Ballistic Element and Lumped Parameter Analyses of the AJ 60A Solid Rocket Motor

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Analysis was conducted on a simplified-geometry version of an Atlas V solid rocket booster (SRB) to obtain pressure and thrust profiles along its intended burn time. Two methods of analysis were employed: the lumped parameter method and the ballistic element method. The lumped parameter method returned meaningful results, determining the maximum thrust of the vehicle to be 1800 kN, with a specific impulse of 191.28 seconds. The thrust chamber reaches pressures of 4.9 MPa as determined by this analysis. The ballistic element method was attempted and partially completed, with results that are not necessarily pertinent to the specific scenario at hand but that do provide an example of the algorithm flow.

Nomenclature

M	=	Mach Number
ε	=	Expansion Ratio
γ	=	Specific Heat Ratio
P_c	=	Combustion Pressure
P_e	=	Exit Pressure
c_f	=	Coefficient of Thrust
A^*	=	Throat Area
A_e	=	Exit Area
F	=	Thrust
c^*	=	Characteristic Velocity
R	=	Gas Constant
T_c	=	Combustion Temperature
$\dot{m}$	=	Mass Flow Rate of Propellant
I_{sp}	=	Specific Impulse
g_0	=	Gravity Constant
D_E	=	Exit Diameter
ρ	=	Density

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I. Introduction

THis analysis examines the pressure and thrust profile of the Atlas V strap-on solid rocket booster during nominal flight, given a simplified fuel geometry and known performance characteristics, included below. Two analyses are performed to obtain this information: a lumped parameter analysis and a ballistic element analysis. The lumped parameter method is more reliant on simplified geometry and studying the entire combustion system as a whole, while the ballistic element method breaks the system up into discrete elements and employs a differential approach to the analysis, more reliant on conservation of mass approaches and more acknowledging of the fact that different areas of the combustion chamber experience the combustion process differently.

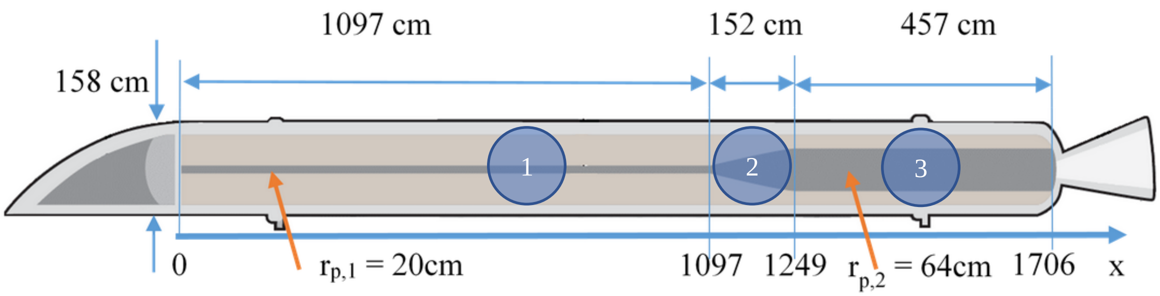

Figure 1 - Simplified geometry of SRB

Table 1 - Given parameters

Propellant density (ρ)	1840	kg/m^3
Characteristic vel. (c^*)	1541	m/s
Combustion temp (T_c)	3636	K
Burn rate average	0.588	cm/s
$\mathbf{r}_b = a\,(P_c)^n$	a	0.415
	n	0.31
Throat diameter (D^*)	0.5	M
Throat area (A^*)	0.196	m^2
Nozzle exit area	2.945	m^2

II. Lumped Parameter Method of Analysis

Given the geometry defined in Figure 1, we break the booster up into three distinct segments as labeled. Initially and as pictured, the dimensions of each section and its contained fuel are given in Table 2 below. Section 2 is a frustum, but for the purposes of this analysis it is considered to be a cylinder with a radius that equals the average of the radii of its two bases.

Table 2- Initial geometry

	Length (cm)	Section Diameter (cm)	Inner Diameter (m)	Initial Inner SA (cm^2)
Section 1	1097	158	20	68926.5
Section 2	152	158	20	64	4925.5
Section 3	457	158	64	91885.3

The time step between recalculations was set to 1 second. For each section, volume of the propellant was calculated at the beginning and at the end of each step, given the average burn rate of 0.588 cm/s. Multiplying this change in volume by the propellant density, the mass of the consumed propellant was found. In this case, the system is assumed to be steady, such that mass flow in is equal to mass flow out.

To find chamber pressure at a given time we start with the conservation of mass equation

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} = r_b \rho_b A_b - \; P_C \frac{A^*}{c^*} \tag{1}$$

For a steady system, $\frac{dm}{dt} = 0$, and we reduce the above equation to

$$P_c = \left(\frac{a \rho_b A_b c^*}{A_t}\right)^{\frac{1}{1-n}} \tag{2}$$

This allows for determination of chamber pressure at each time step, given the burning parameters *a* and *n*, and having solved for the burning surface area. The results for chamber pressure are given in

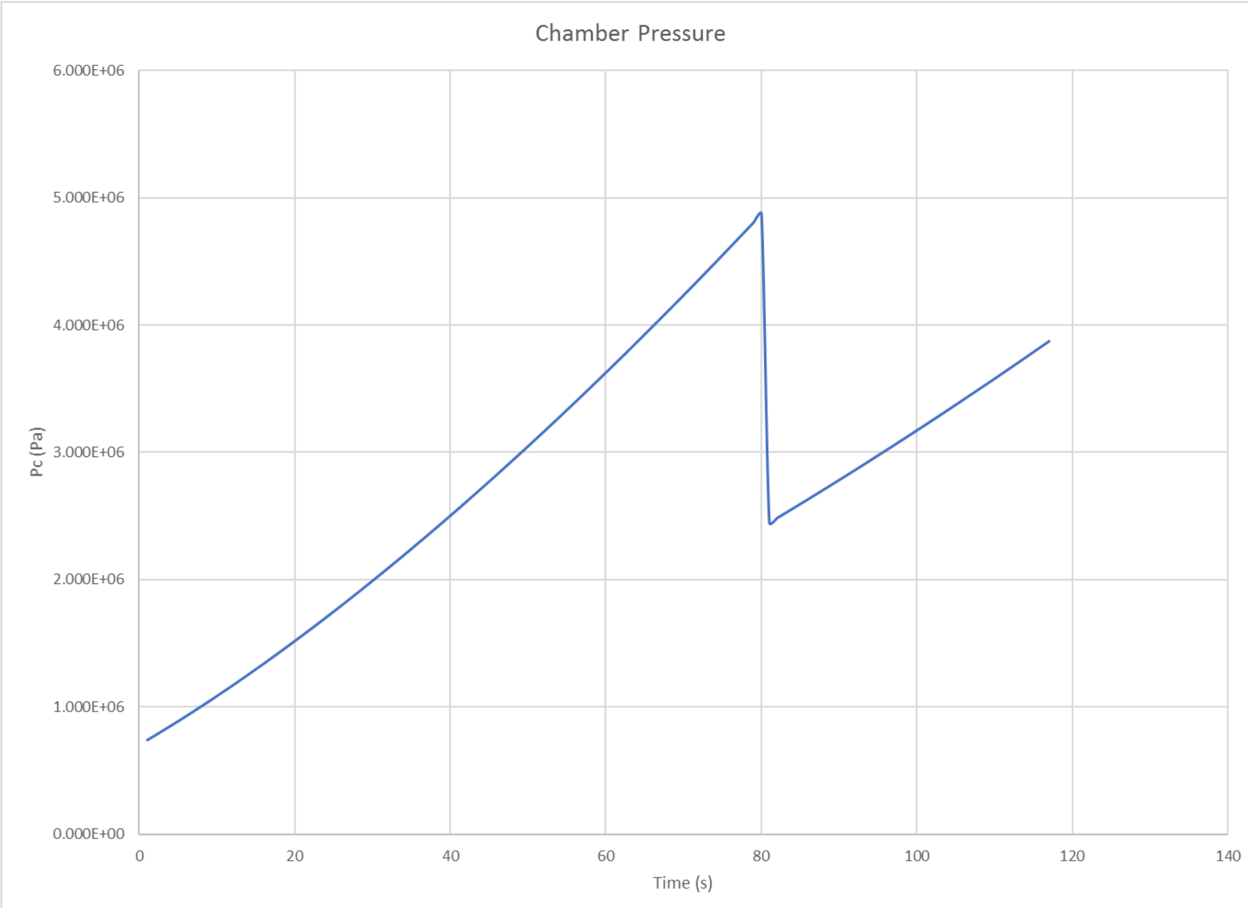

Figure 2 - Chamber pressure over burn time. As the end section of fuel completes burning, the pressure drops temporarily.

When Section 3 has been depleted (t ~ 80 seconds) the chamber pressure drops noticeably in the absence of burning propellant.

To move forward with calculating the thrust profile, we calculate c_F by

$$c_F = \left[\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1-\left[\frac{P_e}{P_c}\right]^{\frac{\gamma-1}{\gamma}}\right)\right]^{\frac{1}{2}} + \frac{(P_e-P_a)A_e}{P_o A_t} \tag{3}$$

where P_a is initially said to be 101325 Pa at sea level and thereafter determined by Equation 4 [1]

$$P = P_o \exp\left(-\frac{M_a g h}{RT}\right) \tag{4}$$

where P_o is the air pressure at sea level, M_a is the molar mass of air (assumed to be constant at 29 g/mol), R is the gas constant, g is the gravitational constant, T is assumed to be a constant 250 K, and h is the height above sea level. c_F and P_c together give us the instantaneous thrust by

$$F = c_F A_T P_c \tag{5}$$

Plotting the thrust over time yields a trend similar to the chamber pressure profile.

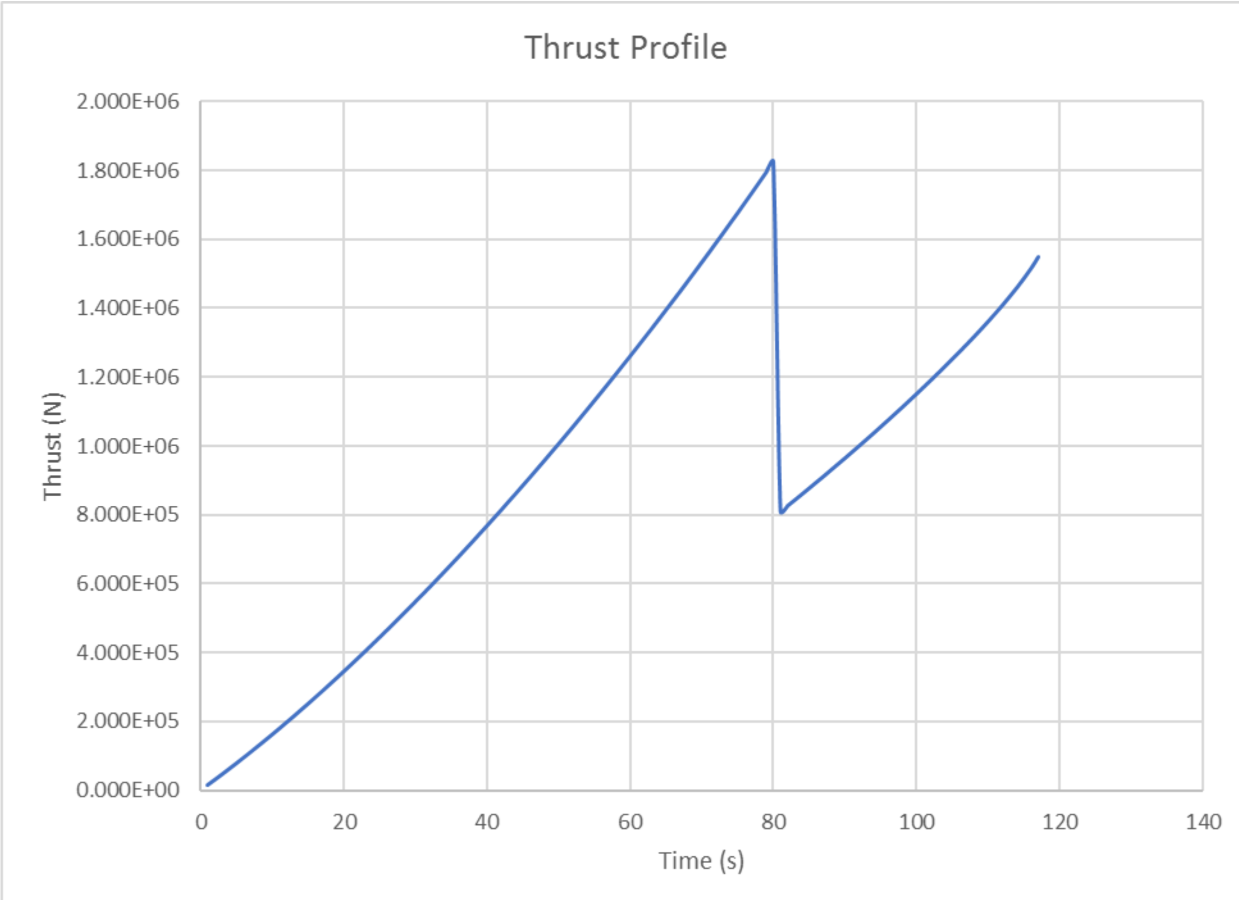

Figure 3 - Thrust over burn time. The same sudden decrease is observed.

Maximum thrust for this profile is a little over 1800 kN. Any deviation from the chamber pressure profile can be attributed to the role of decreasing atmospheric pressure in determining c_F and therefore thrust.

Specific impulse is obtained by summing all the determined impulses at each time step, then dividing by mass and the scaling factor $g_o = 9.81$. For this analysis we find our specific impulse to be 191.28 seconds.

III. Ballistic Element Method of Analysis

We begin by determining the velocity of the propellants inside the chamber, using the ratio of port to throat area, and the specific heat ratio of our propellant. For the purposes of this analysis, the specific heat ratio γ is held at a constant 1.2. Solving for propellant Mach number

$$\frac{A_p}{A_t} = \frac{1}{M_a}\left[\frac{2+(\gamma-1)M_a^2}{1+\gamma}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{6}$$

we find

$$M_a =$$

To determine chamber pressure at a given time, we proceed as follows:

Discretizing the fuel into elements of mass dm, conservation of momentum of the system is represented by

$$\dot{m}v + PA - (\dot{m} + d\dot{m})(v + dv) - (P + dP)A = 0$$
(7)

$$d(m\dot{v}) = d(\rho A v^2) = \text{-AdP} \tag{8}$$

The speed of sound in a calorically perfect fluid being $a = \sqrt{\frac{\gamma P}{\rho}}$, we may write that

$$M = \frac{v}{a} \rightarrow \rho v^2 = \gamma P M^2$$
(9)

and

$$\frac{dP}{P} = \frac{\gamma dM}{1+\gamma M^2} \tag{10}$$

Integrating Equation (5) across the length of the chamber from head to throat,

$$\frac{P_a}{P_h} = \frac{1}{1+\gamma M_a^2} \tag{11}$$

$$\frac{P_{0a}}{P_{0h}} = \phi = \frac{\left(1+\frac{\{\gamma-1)}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}}}{1+\gamma M_a^2} \tag{12}$$

For a steady case, mass flow into an element of volume must equal mass flow out. These are defined by

$$\dot{m}_{in} = aP_{0h}^n \rho_b A_b \tag{13}$$

$$\dot{m}_{out} = \frac{P_{0a}A_t}{c^*} = \frac{\phi P_{0h} A_t}{c^*} \tag{14}$$

The chamber stagnation pressure is initially defined as

$$P_{0h} = P_h = \left(\frac{a\rho_b A_b c^*}{\phi A_t}\right)^{\frac{1}{1-n}} \tag{15}$$

Defining the relationships for each element along the length of the chamber, we obtain the equations

$$P_{0i} = \frac{P_{0(i-1)}}{1+\gamma M^2\left(\frac{\dot{m}_i}{\dot{m}}\right)} \tag{16}$$

$$\dot{m} + \frac{\dot{m}_i}{2} = \frac{M_i P_{0i} A_{pi}}{\sqrt{\frac{RT}{\gamma}}}\left\{1 + \frac{\gamma-1}{2} M_i^2\right\}^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{17}$$

Ideally, at this point, an initial total pressure is chosen for each element and Equations 16 and 17 are solved simultaneously and iterated to find a solution for P_{0i}. This could not be achieved at this time. The iteration process would not yield appropriate results nor converge on a solution. This being the case, a workaround is taken to produce the results below.

The equations used for the workaround are the differential equations for mass flow

$$\frac{dm}{dt} = m\left(\frac{1}{P_{0N}}\frac{dP_{0N}}{dt} + \frac{1}{V}\frac{dV}{dt}\right) \tag{18}$$

and

$$\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \frac{gP_{0n}A_t}{C^*} \tag{19}$$

the initial (t = 0) chamber total pressure at the nozzle is taken to be about 0.8 MPa, given the initial value from the lumped parameter analysis. This value is then plugged in to Equations 18 and 19 and, for every time step, iterated until they produce mass flow rates within 5% of each other. Change in pressure is apparent from the chosen initial pressure, elemental mass flow rates and the throat area have been found previously, and g is equal to 1 because the analysis employs metric units.

Given these iterations, the results below are produced.

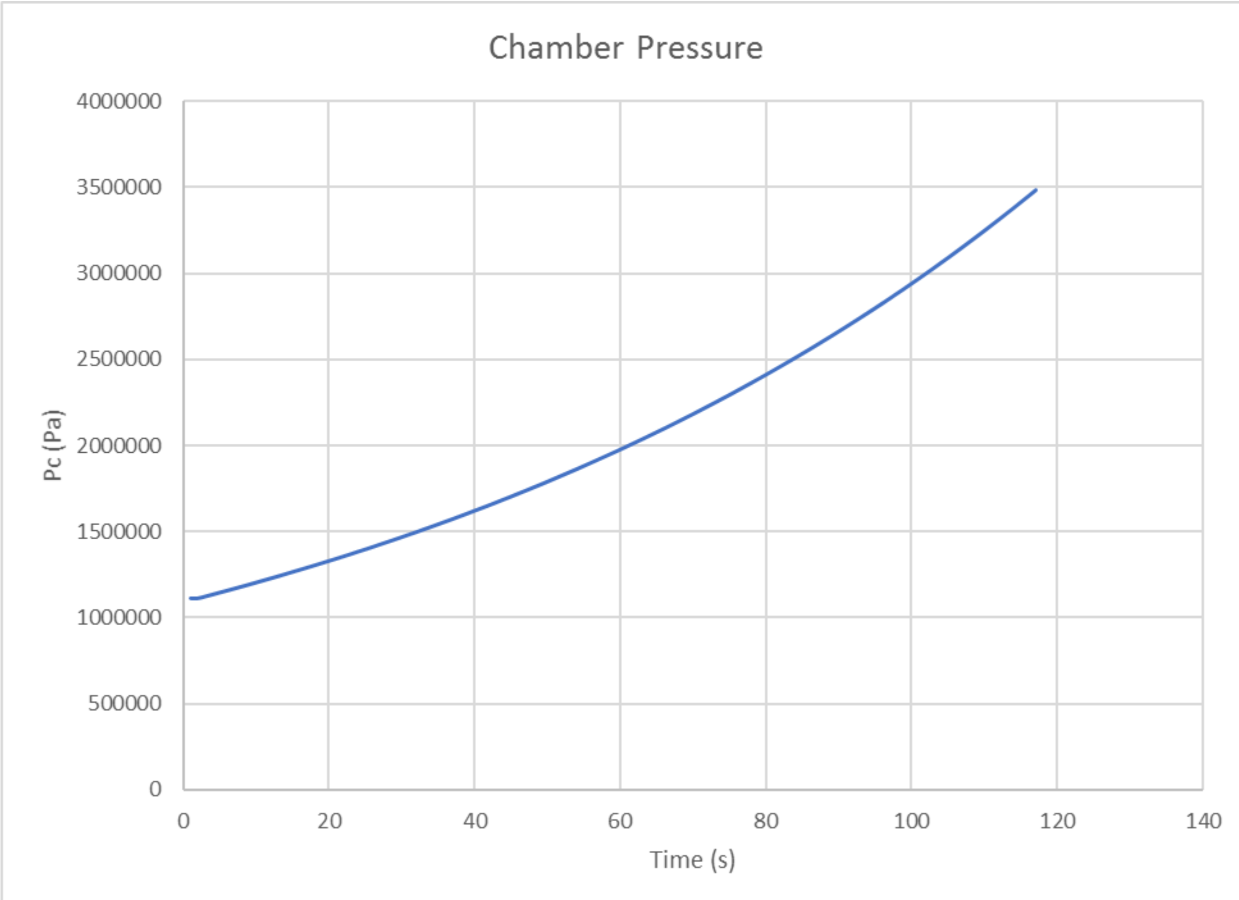

Figure 4 - Chamber pressure over burn time

The burn appears to take the form of a continuous burn, without the decrease in pressure at the depletion of a section. This is due to the total chamber pressure at the nozzle being continuously iterated on in the context of Equations 18 and 19, rather than by 16 and 17. The maximum chamber pressure is about 1.5 MPa lower than that found in the lumped parameter analysis.

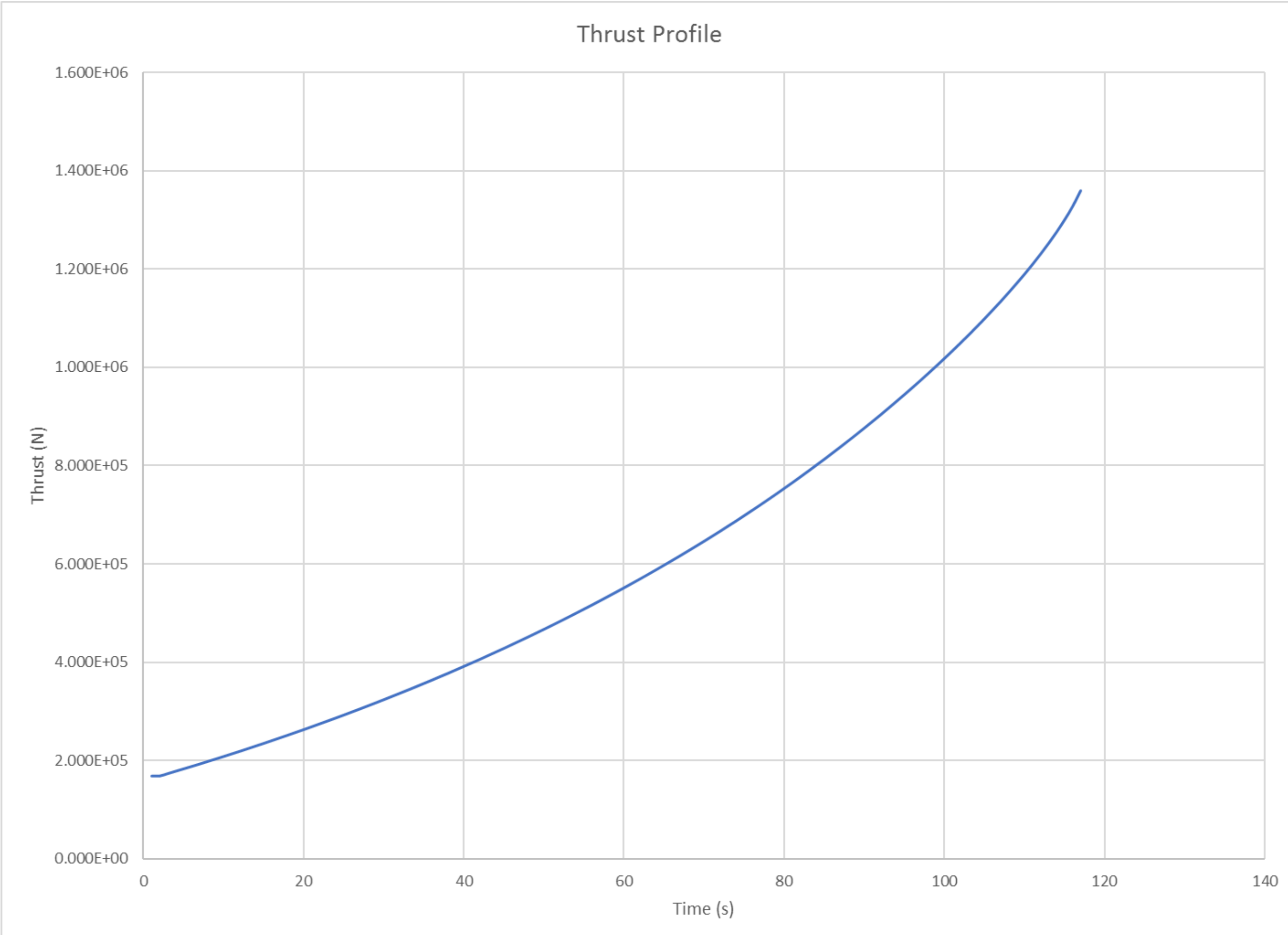

Figure 5 - Thrust as a function of burn time

Thrust, too, shows no decrease at t = 80s, for the same reasons as chamber pressure in Figure 4. This method yields a maximum thrust nearly 400 kN below that found by the lumped parameter analysis. Calculating for specific impulse, it is found to be 126.48 seconds; much lower than the lumped parameter value as well. Clearly more work must be done with the ballistic element analysis method in order to produce meaningful results.

IV. Results and Conclusion

The lumped parameter analysis is the more approximate case, but in this project is taken to be the more accurate method of determining the performance characteristics of the solid rocket booster. The ballistic element method is considered the more accurate…when used correctly. Iterations proved difficult and a suitable initial chamber pressure could not be realized. However, some functionality of the ballistic element method remains on display in this analysis, and results in a characteristic thrust profile as one might expect from such an analysis, though it may be more representative of a basic cylindrical bore than of the less simple geometry given for the SRB. Taking the lumped parameter results to be the more accurate, it is determined that the maximum thrust of the SRB is about 1800 kN, and its specific impulse is 191.28 seconds. The thrust chamber reaches pressures of 4.9 MPa.

V. References

[1] Jacob, Daniel J., *Introduction to Atmospheric Chemistry*, Princeton University Press, 1999. Accessed via http://acmg.seas.harvard.edu/people/faculty/djj/book/bookchap2.html