Chapter 2

Basic Electrical Concepts

In the brief survey of the EEG machine in the last chapter, many terms relating to electrical phenomena were used. We encountered terms like electric current, voltage, AC, DC, and electric circuit, to name a few. Thus far, these terms have gone undefined. To understand how the EEG machine does its job, some knowledge of these and other electrical concepts is essential. This is not to say that the EEG technician or neurology resident need to become adept in the area of physics, electronics, and electrical engineering. On the other hand, without some understanding of these concepts the EEG machine becomes a strange and mysterious device rather than a practical tool for recording a patient's EEG.

But this is not the sole reason for the present chapter. The EEG is itself an electrical phenomenon; an electrical sign of cortical activity. Much of today's knowledge of the EEG and of brain function in general came about as the result of important advances in physics, electronics, and electrical engineering. Indeed, it has been said that significant advances in neurophysiology have gone hand in hand with significant developments in these related areas. This being the case, it is appropriate for those concerned with the technology and practice of clinical electroencephalography to know something about electrical phemonena per se.

Electrical Currents

When we use the term "electrical activity" in speaking of the EEG, what we really are talking about is an *electric current*. What exactly is an electric current? For most of us, an electric current is what makes our lamps light and our motors run. Is this current the same kind of thing that is generated by the brain?

The physicist says that an electric current is a flow of electrons—a flow of negatively-charged particles in a conducting medium. To understand what this means and how it comes about, we need to review briefly some of the basics of atomic structure.

Atomic Structure

We know that all matter is made up of *atoms*. These minute particles, which are arranged in an organized structure called a molecule, consist of various kinds of *elementary particles*. Only the two major kinds of elementary particles, the *protons* and *electrons*, need concern us here. They are largely responsible for the electrical properties of substances.

The atom has an interesting structure. At its center is a relatively small nucleus that contains practically all of the weight of the atom. The particles called protons reside within the nucleus, and they carry a *positive* electrical charge. Surrounding the nucleus are tiny particles bearing a *negative* electrical charge; these are the electrons. Because unlike electrical charges attract each other, the electrons are attracted by and bound to the nucleus. The total negative charge of the electrons in an atom is equal in magnitude to the positive charge of the nucleus. Since equal charges of opposite sign exactly cancel each other, an atom is normally in electrical balance.

The protons and electrons of which all matter is constituted are identical. The fact that different substances display different properties arises in large measure from the circumstance that the number and arrangement of these elementary particles differ from one element to another. It is primarily the different electrical properties of substances that concern us here.

Conductors and Insulators

While the electrons in the atoms of various substances are all alike, the *strength* with which the electrons are bound to the nucleus of an atom differs in different substances. In some substances, the electrons are very tightly bound; in others, they are only loosely bound and free to move about. This difference is responsible for one of the important electrical properties of a substance, namely, its *conductivity*. Substances or materials that have electrons that are loosely bound to the nucleus are electrical *conductors*, while substances whose electrons are all tightly bound and not free to move from their natural positions in an atom are *insulators*. From this it follows that the movement of the electrons in a conductor is what we refer to as an *electric current*.

Most metals are examples of good conductors. Copper and silver are particularly good conductors, and this is the reason why they are used in electrical wiring. Gold is also an excellent conductor; because gold does not readily corrode and is not toxic, it frequently is employed in EEG electrodes. Glass, mica, and porcelain, as well as most plastics, are examples of poor conductors or good insulators. Most persons have seen the white porcelain insulators to which the power lines are attached as they enter a house or building. The insulators prevent the current in the wires from leaking in unwanted directions.

Electric currents also can flow in liquid media such as solutions. In such cases, the particles carrying the electrical charge are *ions*. Ions and the important topic of conduction in liquid media are taken up when we discuss electrodes in a later chapter.

Potential Difference and Voltage

Although electrons in conducting substances are only loosely bound and hence free to move about, they normally do not do so. To get the electrons in a conductor moving, a force has to be applied to them. In other words, we need to apply a *potential difference* between the two ends of the conductor.

An analogy is useful in helping to understand the concept of a potential difference. The electrons in a conductor that are free to move are analogous to water in a long, straight, horizontally positioned pipe. The water has the capability of flowing through the pipe; but it will only dribble out the ends as long as the pipe is exactly level. Only when one end of the pipe is raised above the other end will a flow occur. In the same way, electrons in a conductor will flow only when the electrical charge at the two ends of the conductor differs, that is, when there is a potential difference between the two ends. The potential difference is measured or expressed in volts, after the Italian physicist Volta, whose name was mentioned in the last chapter. Current flow is measured in amperes (A), milliamperes (mA), or microamperes (μA) after the 18th to 19th century French mathematician-physicist André Ampère. Without the presence of a voltage between the two ends of a conductor, there can be no flow of electrons; under such conditions, current flow is equal to zero.

Resistance

How much current flows in a conductor depends upon the voltage applied to it and upon the conductivity of the substance involved. We said earlier that there are good conductors and poor conductors. A good conductor is said to have high conductivity or to have a very low *resistance* to the flow of a current. A poor conductor, on the other hand, has low conductivity or a high resistance to current flow. Resistance, therefore, is inversely related to conductivity, or conductance as it is called. It is a parameter derived from the relation between the voltage applied to the conductor and the current flowing in it. We measure resistance in *ohms*, in honor of the 19th century German physicist George Simon Ohm.

Electrical Circuits

Connecting a voltage between the two ends of a conductor constitutes an electrical circuit. This, of course, represents the simplest kind of electrical circuit. In actual practice, electrical circuits are somewhat more complex; indeed, they frequently can become quite complicated. Nevertheless, the simple circuit does illustrate the important point that an electrical circuit describes a continuous pathway between the two points of a voltage source.

Figure 2.1A shows this circuit with a current-measuring device or ammeter in *series* with the conductor. In this instance a good conductor is connected up, and this is evidenced by the meter showing a relatively large current flowing in the circuit. Figure 2.1B shows the same circuit but with a poor conductor connected; note that considerably less current is flowing than in the previous case. Finally, in Fig. 2.1C, the conductor has been replaced by a good insulator whereupon the meter indicates that no current at all is flowing in the circuit.

We already mentioned in the last section that the current-carrying properties of a conductor are expressed in terms of its resistance and that resistance is measured in ohms. For this reason, a conductor is referred to as a *resistor* and is represented by a zig-zag line in a circuit diagram. This is illustrated in Fig. 2.2 where the circuits shown in Fig. 2.1 are drawn in the conventional manner with the conductors represented by resistors (abbreviated R).

It is obvious from Fig. 2.2 that the current flowing in a circuit decreases as the resistance increases. In other words, with the applied voltage kept constant, the current flowing in a circuit containing a resistor varies *inversely*

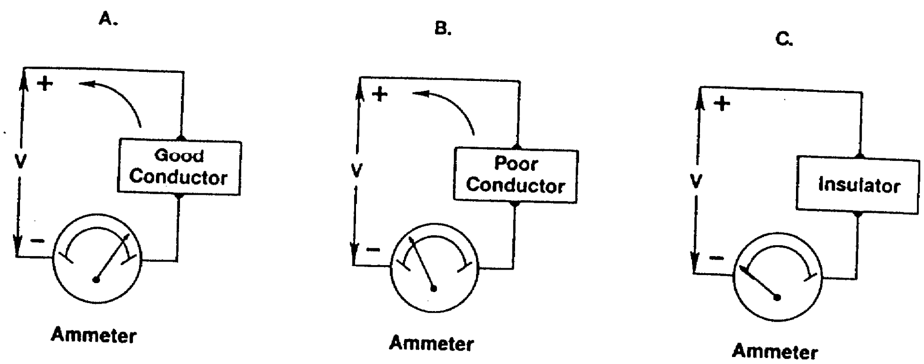


Figure 2.1. Simple electrical circuit containing a good conductor (A), a poor conductor (B), and an insulator (C). The arrows show the direction of current flow.

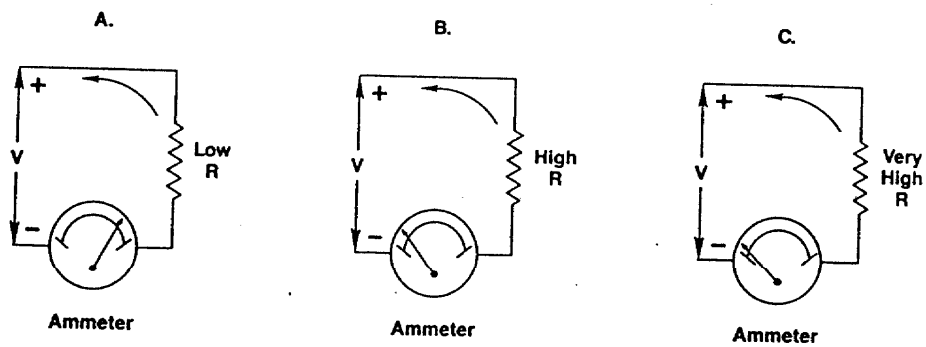


Figure 2.2. Simple electrical circuits shown in Fig. 2.1 drawn in conventional format used in circuit diagrams.

with the magnitude of the resistance. If, on the other hand, we kept the resistance in the circuit constant and allowed the applied voltage to change, we would discover that the current flowing varies *directly* with the magnitude of the voltage. The relationship between current, voltage, and resistance in an electrical circuit is defined by a famous formula referred to as Ohm's law.

Ohm's Law

Ohm's law states that in any electrical circuit,

$$\text{current} = \frac{\text{voltage}}{\text{resistance}}$$

or symbolically,

$$I = \frac{V}{R},$$

where I is current in amperes, V is the potential difference in volts, and R is the resistance in ohms. As with any algebraic formula, if you know the values of any two variables for a particular circuit, you can compute the third or unknown variable from the formula. Thus, for example, if in the circuit shown in Fig. 2.2B, $V = 12$ volts and $R = 50{,}000$ ohms,

$$I = \frac{V}{R},$$

$$I = \frac{12}{50{,}000} = 0.00024 \text{ A}$$

Note that if you doubled V, the current, I, would also be doubled; but if you doubled R instead, the current would be halved.

Ohm's law is a simple yet powerful formal rule that provides a means of analyzing simple circuits as well as many complex circuits. Figure 2.3 shows how the simple circuit in Fig. 2.2 may readily be made more complex by the addition of more resistors and branches with additional such elements. The analysis of these circuits requires two additional rules relating to the way in which separate resistors *in series* and *in parallel* are combined.

Series and Parallel Circuits

When two or more resistors are connected in series—that is, when one is connected to another in a single chain—the total resistance is simply the sum of the resistances of the individual elements. This rule is expressed by the formula

$$R_T = R_1 + R_2 + \cdots R_n,$$

where R_T is the total resistance and R_1, R_2, ... R_n are the resistances of the individual elements. The second rule applies to resistances in parallel. In this case to calculate total resistance you add together the *reciprocals* of the branch elements. This yields the reciprocal of the total

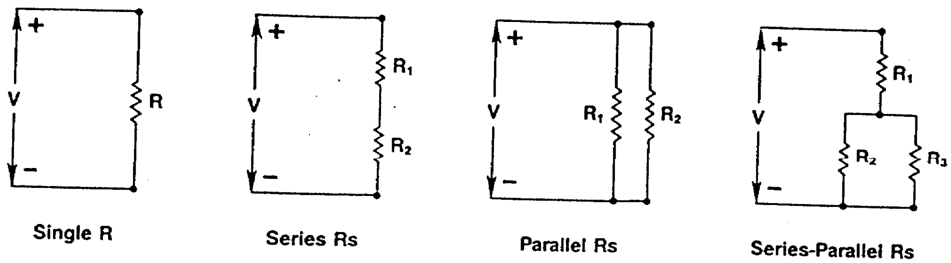


Figure 2.3. Basic forms of series circuits and parallel circuits.

resistance from which total resistance is readily computed. This rule is embodied in the formula

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n},$$

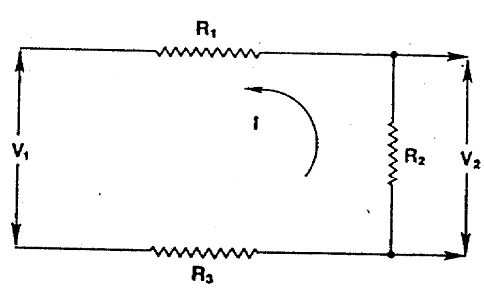
where R_T is the total resistance of the circuit, and R_1, R_2, ... R_n are the resistances of the individual elements in parallel with each other.

It should readily be apparent that when resistors are in series, the total resistance will always be greater than the resistance of any of the individual *R*s. With resistors in parallel, on the other hand, the total resistance will always be less than the resistance of any of the individual *R*s. Why this is so may not immediately be obvious from the formula, although a simple example makes it quite clear. Thus, if $R_1 = 10{,}000$ ohms and $R_2 = 10{,}000$ ohms, then,

$$\frac{1}{R_T} = \frac{1}{10{,}000} + \frac{1}{10{,}000}$$

$$\frac{1}{R_T} = \frac{2}{10{,}000}$$

$$R_T = 5{,}000 \text{ ohms}$$



By Ohm's Law,

$$I = \frac{V_1}{R_T} = \frac{V_1}{R_1 + R_2 + R_3}$$

Again By Ohm's Law,

$$V_2 = R_2 I$$

$$V_2 = R_2 \left(\frac{V_1}{R_1 + R_2 + R_3}\right)$$

Figure 2.4. Application of Ohm's law in analysis of a series circuit.

In this case, R_T is one half the magnitude of either R_1 or R_2. The simplest way of understanding what happens with resistors in parallel is to recognize that by adding a resistor in parallel with another, you provide an additional pathway for current to flow in the circuit. By Ohm's law, you know that the current flowing in a circuit can be increased only by (1) increasing the applied voltage or (2) decreasing total resistance of the circuit. With voltage remaining unchanged, we have to conclude that adding the resistor in parallel results in a decrease in total resistance of the circuit.

The value of Ohm's law and of the rules for combining resistances in series and in parallel will become more apparent when we discuss the topics of electrodes and electrode impedance. In the meantime, the reader will find it helpful for future purposes to analyze the circuit shown in Fig. 2.4. The variable of interest is V_2, the voltage, across R_2. Figure 2.4 shows the general solution for V_2; as may be seen, this involves the use of Ohm's law and a little simple algebra. Now, if R_1 and R_3 are *very small* with respect to R_2, the formula for V_2 is approximated by the expression

$$V_2 = \frac{R_2 V_1}{R_2},$$

and V_2 will very nearly be equal to V_1. As we shall see in later chapters, this simple circuit analysis explains why in EEG work it is necessary for the impedance of the recording electrodes to be low and the input impedance of the EEG amplifiers to be very high. But more about this later.

Circuit Parameters

Earlier, we referred to resistance as a parameter of an electrical circuit. Our discussion turns now to the other parameters that are contained in electrical circuits.

There are but two other circuit parameters. They are called *inductance* and *capacitance*, and the associated elements are termed *inductors* and *capacitors* or *condensers*, respectively. It is surprising and indeed remarkable that electrical circuits consist only of these three parameters, no matter how complex they may become. Of course a circuit can contain other components like diodes, transistors, and vacuum tubes as well; but these are active elements and are not referred to as parameters.

Inductance is a circuit parameter of only minor interest in the area of EEG technology. Aside from the transformers and choke coils of the power supply, and the armature coils of the penmotors, there are no other inductors in an EEG machine. Moreover, inductance does not figure as a parameter in the electrical properties of living tissue. Therefore, no more will be said about inductance in this text.

Capacitance, on the other hand, is a parameter of considerable interest to both the technology and the practice of clinical electroencephalography. Not only are condensers essential components in the power supply and amplifiers of an EEG machine, but they also are the frequency-selective elements that make a filter work. We will hear more about this later in Chapter 4. Finally, living tissue displays electrical characteristics that resemble the electrical properties of a capacitor. For these reasons it is necessary to examine and to understand the electrical properties of this circuit parameter.

Capacitance

The physicist defines a capacitor as two conductors that are separated by an insulator. This, of course, is simply a structural definition that is reflected in the fact that a symbol used for capacitors consists of two short, parallel lines of equal length separated by a narrow space. Of considerably greater interest to the present topic, however, is the functional definition, and here the use of an analogy will be helpful. A capacitor or condenser has the same relationship to electrons that a pail has to water. From this it may be inferred that a condenser is capable of storing electrons. This is indeed the case. A condenser's storage capacity is measured in units called farads (in honor of the 19th century English chemist-physicist, Michael Faraday). Because a farad is an enormous quantity, the practical unit used in the circuits we deal with is the microfarad or millionth of a farad (abbreviated μF or MFD).

To understand the way in which a capacitor affects the functioning of an electrical circuit, it will be useful to return for a moment to the other parameter of electrical circuits that we have already discussed, namely, resistance. For reasons that will later become apparent, we need to address the topic of the *transient response* of electrical circuits.

Transient Response

The transient response of a circuit refers to the behavior of the circuit during the interval of time that a *change* is applied to it and the circuit is still adjusting to the change. This is the opposite of the circuit's *steady-state* response, which refers to the condition of the circuit after it has once again settled down. You can think of transient response and steady-state response in terms of what happens to a person's pulse rate as there are shifts in his or her level of physical activity. While at rest, your pulse rate is, say, 60 to 70 beats per minute. Suppose that at time zero, you begin running. What happens to pulse rate? You would find that pulse rate undergoes a rapid, transient change, shooting up to perhaps 100 to 110 beats per minute. As you continue to run, pulse rate begins to drop, and after a time levels off to perhaps 80 to 90 beats per minute once you attain your normal pace. From then on, until fatigue sets in, it shows only small fluctuations if you maintain a regular pace. This is the steady-state response.

With electrical circuits, the same principles are applicable. The transient response of an electrical circuit is commonly observed by applying an instantaneous change in voltage to the circuit and then measuring the change in current over the interval of time it takes the circuit to adjust to the change. The instantaneous change in voltage is referred to as a *step function.*

Let us begin by examining the transient response of an electrical circuit containing only a resistor. Figure 2.5A illustrates what happens when this is done. With no voltage applied to the circuit, the current flowing, of course, is zero. Now, let us instantaneously change the voltage from zero to some steady, finite value. The plot of voltage versus time in Fig. 2.5A shows this as a step increase in voltage. If the pointer on the current-measuring meter connected in series with the resistor had no inertia, you would see an instantaneous change in its position from zero to some value; there it would remain as long as the step in voltage continued to be applied to the circuit. The plot of current versus time shows this graphically. Note that the transient response of the resistance to a step function is itself a step function. In other words, the changes in current flowing in the circuit follow the changes in applied voltage perfectly. To put it another way, the circuit attains steady-state instantaneously so that, practically speaking, there is no transient response.

What happens when the circuit contains a condenser instead of a resistor is quite different indeed. The outcome is illustrated in Fig. 2.5B. Observe that as with the resistor, the current changes instantaneously from zero to a finite value when the step function is applied. It does not remain there, however, but begins immediately to fall, first rapidly and then slowly until it once again is zero. Note that in doing so the condenser is displaying the characteristics of both a conductor and an insulator. At the instant the voltage is connected to its terminals, the condenser behaves like a good conductor; but then as this voltage remains connected, it becomes a poorer and poorer conductor until current finally ceases to flow, and the condenser displays the properties of an insulator. This outcome is readily

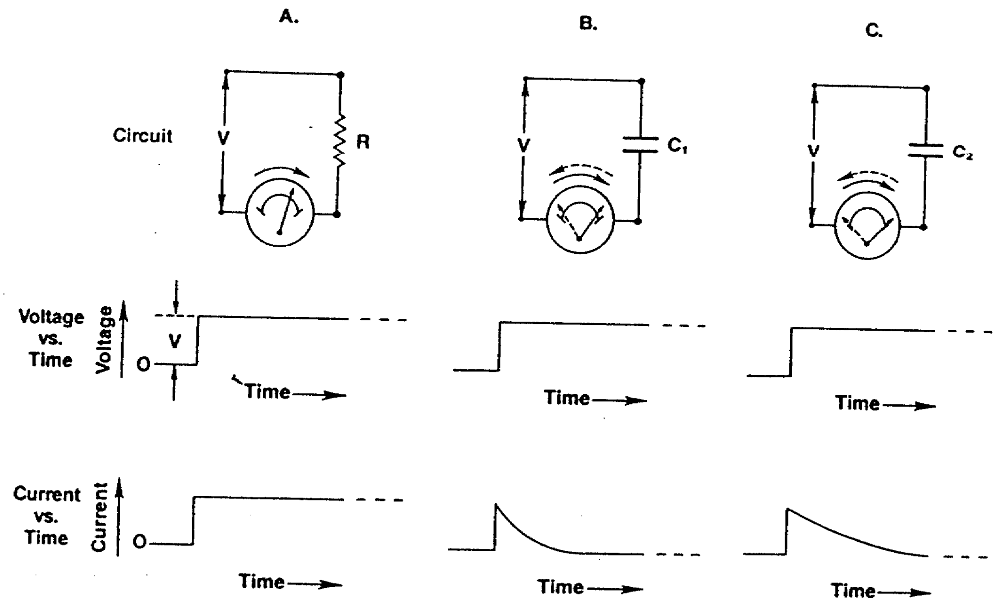


Figure 2.5. Transient response of electrical circuits containing (A) resistance only, (B) capacitance only, (C) capacitance only with C_2 greater than C_1.

understood by referring to our analogy of the water and pail. Assuming that the pail is an enclosed container that cannot overflow, the water will cease to flow in as it becomes filled. In much the same way, the flow of current in the condenser ceases when the condenser becomes filled with electrons or "charged."

The transient response of a capacitor, therefore, is quite different from the transient response of a resistor. While the resistor allows current to flow exactly in phase with the applied voltage, the capacitor exerts a counteracting force upon the flow of current set up by the change in voltage. For this reason, capacitance is considered to be a *reactive* element, and condensers are referred to as *capacitive reactance* in a circuit. We will go into the question of how capacitive reactance is measured later in this chapter in the section on impedance.

After the transient response is over and steady-state has been achieved, note that current through the condenser is zero. In other words, a condenser behaves like an insulator or an "open circuit" to a steady or unchanging voltage. This characteristic is of considerable practical value as it means that a condenser can be used in a circuit to block a steady voltage. As we will see later in a chapter on the differential amplifier, the various stages of amplification in an EEG machine are commonly coupled together by means of condensers. When used in such an application, the condensers are referred to as *blocking capacitors*.

What happens to the transient response as we increase the size of the capacitance? This is shown in Fig. 2.5C. Note that although the current level attained is the same as was the case with C_1 in the circuit, the current falls at a much slower rate. This happens because the larger capacitance has a greater capacity for storing electrons and, hence, takes longer to fill up or become charged. The fact that the condenser is actually charged may be demonstrated by connecting it to a voltmeter and observing that a voltage is present between the two terminals. Since very large condensers can store large numbers of electrons, they are capable of generating large currents when discharged. Charged condensers, therefore, constitute a shock hazard and can be potentially dangerous if you happen to touch their two terminals simultaneously; for this reason, they should be handled very cautiously.

Series R–C Circuit

In the last section we saw that the transient response of a capacitor to a step function was described by a current rising instantaneously to a maximum and then falling off to zero, first rapidly and then more slowly. We now consider the transient response of an R–C circuit—a circuit containing both resistance (R) and capacitance (C) in series. This is an extremely important circuit for the EEG specialist to know about since such circuits are incorporated in the frequency filters on the EEG machine.

Figure 2.6 gives the circuit diagram of a series R–C circuit and shows the response of this circuit to a step function. As in the case of a circuit containing only capacitance, the current rises instantaneously to a maximum value and then falls off, eventually returning to zero. Note in the plot of current versus time that the maximum value of current is equal to V/R, a value that looks like the right side of the equation for Ohm's law. The mathematical function describing the way in which the current varies with time is

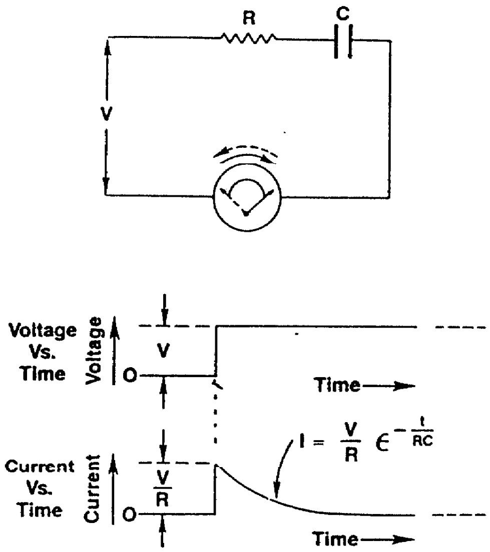


Figure 2.6. Transient response of a series R–C circuit.

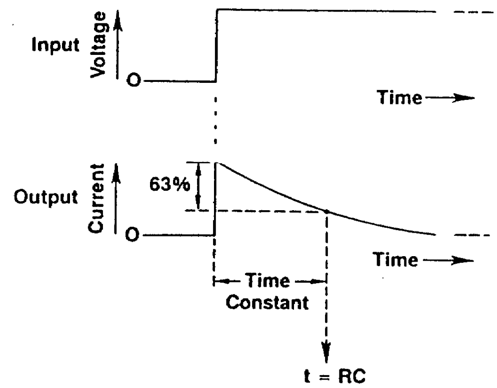


Figure 2.7. Time constant of a series R–C circuit.

referred to as a decaying exponential. It is given by the formula:

$$I = \frac{V}{R}\varepsilon^{-(t/RC)}$$

where V = applied voltage, R = resistance, C = capacitance, t = time in seconds, and ε is the Napierian constant, the number 2.718. . .[1]

While seemingly complex at first glance, this formula is relatively simple when you consider it by parts. Thus, the first part, $I = V/R$, is the now familiar Ohm's law. The rest of the formula corresponds to the quantity ε taken to an exponent, or $2.718^{-(t/RC)}$. Since the exponent is negative, this quantity will always be equal to 1 or <1. Observe that when the time t is equal to zero, the whole exponent also is zero and the quantity $2.718^{-(t/RC)}$ is equal to exactly 1 (remember that any number taken to an exponent that is zero is always equal to 1). Therefore at the instant the voltage is applied to the circuit, $I = (V/R) \times 1$ or $I = V/R$. For values of t greater than zero, the current is equal to some fraction of V/R. Note that the current becomes smaller and smaller as t becomes larger and larger. Theoretically, current approaches but never reaches zero; for practical purposes, however, it is equivalent to zero within the span of several *time constants*. This term is taken up in the next section.

Time Constant

It will be clear from an inspection of the equation for current in the series R–C circuit that the rate at which the decaying exponential falls depends on the value of the exponent, which, in turn, depends on the values of R and C in the circuit. When RC (the product of R and C) is large, the negative exponent is small and the current will fall to zero slowly. On the other hand, when RC is small, the negative exponent is large and the current falls more rapidly.

The rate at which current falls to zero during the course of the transient response is conveniently expressed by the term *time constant*. The time constant of a circuit is the length of time it takes the current to make 63% of its total transition from initial state to steady-state. This is illustrated for the series R–C circuit in Fig. 2.7. Notice that another way of looking at the time constant derives from the fact that it corresponds to the time at which the current is only 37% of its initial value. In terms of the circuit

[1] The mathematically sophisticated reader familiar with the calculus may be interested to learn the origin of this equation. We already know that for a circuit containing resistance alone, the current flowing in the circuit varies directly with the applied voltage and inversely with the resistance. In other words, $I = V/R$. In the case of a circuit containing only capacitance, the current varies directly with the capacitance and with the *rate of change* of the applied voltage. This means that time is a significant variable and current has to be expressed as a *derivative* of voltage. The equation for the current, therefore, is:

$$I = C\,dV/dt$$

Putting the two parameters together, the basic voltage equation for a circuit with R and C in series is:

$$RI + (1/C)\int_0^t I\,dt = V$$

Differentiating this equation with respect to t yields:

$$R\,dI/dt + I/C = 0$$

From which

$$I = (V/R)\,\varepsilon^{-(t/RC)}$$

parameters, the time constant is equal to the product of R and C.[2]

AC and DC

Most everyone has at one time or another noticed the identifying plaques that are affixed to electric irons, hair dryers, and a variety of other electrical appliances. Many of these plaques carry the warning "for use on 120-V AC only." The AC, of course, stands for alternating current, while the 120 V refers to the voltage of the electric power that is almost universally available from the wall, floor, and ceiling outlets in homes, offices, and hospitals. But what exactly is alternating current?

One way of approaching this question is to consider first the opposite of AC, namely, direct current or DC. Direct current is current in which the flow of electrons is in one direction only. It results when you apply a steady voltage to a circuit. Batteries are the most common source of DC; they produce DC electrochemically. Various different kinds of batteries capable of generating a wide range of voltages are available. They are found in automobiles, radios, flashlights, and other appliances. Despite the differences in their physical appearance, all have one characteristic in common: they provide a constant voltage so that the current flowing in the circuit to which they are connected moves in only one direction.

Alternating current or AC is a pulsating or fluctuating electric current that alternately flows in one direction and then in another. It results when an alternating voltage is applied to a circuit. Alternating current is usually produced by electric generators—huge rotating machines—at electrical power stations. As is the case with DC, we also speak of the voltage of an AC source. But because the voltage fluctuates or alternates, it is also necessary to specify the frequency of the alternation. The electricity used every day in homes, hospitals, and industry is 120-V 60 cycle or 60 Hz AC. Because AC is cheaper to produce than DC and also because it can be transmitted by power lines over long distances more efficiently than DC, AC is used in preference to DC as an energy source.

A 60 Hz AC completes one full cycle in 1/60 second (i.e., 0.0167 second). It flows in a positive direction for half the time and a negative direction for the remaining half cycle. The changes in voltage with time are sinusoidal, that is, they are described by a sine curve.

It is important to recognize that AC does not refer only to 60 Hz AC—the electrical energy that lights our homes and powers our appliances. Alternating current has a much broader meaning. The EEG is AC. It is not strictly sinusoidal, but it is AC nonetheless. The electric current generated by a microphone, an audio disc or tape, and a videotape is also AC. As a matter of fact, most sources of electric current are classified as AC sources.

AC Circuits

Thus far in this chapter, we have dealt only with the behavior of electrical circuits when a steady voltage is applied and with the transient response to a single, instantaneous change in voltage. All this falls under the heading of DC circuit analysis. We now turn to the topic of AC circuit analysis to examine the behavior of the same circuits when alternating instead of steady voltages are applied to them. We will deal only with the steady-state response—the response after any transients associated with connecting up the circuit to the source of voltage have subsided.

If all the circuits that we needed to deal with contained only the parameter resistance, our discussion could quickly be terminated. For circuits containing resistance only, Ohm's law applies regardless of whether AC or DC is involved. A resistor behaves in the same way regardless of the frequency of the current flowing. In other words, resistance is independent of frequency of the current flowing in a circuit. If, however, a capacitor is added to the circuit so that C is in series with R as in Fig. 2.6, Ohm's law needs to be modified. The formula in this case is changed to

$$I = \frac{V}{Z}$$

where Z is equal to the *impedance* of the circuit.

Impedance

The concept of impedance is important for the EEG specialist to understand. He or she deals with it on a daily basis whenever EEG leads are attached to a patient

[2] The reader may demonstrate this fact for him or herself by some simple algebra. From the last section we know that current in a series R–C circuit is:

$$I = (V/R)\,\varepsilon^{-(t/RC)}$$

Setting $t = RC$, we have

$$\begin{aligned} I &= (V/R)\,\varepsilon^{-(RC/RC)} \\ &= (V/R)\,\varepsilon^{-1} \\ &= (V/R) \times (1/\varepsilon) \end{aligned}$$

Since $\varepsilon = 2.718$,

$$I = (V/R) \times (1/2.718)$$

$$I = 0.37\,(V/R),$$

which means that when $t = RC$, the current is 37% of its initial value.

or when an EEG recording is interpreted. Thus, the EEG technician measures the impedance of each lead or each pair of leads before starting to take a recording. She/he knows that the impedance has to be low but not too low in order to obtain a satisfactory record. Similarly, the person interpreting the record needs to know that lead impedances were comparable whenever significant amplitude asymmetries show up in the tracings. With all this in mind, let us proceed with a discussion of impedance.

Impedance of an R–C circuit is the combined effect that the two parameters of resistance and capacitance have on the flow of current produced when an alternating voltage is applied to the circuit. Mathematically, impedance is equal to

$$Z = \sqrt{R^2 + \left(\frac{10^6}{2\pi fC}\right)^2}$$

where R is the value of resistance in ohms, C the value of capacitance in μF, f the frequency of the alternating voltage in Hz, and π the familiar constant that is equal to 3.14. . . The term $10^6/2\pi fC$ is referred to as the capacitive reactance.

As is the case with resistance, Z the impedance is also measured in ohms. Let us consider the formula for Z carefully and list what it tells us about the characteristics of Z:

1. The value of Z depends on the values of the three quantities, namely, R, C, and f.
2. If C and f are held constant, Z varies directly with R.
3. If R and f are held constant, the term $10^6/2\pi fC$ or the capacitive reactance increases as C decreases and vice versa, so that Z varies inversely with C.
4. If R and C are held constant, the capacitive reactance increases as f decreases and vice versa, so that Z varies inversely with f. Note particularly that as f approaches zero, Z becomes very large indeed. In the limit when $f = 0$, the applied voltage is no longer alternating but becomes steady; under these conditions we are dealing with DC not AC, and the rules of DC circuit analysis would apply.

Let us summarize. Impedance is a *frequency-sensitive* quantity. Z varies with changes in frequency of the applied voltage as well as with changes in C and R. For this reason it is necessary to specify the particular frequency of the applied voltage whenever we talk about impedance. Thus, for example, we say that a particular circuit has an impedance of 10K ohms at 30 Hz. This property of impedance is uniquely due in the series R–C circuit to the capacitive reactance—to the presence of the capacitor. As we will see in a later chapter, this property of capacitance is the basic principle upon which the operation of the filters on an EEG machine is based.

Computational Example

The fact that differences in frequency have a profound effect on impedance is readily apparent from a simple example. Suppose in series R–C circuit (Fig. 2.6) that R = 10K ohms and C = 1 μF. What is the impedance at 1 Hz? The answer is obtained by substitution of these values in the formula for Z. Thus,

$$\begin{aligned} Z &= \sqrt{R^2 + \left(\frac{10^6}{2\pi fC}\right)^2} \\ &= \sqrt{(10^4)^2 + \left(\frac{10^6}{2 \times 3.14 \times 1 \times 1}\right)^2} \\ &= \sqrt{(10^4)^2 + \left(\frac{10^6}{6.28}\right)^2} \\ &= \sqrt{10^8 + 159{,}236^2} \\ &= 159{,}550 \text{ ohms} \end{aligned}$$

The reader should verify, by similar computation, that impedance at 35 Hz is equal to 10,986 ohms. Note that there is nearly a 15-fold difference in impedance at frequencies of 1 Hz and 35 Hz.

Frequency Response

The fact that impedance of a circuit varies with frequency finds expression in an important measure used to characterize the behavior of electrical circuits. This measure is the *frequency response* of a circuit. Whereas the transient response of a circuit is its response to an instantaneous step change in applied voltage, the frequency response is the response of the same circuit to an alternating applied voltage of constant amplitude that is allowed to vary in frequency. The frequency response of a particular circuit is reported as a *frequency-response curve*, the points for which are obtained by measuring the *amplitude* of the output voltage when voltages of different frequency but the same amplitude are applied to the input of the circuit.

The concept of frequency response will be familiar to readers who own or have used high-fidelity audio reproduction equipment. We know that sounds correspond to mechanical vibrations and that pitch is related to the frequency of these vibrations. Audible sounds have a frequency range of 20 to 20,000 Hz. Audio-reproduction systems simply pick up the mechanical vibrations via a microphone or phono pick-up, convert the vibrations to an alternating electrical voltage, amplify them, and then convert them back to mechanical vibrations in a loudspeaker. The fidelity of such systems is expressed by the frequency-response curve. Systems that reproduce music at very high fidelity have a frequency-response curve that is essentially

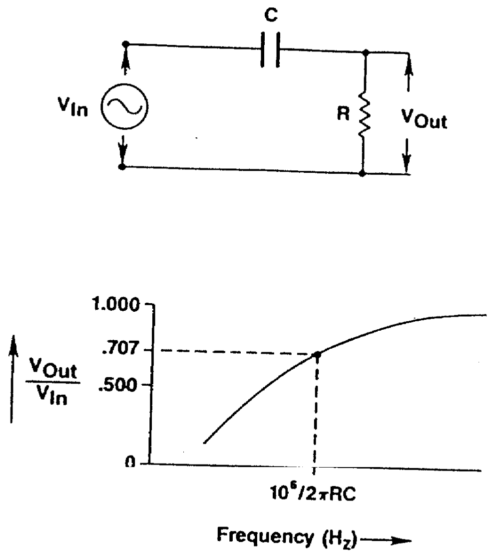


Figure 2.8. Frequency response of a series R–C circuit.

flat from 20 to 20,000 Hz. Low-fidelity systems, on the other hand, severely attenuate frequencies at both ends of the frequency spectrum so that the range of audible frequencies actually transmitted may be limited to only 100 to 8,000 Hz.

Although the frequency response of a circuit may be obtained empirically by applying an alternating voltage of variable frequency but constant amplitude to the input and then measuring the amplitude of the output, we may also calculate the frequency response from the circuit parameters. Figure 2.8 shows the frequency-response curve for the case of the now familiar series R–C circuit. This curve is derived in the following manner: taking the output of the circuit as the voltage across the resistor, we have from Ohm's law:

$$I = \frac{V_{in}}{Z} = \frac{V_{in}}{\sqrt{R^2 + (10^6/2\pi f C)^2}}$$

and again by Ohm's law

$$V_{out} = RI = \frac{R}{\sqrt{R^2 + (10^6/2\pi f C)^2}} V_{in}$$

Observe in the above formula that, when f is allowed to increase so that it becomes large, the quantity $10^6/2\pi f C$ will approach zero. This makes the fraction

$$\frac{R}{\sqrt{R^2 + (10^6/2\pi f C)^2}}$$

very nearly equal to

$$\frac{R}{\sqrt{R^2}} = 1,$$

so that V_{out} is almost equal to V_{in}. If we go in the opposite direction, i.e., allow f to decrease and become small, the denominator of the fraction becomes much larger than R the numerator, and V_{out} is a progressively smaller fraction of V_{in}. This is shown graphically by the frequency-response curve in the lower part of Fig. 2.8. Amplitude is plotted as the ratio of V_{out}/V_{in}. Since this ratio is always less than 1, we say that this circuit attenuates the input, with lower frequencies being attenuated more than higher frequencies. The dividing point between low and high frequency is arbitrarily taken to be the point at which frequency is equal to $10^6/2\pi f C$ Hz. This point is designated the *cutoff frequency* and corresponds to a 30% attenuation of V_{in}; in other words, $V_{out}/V_{in} = 0.707$ (see Fig. 2.8).[3]

What is the cutoff frequency for the series R–C circuit considered earlier in the computational example when R = 10K ohms and C = 1 μF?

$$\text{Cutoff frequency} = \frac{10^6}{2\pi RC}\,\text{Hz}$$

$$= \frac{10^6}{2 \times 3.14 \times 10{,}000 \times 1}$$

$$= \frac{10^6}{6.28 \times 10^4} = \frac{100}{6.28}$$

$$= 15.9\ \text{Hz}$$

This means that a 15.9 voltage applied to the input will be attenuated by 30% when it appears at the output of the circuit. The practical significance of this finding will become apparent in Chapter 4.

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[3]This is readily confirmed by some simple algebra. We have shown that

$$V_{out} = \frac{R}{\sqrt{R^2 + (10^6/2\pi f C)^2}} V_{in}$$

At the cutoff point,

$$f = 10^6/2\pi RC$$

Substituting this in the equation for V_{out}, we have

$$V_{out} = \frac{R}{\sqrt{R^2 + \left(\dfrac{10^6}{2\pi \times 10^6/2\pi RC \times C}\right)^2}} V_{in}$$

$$= \frac{R}{\sqrt{R^2 + R^2}} V_{in}$$

$$= \frac{R}{\sqrt{2R^2}} V_{in} = \frac{R}{\sqrt{2}\,R} V_{in}$$

$$= \frac{R}{1.414\,R} V_{in}$$

so that

$$\frac{V_{out}}{V_{in}} = 0.707$$