Analysis of a Magnetoplasmadynamic Thruster

John J. Murray[1]

The University of Alabama, Tuscaloosa, AL, 35487

Thrust, specific impulse, and efficiency of a magnetoplasmadynamic thruster are calculated across a range of operating currents. All are found to increase with increasing current.

Nomenclature

η	=	efficiency
F_{em}	=	electromagnetic force
g_0	=	constant acceleration due to Earth's gravity, 9.81 m/s^2
I_{sp}	=	specific impulse
μ_0	=	permittivity of free space, 1.26 x 10^{-6} N/A^2
u_e	=	exit velocity

I. Background

Magnetoplasmadynamic thrusters are gaining traction in the propulsion realm as the most powerful electromagnetic engines yet designed. Much like other forms of electric propulsion, their operation relies on the ionization of a gas into a plasma and its acceleration across some electromagnetic field to provide a thrust reaction to the mass flow. In the case of an MPD, a hollow, cylindrical anode holds within it a rod-shaped cathode. An electrical arc is generated between the two, causing the cathode to heat and release electrons. Within the void between the two surfaces, a working gas is heated and ionized by the electrons, becoming a plasma. As current passes through the cathode, a magnetic field is generated, curling around the rod concentrically. The ionized plasma experiences an acceleration in the presence of this magnetic field (the Lorentz force phenomenon of magnetic fields on charged masses) and is ejected from the cavity at high velocity. This mass flow produces a thrust reaction on the thruster and in turn accelerates the attached vehicle.[2]

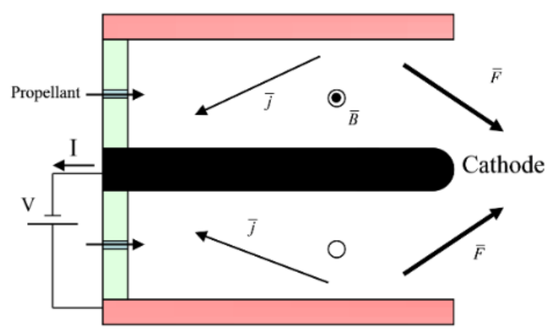

Figure 1 – Basic design of an MPD (Source: R. Branam, Dept. of Aerospace Engineering & Mechanics, The University of Alabama)

---

[1] M.S. Candidate, Department of Aerospace Engineering and Mechanics, The University of Alabama.

[2] Source of background information: NASA Glenn Research Center, https://www.nasa.gov/centers/glenn/about/fs22grc.html

II. Requirements & Design Parameters

The following requirement is given for the engine:

Output power : *30,000 W*

and the design parameters are as follows:

Mass flow :	*92 mg/s*
R_a :	*10 cm*
R_c :	*2 cm*

Operating current limits can vary from 2600 – 3800 A.

III. Performance Analysis

Starting with a simplified form of the electromagnetic force (having taken into account the constant strength of the magnetic field along its own direction, and employing Gauss's theorem to reduce a volume integral for the total magnetic flux to a surface integral that sums the magnetic flux through discrete area elements), the thrust in the axial direction is found to be

$$F_{em,x} = -\frac{1}{\mu_0} \oiint \frac{B^2}{2} dA_x \qquad \text{(Equation 29 in the notes)}$$

The outer cylindrical surface of the cathode contributes no net surface area with a normal in the axial direction of the mass flow, and neither does the inside of the concentric anodic shell, so these surfaces are neglected in this calculation. The anode rim and the cathode tip provide a small amount of surface area normal to the axial direction, but their contributions to the force formulation are neglected in this case in favor of focusing on the largest surface normal to the thrust direction, the backplate of the thruster. Knowing from Ampere's law that the magnetic field strength contributed by any given point of the backplate is related to the current passing across it,

$$B(P) = \mu_0 \frac{I'}{2\pi r} \tag{24}$$

the resultant electromagnetic force on the plasma can be written as

$$F_{em} = +\frac{1}{\mu_0} \int_{R_c}^{R_a} \frac{1}{2} \left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{R_a}{R_c}\right) \tag{30}$$

where R_a and R_c are the radii of the anodic and cathodic cylinders, and the current *I'* is set to *I* across the backplate.

Given this force on the plasma, the exit velocity of the ionized stream may be calculated using a design mass flow

$$u_e = \frac{F_{em}}{\dot{m}} = \left(\frac{\mu_0}{4\pi} \ln\left(\frac{R_a}{R_c}\right)\right) \frac{I^2}{\dot{m}} \tag{32}$$

which can then be used to calculate the specific impulse of the thruster

$$I_{sp} = \frac{u_e}{g_0}$$

The efficiency of the thruster is the ratio of the output power to the input power

$$\eta = \frac{\frac{1}{2}\dot{m}u_e^2}{P_{in}}$$

IV. Results of Analysis for Given Current Range

Calculating the thrust, specific impulse, and efficiency of an MPD thruster operating across the range of amperages given (in 50 A increments), the following results are tabulated and plotted. Input power is assumed to remain constant, as is mass flow. All increase with increased current.

Table 1 - Thrust, specific impulse, and efficiency of the thruster across current range

I (A)	Thrust (N)	Exit Velocity (m/s)	Isp (s)	Efficiency
2600	1.09	11826	1205	0.214
2650	1.13	12285	1252	0.231
2700	1.17	12753	1300	0.249
2750	1.22	13229	1349	0.268
2800	1.26	13715	1398	0.288
2850	1.31	14209	1448	0.310
2900	1.35	14712	1500	0.332
2950	1.40	15224	1552	0.355
3000	1.45	15744	1605	0.380
3050	1.50	16273	1659	0.406
3100	1.55	16811	1714	0.433
3150	1.60	17358	1769	0.462
3200	1.65	17913	1826	0.492
3250	1.70	18477	1884	0.524
3300	1.75	19050	1942	0.556
3350	1.81	19632	2001	0.591
3400	1.86	20222	2061	0.627
3450	1.92	20821	2122	0.665
3500	1.97	21429	2184	0.704
3550	2.03	22046	2247	0.745
3600	2.09	22671	2311	0.788
3650	2.14	23306	2376	0.833
3700	2.20	23948	2441	0.879
3750	2.26	24600	2508	0.928
3800	2.32	25260	2575	0.978

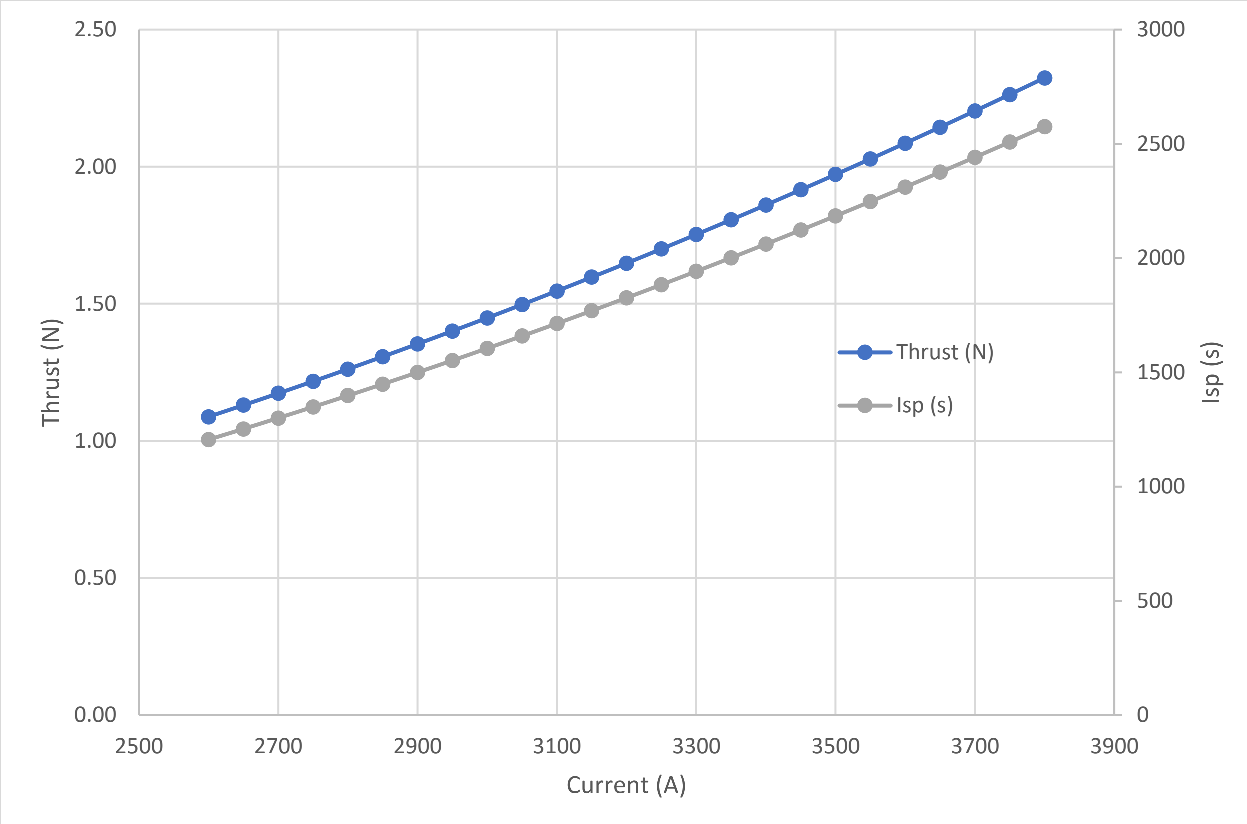

Figure 2 - Thrust and specific impulse for various operating currents

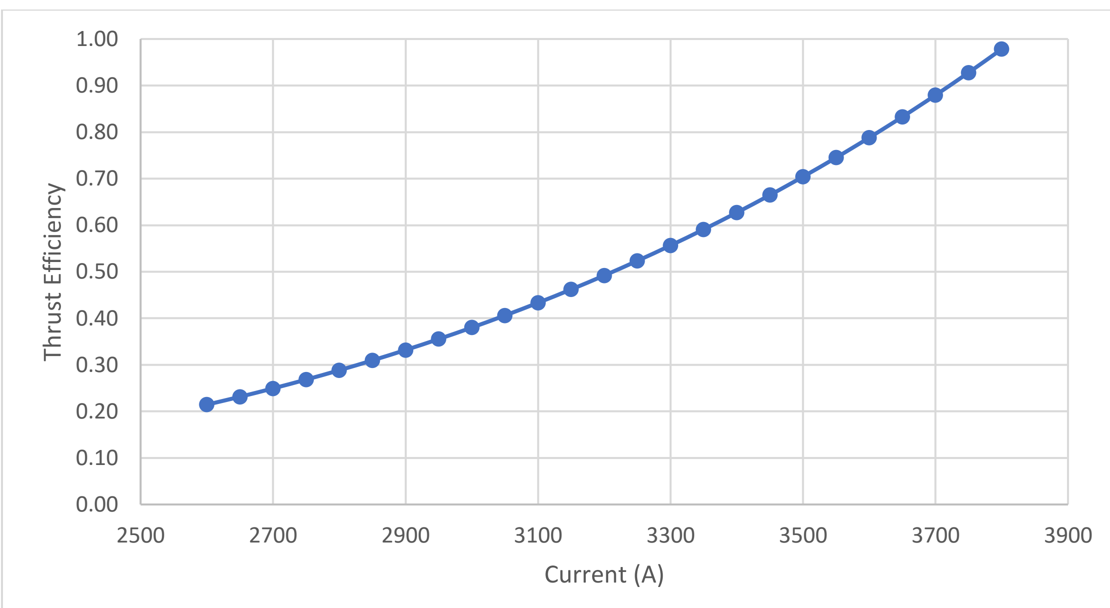

Figure 3 - Thruster efficiency for various operating currents