Valbal Trajectory Planning

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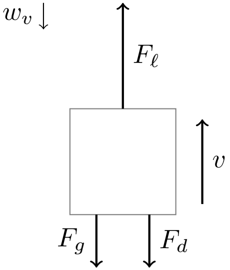
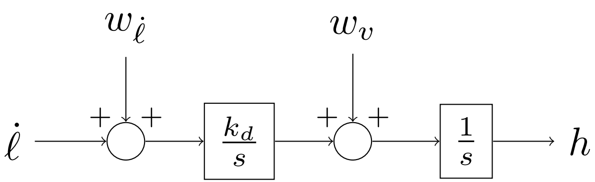
System Dynamics

- Assumptions
  - $F_d \propto v$ i.e. drag is linear.
  - $F_l - F_g = F_d$ i.e. the balloon is always at terminal velocity
- Equations of motion
  - let $\ell = F_\ell - F_g$ be the net lift on the balloon
  - $\dot{\ell}$ is commanded by controller
  - $\dot{v}(t) = k_d(\dot{\ell}(t) + w_{\dot{\ell}}(t))$
  - $\dot{h}(t) = v(t) + w_v(t)$
  - $\mathcal{L}\{h(t)/\dot{\ell}(t)\} = k_d/s^2$



F_d: Force of drag
F_g: Gravity
F_ℓ: Buoyant force
v: vertical velocity of balloon
w_v: vertical velocity of surrounding air

Plant Block Diagram

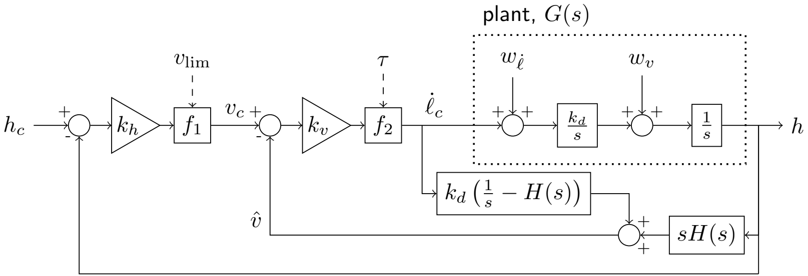


$\dot{\ell}$: commanded change in lift (valve and ballast actions)
$w_{\dot{\ell}}$: atmospheric lift disturbance
w_v: atmosphereic velocity disturbance
h: altitude

$$x = \begin{bmatrix} h \\ v \end{bmatrix} \qquad u = \dot{\ell}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ k_d \end{bmatrix} u + \begin{bmatrix} w_v \\ k_d w_{\dot{\ell}} \end{bmatrix}$$

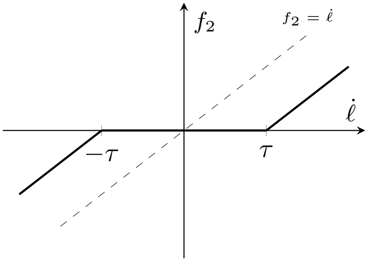
Controller Block Diagram



$H(s)$	low-pass filter
$f_1(v\,; v_{\lim})$	clamp on the velocity commanded by the altitude loop set by $v_{\lim}$
$f_2(\dot{\ell}\,; \tau)$	deadband on the controller effort set by τ
h_c	commanded altitude (set by Flight Controller)
v_c	commanded velocity (output of position loop)
$\dot{\ell}_c$	commanded change in lift per unit time (output of velocity loop)
$w_{\dot{\ell}}$	atmospheric disturbances that change balloon lift (heating/cooling)
w_v	atmospheric disturbances that change balloon velocity (turbulence)
h	balloon altitude
$\hat{v}$	estimate of velocity

Controller Deadband

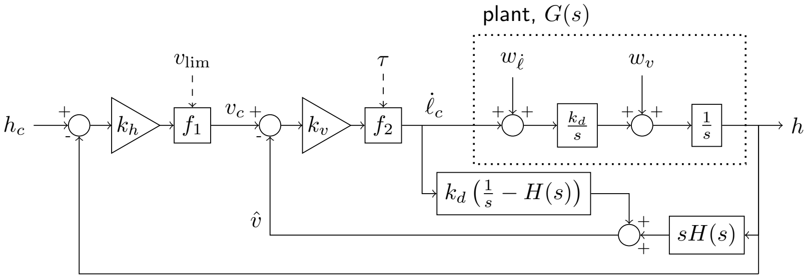
Since we typically command a target altitude and an allowable region, we add a deadband to the controller output. Let $\dot{l}_o$ be the output of the nonlinearity. Deadband:



$$f_2(\dot{\ell}\,;\tau) = \begin{cases} 0 & |\dot{\ell}| < \tau \\ \textbf{sign}(\dot{\ell})(|\dot{\ell}| - \tau) & |\dot{\ell}| > \tau \end{cases}$$

To set bounds on the altutude, we set $\tau = e_{\mathrm{tol}} k_v k_h$, where e_{tol} is the allowable distance from the altitude command.

Controller Block Diagram



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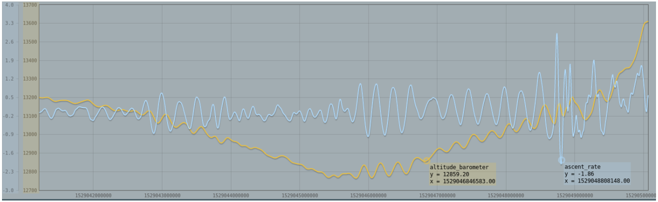
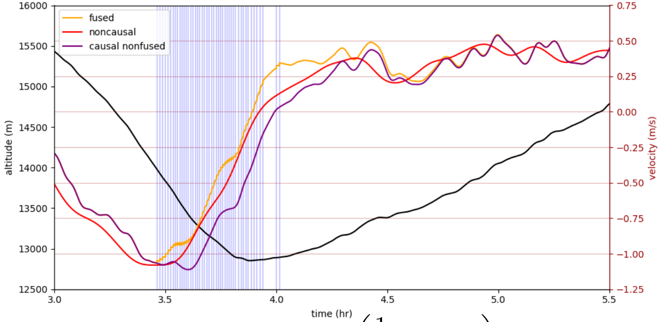
Atmosphere Waves



Velocity Estimator

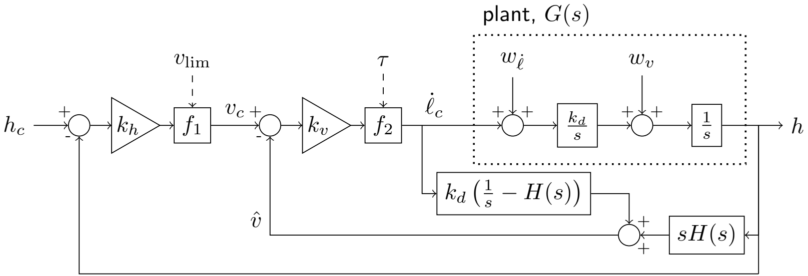
Low pass filtered velocity estimate uses a 2nd order filter to remove the effect of atmospheric waves



$$\mathcal{L}\{\hat{v}\} = sH(s)\mathcal{L}\{h\} + \left(\frac{1}{s} - H(s)\right)\mathcal{L}\{\dot{l}_c\}$$

$H(s)$ a 2nd order lowpass filter

Controller Block Diagram



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Picking gains

note: while the deadband makes the controller non-linear, it still peicewise linear, thus linear analysis can be used.
Transfer function for the linear system is

$$T(s) = \frac{k_h k_v k_d}{s^2 + k_v k_d s + k_h k_v k_d}.$$

So damping ratio is $\zeta = \frac{1}{2}\sqrt{\frac{k_d k_v}{k_h}}$.

- We choose gains such that $\zeta > 1$ and we have over damping.
- This gives ratio between k_v and k_h, but what about magnitiude?
- high gain $\rightarrow$ controller waits and acts agressively near e_{tol}
- low gain $\rightarrow$ controller acts cautiously before e_{tol}

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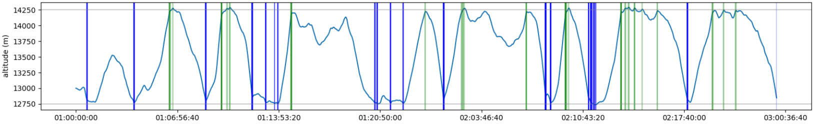
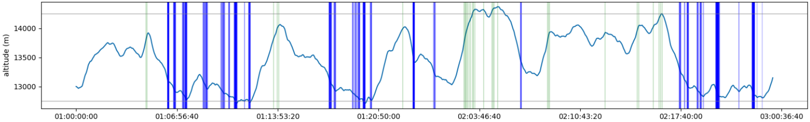
High vs Low Gain

Plots of simulation shown

high gain

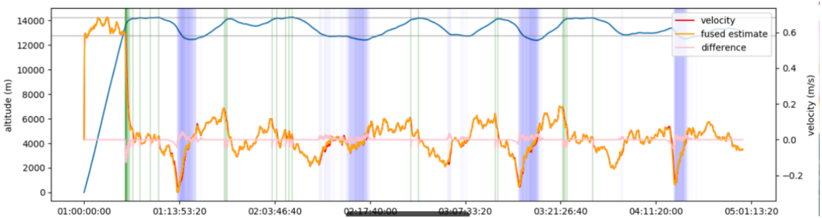


low gain

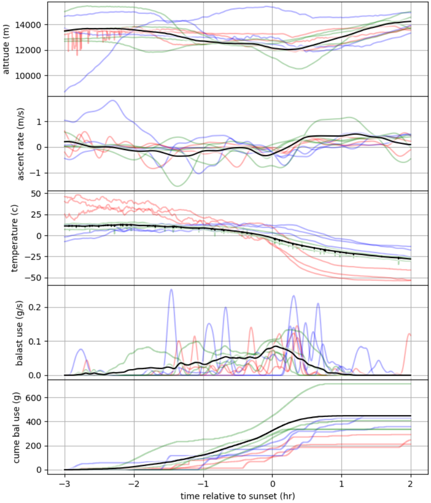


High gain performs better but can't tolerate uncertainty, low gain is worse but performs better under uncertainty

Simulations



Nightfall



- Left plot shows 10 sunsets across various flights (each flight different color).
- plot blow shows a fit to the data using convex regularization and contraints