System | $P_{available}$ | = | Power Available to Turbine |
| $\eta_p$ | = | Pump Efficiency | $c_p$ | = | Specific Heat |
| $\eta_T$ | = | Turbine Efficiency | $T_{T,i}$ | = | Temperature at Turbine Inlet |
| ρ | = | Density | $P_{trat}$ | = | Pressure Ratio across Turbine |
| $P_{Required}$ | = | Power Required by Pump | | | |

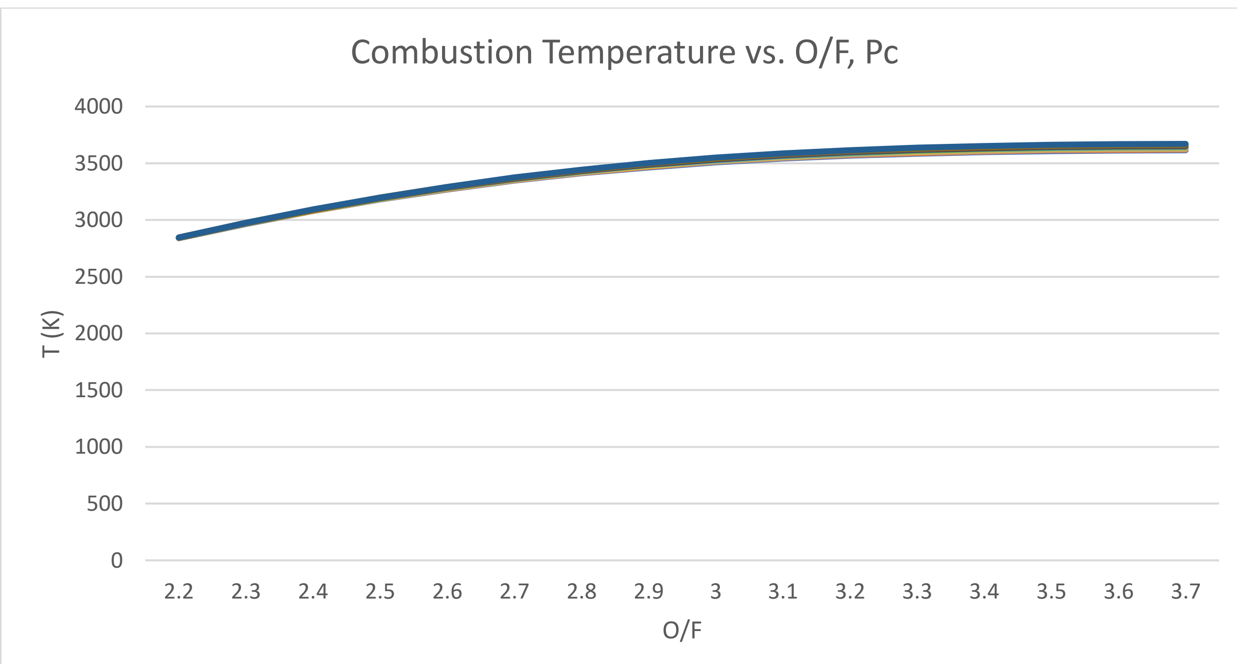
## I. Introduction

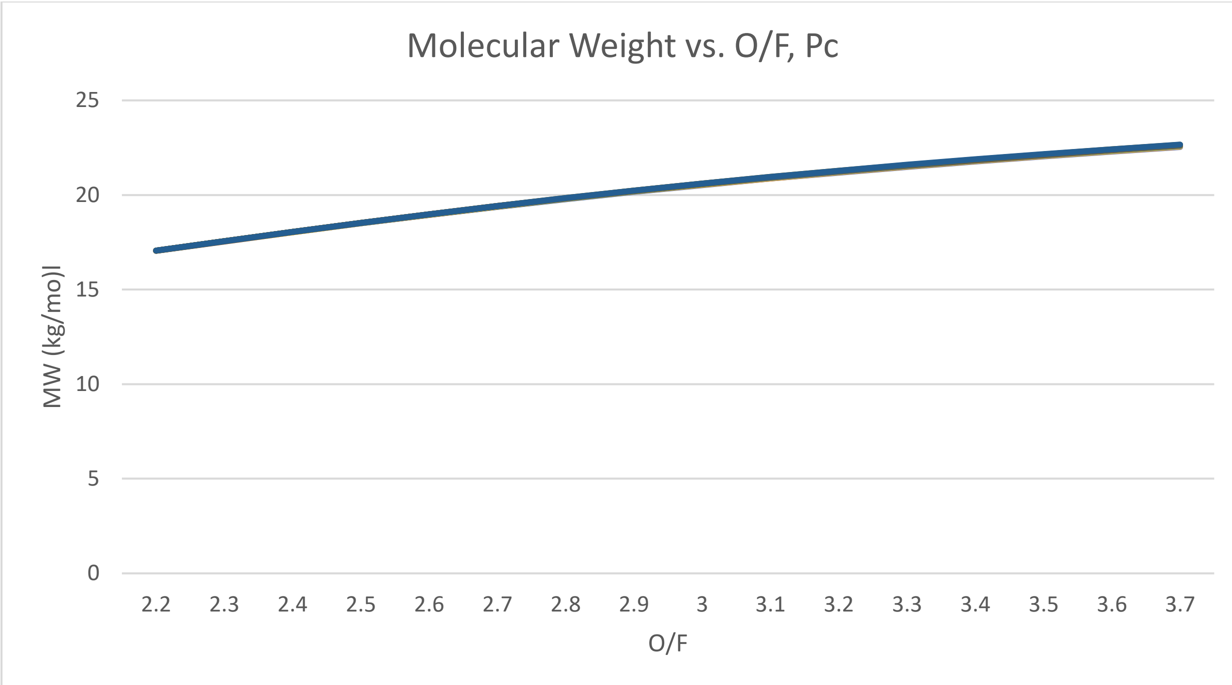
THis analysis examines the design of a liquid rocket engine to replace the RL10. While the RL10 is a robust and respectable design, the advancements in technology since its original development warrant a redesign. Furthermore, choosing a different propellant, such as methane, increases the potential mission the RL10 can complete by increasing the lifetime the propellant can survive in orbit. The mission analysis defines the ΔV requirements for the upper stage of the Atlas V as 4,268 m/s, the thrust for the engine design at 220 kN, a minimum specific impulse ($I_{sp}$) of 375 seconds is needed, engine max length 228.6 cm and max diameter is 190.5 cm. To meet these mission requirements, the design process needs to minimize mass of the engine while meeting minimum performance parameters.

## II. Choose Propellants, Chamber Pressure and O/F Ratio

The propellant choice has been defined as a liquid methane fuel with liquid oxygen as the oxidizer. The feasibility of the specific impulse requirement of 375 s is at this point determined by approximation at various O/F ratios (2.2 – 3.7) and chamber pressures of 10-15 MPa. Using the Chemical Equilibrium with Applications tool developed by NASA, the following characteristics of methane at combustion were produced. The expansion ratio is held at a constant 300.

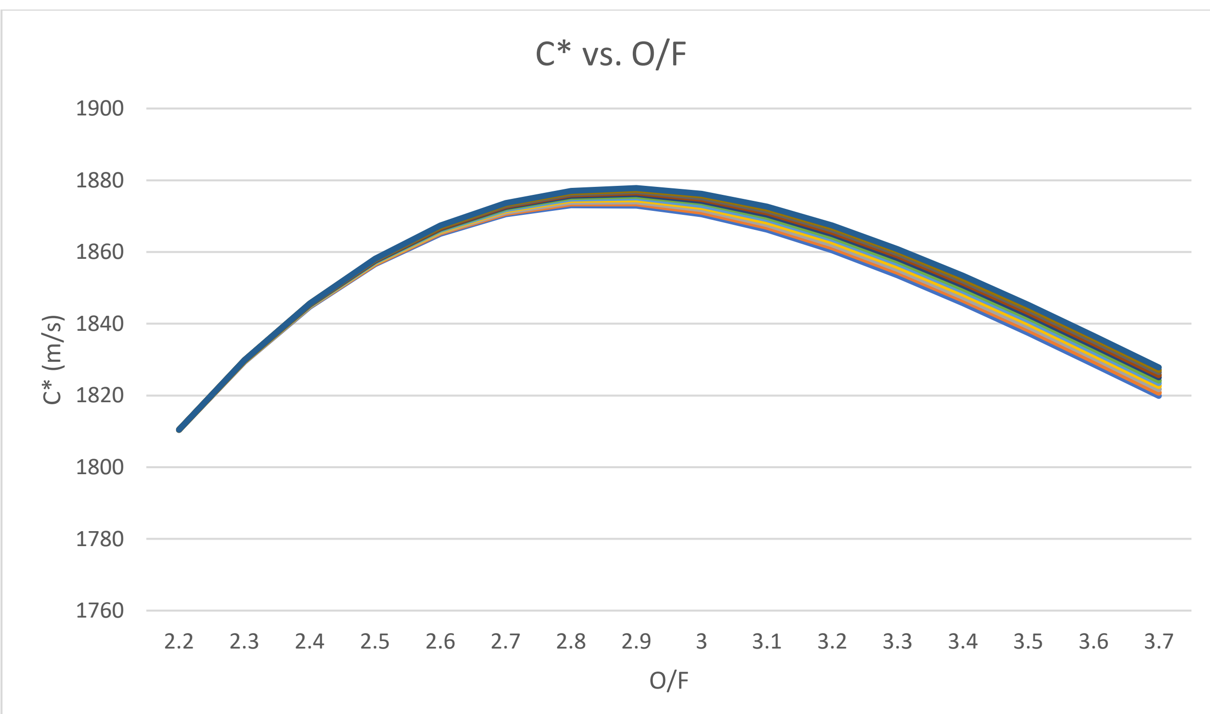




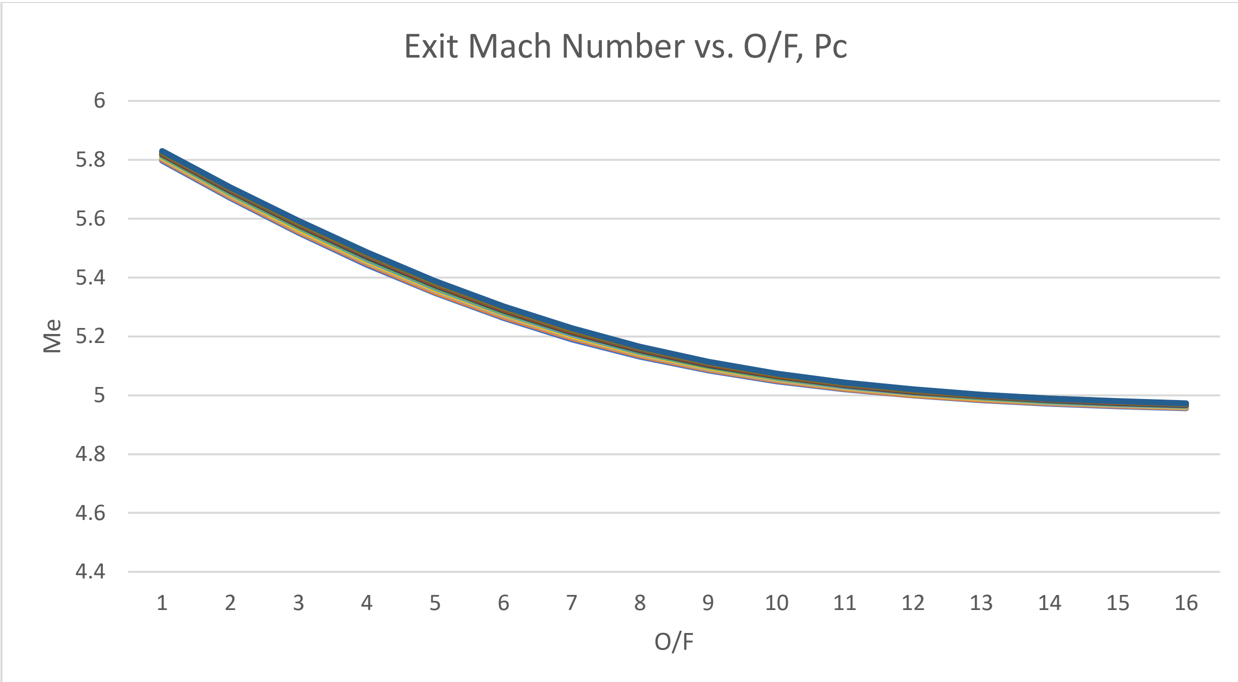

Given combustion temperature, specific heat ratio, and molecular weight, it is possible to find the characteristic velocity by

$$c^* = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{(\gamma+1)}{(\gamma 11)}}\frac{\Re T_C}{\mathcal{M}}} = \frac{A^* P_c}{\dot{m}}$$



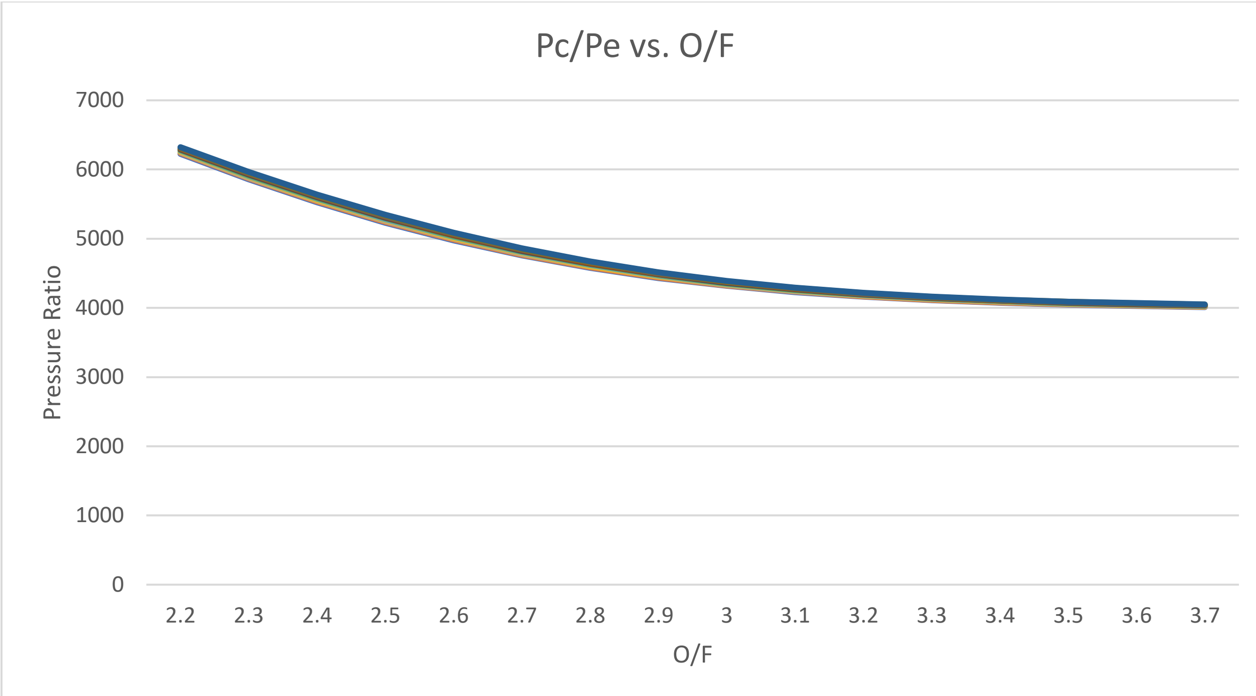

Solving iteratively for the exit Mach number at a constant expansion ratio and at the various specific heat ratios,

$$\varepsilon = \frac{1}{M_e}\left[\left(\frac{2}{(\gamma+1)}\right)\left(1+\frac{(\gamma-1)}{2}M_e^2\right)\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$



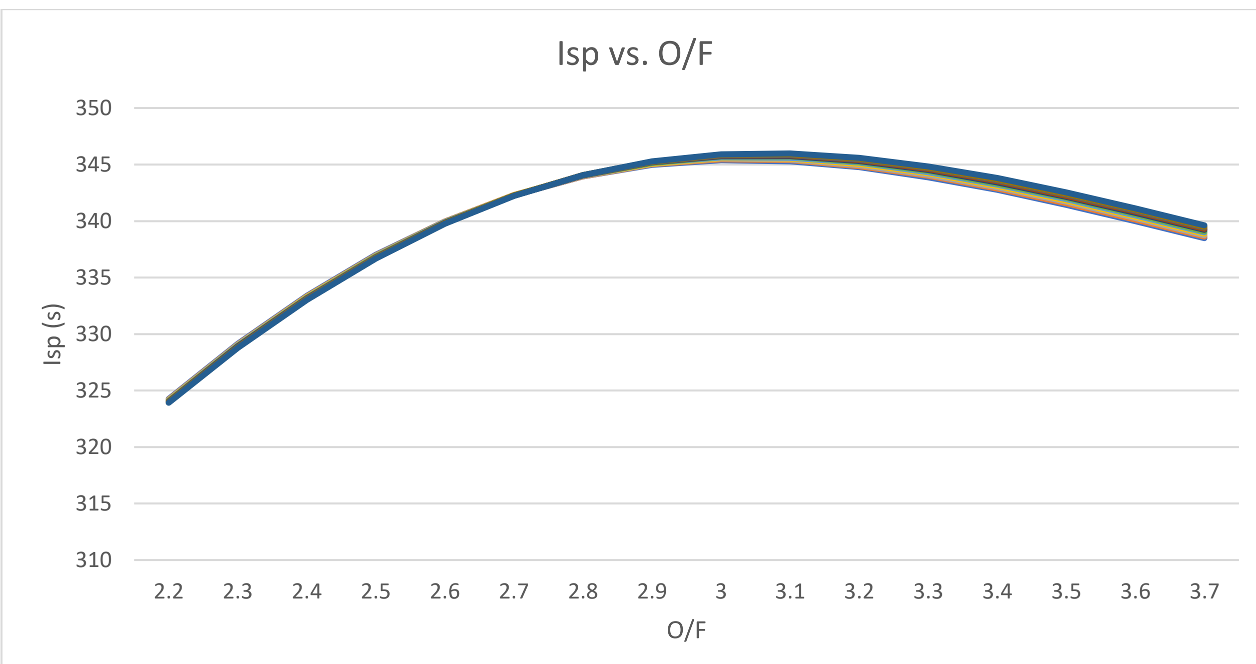

The exit Mach numbers give us a pressure ratio across the nozzle by

$$\frac{P_C}{P_e} = \left[1+\frac{(\gamma-1)}{2}M_e^2\right]^{\frac{\gamma}{(\gamma-1)}}$$




Finally, solving for specific impulse by

$$I_{sp} = \frac{1}{g_c}\sqrt{\frac{2\gamma}{(\gamma-1)}\frac{\Re T_C}{\mathcal{M}}\left[1-\left(\frac{P_{exit}}{P_C}\right)^{\frac{(\gamma-1)}{\gamma}}\right]} + \frac{c^*\varepsilon}{g_c}\left(\frac{P_{exit}-P_a}{P_c}\right)$$



we find that the maximum $I_{sp}$ we can expect tops out at under 350 seconds (after accounting for an 80% engine efficiency). Given that the expansion ratio cannot be raised much higher than 300 and the throat area cannot be decreased much further for significant improvement, the requirement of 375 s is already in jeopardy.

The O/F ratio has been initially chosen to be 3.0. It is apparent that varying the chamber pressure from 10-15 MPa does not have a significant impact on performance. A chamber pressure at the low end of the range, 11.5 MPa, is then selected.

## III. Nozzle/Cooling Jacket Dimensions

The shape of the parabolic nozzle is defined by the maximum turning angle possible for the exhaust gases at the throat (the Prandtl angle $\boldsymbol{\nu}$, here equaling $2\theta_B$), as well as the exit angle, both of which are determined by the exit Mach number and the specific heat ratio at the exit.

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\,tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - tan^{-1}\left(\sqrt{M^2-1}\right) = 123.3°$$

$$\theta_B = \frac{\nu(M)}{2} = 61.62°$$

$$\theta_E = \frac{1}{2}\arcsin\left[\frac{2}{\gamma M_E^2}\cot\left(\arcsin\left(\frac{1}{M_E}\right)\right)\right] = 9.83°$$

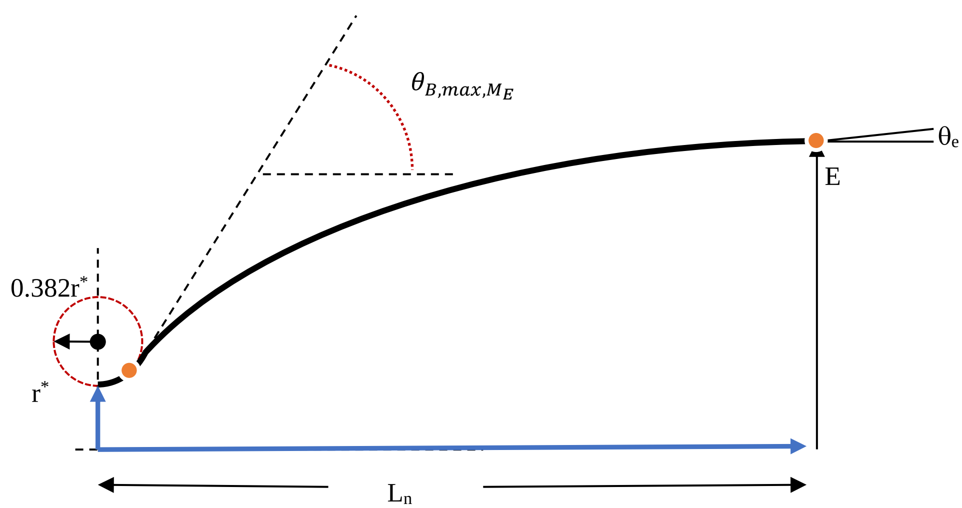



*Figure 1: Nozzle Contour, Method of Characteristics (courtesy Dr. Richard Branam, The University of Alabama)*

The throat region is modeled by a circle with radius 0.382r*. The intersection point of the downstream circle and the parabola can be determined analytically by

$$\frac{dy}{dx} = \frac{-(x - x_0)}{(y - y_0)} = \tan\theta \qquad (x - x_0) = -\tan\theta\,(y - y_0)$$

$$(-\tan\theta\,(y - y_0))^2 + (y - y_0)^2 = R^2$$

$$(y_B - y_0) = \sqrt{\frac{R^2}{1 + tan^2(\theta_B)}} \qquad (x_B - x_0) = -\tan\theta_B\sqrt{\frac{R^2}{1 + tan^2(\theta_B)}}$$

$$y_B = \sqrt{\frac{(0.382\,r^*)^2}{1 + tan^2(\theta_B)}} + 1.382\,r^* = 0.052\,m \qquad x_B = -\tan\theta_B\sqrt{\frac{R^2}{1 + tan^2(\theta_B)}} + x_0 = 0.015\,m$$

The radius of the parabola at the intersection point $r_B = y_B$. To find the focal point of the parabola, we employ the equations

$$r_0 = \frac{\tan\theta_E \frac{(y_B - y_0)}{(x_0 - x_B)} r_E - r_B}{\left(\tan\theta_E \frac{(y_B - y_0)}{(x_0 - x_B)} - 1\right)} \qquad r_E = \sqrt{\frac{\varepsilon A^*}{\pi}}$$

$$p = \tan\theta_E\left( r_E - \frac{\tan\theta_E \frac{(y_B - y_0)}{(x_0 - x_B)} r_E - r_B}{\left(\tan\theta_E \frac{(y_B - y_0)}{(x_0 - x_B)} - 1\right)_0}\right)$$

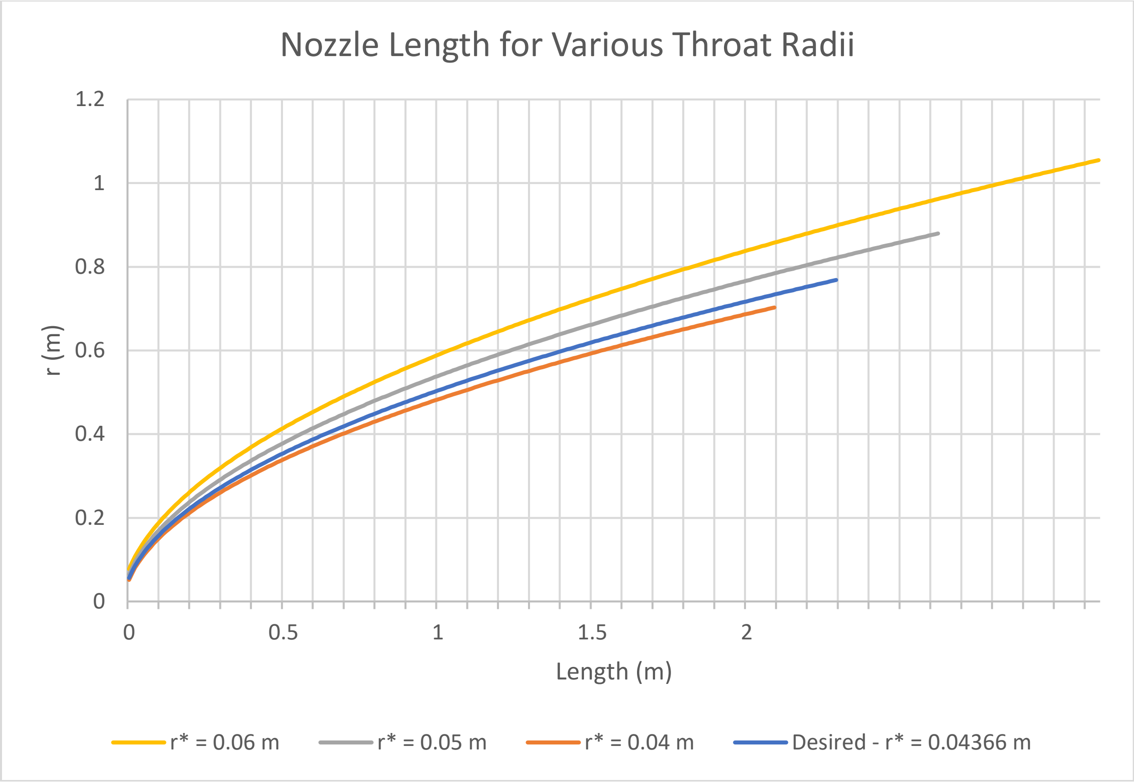
$$x_0 = x - \frac{(r - r_0)^2}{2p}$$

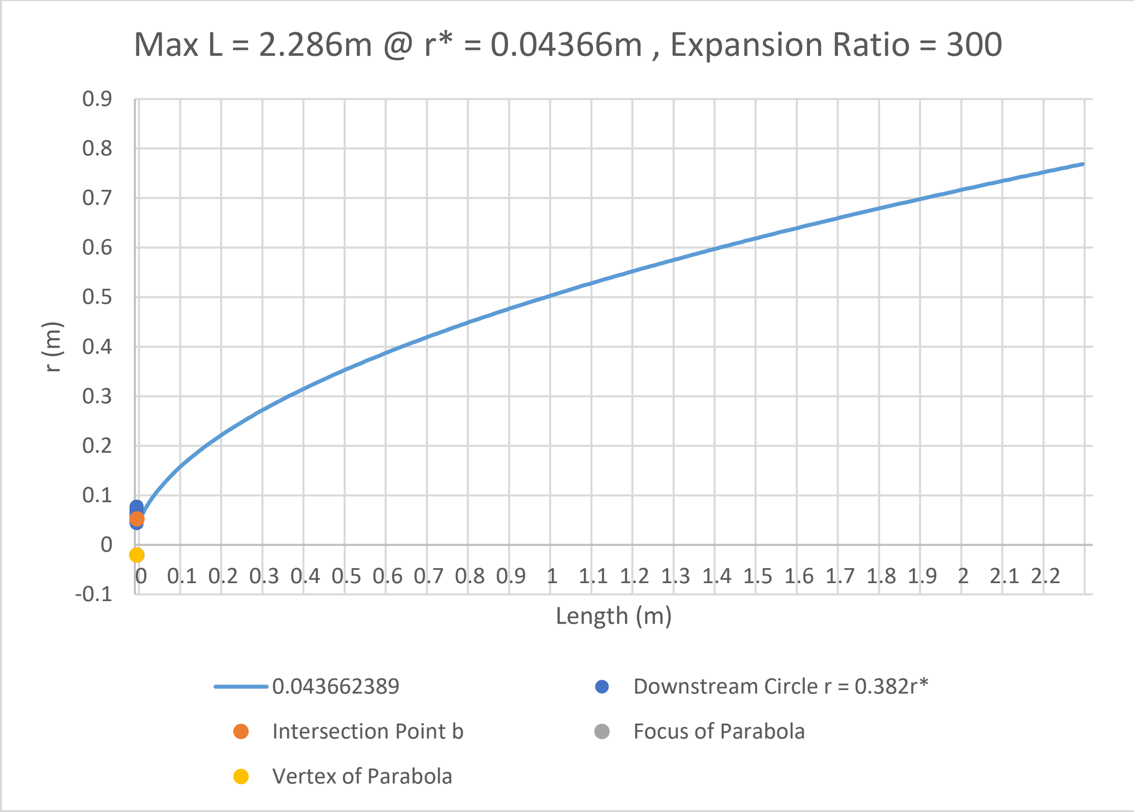
Which gives us our length of the nozzle,

$$L_N = x_0 + \frac{(r_E - r_0)^2}{2p}$$

Performing a what-if analysis on this value in Excel and using the goal seek to reach the maximum length allowed (2.286 m), we find the following parameters for our nozzle shape.

| **$A^*$ cm$^2$** | **$r_E$ max (cm)** | **p (cm)** | **$r_B$ (cm)** | **$x_B$ (cm)** | **Length (cm)** |
|---|---|---|---|---|---|
| **59.9** | **75.6** | **13.5** | **5.24** | **1.46** | **228.6** |


Nozzle Length for Various Throat Radii
1.2
1
0.8
0.6
0.4
0.2
0
r (m)
0
0.5
1
1.5
2
Length (m)
r* = 0.06 m
r* = 0.05 m
r* = 0.04 m
Desired - r* = 0.04366 m


Max L = 2.286m @ r* = 0.04366m , Expansion Ratio = 300
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
r (m)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2
Length (m)
0.043662389
Downstream Circle r = 0.382r*
Intersection Point b
Focus of Parabola
Vertex of Parabola

Taking our envelope constraints as the starting requirement for our nozzle, setting our cooling channel width at the throat to 0.005 m and maintaining 0.002 m of structural land between them, we find that for the maximum length of 2.286 m we will require a throat radius of 0.0437 m, which, given by

$$n = \frac{\pi}{w + a}\sqrt{\frac{4A^*}{\pi}}$$

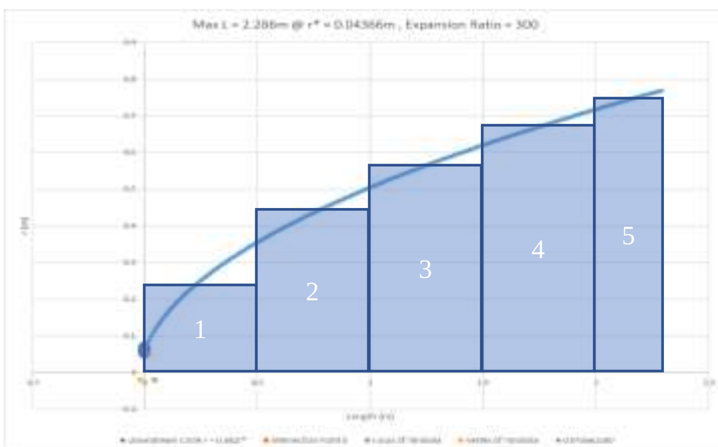
results in 39 cooling channels around the circumference. Channel height at the throat, determined by

$$h = \frac{\dot{m}}{w\ \rho\ (\frac{1}{2}a_s)}\ ; a_s = \sqrt{\gamma RT}$$

comes out to 0.0011 m.

Breaking up the nozzle section into five sections so as to estimate the pressure loss through this portion of the cooling jacket, we take the following approach: The average radius of each section is estimated by treating each as a truncated cone, and the channels are treated as discretely-sized based on this value. The length of each section is determined analytically by the arc length integral

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$




| Average Nozzle Section Dimensions | | | | Milled Channel Dimensions | | | | |
|---|---|---|---|---|---|---|---|---|
| Section | r (m) | D (m) | C (m) | Length (m) | w (m) | a (m) | h (m) | D(h,i) |
| 1 | 0.25 | 0.5 | 1.570796 | 0.648 | 0.028769 | 0.011508 | 0.028769 | 0.028769 |
| 2 | 0.45 | 0.9 | 2.827433 | 0.523 | 0.051784 | 0.020714 | 0.051784 | 0.051784 |
| 3 | 0.57 | 1.14 | 3.581416 | 0.513 | 0.065594 | 0.026237 | 0.065594 | 0.065594 |
| 4 | 0.68 | 1.36 | 4.272566 | 0.509 | 0.078252 | 0.031301 | 0.078252 | 0.078252 |
| 5 | 0.74 | 1.48 | 4.649557 | 0.29 | 0.085157 | 0.034063 | 0.085157 | 0.085157 |

## IV. Pressure Budget Calculations

All pressure budget/power balance calculations performed in this section are performed starting from a throat radius of 0.055 m, a chamber pressure of 11.6 MPa, a turbine inlet temperature of 900 K and a TPR of 1.3. These starting parameters were chosen along with an expansion ratio of 300 to ensure the maximum possible engine size within the required envelope. The exit diameter of the nozzle is here 1.905 m.

If we were to take the parabolic nozzle shape from the previous section as our starting condition, the chamber pressure would reach higher than 18 MPa, with a turbine inlet temperature of 1000 K and a TPR of 1.5 to reach the required pump pressure of over 30 MPa. Methane does not exist in a useful form at these temperatures and pressures, making it (among other things) difficult to study and characterize at the high pressure areas of the engine. For the purposes of moving forward with the design, the aforementioned parameters are used.

Iterating through chamber pressures to find an appropriate starting point, we find the following:

| Iteration | Pc (MPa) | Tc (K) | gamma | Me | MW | **Isp** | m_dot_F | m_dot_O |
|---|---|---|---|---|---|---|---|---|
| 1 | 11.50 | 3522 | 1.1438 | 5.094 | 20.56 | **345.58368** | 14.78 | 44.34 |
| 2 | 11.65 | 3523 | 1.1438 | 5.095 | 20.56 | **345.60072** | 14.78 | 44.34 |
| 3 | 11.65 | 3523 | 1.1438 | 5.095 | 20.56 | **345.60088** | 14.78 | 44.34 |
| 4 | 11.65 | 3523 | 1.1438 | 5.095 | 20.56 | **345.60089** | 14.78 | 44.34 |

Through the feed system, pressure loss due to friction was modeled by the frictional flow equations

$$Re_{h,i} = \frac{\rho V D_{h,i}}{\mu} = \frac{4(\dot{m}/n)}{\mu \pi D_{h,i}}$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{7.4D_h} + \frac{2.51}{Re_h\sqrt{f}}\right)$$

$$\Delta P = \frac{\dot{m}^2}{D_{h,i}^5}\frac{8fL}{\rho(n\pi)^2}$$

The following is a pressure loss estimation done for the cooling jacket based on the milled channel dimensions found previously. Following this is an estimation of the losses through the feed system that may be expected, using copper as the feed material, with a roughness coefficient of 6.10E-04.

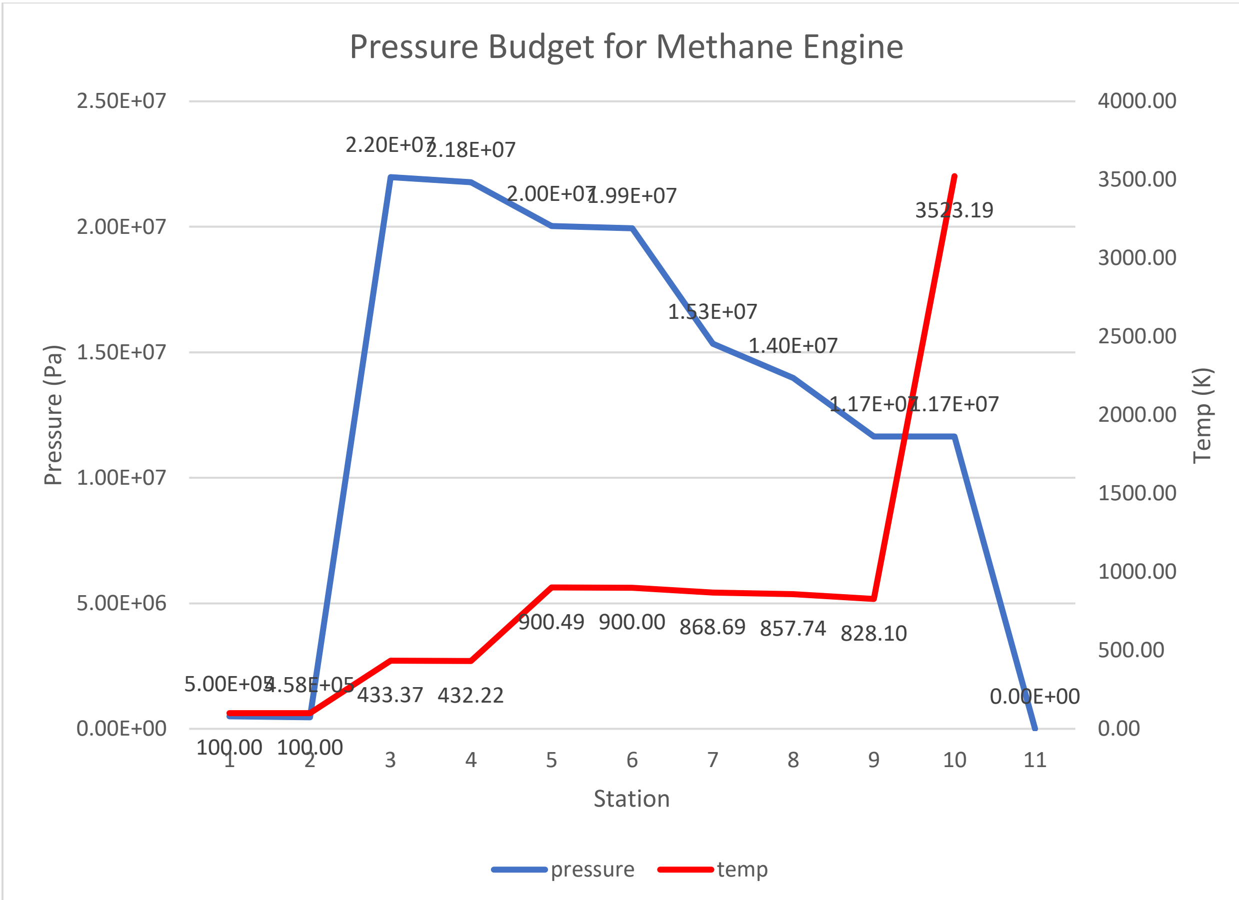
| **Section** | **D(h,i)** | **Fluid velocity (m/s)** | **Re** | **f** | **delP (Pa)** |
|---|---|---|---|---|---|
| Chamber | 0.028769 | 8.3 | 9.22E+05 | 0.0387 | **3666.6** |
| Throat | 0.001864 | | | | |
| 1 | 0.028769 | 8.3 | 9.22E+05 | 0.0387 | **3666.6** |
| 2 | 0.051784 | 2.6 | 5.12E+05 | 0.0321 | **289.1** |
| 3 | 0.065594 | 1.6 | 4.04E+05 | 0.0299 | **104.9** |
| 4 | 0.078252 | 1.1 | 3.39E+05 | 0.0285 | **49.4** |
| 5 | 0.085157 | 1.0 | 3.11E+05 | 0.0279 | **34.4** |

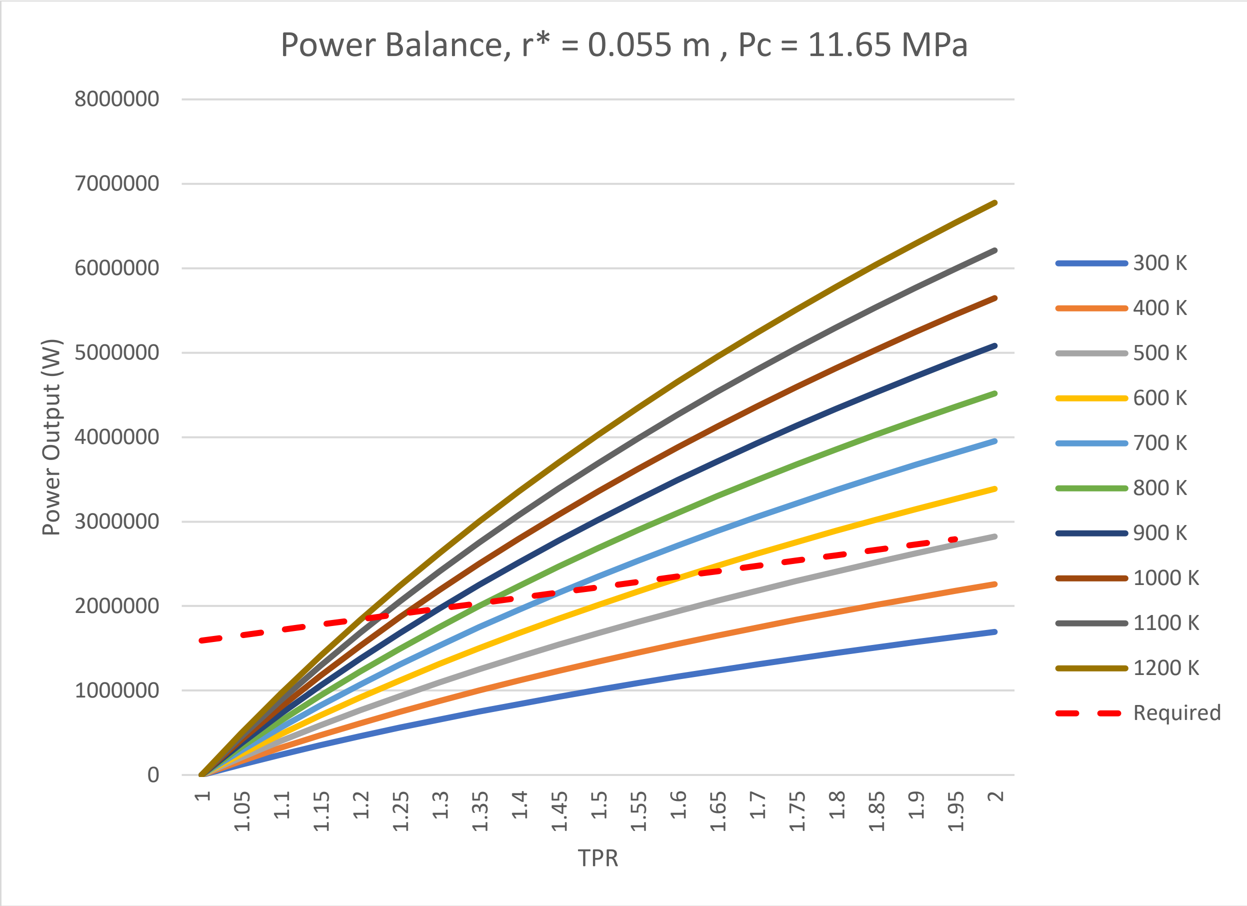
| CH4 | Feed | Length (m) | Inner Diameter (m) | Wall Thickness (m) | Fluid velocity (m/s) | Re | f | delP (Pa) |
|---|---|---|---|---|---|---|---|---|
| 1 | Tank to pump | 1 | 0.05 | 0.005 | 17.1 | 2.48E+06 | 0.0323 | **41697.4** |
| 2 | Pump to cooling jacket | 3 | 0.06 | 0.006 | 51.9 | 1.58E+07 | 0.0306 | **207092.9** |
| 3 | Cooling Jacket to turbine | 0.8 | 0.05 | 0.005 | 83.4 | 2.14E+07 | 0.0323 | **80417.4** |
| 4 | Turbine to injector | 0.8 | 0.04 | 0.004 | 334.0 | 5.87E+06 | 0.0347 | **1362078** |
| **O2** | | | | | | | | |
| 1 | Tank to pump | 1 | 0.06 | 0.005 | 14.4 | 6.15E+06 | 0.0305 | **57264.4** |
| 2 | Pump to injector | 1 | 0.06 | 0.006 | 145.3 | 2.78E+07 | 0.0305 | **579322.9** |

Injector losses are given as 20% of the chamber pressure.

Tracing our way through the engine, we find the following pressures and temperatures at various points of the fuel feed system, visualized below. Density, viscosity, and specific heat ratios of methane at various temperatures and pressures were sourced from NIST data, accessed via Wolfram|Alpha.

| | | Pressure (Pa) | Temp (K) | Density (kg/m3) | Viscosity (Pa s) | Gamma |
|---|---|---|---|---|---|---|
| 1 | Tank | 500000.00 | 100.00 | 439.2 | *1.52E-04* | 1.61 |
| 2 | Fuel Pumps Inlet | 458302.62 | 100.00 | 439.2 | *1.52E-04* | 1.61 |
| 3 | Fuel Pumps Outlet | 21980134.62 | 433.37 | 100.8 | *1.98E-05* | 1.387 |
| 4 | Cooling Jacket Inlet | 21773041.75 | 432.22 | 90.31 | *1.76E-05* | 1.46 |
| 5 | Cooling Jacket Outlet | 20025504.75 | 900.49 | 44.97 | *2.71E-05* | 1.156 |
| 6 | Fuel Turbine Inlet | 19945087.33 | **900.00** | 44.81 | *2.71E-05* | 1.156 |
| 7 | Fuel Turbine Outlet | 15342374.87 | 868.69 | 35.21 | *2.50E-05* | 1.158 |
| 8 | Fuel Injector Inlet | 13980295.98 | 857.74 | 32.93 | *2.58E-05* | 1.239 |
| 9 | Fuel Injector Outlet | 11650246.65 | 828.10 | 38.71 | *2.05E-05* | 1.242 |
| 10 | Chamber | **11650246.65** | 3523.19 | | | 1.13 |
| 11 | Exit | 0.00 | | | | |


Pressure Budget for Methane Engine
Pressure (Pa)
Temp (K)
Station
2.50E+07
2.00E+07
1.50E+07
1.00E+07
5.00E+06
0.00E+00
4000.00
3500.00
3000.00
2500.00
2000.00
1500.00
1000.00
500.00
0.00
1
2
3
4
5
6
7
8
9
10
11
5.00E+05
4.58E+05
2.20E+07
2.18E+07
2.00E+07
1.99E+07
1.53E+07
1.40E+07
1.17E+07
1.17E+07
0.00E+00
100.00
100.00
433.37
432.22
900.49
900.00
868.69
857.74
828.10
3523.19
pressure
temp




When the throat radius is constrained to the 0.0437 m given by the nozzle sizing conditions, the chamber pressure is determined to be over 18 MPa, the minimum turbine inlet temperature is around 1000 K, and the required turbine pressure ratio to match the necessary pump power is 1.5, bringing the pressure of the fluid at the pump outlet to over 30 MPa. At these pressures and temperatures, methane cannot exist in a useful form.

## V. Mass Budget

The mass of the feed system can be estimated as follows.

| CH4 | Feed | Length (m) | Inner Diameter (m) | Wall Thickness (m) | Volume of Material (cu. m) | Material | Density (kg/cu m) | Mass (kg) |
|---|---|---|---|---|---|---|---|---|
| 1 | Tank to pump | 1 | 0.05 | 0.005 | 0.0009 | Copper | 8960 | **7.74** |
| 2 | Pump to cooling jacket | 3 | 0.06 | 0.006 | 0.0037 | | | **33.44** |
| 3 | Cooling Jacket to turbine | 0.8 | 0.05 | 0.005 | 0.0007 | | | **6.19** |
| 4 | Turbine to injector | 0.8 | 0.04 | 0.004 | 0.0004 | | | **3.96** |
| O2 | | | | | | | | |
| 1 | Tank to pump | 1 | 0.06 | 0.005 | 0.0010 | Copper | 8960 | **9.15** |
| 2 | Pump to injector | 1 | 0.06 | 0.006 | 0.0012 | | | **11.15** |

The mass of the fuel and oxidizer tanks can be found as follows, assuming the following;

| | | |
|---|---|---|
| Payload Mass (kg) | **5000** | assumed |
| delta-v | 4268 | requirement |
| $I_{sp}$ | 345.60089 | calculated |
| $g_0$ | 9.81 | constant |
| $f_{inert}$ | 0.03 | assumed |
| O/F | 3 | calculated |
| Propellant mass | **13673.476** | |

| **Component** | **Mass (kg)** | **Density in Tank (kg/m3)** | **Volume (m3)** | **Tank wall thickness (m)** | **Tank Material** | **Tank Material Density (kg/m3)** | **Cylindrical height** | **Cylindrical Radius (m)** | **Tank Material Volume (m3)** | **Tank Mass (kg)** |
|---|---|---|---|---|---|---|---|---|---|---|
| Methane (liquid) | **3,418** | 439.2 | 439.2 | 0.05 | Steel | 8050 | 3.871 | 0.8 | 1.117 | **8,990.2** |
| Oxygen (liquid) | **10255** | 1092 | 1,092.0 | 0.1 | Steel | 8050 | 3.690 | 0.9 | 2.517 | **20,262.0** |

The RL-10 currently uses a nickel alloy for its nozzle material[2], but for the purposes of this mass budget, stainless steel (maximum density: 8000 kg/m$^3$) will be chosen as the most abundant material in the nozzle wall. We estimate this by integrating the total length of the parabolic shape around its circumference,

[2] Smithsonian National Air and Space Museum, https://airandspace.si.edu/collection-objects/rocket-engine-liquid-fuel-rl-10-0

$$\int_{x_0}^{L_N} 2\pi\sqrt{2px}\sqrt{1+\left(\frac{p}{\sqrt{2px}}\right)^2}\,dx = 7.808\,m^2$$

and estimating an average wall thickness of 2 cm, we approximate the mass of the nozzle to be

$$7.808\,m^2(0.02\,m)\left(8000\frac{kg}{m^3}\right) = \mathbf{1250\,kg}$$

All other component masses are estimated from the RL-10 Design Report[3], the major contributors being as follows:

| **Component** | **Mass (kg)** |
|---|---|
| Thrust chamber | **46.5** |
| Turbopump | **34.1** |
| Engine mount | **4.9** |
| Injector assembly | **6.7** |
| Ignition system | **3.2** |
| Various valves & connectors | **73.2** |

[3] Pratt & Whitney, accessed via https://pslhistory.grc.nasa.gov/PSL_Assets/History/C%20Rockets/Design%20Report%20for%20RL-10-A-3-3.pdf

## VI. Results and Conclusion

Starting the design process from different requirements (envelope, thrust requirement, etc.) yields different final results for the design, and not always in reconcilable ways. This was most apparent in the envelope constraint leading to an impossible chamber pressure, and a still insufficient specific impulse value. The turbine inlet temperature, too, was at the upper limit of material constraints. Abandoning these and fixing a throat area to start would have lowered the pressure on the propellant, but yielded a nozzle far too long to meet the envelope constraints, and again, $I_{sp}$ still would not have been sufficient.