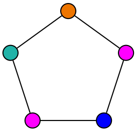
Kempe recoloring of planar graphs

Clément Legrand-Duchesne

LaBRI, Bordeaux

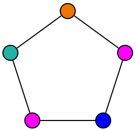
June 17, 2022

Joint work with Quentin Deschamps, Carl Feghali, František Kardoš and Théo Pierron



Kempe chain (1879)

Maximal bichromatic connected component in G



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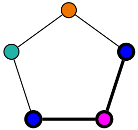

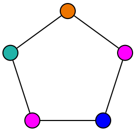

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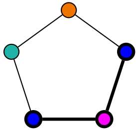

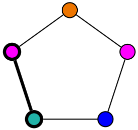

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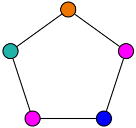

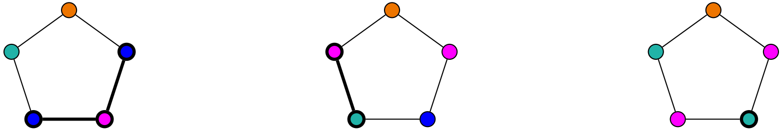

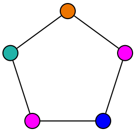

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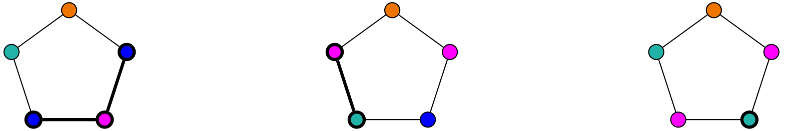


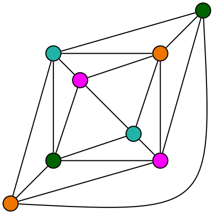
Reconfiguration graph

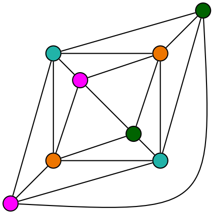
- $V(R^k(G)) = k$-colorings of G.
- α and β adjacent if $\alpha \underset{\text{Kempe}}{\longleftrightarrow} \beta$

Recoloring planar graphs

Meyniel 1978

- All 5-colorings of a planar graph are Kempe equivalent
- False for 4-colorings:





Las Vergnas and Meyniel 1981

All the k colorings of a d-degenerate graph are Kempe equivalent, for $k > d$

Bounded diameter

Conjecture of Bonamy, Bousquet, Feghali, Johnson 19

$R_k(G)$ has diameter $O(n^2)$, if G is d-degenerate and $k > d$

Bonamy, Delecroix, L. 22+

$R_k(G)$ has polynomial diameter for:

- $k \geq \Delta$ (unless, $k = 3$ and $G = K_2 \square K_3$),
- $k \geq \mathrm{mad}(G) + \varepsilon$
- $k \geq \mathrm{tw}(G)$

Journées GrR 2021

What about planar graphs with $k = 5$ or $k = 6$?

Deschamps, Feghali, Kardoš, L., Pierron 22+

$R_k(G)$ has polynomial diameter if G is planar and $k \geq 5$

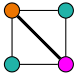
3-colorable planar graphs

Mohar 2007

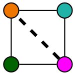
$R_4(G)$ is connected if G is a 3-colorable planar graph

Fisk 1977

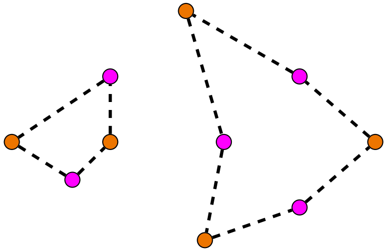
$R_4(G)$ has polynomial diameter if G is a triangulation of the plane



singular



non-singular



Sketch of proof

Theorem

For every 5-coloring α and every 4-coloring β, there is a sequence of Kempe changes going from α to β recoloring each vertex at most $f(n)$ times

1. Find an independent set I of linear size, of degree at most 6 in G, monochromatic in α and β,

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4. In β, I belongs to a color class B, recolor B with color 5. Now $G \setminus B$ is 3-colorable, recolor $\gamma_{|G \setminus B}$ to $\beta_{|G \setminus B}$

Collapsing I