Consumer Demand with Social Influences: Evidence from an E-Commerce Platform

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Abstract

For some kinds of goods, rarity itself is valued. "Fashionable" goods are demanded in part because they are unique. In this paper, we explore the economics of rare goods using auctions of limited-edition shoes held by an e-commerce platform. We model endogenous entry and bidding in multi-unit auctions and construct demand curves from realized bids. We find that doubling inventory reduces willingness to pay by 7-15%. From the monopolist perspective, ignoring the value of rarity leads to substantial overproduction: auctioned quantities are 82% above the profit-maximizing level. From the consumer perspective however, the negative spillovers of restricting quantity more than offset the benefits of rarer goods.

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1 Introduction

Economists have long-recognized the fact that for some goods, demand does not depend solely on the price and the functional properties of the good, but rather is subject to "social influence" (Pigou, 1913; Leibenstein, 1950; Becker, 1991; Krueger, 2013). Examples include fashionable clothing, jewelry, artwork, collectibles, concert and event tickets, cars, and restaurants. For this kind of goods, the conjectured primary social influence is for the (unmet) demand of others to increase individual valuation—consumers gain utility from obtaining a good that few others also consume or possess because it signals high social status. Sociologists of fashion have characterized this as a taste for distinction (Simmel (1957)), and economists describe it as conspicuous consumption (Veblen (1899)). We call goods with these characteristics "fashion" goods. Despite the many theoretical contributions studying social influences in demand, there is little empirical evidence measuring their importance in practice and the related implications for firm strategy and consumer welfare.

In this paper, we consider a firm selling substitutable fashion goods whose consumers derive utility from consuming those goods and from their rarity. For each good taken in isolation, unlike a firm offering a conventional good, the firm faces a demand curve that depends on the total quantity produced. Increasing quantity causes movement along the demand curve, but also shifts the demand curve downward as consumers' valuations for the good decrease. Although rarity directly increases consumer valuation for a fashion good, it also makes it harder to purchase the good. The risk of rationing shifts consumers towards alternative fashion substitutes that are relatively more widely available, thus decreasing demand for the rarer good. We explore how these two opposed forces jointly affect the optimal inventory choices of a profit-maximizing firm.

We exploit a particular sale of customized outdoor slippers, or "slides", shown in Figure A2 (Appendix). The slides were manufactured by Straye, but they were customized by Ben Baller, a celebrity jeweler particularly popular in the hip hop community. The slides had the expression "Ben Baller did the chain" printed on the upper layer. This expression is a lyric from the rapper A$AP Ferg, who rapped "Ferg is the name, Ben Baller did the chain" on the track "Plain Jane."[1] In the song, the expression carries a meaning of exclusivity.

The product was offered in two colors (red and black) that consumers perceive as substitutes and was available in nine different adult sizes, creating nine distinct markets. To sell the slides, Mr. Baller partnered with StockX, an e-commerce platform for branded shoes, handbags, and watches. The slides were sold directly to customers in an auction that StockX called "IPO," using the same acronym as for Initial Public Offering. The IPO was effectively a sealed-bid uniform price auction with a $50 reserve price. Total quantities for each color and size combination were announced *ex ante*, and the auctions for each color-size product were run independently. Customers could submit at most one bid for each color-size combination. Slides were allocated to the highest bidders. The lowest winning bid determined the clearing price for all winning bidders.

The red slide was considerably rarer than the black slide, despite being functionally equivalent. According to internal discussions, the rareness of the red slide was not due to any perceived or anticipated difference in demand or difference in production cost for red

[1] `https://genius.com/12495999`, accessed June 2022.

versus black, which both cost \$30 to manufacture. In fact, Mr. Baller reportedly did not know beforehand which color would prove more popular. The product had never been sold before, and the IPO was also one of the first that StockX planned, making it unlikely that inventory levels by shoe size and color were optimized.

We have access to the full collection of bids and (anonymous) bidder identifiers, which we use to generate stylized facts. One strong piece of evidence for rarity entering directly into consumer valuation is that bids for the rarer red slides were 3.9% higher on average. Although rarity and redness are confounded, we (a) do have variation in relative inventory by size, and (b) we observe that many bidders bid on both the red and black slides in the same size. Once the effect of color is controlled for, we find that doubling the total quantity offered reduces bids—and hence shifts the demand curve downward—by 7.8%.

We use the descriptive facts to motivate a structural model of entry and bidding in multi-unit auctions of substitutable products. The IPO rules and the large number of entrants make bidding truthfully an approximately optimal strategy, conditional on entering an auction. To rationalize heterogeneity in entry, we allow for two types of consumers: global bidders, who, given their shoe size, bid on any color for which their valuation exceeds the reserve price; and local bidders, who bid on the color that maximizes their expected utility. The setup allows us to back out the distributions of bidder types and willingness to pay as a function of color and available inventory.

Using the estimated model, we investigate the role of rarity in determining the auctioneer's optimal choice of inventory levels. Increasing auctioned quantities of red slides not only reduces valuations ("rarity") but also affects bidders' entry choices ("substitution"). Rarity and substitution lead to countervailing effects on firm profits from an increase in inventory. On the one hand, making the red slides more widely available increases the winning probability when entering the red slide auction, inducing some bidders to substitute away from the black slide auction. On the other hand, making red slides more widely available reduces valuations upon winning, which depresses bids in the red slide auction. Overall, we find that ignoring the role of rarity in demand leads to substantial over-production, even when substitution effects are accounted for: chosen quantities without considering rarity are 82% above the profit-maximizing levels.

Our paper provides empirical evidence that firms should consider social influences and substitution among fashion goods when choosing inventory levels. Given how many goods are subject to social influences, understanding this feature of firm decision-making is consequential. As our exercise showcases, the increasing use of auctions to sell limited edition goods can provide valuable customer insights to inform these decisions.

Our work is related to the literature on conspicuous consumption (Bernheim (1994), Bagwell and Bernheim (1996)), in which consumer utility is a function of both consumption of a product and status signaled by that product. When status is taken into account, demand curves can exhibit positive slopes (Corneo and Jeanne (1997)) and even perfect competition can give rise to positive mark-ups (Pesendorfer (1995)). In our setup, status comes from the rarity of an item, which is measured by the total quantity supplied. The marketing literature has long recognized the social needs of uniqueness and exclusivity that limited edition products can fulfill (Amaldoss and Jain (2005a), Amaldoss and Jain (2005b), Amaldoss and Jain (2008), Amaldoss and Jain (2010), Balachander and Stock (2009)), especially for products such as cars and clothing that are consumed publicly (Chao and Schor (1998)).

The goods sold on StockX tend to be fashion goods, with an active secondary market where resale prices often exceed retail prices. Manufacturers of these goods may under-price their products in the primary market due to fairness constraints (Kahneman et al. (1986)). Rather than price, other ordeals, such as waiting in line, help allocate the goods to consumers (Nichols and Zeckhauser, 1982; Alatas et al., 2012). However, under-pricing leads to demand rationing and often induces inefficient rent-seeking behavior by brokers and scalpers (Leslie and Sorensen (2014)). Bhave and Budish (2018) study how introducing auctions to sell otherwise under-priced items helps reduce the arbitrage profits enjoyed by brokers.

Anecdotal evidence, such as the stock-outs of the Xbox and the PlayStation and the lines at Apple stores preceding iPhone launches,[2] suggests that scarcity may be an intentional strategy to induce willingness to pay and increase sales (DeGraba (1995), Debo and van Ryzin (2011)). Balachander et al. (2009) provide rare empirical evidence that in the US automobile industry, low introductory inventory levels are associated with higher consumer preferences and this is likely due to the signaling value of supplier-induced scarcity. Like Balachander et al. (2009) we directly include inventory levels into the utility function. Unlike Balachander et al. (2009), we leverage a more direct measure of consumers' willingness to pay (i.e., submitted bids) and study how a multi-product firm should set inventory levels when the fashion goods it sells are substitutable.

The paper is organized as follows. Section 2 presents our empirical context and provides simple stylized facts that motivate our structural model. We present our model in Section 3 and its estimation in Section 4. Section 5 describes the counterfactual analysis and Section 6 concludes.

2 Empirical Context and Stylized Facts

The limited-edition product we consider was sold on StockX.com. StockX is an online marketplace founded in 2015 for the resale of branded sneakers, streetwear, handbags, and watches.[3] At least for streetwear and sneakers, the marketplace only sells new goods, and StockX verifies that the goods bought and sold are unused, authentic and defect-free.[4] The marketplace guarantees product quality by having sellers ship tentatively sold items to a StockX authentication center, where each item is physically inspected. If the good fails inspection, it is sent back to the seller and the transaction is canceled; if the good passes, it is shipped to the buyer.

Direct authentication takes time and effort, but the fact that StockX is popular despite these delays reflects the limitations of the non-intermediated secondary market in these brand goods. The quality of counterfeit goods is often extremely high, and only the best trained individuals would be able to spot a fake. Additionally, many of these goods are extremely rare and high value so the extra cost of authentication is small compared to its benefits.

[2] `https://www.inc.com/business-insider/massive-lines-iphone-x-apple-store-people-queue-days-tim-cook.html` and `https://money.cnn.com/2013/12/24/technology/xbox-playstation-supply-shortages/`, accessed June 2022.

[3] Since the time of our data, the platform has expanded to include trading cards, electronics, collectibles, and Non-fungible tokens (NFTs).

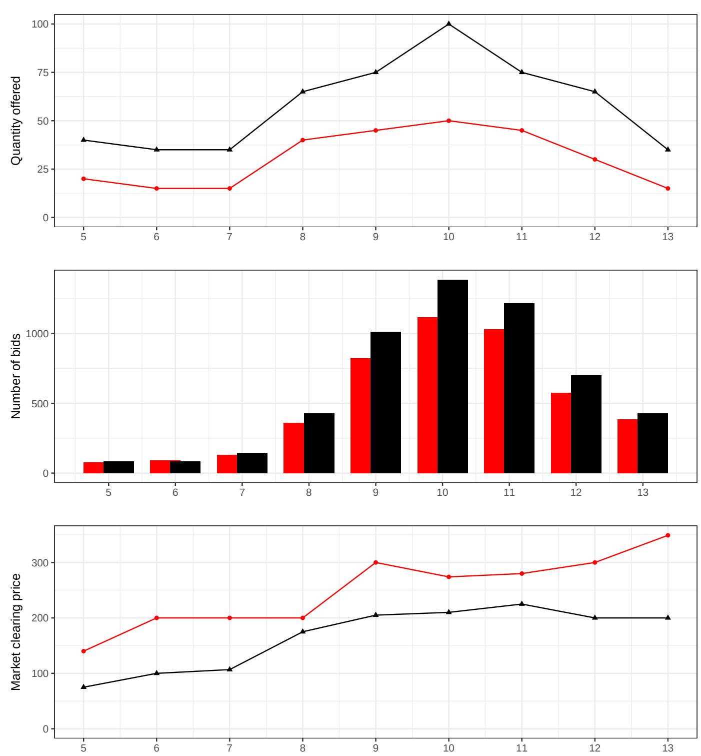
[4] Handbags and watches can be used but in great conditions.

Because the platform verifies that all goods sold are of the same high quality, StockX can sell them by aggregating products of the same type—e.g. pairs of Air Jordan 1 Nike shoes—into a single product page. Separately for each shoe size, StockX uses a continuous double auction to determine the transaction price, with buyers and sellers submitting time-limited bids and asks. Either side can observe the order book and buy or sell at the bid or ask price. StockX makes the order book and transaction history public, so buyers and sellers have access to market-level information before placing their bids.

Just like StockX mimics the stock market for the resale of shoes, the initial sale of Ben Baller slides mimicked an initial public offering. A limited number of red and black slides were available for sale in a sealed-bid uniform price auction with a $50 reserve price. The auctions were run independently for each size and color combination, and each bidder was allowed to place at most one bid per auction, even if they could bid across multiple auctions. At the end of the auction, all the available pairs of shoes were allocated to the highest bidders, and the market clearing price was defined by the lowest winning bid.[5]

We have internal company data on the bids that were placed during the Ben Baller auctions, with anonymous identifiers for each individual user. We also have the inventory available by shoe color and size, which allows us to derive market-clearing prices, and to determine the winners and the losers of each auction.

Figure 1: Ben Baller Slide IPO: Inventory, Bids, and Prices (in $)



Notes: The figures show the quantity of shoes auctioned (top panel), the number of bids received (middle panel), and the market clearing price (bottom panel) by color-size combination.

[5]The full rules are available here: `https://stockx.com/news/Ben-baller-ipo-official-rules/`.

The chosen production quantities by color and size are shown in the top panel of Figure 1. The relative rarity of red slides is readily apparent, though we also see substantial variation in inventory across shoe sizes. In part, different inventories likely reflect the distribution of shoe sizes in the population.

A total of $10,075$ bids were submitted by $6,936$ distinct bidders. 37.5% of bidders placed bids in multiple auctions, accounting for 56.9% of bids. Bidders placing multiple bids overwhelmingly bid twice, once on a red slide, once on black slide of the same size—85.9% of multi-auction bidders did that.

The total number of bids submitted by color-size combination are shown in the middle panel of Figure 1. The market clearing prices are shown in the bottom panel of Figure 1. Despite the fact that black slides received on average more bids than red slides of the same size, market clearing prices are higher for red slides. In the next section, we argue that these differences reflect the fact that red slides were rarer and thus expected to go for a higher clearing price, making entry less attractive for red slides compared to black slides.

Close examination of the data reveals the presence of a small number of outliers: in many auctions, the top bids are much higher than the remaining ones and in the range of one million dollars. We believe these bids are unlikely to reflect bidders' true willingness to pay. Since the presence of outliers can bias or distort estimates of interest, we handle outliers via trimming. In each auction, we trim the top 5% of bids before conducting our analysis. We provide robustness checks to this choice in Table A1 (Appendix).

To evaluate the reduced-form relationship between rarity and bidding behavior, we run regressions of the form

$$\log b_i = \alpha \textsc{Red}_i + \beta \log Q_i + \gamma_{s(i)} + \epsilon_i, \tag{1}$$

where i denotes a bid. RED_i is a dummy variable for the red slides. We also control for shoe size by including fixed effects for three size categories: small (5-7), medium (8-10), large (11-13).[6]

Results are presented in Table 1.[7] Specifications (1) to (4) show the results based on Ordinary Least Squares (OLS) regressions. Column (1) only contains a constant and the dummy variable for red slides. On average, red slides receive bids that are 3.9% higher than black slides. Column (2) replaces the red dummy with the log of the inventory available for the shoe size-color combination corresponding to the bid. The estimated elasticity of bids to inventory levels is 4.8: doubling inventory levels reduces bids by 4.8% on average. This elasticity remains similar when we control for color (column (3)) and fixed effects for the small, medium, and large shoe sizes (column (4)).

The OLS estimates do not account for truncation of bids at the reserve price. As a consequence, they should be interpreted as the effect on the mean bid *conditional* on the

[6]Separate fixed effects for each shoe size tend to absorb a lot of variation and make it harder to identify the effect of quantity.

[7]There is a mass of bidders at the reserve price. While it seems improbable that a large number of bidders have a valuation right at the reserve price, those bidders likely anticipated that StockX might still reward losing bidders. In the past, StockX has rewarded participants with discounts like free shipping on future orders. As such, we view the mass of bids right at the reserve price as likely reflecting factors other than a *bona fide* attempt to win the slides. We thus remove bids right at the reserve price when running the regressions.

Table 1: Effect of Rarity on Bids

	Dependent variable: Bid (log)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	4.577	4.785	4.744		4.282	4.672	4.593	
	(0.008)	(0.047)	(0.068)		(0.023)	(0.088)	(0.128)	
Red (indicator)	0.039		0.013	0.011	0.073		0.025	0.022
	(0.011)		(0.016)	(0.018)	(0.022)		(0.029)	(0.035)
Quantity (log)		-0.048	-0.039	-0.042		-0.090	-0.073	-0.078
		(0.012)	(0.016)	(0.022)		(0.022)	(0.030)	(0.042)
Shoe size FE	No	No	No	Yes	No	No	No	Yes
Model	OLS	OLS	OLS	OLS	MLE	MLE	MLE	MLE
Observations	6,467	6,467	6,467	6,467	6,467	6,467	6,467	6,467

Notes: Standard errors are in parenthesis. Shoe size FE are dummies for three size categories: small (5-7), medium (8-10), large (11-13). MLE gives the maximium likelihood estimator assuming log-normally distributed bids and accounts for truncation at the reserve price.

valuation being greater than the reserve price. We account for truncation using a Maximum Likelihood estimator (MLE), in which bids are assumed to be distributed according to a log-normal distribution truncated at the reserve price of \$50. Specifications (5) through (8) in Table 1 present the MLE results, which support an even bigger elasticity than the OLS estimates. Indeed, the coefficient on quantity in column (8) implies that doubling inventory is associated with a reduction in the bid amount of 7.8%.

What does the value of rarity imply for the seller's optimal inventory choices? In what follows, we investigate this question via counterfactual analysis of a structural model.

3 Model of Entry and Bidding Behavior

This section presents a model of entry and bidding behavior motivated by the descriptive evidence above. We consider each shoe size as its own separate market, where two products are available: red and black slides. Bidders draw valuations for the two products and make their entry decisions as a function of those valuations: in this sense, entry is "selective" as in Samuelson (1985). We allow for bidder heterogeneity, in that some bidders may bid on at most one color, whereas others can bid on both colors.

We consider the allocation of two products, B (black) and R (red), whose inventory levels are denoted Q_B and Q_R. The products are sold via two simultaneous multi-unit uniform price auctions with a public reserve price P_0 equal to \$50. There are N symmetric potential bidders, which is common knowledge. A share p of those bidders are *global bidders*, who are deep-pocketed and can bid on both products. The rest are *local bidders*, who are budget-constrained and can bid on at most a single product. There are no costs to bidding in an auction.

Each bidder i privately draws a pair of valuations (v_{iB}, v_{iR}). We assume that the pair (v_{iB}, v_{iR}) is drawn from a joint distribution $F(.,.)$, continuous in both arguments, with support $[\underline{v}_B, \overline{v}_B] \times [\underline{v}_R, \overline{v}_R]$, and with marginal densities strictly positive on the interior of the support. Valuations are independent and identically distributed across bidders. We allow for the joint distribution $F(.,.)$ to depend on the quantities (Q_B, Q_R) to capture the effect of rarity on valuations. The reserve price is binding, that is, $P_0 > \underline{v}_R$ and $P_0 > \underline{v}_B$.

Bidders make their entry and bidding decisions simultaneously based on their type– global or local–and valuations. Outcomes are determined by the rules of the auction mechanism: the available inventory is sold to the highest bidders, and the clearing price is determined by the lowest winning bid.

We seek to characterize a Bayesian Nash Equilibrium of the entry-bidding game. In the bidding stage, the fact that bidders cannot place more than one bid greatly simplifies the analysis because unit-demand limits bidders' ability to influence clearing prices via demand reduction or bid shading strategies (Vickrey (1961)). However, there is a chance for a bidder to affect the price they pay, which happens when their bid is exactly the Q_R^{th}-highest bid in the auction R or the Q_B^{th}-highest bid in auction B. Since the probability that this occurs and bid-shading incentives both decrease in N, for simplicity we assume that all participants have a dominant strategy of bidding their valuation if they enter the auction.[8]

When choosing whether to enter, global bidders enter any auction for which their valuation is above the reserve price: There are four possible cases: if $v_B \geq P_0$ and $v_R \geq P_0$, the global bidder enters both auctions; if $v_B \geq P_0$ and $v_R < P_0$, they enter auction B only; if $v_B < P_0$ and $v_R \geq P_0$, they enter auction R; and if $v_B < P_0$ and $v_R < P_0$ they do not enter any auctions.

Local bidders enter at most a single auction, as long as their valuation is above the reserve price. If $v_B \geq P_0$ and $v_R < P_0$, then the local bidder enters auction B only. Similarly, if $v_R \geq P_0$ and $v_B < P_0$, then the local bidder enters auction R only. In the case where $v_R \geq P_0$ and $v_B \geq P_0$, a local bidder enters the auction with the highest expected payoff. In the remainder of this section, we focus on characterizing the entry equilibrium for local bidders when $v_R \geq P_0$ and $v_B \geq P_0$.

Let us consider, temporarily, cut-off entry strategies of the following type: a local bidder enters auction R if and only if $v_R \geq c(v_B)$ for some endogenous function $c(.)$. They enter auction B otherwise. We show below that any entry equilibrium of this game has a payoff-equivalent representation in cut-off entry strategies. Let $\pi_R(v_R|c)$ and $\pi_B(v_B|c)$ denote bidder i's expected payoffs if they enter auction R and B respectively, when all local bidders follow the entry cut-off strategy $c(.)$. Local bidder i enters auction R if and only if $\pi_R(v_R|c) \geq \pi_B(v_B|c)$, so the optimal cut-off rule $c^*(.)$ must satisfy the following indifference condition:

$$\pi_R(c^*(v_B)|c^*) = \pi_B(v_B|c^*) \quad \text{for all } v_B \in [\underline{v}_B, \overline{v}_B] \tag{2}$$

The next proposition proves the existence of an equilibrium in cut-off strategies.

[8] In a multi-unit auction where k items are sold to n bidders at the lowest winning bid, the equilibrium bidding strategy takes the form

$$\beta(x) = x - \int_0^x \left[\frac{F(u)}{F(x)}\right]^{n-k} du$$

In our application, the second (bid-shading) term will be close to zero as the number of bidders per auction ranges from 77 to 1,386.

Proposition 1 *There exists a symmetric equilibrium of the entry stage in cut-off strategies $c^*(.)$, such that local bidder i, with valuations $v_{iR} \geq P_0$ and $v_{iB} \geq P_0$, enters auction R iff $v_{iR} \geq c^*(v_{iB})$ and enters auction B otherwise. The function $c^*(.)$ is increasing on $[P_0, \overline{v}_B]$, takes values in $[P_0, \overline{v}_R]$, and satisfies the boundary condition $c^*(P_0) = P_0$. Additionally, any pure strategy equilibrium of the entry stage has a payoff-equivalent representation in cut-off strategies.*

Proof: Available in Appendix A.

Note that while at least one such equilibrium exists, there may be multiple equilibria in cut-off strategies. We verify the uniqueness of the equilibrium numerically within the estimation routine.

4 Estimation

For each shoe size, we observe the bid, or pair of bids, submitted by each entrant, and the quantities sold in red and black. The primitives to recover are the distribution of valuations $F(.,.)$ and the share of local bidders p. The main identification challenge is that bidder types are not directly observed. Due to the binding reserve price, one cannot distinguish between a global bidder i who bid in auction R only because $v_{iR} \geq P_0$ and $v_{iB} < P_0$, from a local bidder j who preferred to bid on R because $v_{jR} \geq c^*(v_{jB})$ and $v_{jB} \geq P_0$. Nonetheless, Appendix B shows that the model primitives are non-parametrically identified from the observed data.

Due to our limited sample size, we impose parametric restrictions on the distribution of valuations $F(.,.)$. Estimation proceeds in two steps. First, we note that global bidders can be identified in the data when they enter both auctions. Therefore, we estimate the joint distribution of valuations $F(.,.)$ using bids submitted by global bidders who bid in both auctions B and R. Pairs of valuation (v_{iB}, v_{iR}) are assumed to be drawn from a bi-variate log-normal distribution with means that depend on the color and the inventory:

$$\begin{bmatrix} \log(v_{iB}) \\ \log(v_{iR}) \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_B + \beta \log Q_B \\ \mu_B + \mu_R + \beta \log Q_R \end{bmatrix}, \begin{bmatrix} \sigma_B & \rho \\ \rho & \sigma_R \end{bmatrix} \right). \tag{3}$$

σ_R, σ_B, and ρ denote the standard deviations of $\log(v_{iB})$, $\log(v_{iR})$, and the correlation coefficient between these two variables. When constructing the likelihood, we account for the fact that bids are truncated at P_0.

Second, for each shoe size, we calibrate the total number of potential entrants N and the share of local bidders p to match moments given by the entry rates in the data. Denote by n_{BR}, n_B, and n_R the observed number of bidders who entered both auctions of a given size, auction B only, and auction R only, respectively. N is set such that the share of entrants $\frac{n_B+n_R+n_{BR}}{N}$ equals the probability that a given bidder enters:

$$\frac{n_B + n_R + n_{BR}}{N} = 1 - F(P_0, P_0) \tag{4}$$

p is set such that the share of bidders who submit two bids equals the probability that a bidder is global and draws both valuations above the reserve price:

$$\tfrac{n_{BR}}{N} = (1-p)\left[1 - F_B(P_0) - F_R(P_0) + F(P_0, P_0)\right] \tag{5}$$

Two main advantages of this estimation approach are that: (i) it does not require knowledge of local bidders' entry equilibrium strategy $c^*(.)$; and (ii) it does not require knowledge of the total number of potential entrants when estimating valuations.

Results. Table 2 shows Maximum Likelihood estimates of the parameters governing the distribution of valuations. The estimate for ρ indicates that valuations for B and R are highly correlated within bidder. The quantity auctioned in a given color negatively impacts valuations for that color (β is negative and statistically significant), confirming that bidders exhibit a preference for rarity. Finally, we do not find any significant ex ante differences in valuations across colors, given that μ_R is indistinguishable from zero.

Table 2: MLE Estimates

	Estimate	Std. Error	t-value
μ_B	4.254	0.036	118.511
μ_R	-0.026	0.032	-0.821
β	-0.150	0.007	-21.207
σ_R	1.127	0.042	26.665
σ_B	1.136	0.023	50.176
ρ	0.971	0.002	425.272
Observations		$3,542$	
- Log Likelihood		$1,340$	

Notes: MLE Estimates of the model primitives contained in Equation 3.

The share of local bidders p is similar across sizes and close to 58%. The distribution of the number of potential entrants mirrors the distribution of number bids received (middle panel in Figure 1), with medium sizes having the largest populations of potential entrants. For completeness, we include the estimates of the number of potential entrants N (Equation 4) and the share of local bidders p (Equation 5) in Figure A3 (Appendix).

Model Fit. Before using these estimates for counterfactuals, we examine how well the estimated model fits the data. To do so, we solve for the Bayesian Nash Equilibrium for each shoe size separately. In particular, we find local bidders' entry strategy $c^*(.)$, simulate auction outcomes, and compare these to the data. We solve the game by searching for a fixed point of the best-response mapping. Since we cannot search over the infinite-dimensional space of increasing functions satisfying $c(P_0) = P_0$, we restrict our search to the class of piece-wise linear functions of the form

$$c(v_B) = \begin{cases} k_0 v_B + k_1 & \text{if } v_B \leq \tilde{v} \\ v_B + k_2 & \text{if } v_B > \tilde{v} \end{cases}, \tag{6}$$

where for a given choice of $(\tilde{v}, k_2)$, (k_0, k_1) are set to satisfy the restrictions: $c(P_0) = k_0 P_0 + k_1 = P_0$ and $k_0\tilde{v} + k_1 = \tilde{v} + k_2$. These two constraints ensure that the function satisfies the constraint $c(P_0) = P_0$ and is continuous.

Our choice for the shape of $c(.)$ is motivated by the interim payoffs. We provide intuition for the linear segment $v_B + k_2$ in Equation 6, noting that the other segment $k_0 v_B + k_1$ is uniquely determined by the restrictions above. When valuations are large enough relative to the expected market clearing prices, the probability of winning is approximately one, and interim payoffs satisfy

$$\pi_R(v_R|c) \underset{v_R \to \infty}{\longrightarrow} v_R - E[p_R] \text{ and } \pi_B(v_B|c) \underset{v_B \to \infty}{\longrightarrow} v_B - E[p_B], \tag{7}$$

where p_B and p_R are the prices bidder i expects to pay in case of winning. At the limit, the optimal cutoff strategy that makes bidder i indifferent between entering auction B and R must satisfy

$$c(v_B) - E[p_R] = v_B - E[p_B].$$

Hence, $c(.)$ will be of the form $v_B + k_2$ where $k_2 = E[p_R] - E[p_B]$ when v_B is large relative to the expected market clearing prices. How large does v_B need to be, i.e., what is the value of $\tilde{v}$ in Equation 6? With N large, the distributions of market clearing prices are tightly centered around the mean of the Q_B^{th}-order and Q_R^{th}-order statistics out of N draws. In practice, we start with $\tilde{v} = E[p_B]$, and iterate on both $(\tilde{v}, k_2)$ in the best-response mapping. That is, for a given candidate cutoff rule $c^{(k)} = (\tilde{v}^{(k)}, k_2^{(k)})$ in iteration k, we numerically compute the expected market-clearing price in auction B given entry strategy $c^{(k)}$, and set this value as the updated $\tilde{v}^{(k+1)}$. Next, we update $k_2^{k+1} = E[p_R|p_R < v_B + k_2, c^{(k)}] - E[p_B|p_B < v_B, c^{(k)}]$. Iterations proceed until convergence.[9] Finally, we initialize the iteration procedure at different starting points and verify that the algorithm converges to the same equilibrium $c^* = (\tilde{v}^*, k_2^*)$.

Given the equilibrium entry strategies found above, we simulate 10,000 auctions of red and black slides for each shoe size. We compare the average simulated numbers of entrants into the R auction, B auction, and both auctions to the observed entry levels. Similarly, we compare the average simulated prices to the observed market clearing prices.

The first three panels of Figure 2 compare the realized number of entrants with the results of model simulations (mean predictions are shown), by size and entry type, i.e., entry into B only, R only, or both. The results confirm that the model performs well. Importantly, the model replicates the feature that there are fewer entrants into auction R relative to auction B due to the fact that $Q_R < Q_B$ for all shoe sizes (see Figure 1).

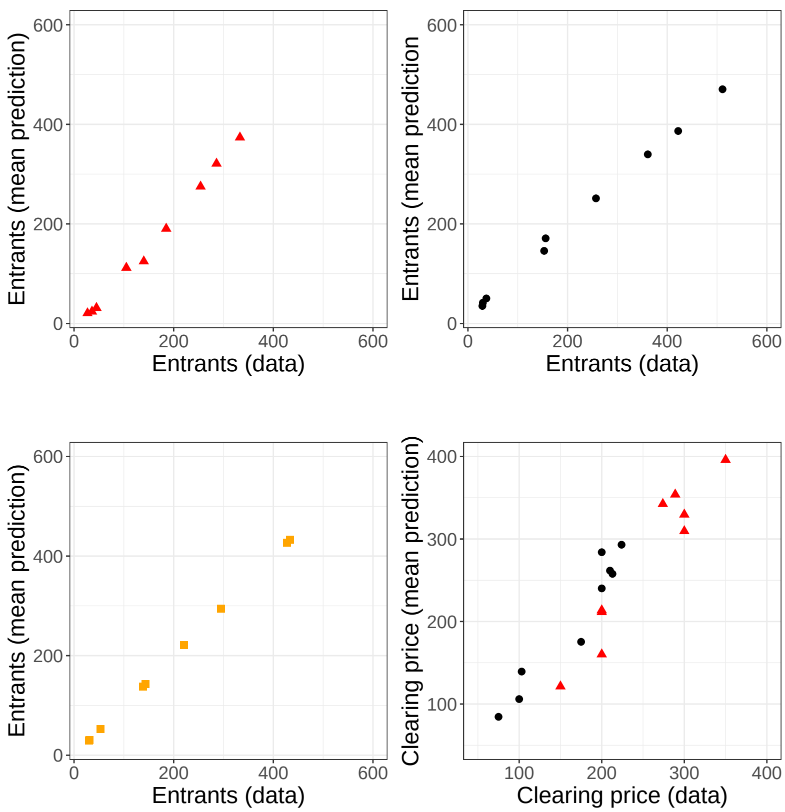
The bottom-right panel of Figure 2 compares the 18 realized market clearing prices by color and size to the mean simulated prices of our model. Although the model slightly over-predicts prices at the top, the model prediction of market clearing prices are overall quite close to the data given the restrictions imposed by our parametric assumptions. Overall, the model captures the heterogeneity in prices and entry rates across sizes and colors quite well.

5 Counterfactual Analysis

In this section, we investigate how preferences for rarity and substitution patterns of local bidders affect the optimal quantity sold by the auctioneer. To implement our counterfactuals,

[9] Convergence is reached when the cut-off rule satisfies $|\pi_R(v_B + k_2^{(k+1)}|c) - \pi_B(v_B|c)| < \$0.01$, $\forall v_B \geq \tilde{v}^{(k+1)}$.

Figure 2: Comparison of Model Simulations to Data: Entry and Clearing Prices



Notes: The figures display the simulated and observed number of bidders who enter auction R only (top left), auction B only (top right), and both auctions (bottom left). The bottom right panel displays the simulated and observed market clearing prices.

we use the estimated model and vary the quantity of red slides, which are scarcer than black slides for all shoe sizes. The auction outcomes of interest include bidders' entry rates, market clearing prices, seller's profits, and consumer surplus.

Varying the quantity Q_R will affect not only valuations for R but also bidders' entry choices. Indeed, local bidders make their entry decisions by comparing the expected payoffs from entering auctions B and R. The payoffs depend on the inventory levels (Q_B, Q_R), which influence the expected market clearing price and likelihood of winning in each auction. Increasing Q_R, all else equal, raises the expected payoff from entering auction R and may induce bidders to substitute away from auction B to R. This shift towards auction R will increase market clearing prices for R at the expense of auction B. However, increasing the R inventory also has a direct negative effect on willingness to pay, which will decrease the market clearing price for R.

In order to disentangle the effect of Q_R on valuations ("rarity") from its effect on entry ("substitution"), we consider alternative quantities $\widetilde{Q}_R = \gamma Q_R$, where Q_R is the baseline inventory and γ is a multiplier between 0.5 and 10. For each γ and shoe size, we compute outcomes under four counterfactual scenarios:

1. No rarity, No substitution. In this counterfactual, we assume that valuations and local bidders' entry decisions (the cut-off function $c^*(.)$) do not vary with quantity and are kept fixed at their baseline levels.

2. Rarity, No substitution. This counterfactual allows valuations to depend on quantities but entry decisions remain fixed at their baseline level.

3. No Rarity, Substitution. Valuations do not vary with quantity but local bidders make optimal entry decisions given auctioned quantities.

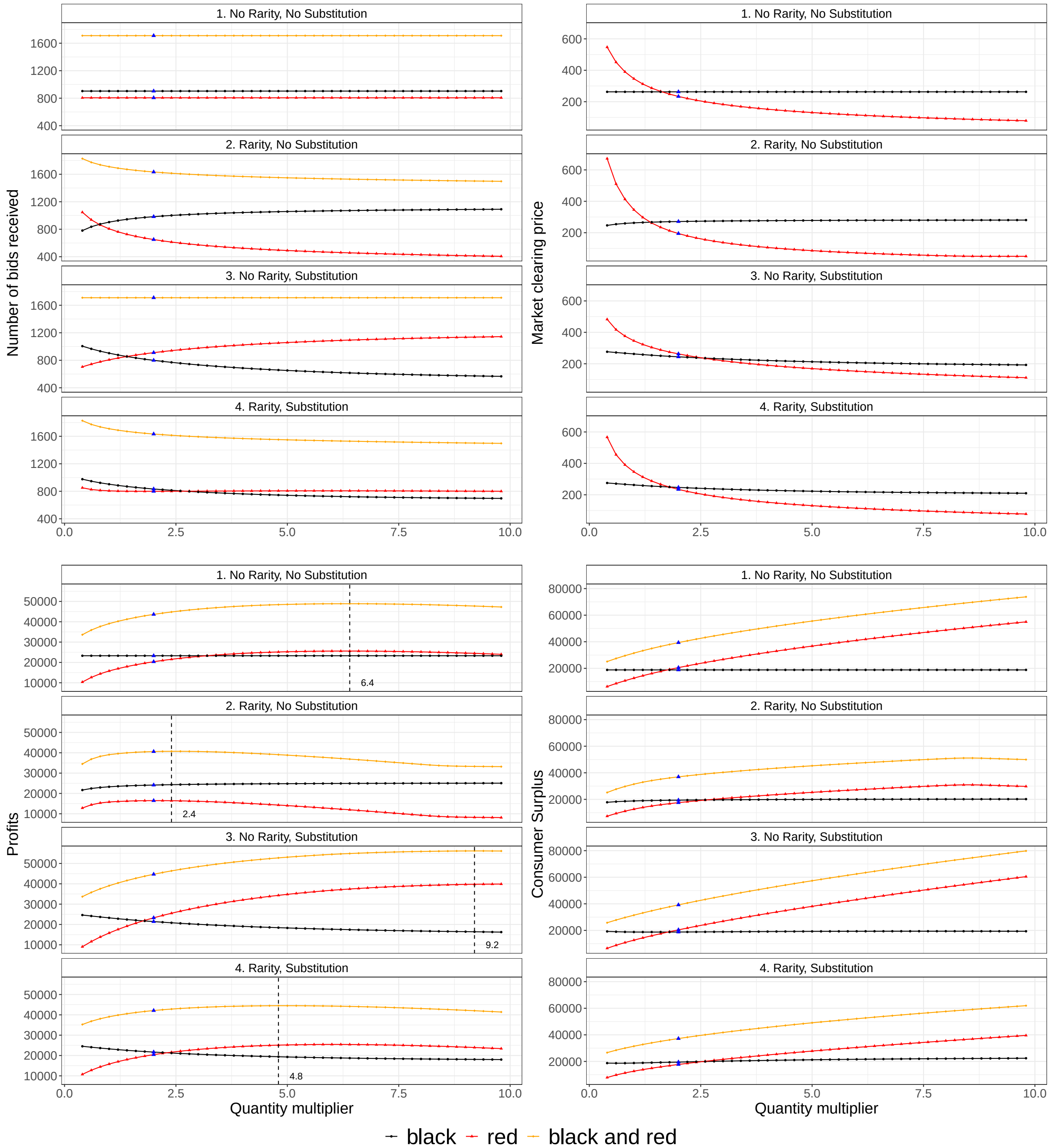
4. Rarity, Substitution. In this counterfactual, valuations depend on auctioned quantities and bidders make optimal entry decisions.

Scenarios 3 and 4 require us to recompute the Bayesian Nash Equilibrium entry strategies for each choice of inventory $(Q_B, \widetilde{Q}_R)$.[10]

We present results for size 10 shoes, which received the highest number of bids. Each panel in Figure 3 presents the counterfactual outcomes—number of bids, market clearing prices, profits, and consumer surplus—as a function of the quantity multiplier γ. Each row corresponds to one of the four scenarios. In scenario 1, the number of bids in each auction (top-left panel in Figure 3) does not change because valuations are constant and entry decisions are kept fixed. The market clearing price (top-right panel) for red shoes decreases with inventory, as the auctioneer goes down the demand curve. However, revenues monotonically increase with γ because the price reduction is more than offset by more units sold. To compute profits (bottom-left), we use a constant marginal cost of \$30 that StockX confirmed was the production cost. The profit-maximizing inventory level is denoted with a dotted vertical line in the figure. Holding constant the quantity of black slides, profits across both black

[10]In the extreme case where the counterfactual quantity Q_R is higher than the number of entrants, we assume that the auctioneer sells to all entrants at the reserve price $P_0 = \$50$.

Figure 3: Outcomes Under Four Counterfactual Scenarios



Notes: The figures plot 4 outcomes (number of bids, market clearing prices, profits, and consumer surplus) under the 4 different counterfactual scenarios described in Section 5. The blue triangles mark the value of γ where quantities auctioned in red and black are equal.

and red slides are maximized with an inventory of red slides equal to 6.4 times the baseline inventory.

In scenario 2, willingness to pay decreases with γ, which translates into a smaller number of bidders entering auction R, because they either prefer entering auction B instead or stay out altogether due to the binding reserve price. Note that, although bidders are not allowed to re-optimize their entry decisions in scenario 2, some bidders still substitute from R to B based on the baseline entry equilibrium whenever $v_{iR} < c^*(v_{iB})$. The lower number of entrants into auction R leads to a sharper drop in the market clearing price for R (relative to scenario 1), while the clearing price for B increases slightly with γ. Revenues increase less than in scenario 1, which translates into an profit-maximizing quantity reached when γ equals 2.4. As expected, preferences for rarity lead the seller to contract inventory levels to keep valuations, and thus prices, high.

Scenario 3 has the opposite effect on the number of bids compared to scenario 2. As γ increases, the expected payoff from entering auction R increases which draws bidders away from B towards R. This shift in entry rates dampens the reduction in the market clearing price in auction R and implies reduced revenues in auction B. The substitution from auction B to R has a much smaller (negative) effect on revenues in B than the (positive) effect on R, so a profit-maximizing monopolist would in fact expand inventory to 9.2 times the baseline, larger than both the optimal level in scenarios 1 and 2. This result is perhaps counter intuitive, because one would think that substitution leads to cannibalization. However, allowing bidders to switch from the black to the red shoe auction as the red inventory increases leads to a less drastic reduction in the red shoe market clearing price compared to scenario 1, without greatly affecting the black shoe price. This allows the seller to increase inventory more than in scenario 1.

Finally, in scenario 4, the two forces (rarity and substitution) somewhat compensate each other: the first reduces participation in auction R, whereas the second increases participation in auction R. Overall, the total number of entrants in B and R decreases because with lower valuations a fraction of bidders no longer meets the reserve price. Both prices monotonically decrease, but similarly to scenario 3 increasing inventory has opposite effects on the revenues for B and R. The resulting profit-maximizing γ is between the two extreme scenarios 2 and 3, at 4.8.

From the seller's perspective, our main finding from these four scenarios is that preference for rarity leads to substantially lower profit-maximizing quantities. This can be seen whether substitution is shut down (scenario 1 versus 2) or accounted for (scenario 3 versus 4). Ignoring the effect of rarity leads to quantity choices that are approximately 92% above the theoretical optimum. Although Figure 3 focuses on shoe size 10, our main prediction is robust across sizes: if preferences for rarity are ignored, the quantity chosen is on average 82% higher than the optimum.

We also note that when the auctioneer accounts for the substitution between black and red shoes (scenario 3 versus 1), the profit-maximizing quantity is higher. We think this finding is interesting in its own right, as it stands in contrast to economic intuition for a multi-product monopolist, which would suggest that optimal quantities should decrease when cannibalization is accounted for. For simultaneous auctions with endogenous entry, however, raising the quantity in one auction changes the residual demand curve (and, consequently, the marginal revenue curve) by affecting bidders' entry decisions. To keep exposition concise, we defer a detailed discussion of this point to Appendix C.

In all scenarios, consumer surplus in the red slide auction as well as in aggregate increase, reflecting the fact that for most of the range of values of $\widetilde{Q}$ considered (i.e., for $\gamma \leq 10$), the reduction in valuations due to the product being more available is more than compensated by consumer gains from lower market clearing prices (infra-marginal consumers) and larger quantities sold (marginal consumers). This result emphasizes that even with fashion goods, the negative spillovers of restricting quantity more than offset the individual benefits of rarer goods.

6 Conclusion

Most economists who have examined fashion typically lament how under-studied it is despite its clear importance (Robinson (1961)). In this paper, we examined how rarity impacts consumers willingness to pay. Using auction data from the sale of limited-edition fashion goods on StockX, an e-commerce marketplace, we find that limiting the number of shoes available for sale increases consumers bids. This increase translates into a higher sale price, not just because of the lower available supply, but also because rarity induces higher bidder valuations. Our model also explicitly considers the effect of substitution among similar products in bidders' entry behavior, which in turn affects inventory levels of multi-product firms. Preference for rarity and substitution have opposite effects on firms' inventory choices. While preference for rarity pushes profit-maximizing firms to constrain inventory levels to keep valuations high, substitution pushes them to increase inventory. When focusing on consumer surplus, although the directions of the effects are similar, we find that consumers' higher utility from rarer goods is not enough to counterbalance the negative effects of higher prices and fewer purchases arising from a reduction in inventory levels.

Many goods share the properties that make this "rarity" consideration consequential, from branded clothing to trading cards. In these cases, when setting production quantities, it is important for firms to take into account the impact of rarity and exclusivity on consumers' preferences. As our analysis showcases, the use of auctions to sell limited edition goods can provide valuable customer insights to inform these decisions. While we focus here on a multi-product monopolist, more work is needed to understand how rarity affects competition and market structure, which is a promising avenue for future research.

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A Proof of Proposition 1

Let $\mathcal{C}$ denote the set of increasing functions from $[P_0, \overline{v}_B]$ to $[P_0, \overline{v}_R]$, satisfying $c(P_0) = P_0$. We will show that the best-response mapping is a continuous function that maps the compact and convex set $\mathcal{C}$ into itself. Then, the Brouwer fixed-point theorem guarantees that an equilibrium exists.

Let $c_0 \in \mathcal{C}$ be a given cut-off entry strategy used by local bidders when $v_R \geq P_0$ and $v_B \geq P_0$. Denote by p_B and p_R the random variables corresponding to the payment bidder i pays if they enter and win in auctions B and R respectively: e.g., p_B is the Q_B-th highest bid among the $n_B - 1$ rival entrants in auction B. The number of entrants n_B (and their types) is itself random and depends on the primitives ($F(.,.)$, p, N, Q_B, and Q_R) as well as on the endogenous entry strategy c_0 which creates selection on values for local bidders. To make the latter dependence explicit we denote the payments $p_B(c_0)$ and $p_R(c_0)$. From the regularity assumptions imposed on $F(.,.)$ and the presence of global bidders, $p_B(c_0)$ and $p_R(c_0)$ have continuous cumulative distribution functions with supports $[P_0, \overline{v}_B]$ and $[P_0, \overline{v}_R]$.

Interim payoffs can be expressed as

$$\pi_B(v_B|c_0) = \int_0^{v_B} (v_B - p) dF_{p_B(c_0)}(p) = \int_0^{v_B} F_{p_B(c_0)}(p) dp, \tag{8}$$

$$\pi_R(v_R|c_0) = \int_0^{v_R} (v_R - p) dF_{p_R(c_0)}(p) = \int_0^{v_R} F_{p_R(c_0)}(p) dp. \tag{9}$$

where $F_{p_B(c_0)}$ and $F_{p_R(c_0)}$ are the cumulative distribution functions of $p_B(c_0)$ and $p_R(c_0)$. Bidder i's best-response to entry strategy $c_0(.)$, denoted $c_0^{BR}(.)$ must satisfy the indifference condition for all $v_B \in [P_0, \overline{v}_B]$

$$\pi_R(c_0^{BR}(v_B)|c_0) = \pi_B(v_B|c_0) \tag{10}$$

From Equation (9), $\pi_R(x|c_0)$ is continuous and strictly increasing in x and satisfies the boundary condition: $\pi_R(P_0|c_0) = 0 \leq \pi_B(v_B|c_0)$. Moreover, the extended function $\pi_R(x|c_0)$ converges to $\bar{v}_R - p_R$. For v_B such that $\pi_B(v_B|c_0) > \bar{v}_R - p_R$ (i.e., the highest possible valuation for R gives a payoff that is lower than the payoff from bidding in auction B), the bidder will always enter auction B. For v_B such that $\pi_B(v_B|c_0) \leq \bar{v}_R - p_R$, by the Intermediate Value Theorem, there exists a unique solution x^* to the equation $\pi_R(x|c_0) = \pi_B(v_B|c_0)$. If $x^* \in [P_0, \overline{v}_R]$, denote $c_0^{BR}(v_B) \equiv x^*$, otherwise, if $x^* > \overline{v}_R$, denote $c_0^{BR}(v_B) \equiv \overline{v}_R$. In the latter case, the bidder never enters auction R when drawing value v_B. Because $\pi_B(P_0|c_0) = \pi_R(P_0|c_0) = 0$, we have that $c_0^{BR}(P_0) = P_0$. Since $\pi_B(v_B|c_0)$ is increasing in v_B, the implicit function theorem implies that c_0^{BR} is increasing in v_B. Therefore $c_0^{BR} \in \mathcal{C}$.

To prove continuity of the best-response mapping at c_0, we show that for every $\tilde{\epsilon} > 0$, there exists $\delta > 0$, s.t. for every $c \in \mathcal{C}$, $||c - c_0|| < \delta$ implies that $||c^{BR} - c_0^{BR}|| < \tilde{\epsilon}$.

Define the function

$$H(x, c, v_B) = \pi_R(x|c) - \pi_B(v_B|c)$$

for $x \in [P_0, \overline{v}_R]$, $c \in \mathcal{C}$, and $v_B \in [P_0, \overline{v}_B]$.

From Equations (8) and (9), we have that $H_x(x, c, v_B) = F_{p_R(c)}(x)$. The latter function is continuous in x and strictly positive for $x > P_0$. Moreover, for any sequence $c_n \in \mathcal{C}$ that converges to c, $p_R(c_n)$ converges in distribution to $p_R(c)$: i.e., $F_{p_R(c)}(x)$ is continuous in c at all x.

We have that $c_0^{BR}(v_B)$ defined above satisfies, for all $v_B \in [P_0, \overline{v}_B]$,

$$H(c_0^{BR}(v_B), c_0, v_B) = 0.$$

By continuity of $H_x(x, c, v_B) = F_{p_R(c)}(x) > 0$ in (x, c), for any ϵ small enough, there exists δ such that for $||c - c_0|| < \delta$, $H(c_0^{BR}(v_B) + \epsilon, c, v_B) > 0$ and $H(c_0^{BR}(v_B) - \epsilon, c, v_B) < 0$. Importantly, $H_x(x, c, v_B)$ does not depend on v_B, therefore, δ holds for all v_B. The Intermediate Value Theorem guarantees that there exists a unique x^* with $|x^* - c_0^{BR}(v_B)| < \epsilon$, such that $H(x^*, c, v_B) = 0$. By definition of the best-response mapping, $x^* = c^{BR}(v_B)$. Hence, for any ϵ and v_B, we have shown that there exists δ such that $||c - c_0|| < \delta$ implies $|c^{BR}(v_B) - c_0^{BR}(v_B)| < \epsilon$. This shows continuity at c_0, for every v_B, and a fortiori, uniformly. The best-response mapping is a continuous function that maps $\mathcal{C}$ into itself. By Brouwer fixed-point theorem, an equilibrium $c^* \in \mathcal{C}$ exists.

Finally, for any symmetric pure strategy entry equilibrium governing local bidders' entry decisions when $v_B \geq P_0$ and $v_R \geq P_0$,

$$m : (v_B, v_R) \mapsto \{\text{enter R, enter B}\}, \tag{11}$$

let $S(v_B)$ denote the set of valuation for R such that local bidder i with valuation v_B enters R, that is,

$$S(v_B) = \{v_R | m(v_B, v_R) = (\text{enter R})\}. \tag{12}$$

Let $\underline{S(v_B)}$ denote the smallest element of this set (keeping v_B fixed). Suppose there exists some $\widetilde{v}_R > \underline{S(v_B)}$ such that a bidder drawing $(v_B, \widetilde{v}_R)$ would prefer to enter auction B. By monotonicity of interim payoffs we have that

$$\pi_R(\widetilde{v}_R | m) \geq \pi_R(\underline{S(v_B)} | m) \geq \pi_B(v_B | m) \tag{13}$$

If any inequality is strict, a bidder drawing $(v_B, \widetilde{v}_R)$ would strictly prefer entering auction R, which contradicts equilibrium. Hence, it must be that

$$\pi_R(\widetilde{v}_R | m) = \pi_R(\underline{S(v_B)} | m) = \pi_B(v_B | m)$$

A bidder drawing either $(v_B, \widetilde{v}_R)$ or $(v_B, \underline{S(v_B)})$ is indifferent between entering auction R and B. Therefore, there exists a payoff-equivalent representation of entry equilibrium m, where bidder i enters auction R under $(v_B, \widetilde{v}_R)$ and auction B under $(v_B, \underline{S(v_B)})$. Iterating this argument, we can conclude that any pure strategy equilibrium has a payoff-equivalent representation using cut-off strategies. ∎

B Nonparametric Identification

This section considers the identification of the model. The identification strategy assumes the econometrician has access to an infinitely large sample of auctions with exogenous

(N, Q_R, Q_B); i.e., the asymptotics are in the number of auctions rather than in the number of bidders per auction.[11]

For each auction in the sample, we assume that the econometrician observes the parameters (N, Q_B, Q_R) and bids submitted by each entrant. In particular, the numbers of bidders who enter auction B only, auction R only, or both are observed and denoted n_B, n_R, and n_{BR}. The primitives to identify are the joint distribution of valuation $(v_{iB}, v_{iR}) \sim F(.,.)$ and the probability p that a bidder is local.

First, the joint distribution of (v_{iB}, v_{iR}) conditional on $(v_{iB} \geq P_0, v_{iR} \geq P_0)$ is directly identified from pairs of bids submitted by global bidders.

Second, because bidders bid their valuation, expected payoffs $\pi_R(v_R|c^*)$ and $\pi_B(v_B|c^*)$ from entering auction B and R are directly identified from the infinitely large sample of auctions. Using the expected payoffs, one can identify the equilibrium entry strategy $c^*(.)$ used by local bidders with $(v_{iB} \geq P_0, v_{iR} \geq P_0)$ by imposing the indifference condition[12]

$$\pi_R(c^*(v_B)|c^*) = \pi_B(v_B|c^*).$$

To identify the distribution of valuations when bidders enter only one auction, we introduce the following short-hands for the events partitioning the valuation space, represented in Figure A1

$$\begin{aligned}
A &= \{v_{iB} < P_0, v_{iR} \geq P_0\} \\
B &= \{v_{iB} > P_0, v_{iR} \geq c(v_{iB})\} \\
C &= \{v_{iB} > P_0, P_0 < v_{iR} < c(v_{iB})\} \\
D &= \{v_{iB} \geq P_0, v_{iR} < P_0\} \\
E &= \{v_{iB} < P_0, v_{iR} < P_0\}
\end{aligned}$$

The (observed) probability distribution of bids submitted by bidders who entered auction B only, denoted $g_B(v)$, must satisfy

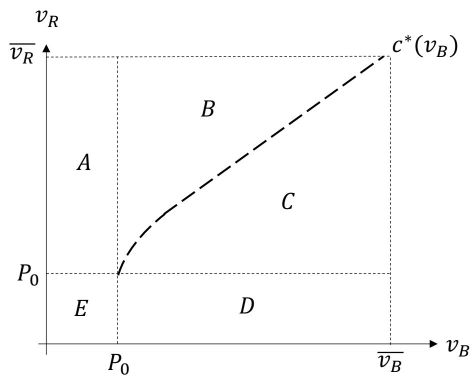
$$g_B(v) = \frac{Pr(D) f_B(v|D) + pPr(v_{iR} < c(v)|v \geq P_0, v_{iR} \geq P_0) f_B(v|P_0 \leq v_{iR} \leq c(v))}{Pr(D) + p \cdot Pr(C)} \tag{14}$$

We show that all elements in Equation (14) are known except $f_B(v|D)$ and p. $g_B(v)$ is identified from the data. $Pr(v_{iR} < c(v)|v \geq P_0, v_{iR} \geq P_0)$ and $f_B(v|P_0 \leq v_{iR} \leq c(v))$ are identified from the first step and knowledge of $c(.)$. The probability of events D and C are obtained by solving the system of four equations in four unknowns $(Pr(B), Pr(C), Pr(A), Pr(D))$,

[11] With an infinite number of bidders—keeping Q_R/N and Q_B/N fixed—the market clearing price will be a deterministic function of the primitives. As a consequence, a bidder can perfectly predict whether they will win the auction if they enter, this will lead to unravelling of the entry equilibrium.

[12] In case of equilibrium multiplicity, we assume the same equilibrium c^* is played in all auctions.

Figure A1: Partition of the Valuation Space



Notes: The figures plots the five areas, A through E, corresponding to different combinations of (v_{iB}, v_{iR}) that lead to different entry and bidding outcomes.

$$\begin{cases} E[n_{BR}]/N = (1-p)(Pr(B) + Pr(C)) \\ E[n_B]/N = Pr(D) + pPr(C) \\ E[n_R]/N = Pr(A) + pPr(B) \\ Pr(v_{iR} \leq c(v_{iB})|v_{iB} \geq P_0, v_{iR} \geq P_0) = Pr(C)/(Pr(C) + Pr(B)) \end{cases} \tag{15}$$

For the last equation, we leverage knowledge of $c(.)$ and the distribution of (v_{iB}, v_{iR}) conditional on $(v_{iB} \geq P_0, v_{iR} \geq P_0)$ identified in the first step.

This shows that, given p, the density function $f_B(v_B|D)$ is identified. By following a similar approach, we can show that the density function $f_R(v_R|A)$ is identified. Therefore, the distribution of valuations is identified for all relevant portions of the space: the joint distribution $F(.,.)$ is identified conditional on events B and C, the distribution of valuations for R is identified conditional on event A, and the distribution of valuations for B is identified conditional on event D.

Finally, the probability p that a bidder is local is identified from exogenous variation in the number of bidders N. The approach relies on the fact that an increase in the number of bidders N is more detrimental to bidders' payoffs if p is small. Intuitively, adding a global bidder increases competition in both auctions B and R whereas adding a local bidder increases competition in (at most) a single auction. Therefore, an increase in N will be more detrimental to bidders' payoff when the probability $(1-p)$ a bidder is global is larger. Since expected payoff are directly identified in the data, we can identify the parameter p from the change in expected payoffs as N increases.

C Counterfactual Analysis: The Effect of Substitution

When the effect of substitution between B and R is considered (scenario 3 versus 1), the profit-maximizing quantity is higher. This finding stands in contrast to economic intuition for a multi-product monopolist: in general, when accounting for cannibalization across products, a monopolist should reduce the quantity produced relative to a status-quo where cannibalization is ignored.

In our environment (simultaneous auctions with endogenous entry), however, the marginal revenue from increasing Q_R is higher under scenario 3 than 1 for two reasons: first, for most values of γ, entry into R is higher under scenario 3, which raises the marginal revenue of Q_R because higher entry counters the drop in market clearing prices; second, the marginal revenue from an additional bidder differs across auction B and R. In our case, it is higher for the larger auction (in most cases $Q_R > Q_B$), so substitution away from B to R raises marginal revenue and leads to a higher profit-maximizing quantity.

To gain intuition, we can compare the F.O.C. of a multi-product monopolist choosing quantities (Q_B, Q_R) to that of the auctioneer in our setting. Denote $R_B(Q_B, Q_R)$ and $R_R(Q_B, Q_R)$ the revenue obtained by the monopolist from selling goods B and R in the spot market, the F.O.C. for Q_R is

$$\frac{\partial R_R}{\partial Q_R} + \frac{\partial R_B}{\partial Q_R} = c$$

where the second term on the left-hand side represents cannibalization and reduces the marginal revenue from increasing Q_R.

In multi-unit auctions with endogenous entry (assuming here that the number of bidders is continuous), revenue depends on the quantity auctioned and the number of bidders: $R_j(Q_j, n_j)$ for $j \in \{B, R\}$. In scenario 3, the number of bidders n_j^* is endogenous and a function of auctioned quantities. The corresponding F.O.C. is

$$\frac{\partial R_R(Q_R, n_R^*)}{\partial Q_R} + \frac{\partial R_R}{\partial n_R}\frac{\partial n_R^*}{\partial Q_R} + \frac{\partial R_B}{\partial n_B}\frac{\partial n_B^*}{\partial Q_R} = c \tag{16}$$

Raising quantity Q_R changes the marginal revenue (first term), but also attracts additional competition into auction R (second term), at the cost of reducing competition in auction B (third term). Because the total number of bidders is fixed: $\frac{\partial n_R}{\partial Q_R} = -\frac{\partial n_B}{\partial Q_R}$. The optimality condition simplifies to

$$\frac{\partial R_R}{\partial Q_R} + \frac{\partial n_R}{\partial Q_R}\left[\frac{\partial R_R}{\partial n_R} - \frac{\partial R_B}{\partial n_B}\right] = c$$

If substitution is ignored, as in scenario 1, the corresponding optimality condition reads

$$\frac{\partial R_R(Q_R, n_R)}{\partial Q_R} = c \tag{17}$$

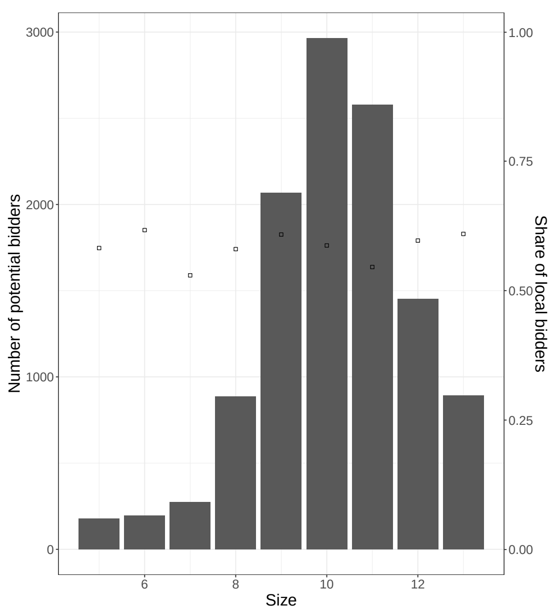
where n_R is fixed. The marginal revenue from Q_R can be either higher or lower in scenario 3 relative to 1 depending on the shape of the demand curve. In our specific setting, the marginal revenue is higher under scenario 3 because (i) $\frac{\partial R_R(Q_R, n_R^*)}{\partial Q_R} > \frac{\partial R_R(Q_R, n_R)}{\partial Q_R}$ if $n_R^* > n_R$ and (ii) $\frac{\partial R_j}{\partial n_j}$ is increasing in Q_j.

D Supplementary Tables and Figures

Figure A2: Ben Baller Slides in Red and Black

Notes: The figure displays the shoes sold in the IPO.

Figure A3: (N, p) by shoe size



Notes: This figure shows the number of potential entrants N (bars) and the share of local bidders p (squares) by size.

Table A1: Reduced Form Results: Alternative Treatments of Outliers

	Dependent variable: Bid (log)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Top 7 percent of bids trimmed								
Constant	4.530	4.779	4.687		4.284	4.727	4.561	
	(0.007)	(0.044)	(0.064)		(0.020)	(0.078)	(0.114)	
Red (indicator)	0.053		0.029	0.030	0.096		0.053	0.055
	(0.011)		(0.015)	(0.017)	(0.020)		(0.026)	(0.031)
Quantity (log)		-0.057	-0.037	-0.035		-0.101	-0.065	-0.061
		(0.011)	(0.015)	(0.021)		(0.020)	(0.027)	(0.037)
Observations	6,207	6,207	6,207	6,207	6,207	6,207	6,207	6,207
Panel B. Top 3 percent of bids trimmed								
Constant	4.597	4.848	4.770		4.247	4.738	4.583	
	(0.008)	(0.049)	(0.071)		(0.026)	(0.095)	(0.138)	
Red (indicator)	0.051		0.024	0.022	0.102		0.049	0.045
	(0.012)		(0.016)	(0.019)	(0.024)		(0.032)	(0.038)
Quantity (log)		-0.058	-0.041	-0.045		-0.112	-0.079	-0.086
		(0.012)	(0.017)	(0.023)		(0.024)	(0.032)	(0.045)
Observations	6,618	6,618	6,618	6,618	6,618	6,618	6,618	6,618
Panel C. Bids capped at $1,000								
Constant	4.664	4.940	4.846		3.856	4.593	4.337	
	(0.010)	(0.058)	(0.084)		(0.059)	(0.156)	(0.227)	
Red (indicator)	0.058		0.030	0.031	0.156		0.082	0.085
	(0.014)		(0.019)	(0.023)	(0.039)		(0.052)	(0.061)
Quantity (log)		-0.063	-0.043	-0.039		-0.168	-0.112	-0.106
		(0.015)	(0.020)	(0.028)		(0.039)	(0.052)	(0.074)
Observations	6,909	6,909	6,909	6,909	6,909	6,909	6,909	6,909
Shoe size FE	No	No	No	Yes	No	No	No	Yes
Model	OLS	OLS	OLS	OLS	MLE	MLE	MLE	MLE

Notes: Standard errors are in parenthesis. Shoe size FE are dummies for three size categories: small (5-7), medium (8-10), large (11-13). MLE gives the maximium likelihood estimator assuming log-normally distributed bids and accounts for truncation at the reserve price.