Άσκηση 1

a)

$$\underbrace{\begin{bmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 11 \end{bmatrix}$$

$$A = L \cdot U = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$4 = l_{11}$ | $-1 = l_{11} u_{12} + 0 \cdot 1 \Rightarrow u_{12} = -\frac{1}{4}$

$2 = l_{21}$ | $5 = l_{21} u_{12} + l_{22} \Rightarrow l_{22} = 5 - 2 \cdot \left(-\frac{1}{4}\right) = \frac{11}{2}$

$1 = l_{31}$ | $2 = l_{31} u_{12} + l_{32} \Rightarrow l_{32} = 2 - 1 \cdot \left(-\frac{1}{4}\right) = \frac{9}{4}$

$1 = l_{11} u_{13} + 0 \cdot u_{23} + 0 \cdot 1 \Rightarrow u_{13} = \frac{1}{4}$

$2 = l_{21} u_{13} + l_{22} u_{23} + 0 \cdot 1 \Rightarrow u_{23} = \frac{1}{\frac{11}{2}} \left(2 - 2 \frac{1}{4}\right) = \frac{2}{11} \cdot \frac{3}{2} = \frac{3}{11}$

$4 = l_{31} u_{13} + l_{32} u_{23} + l_{33} \cdot 1 \Rightarrow l_{33} = 4 - 1 \cdot \frac{1}{4} - \frac{9}{4} \cdot \frac{3}{11} = \frac{138}{44} = \frac{69}{22}$

Apa

$$L = \begin{bmatrix} 4 & 0 & 0 \\ 2 & \frac{11}{2} & 0 \\ 1 & \frac{9}{4} & \frac{69}{22} \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{3}{11} \\ 0 & 0 & 1 \end{bmatrix}$$

$$L \cdot b' = b \Rightarrow \begin{bmatrix} 4 & 0 & 0 \\ 2 & \frac{11}{2} & 0 \\ 1 & 9/4 & \frac{69}{22} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 11 \end{bmatrix}$$

$$4a = 8 \Rightarrow a = 2$$

$$2a + \frac{11}{2} b = 3 \Rightarrow b = \frac{2}{11}(3-4) = -\frac{2}{11}$$

$$a + \frac{9}{4} b + \frac{69}{22} c = 11 \Rightarrow c = \frac{22}{69}\left(11 - 2 - \frac{9}{4}\cdot\left(-\frac{2}{11}\right)\right)$$

$$c = 3$$

$$U \cdot x = b' \Rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{3}{11} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{2}{11} \\ 3 \end{bmatrix}$$

$z = 3$

$y + \frac{3}{11} z = -\frac{2}{11} \Rightarrow y = -\frac{2}{11} - \frac{3}{11} \cdot 3 = -1$

$x - \frac{1}{4} y + \frac{1}{4} z = 2 \Rightarrow x = 2 - \frac{1}{4} - \frac{3}{4} = 1$

Άρα

$x = 1$

$y = -1$

$z = 3$

b)

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & -3 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 3 & -3 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

$l_{11} = 3$

$l_{21} = 1$

$l_{31} = 1$

$u_{12} = \frac{a_{12}}{l_{11}} = -\frac{3}{3} = -1$

$u_{13} = \frac{a_{13}}{l_{11}} = \frac{1}{3}$

$l_{22} = a_{22} - l_{21} u_{12} = 1 - 1(-1) = 2$

$l_{32} = a_{32} - l_{31} u_{12} = -1 - 1 \cdot (-1) = 0$

$u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}} = \frac{0 - 1\frac{1}{3}}{2} = -\frac{1}{6}$

$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{32} = 3 - 1\frac{1}{3} - 0 = \frac{8}{3}$

Apa

$$L = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 8/3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1 & 1/3 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L \cdot b' = b \Rightarrow \cancel{\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 8/3 \end{bmatrix}} \Rightarrow$$

$$b'_1 = \frac{b_1}{l_{11}} = \frac{-1}{3}, \quad b'_2 = \frac{b_2 - l_{21} \cdot b'_1}{l_{22}} = \frac{3 - 1 \cdot \left(-\frac{1}{3}\right)}{2},$$

$$\Rightarrow b'_2 = \frac{10}{6} = \frac{5}{3}, \quad b'_3 = \frac{b_3 - l_{31} b'_1 - l_{32} b'_2}{l_{33}} = \frac{2 - 1 \cdot \left(-\frac{1}{3}\right) - 0}{\frac{8}{3}}$$

$$b'_3 = \frac{7}{8}$$

Apa $b' = \begin{bmatrix} -\frac{1}{3} \\ \frac{5}{3} \\ \frac{7}{8} \end{bmatrix}$

$$U \cdot x = b' \Rightarrow \begin{bmatrix} 1 & -1 & 1/3 \\ 0 & 1 & -1/6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/3 \\ 5/3 \\ 7/8 \end{bmatrix}$$

$$z = \frac{7}{8}$$

$$y = \frac{5}{3} + \frac{1}{6} \cdot z = \frac{5}{3} + \frac{7}{48} = \frac{87}{48}$$

$$x = -\frac{1}{3} + 1y - \frac{1}{3}z = -\frac{1}{3} + \frac{87}{48} - \frac{7}{24} = \frac{57}{48}$$

Άσκηση 2

α) $x = \frac{24-3y}{4}$

$y = \frac{30-3x+z}{4}$

$z = \frac{-24+y}{4}$

1η επαν

$x = \frac{24-3\cdot 1}{4} = \frac{21}{4} = 5{,}250$

$y = \frac{30-3\cdot 5{,}250+1}{4} = 3{,}812$

$z = \frac{-24+3{,}812}{4} = -5{,}047$

2η επαν

$x = \frac{24-3\cdot 3{,}812}{4} = 3{,}141$

$y = \frac{30-3\cdot 3{,}141+(-5{,}047)}{4} = 3{,}883$

$$z = \frac{-24 + 3,883}{4} = -5,029$$

3η επαν

$$x = \frac{24 - 3 \cdot 3,883}{4} = 3,088$$

$$y = \frac{30 - 3 \cdot 3,088 + (-5,029)}{4} = 3,927$$

$$z = \frac{-24 + 3,927}{4} = -5,018$$

B)

- Για 3 δεκαδικά χρειαστηκαν 13 επαναληψεις
- Για 9 δεκαδικα 43 επαναληψεις
- Για 12 δεκαδικα 56 επαναληψεις

Άσκηση 3

a) $P_0(x) = f_0 L_0 + f_1 L_1 + f_2 L_2$

$$L_0(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} = -\frac{x^2}{2} + \frac{3}{2}x - 1$$

$$L_1(x) = \frac{(x-0)(x-2)}{(1-0)(1-2)} = -x^2 + 2x$$

$$L_2(x) = \frac{(x-0)(x-1)}{(2-0)(2-1)} = \frac{x^2}{2} - \frac{x}{2}$$

$$P_0(x) = -6{,}3977x^2 + 8{,}1634x - 1$$

в)

x	1	2
y	$0,765189$	$-10,263811$
y'	$1,53158$	$-42,69479$

$$L_1(x) = \frac{x-2}{1-2} = -x+2 \qquad L_1'(x) = -1$$

$$L_2(x) = \frac{x-1}{2-1} = x-1 \qquad L_2'(x) =$$

$$A_1(x) = [1 - 2(x-1)(-1)](-x+2)^2$$

$$B_1(x) = (x-1)(-x+2)^2$$

$$A_2(x) = [1 - 2(x-2)(1)][x-1]^2$$

$$B_2(x) = (x-2)(x-1)^2$$

$$H(x) = A_1(x)y(1) + A_2(x)y(2) + B_1(x)y'(1) + B_2(x)y'(2)$$

$$H(x) = -19,104x^3 + 63,8549x^2 - 68,8661x + 24,8811$$

δ) Στο παρακάτω σχήμα βλέπουμε την πραγματική καμπύλη με κόκκινο χρώμα, με μπλε η προσέγγιση με πολυώνυμο Lagrange και με μαύρο η προσέγγιση με πολυώνυμο Hermite

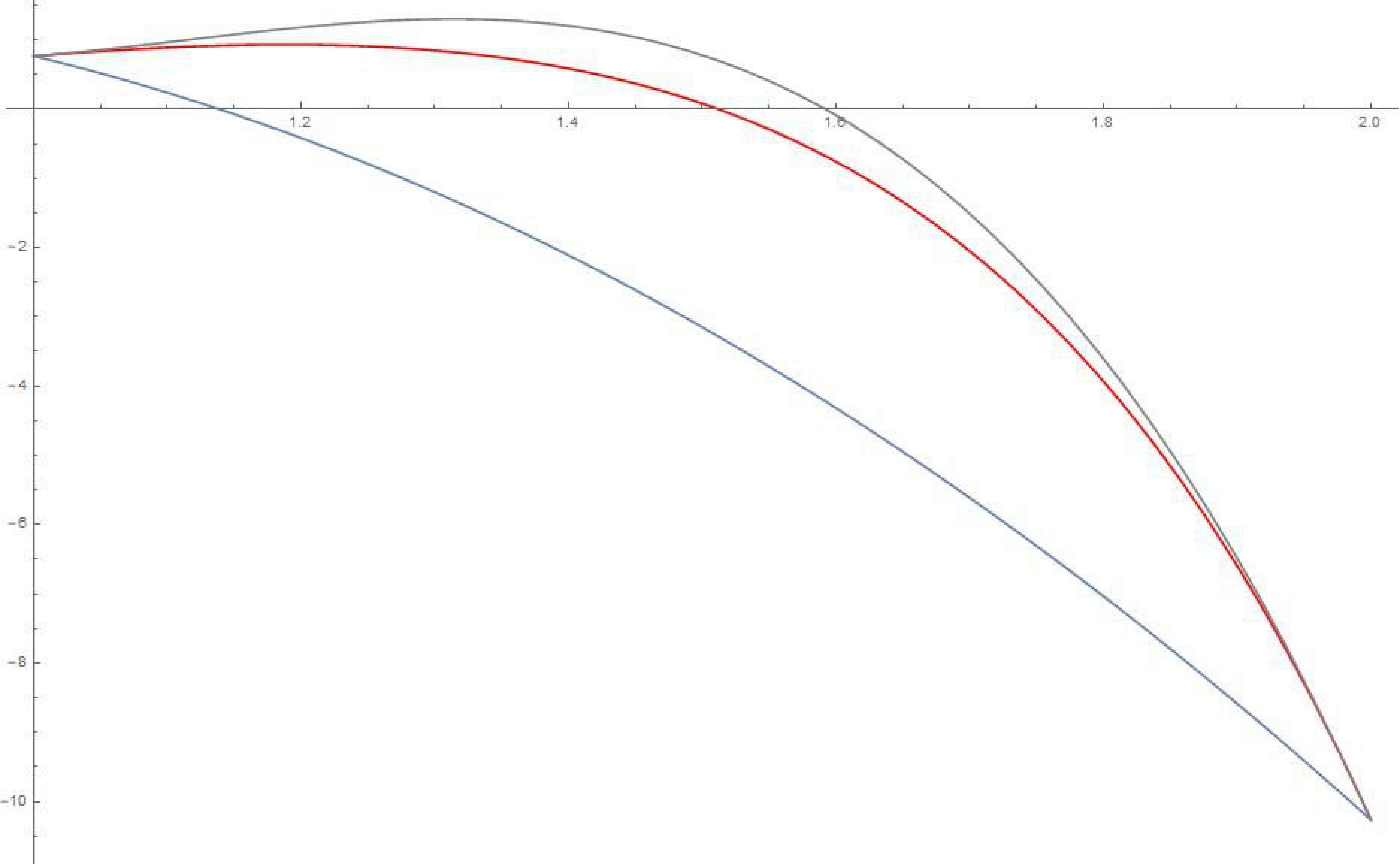
Στο $x = 1,2$ έχουμε

$y(1,2) = 0,9292$

$P(1,2) = -0,4165$

$H(1,2) = 1,1810$

Παρατηρούμε ότι η καμπύλη με την προσέγγιση του Hermite ανταποκρίνεται καλύτερα στην συμπεριφορά της πραγματικής από ότι η προσέγγιση Lagrange. Το ίδιο συμβαίνει και στο σημείο $x = 1,2$



1.2
1.4
1.6
1.8
2.0
-2
-4
-6
-8
-10