ΤΥΠΟΛΟΓΙΟ ΑΡΙΘΜΗΤΙΚΗΣ ΑΝΑΛΥΣΗΣ

- $$x_3 = \frac{x_1 + x_2}{2}$$

$$x_{n+2} = x_{n+1} - f(x_{n+1}) \frac{x_{n+1} - x_n}{f(x_{n+1}) - f(x_n)}$$

$$r = x_{n+2} - \frac{(x_{n+2} - x_{n+1})^2}{x_{n+2} - 2x_{n+1} + x_n}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'^2(x_n) - f''(x_n)f(x_n)}$$

$$x_{n+1} = x_n - \frac{fg_y - gf_y}{f_x g_y - g_x f_y} \qquad y_{n+1} = y_n - \frac{gf_x - fg_x}{f_x g_y - g_x f_y}$$

$$\epsilon_{n+1} = \epsilon_n / 2 \qquad \epsilon_{n+1} \approx \left(\frac{f''(\xi)}{2f'(\xi)} \right)^{1/1.618} \epsilon_n^{1.618}$$

$$\epsilon_{n+1} = g'(r)\epsilon_n \qquad \epsilon_{n+1} = -\frac{f''(\xi)}{2f'(\xi)} \epsilon_n^2$$

- $$x_i = \frac{1}{a_{ii}^{(i-1)}} \left[b_i^{(i-1)} - \sum_{k=i+1}^{n} a_{ik}^{(i-1)} x_k \right]$$

$$\ell_{i1} = a_{i1} \qquad \ell_{ij} = a_{ij} - \sum_{k=1}^{j-1} \ell_{ik} u_{kj}, \quad j \leq i, \quad i = 1, 2, \cdots, n$$

$$u_{1j} = \frac{a_{1j}}{a_{11}} \qquad u_{ji} = \frac{1}{\ell_{jj}} \left[a_{ji} - \sum_{k=1}^{j-1} \ell_{jk} u_{ki} \right], \quad j \leq i, \quad j = 1, 2, \cdots, n$$

$$X^{(k+1)} = D^{-1}B - D^{-1}CX^{(k)}$$

$$X^{(k+1)} = D^{-1} \left[B - LX^{(k+1)} - UX^{(k)} \right]$$

- $$p(x) = \sum_{i=0}^{n} L_i(x) y_i$$

$$L_i(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_{i-1})(x - x_{i+1}) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}$$

$$P_n(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!} \Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!} \Delta^3 f_0 + \cdots$$

$$P_n(x_s) = f_0 + \begin{pmatrix} s \\ 1 \end{pmatrix} \Delta f_0 + \begin{pmatrix} s \\ 2 \end{pmatrix} \Delta^2 f_0 + \cdots + \begin{pmatrix} s \\ n \end{pmatrix} \Delta^n f_0$$

$$P_n(x_s) = f_0 + \begin{pmatrix} s \\ 1 \end{pmatrix} \Delta f_{-1} + \begin{pmatrix} s+1 \\ 2 \end{pmatrix} \Delta^2 f_{-2} + \cdots + \begin{pmatrix} s+n-1 \\ n \end{pmatrix} \Delta^n f_{-n}$$

$$p(x) = \sum_{i=1}^{n} A_i(x) f(x_i) + \sum_{i=1}^{n} B_i(x) f'(x_i)$$

$$A_i(x) = \left[1 - 2L_i'(x_i)(x - x_i)\right] \left[L_i(x)\right]^2 \qquad B_i(x) = (x - x_i) \left[L_i(x)\right]^2$$

$$f(x) - p(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \pi_n(x)$$

$$f(x) - p(x) = \frac{f^{(2n+2)}(\xi)}{(2n+2)!} \pi_n(x)^2$$

$$\pi_n(x) = (x - x_0)(x - x_1)(x - x_2) \cdots (x - x_n)$$

$$h_{i-1} S_{i-1} + 2\left(h_{i-1} + h_i\right) S_i + h_i S_{i+1} = 6 \left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \right)$$

$$a_i = \frac{S_{i+1} - S_i}{6h_i} \qquad b_i = \frac{S_i}{2}$$

$$c_i = \frac{y_{i+1} - y_i}{h_i} - \frac{2h_i S_i + h_i S_{i+1}}{6} \qquad d_i = y_i$$

$$p(x) = \sum_{i=0}^{n} \frac{f^{(i)}(x)}{i!} (x - x_0)^i$$

$$f(x) - p(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x - x_0)^{n+1}$$

- $$y'(x) \approx \frac{y(x+h) - y(x)}{h} + O(h) \qquad y'(x) \approx \frac{y(x+h) - y(x-h)}{2h} + O(h^2)$$

$$y_0' \approx \frac{y_1 - y_0}{h} + O(h) \qquad y_0' \approx \frac{y_1 - y_{-1}}{2h} + O(h^2)$$

$$y_0'' \approx \frac{y_{-1} - 2y_0 + y_1}{h^2} + O(h^2) \qquad y_0' \approx \frac{-y_2 + 8y_1 - 8y_{-1} + y_{-2}}{12h} + O(h^4)$$

$$\int_{x_0}^{x_n} y(x) \mathrm{d}x \approx \frac{h}{2} \left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)$$

$$\int_{x_0}^{x_n} y(x) \mathrm{d}x \approx \frac{h}{3} \left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$\epsilon = -\frac{(b-a)h^2}{12} y^{(2)}(\xi) \qquad \epsilon = -\frac{(b-a)h^4}{180} y^{(4)}(\xi)$$

$$
\begin{aligned}
\int_{x_0}^{x_n} y(x)\mathrm{d}x \;\approx\; & \frac{h}{2}\left(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right)-\frac{h^2}{12}\left[f'(x_n)-f'(x_0)\right] \\
& +\frac{h^4}{720}\left[f^{(3)}(x_n)-f^{(3)}(x_0)\right]-\frac{h^6}{30240}\left[f^{(5)}(x_n)-f^{(5)}(x_0)\right]
\end{aligned}
$$

$$\int_{-1}^{1} y(x)\mathrm{d}x \approx \sum_{i=1}^{n} A_i y(x_i)$$

- 1.

$$y_{k+1}=y_k+hy'(x_k)+\frac{h^2}{2}y''(\xi)$$

$$y_{k+1}=y_k+\frac{h}{2}\left[y'(x_k)+f(x_{k+1},y_k+hy_k')\right]$$

2.

$$
\begin{aligned}
k_1&=hf(x_k,y_k),\\
k_2&=hf(x_k+h/2,y_k+k_1/2)\\
k_3&=hf(x_k+h/2,y_k+k_2/2)\\
k_4&=hf(x_k+h,y_k+k_3)\\
y_{k+1}&=y_k+\frac{1}{6}\left(k_1+2k_2+2k_3+k_4\right)
\end{aligned}
$$

3.

$$y_{n+1}=y_n+\frac{h}{12}\left[23f_n-16f_{n-1}+5f_{n-2}\right]+O\left(h^4\right)$$

4.

$$y_{k+1}=y_k+\frac{h}{24}\left(55f_k-59f_{k-1}+37f_{k-2}-9f_{k-3}\right)+\frac{251}{720}h^5y^{(5)}(\xi_1)$$

$$y_{k+1}=y_k+\frac{h}{24}\left(9f_{k+1}+19f_k-5f_{k-1}+f_{k-2}\right)-\frac{19}{720}h^5y^{(5)}(\xi) \tag{1}$$

5.

$$y_{k+1}=y_{k-3}+\frac{4h}{3}\left(2f_{k-2}-f_{k-1}+2f_k\right)$$

$$y_{k+1}=\frac{1}{8}\left(9y_k-y_{k-2}\right)+\frac{3h}{8}\left(-f_{k-1}+2f_k+f_{k+1}\right)$$

6.

$$\mathrm{y}_{\mathrm{k+1}}=y_{k-3}+\frac{4h}{3}\left(2f_{k-2}-f_{k-1}+2f_k\right)+\frac{28}{90}h^5y^{(5)}(\xi_1)$$

$$y_{k+1}=y_{k-1}+\frac{h}{3}\left(f_{k-1}+4f_k+f_{k+1}\right)-\frac{1}{90}h^5y^{(5)}(\xi_2)$$

- $$u_i^{n+1} = u_i^n + \kappa\frac{\Delta t}{\Delta x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

$$u_i^{n+1} = u_i^{n-1} + 2\kappa\frac{\Delta t}{\Delta x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

$$u_i^{n+1} = u_i^n + \kappa\frac{\Delta t}{\Delta x^2}(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1})$$

$$u_i^{n+1} = u_i^n + \kappa\frac{\Delta t}{2\Delta x^2}\left[u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} + u_{i+1}^n - 2u_i^n + u_{i-1}^n\right]$$

$$u_i^{n+1} = u_i^n + \kappa\frac{\Delta t}{\Delta x^2}\left[\theta(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}) + (1-\theta)(u_{i+1}^n - 2u_i^n + u_{i-1}^n)\right]$$

$$u_{i,j}^{n+1} = \frac{1}{4}\left[u_{i-1,j}^n + u_{i+1,j}^n + u_{i,j-1}^n + u_{i,j+1}^n\right]$$

$$u_i^{n+1} = \gamma u_{i-1}^n + 2(1-\gamma)u_i^n + \gamma u_{i+1}^n - u_i^{n-1} + \Delta t^2 f_i, \quad \gamma = \frac{\alpha^2\Delta t^2}{h^2}$$

$$u_i^{n+1} = \frac{1}{2}(u_{i+1}^n + u_{i-1}^n) - \frac{c}{2}(u_{i+1}^n - u_{i-1}^n), \quad c = \alpha\frac{\Delta x}{\Delta t}$$

$$u_i^{n+1} = u_i^n - \frac{c}{2}(u_{i+1}^n - u_{i-1}^n) + \frac{c^2}{2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n), \quad c = \alpha\frac{\Delta x}{\Delta t}$$