WHY ALPHA VARIABLES AREN'T ALWAYS SIMPLE

By

J. M. LIPSKI

(Edmonton)

There are many goals which a linguist can set for himself in studying language and languages, and among these goals there are two which, both from a philosophical and from a methodological viewpoint, may be considered as focal points for modern linguistic investigation. The first goal is to *describe* a particular language or group of languages, in greater or lesser detail, as the occasion demands. This description may be a synchronic one, demonstrating the manner in which the sounds and forms of a language combine and interact, or it may be diachronic, describing, often in minute detail, the processes by which historical changes have been effected. The second major goal which serves to roughly divide contemporary linguistic thought, and which many scholars feel is unattainable in practice, is to *explain* linguistic data; i.e. to show why given changes occurred, or did not occur, and why languages are what they are.

In dealing with situations of pure description, matters proceed rather smoothly, for the only possible points of controversy lie in the data themselves, which are available for all to see, and to interpret at will. The only limitations to be encountered are practical ones, including such factors as availability and reliability of data, preconceived opinions of the investigator, etc. In the case of linguistic theories containing attempts at explanation, however, there are, in addition to the usual array of methodological problems, a number of philosophical questions involved which, being basically unresolvable, divide linguists into two groups: those who believe that explanation is possible and those who, for all intents and purposes, believe that it is not. Structural linguists have generally exhibited no qualms about seeking explanation for linguistic data, but within the more recent framework of generative transformational grammar a curious ambivalence towards this question has become manifest. In the first concise statement of the goals of transformational grammar, published by Chomsky (1957: Chap. 6), the goal of explanation in linguistics as discounted as follows: "I think that it is very questionable that this goal is attainable in any interesting way, and I suspect that any attempt to meet it will lead into a maze of more and more elaborate and complex analytic proce-

dures that will fail to provide answers for many important questions about the nature of linguistic structures." (pp. 52—3). Chomsky's opinions have not, however, been shared by all his followers, many of whom have been very interested in the way in which the goal of explanation might be approached. In the realm of diachronic linguistics, especially phonology, the search for criteria of explanatory value becomes most evident. Taking as a point of departure the formalization of language as a system of ordered rules, historical change has been characterized by Kiparsky (1965, 1968) in terms of rules evolving so as to become maximally effective, while the notions of rule simplification and regularization of paradigms have been explored by Kiparsky (1971), King (1969a, b), and many others. Even Chomsky himself seems to have at least partially changed his mind as regards explanation in linguistics, for in Chomsky and Halle (1968) we are presented with the theory of "marking", which, both further on in the same book, and in numerous subsequent studies, has been cited as having provided the motivation for various sound changes, as well as determining the "correct" synchronic form of grammars.

Although having its roots in earlier philosophical and scientific theories, generative grammar brought with itself several innovations for the methodology of linguistic investigation, among which was an array of formal and mathematical terms and descriptive devices never before applied to the study of language. Within the domain of phonology, these devices consist, among other things, of various types of parenthesis, brackets, subscripts, arrows, as well as a number of axiomatic conventions as to how these devices are to be employed; the primary purpose of such a system is to describe phonological processes in as concise and exact a form as possible. Since generative grammar has adopted "simplicity" as one of its corner-stones, the notational devices in phonology are often used to collapse a number of rules into a single rule, thereby "capturing" the greatest possible number of "generalizations". While such an approach is unquestionably valid for a pure description, where maximal efficiency of presentation is often desirable, it is not at all certain that the "generalities" expressed by certain notational conventions possess empirical validity when used as explanations, and it is this uncertainty among many modern linguists which has led, for example, to Robert Hall's (1970) stigmatization of generative grammarians: " 'Generality' has come, among some followers of Chomsky, to be prized as a virtue in itself, to be sought for even at the expense of accuracy or fidelity to detail." (pp. 215—6).

Among the practitioners of generative grammar, however, restraint has often been called for; thus Harms (1968: 57) states: "The goal is not to collapse rules in order to save ink. Any device that leads to a more economical description must be shown to lead, at the same time, to desirable generalizations in terms of how that device is to be counted. Outside of the simplicity metric, economy in the description has no real meaning." Kiparsky (1968: 172)

warns: "Nor is the fact that a generalization can be stated enough to show that it is real. All sorts of absurd notational conventions can easily be dreamed up which would express the kinds of spurious generalizations that we would want to exclude from grammars." More specifically, one may discover in the literature attempts at eliminating many of the spurious simplifications made possible by overzealous use of certain notational devices, among which are those of Lightner (1963), Stanley (1967), and Cressey (1970). Such restrictions are necessary because, as noted by Zwicky (1970: 553): "Virtually any two phonological rules, however unrelated their nature or effect, have sufficient formal similarity to be consolidated by the notational conventions of Chomsky and Halle."

One descriptive device which has enjoyed great popularity, but whose exact mechanism has seldom been questioned or investigated, is the so-called "Greek-letter" or "alpha" variable, first introduced by Halle (1962) as a means of describing various sorts of assimilation. As subsequently used, alpha variables serve to formally unite the coefficients of two or more distinctive features by showing not only that, under certain specified conditions, these features take either lthe same or opposite coefficients, but also that this formal relationhip hods for any value which the variable may assume; all depending, of course, on the configuration of the coefficients of the alpha variables themselves. In the long run, it seems that alpha variables have been a mixed blessing to linguistics, for while, on the one hand, they can conveniently symbolize many undeniably valid processes, they may also be used to form all manner of strange rules, due to the inherent power contained in such notational devices. As found in current linguistic descriptions, the use of variables as feature coefficients may be roughly divided into three broad and partially overlapping categories. The first category consists of the use of variables to specify the interrelationship of the coefficients of distinctive features within the same segment. Quite often, this usage leads to very natural and straightforward descriptions; for example, the fact that a language contains only front unrounded and back rounded vowels may be easily depicted by a statement such as:

(1)
$$V \rightarrow \begin{bmatrix} \alpha \text{front} \\ -\alpha \text{ round} \end{bmatrix}$$

On the other hand, an injudicious use of variables within the same segment can lead to such "unnatural classes" as:

(2)
$$\begin{bmatrix} \alpha \text{ vce} \\ -\alpha \text{ round} \end{bmatrix} \begin{bmatrix} \alpha \text{ palatal} \\ \alpha \text{ nasal} \end{bmatrix} \begin{bmatrix} \alpha \text{ back} \\ -\alpha \text{ vocalic} \end{bmatrix}$$

Formally, there is no way of discriminating between these various descriptions;

rather, one has to consider the intrinsic content of the individual distinctive features involved. One possibility has been considered by Zwicky (1970: 552): "It appears that variables used to specify classes must relate features of the same type — either two cavity features (back and round, grave and compact, round and low, diffuse and grave, coronal and anterior, or diffuse and compact) or two manner features (vocalic and consonantal, vocalic and continuant, or continuant and striden)." An even stronger objection to the idea that using alpha variables to represent feature similarity in the same segment constitutes a true linguistic simplification has been made by Wheeler (1972: 90): "I believe this use of Greek-letter variables is quite mistaken . . . It is proper to present natural classes by means of *properties* which they share, not by making use of the fortuitous circumstance that they have in common merely *agreement between the values*, be it 'plus' or 'minus', of certain features, or . . . disagreement between the values of those features." This topic will be briefly returned to at a later point in the discussion.

The second use of alpha variables is to specify the identity or non-identity of the coefficients of the same distinctive feature in two or more segments. This is the use for which variables were originally proposed, and it is here that the most unquestionably "natural" employment of alpha variables may be found; i.e. in specifying cases of local assimilation. This category may be further subdivided into redundancy rules and true phonological change rules. As an example of the former, one might consider the common situation whereby the first member of a consonant cluster agrees in voicing with the following member, roughly:

$$(3) \qquad C \rightarrow [\alpha \, vce]/- \begin{bmatrix} C \\ \alpha \, vce \end{bmatrix}$$

Rule (3) may also be considered as an example of a phonological change rule, to describe the situation in a language (for example, Italian) whereby consonant clusters originally differing in voicing were later assimilated to a single voicing value.

The final possibility in the use of alpha variables, and by far the most controversial, involves the use of variables to unite the coefficients of *different* distinctive features on either side of the equation. Again, this category may be divided into redundancy and change rules. As an example of a redundancy rule, one may take the following from Chomsky and Halle (1968: 352):

$$(4) \qquad \begin{bmatrix} +voc \\ -cons \\ -back \end{bmatrix} \rightarrow [\alpha \, rnd]/- \begin{bmatrix} +voc \\ +cons \\ \alpha \, lateral \end{bmatrix}$$

An example of an analogous rule involving phonological change may be found in King (1969b: 14):

$$(5) \qquad \begin{bmatrix} -cons \\ \alpha \, high \\ +rnd \\ +long \end{bmatrix} \rightarrow \begin{bmatrix} -\alpha \, back \\ +high \end{bmatrix}$$

Initially, some doubt was expressed as to the validity of such usage, which literally opens the door to an unlimited number of weird and unnatural rules. Chomsky and Halle (1968: 352) note the following, in justification of their own descriptions: "There is empirical evidence in favor of imposing a limitation on the use of variables with different features in different segments. The great majority of examples involve only a single feature, and in other cases there clearly seems to be some intrinsic connection between the features involved in the process of assimilation." As regards the last assertion, it is often difficult, at least for some people, to perceive the "intrinsic connection" between such feature pairs as *round-lateral*, *long-voice* (Harms 1968: 62), etc. However, regarding such a usage of alpha variables merely as a descriptive convention, there is no pressing reason which calls for their abandonment, other than the fact that they may perhaps hint at a greater generality than is reflected by the data under consideration. The problem is that such rules are often used in attempts at *explaining* phonological changes, and since, due to their inherent nature, as well as to the way in which they have been traditionally interpreted, variables effect a formal simplification in rule structures, such "simplified" rules are used as proof that the sound changes in question occurred through a drive for simplicity. A case in point of this kind of reasoning is offered by King (1969b), who gives a number of examples of covariant sound shifts, the description of which may be simplified by using alpha variables for different features in different segments.[1] Since in order to qualify as an explanation, a description must be shown to be valid on all accounts, any descriptive device used in explaining a phonological process must be thoroughly examined in all its possible uses. Such an examination has never been formally carried out in the case of alpha variables, however, since almost from the very start they were assumed to possess universal validity in expressing linguistically significant generalizations. It has been almost universally assumed that variables "count" less in the evaluation of rules than do plus and minus (or scalar) values; in the first concrete proposal to this effect Contreras (1969) suggested that variables count half the value of pluses and minuses.[2] However, the various contexts

---

[1] It appears, in fact, that *all* examples of chain-type rules involve this usage of variables; see, for instance, Newton (1972).

[2] Some aspects of this proposal have been criticized by Harris (1970), among others. However, inasmuch as any purely formal counting procedure is valid in determining the desirability of phonological rules, Contreras' proposal seems as good as any other that might be offered.

in which variables may be used are all treated equally by this evaluation procedure, with the result that the statements in (2) are as "simple" as those of (1), while rules like (4) and (5) are counted the same as purely assimilatory rules with the same number of features mentioned. It has already been noted that there is no formal way of expressing the discrepancies between (1) and (2), but that these cases must instead be handled by consideration of specific information. In the case of variables used on both sides of the equation, however, it is possible to construct a mathematical model which will duplicate the mechanism of such rules. The remainder of this paper will be devoted to the construction of such a model, in order to demonstrate what may already be obvious to many investigators: that alpha variables do not always represent true simplifications, formal or otherwise, in all contexts in which they may appear. An attempt at formalizing some of the notions inherent in the concept of phonological rules may be found in Chomsky and Halle (1968: 390—99). Due to the somewhat different approach adopted by this paper, however, the notation developed by Chomsky and Halle will not be adhered to. For simplicity of presentation, the exposition below will be confined to the case of strictly binary distinctive features. It is easily seen that the theoretical framework deyeloped below may be extended intact to a non-binary system, by modifying only the appropriate numerical values.

For the purpose at hand, it will be assumed that there exists a universal set of n distinctive features $F_1, \ldots, F_n$, to be used in all rules.[3] The coefficients of these features, strictly binary as noted above, are chosen from the set $\langle +1, -1 \rangle$. One may consider the completely unspecified matrix of distinctive features, henceforth $M_\emptyset$, to be of the form:

$$(6) \qquad \begin{vmatrix} \langle +1, -1 \rangle_1 \, F_1 \\ \cdot \\ \cdot \\ \cdot \\ \langle +1, -1 \rangle_n \, F_n \end{vmatrix}$$

Taking the $n \times 1$ matrix $\{a_i: 1 \leq i \leq n\}$, where each a_i is taken from the set $\langle +1, -1 \rangle$, the fully specified matrix Ma_i may be defined as:

$$(7) \qquad \begin{vmatrix} a_1 \, F_1 \\ \cdot \\ \cdot \\ \cdot \\ a_n \, F_n \end{vmatrix}$$

[3] For those who might object to this purely pragmatic idealization, it should be noted that, in describing any given language, only a certain finite set of distinctive features is employed, and this set may therefore be considered to represent the hypothetical 'universal' set of distinctive features whose discovery remains one of the unattained goals of modern linguistics.

This corresponds to a fully specified segment. One must now consider the manner in which the fully specified matrix Ma_i is derived from the two matrices $M_\emptyset$ and $\{a_i\}$; i.e. how the distinctive features are paired off with their corresponding coefficients. For each i in the set $\langle 1 \ldots n \rangle$ one may define a specification mapping g_i on the product set $\langle \{a_i\} \, XM_\emptyset \rangle$:

$$(8) \qquad g_i(a_j, \langle +1, -1 \rangle_j) = I, \qquad i \neq j; \; g_i(a_i, \langle +1, -1 \rangle_i) = a_i$$

where I is the identity mapping defined as $I(x) = x$, for all x. The fully specified matrix Ma_i may then be considered as the composition of the various functions g_i:

$$(9) \qquad g_n \circ \ldots \circ g_1 \, (\{a_i\} XM_\emptyset)$$

where the composition $g \circ f$ of two mappings g and f is defined as $g(f)$. A matrix with the coefficients of all features specified except one, say F_j, is then of the form:

$$(10) \qquad g_n \circ \ldots \circ g_{j+1} \circ g_{j-1} \circ \ldots \circ g_1 \, (\{a_i\} XM_\emptyset)$$

A matrix with more than one unspecified coefficient may be similarly represented by deleting the appropriate specification mappings.

At this point we should digress for a moment in order to consider the manner in which ordinary phonological rules operate within the formal framework defined above. It is immediately noticed that all phonological rules merely switch the coefficients of particular features. A deletion may be considered as a switch from [+seg] to [−seg], while an insertion is similarly a switch from [−seg] to [+seg], together with the appropriate additional changes. One may therefore define a coefficient switching map s_i:

$$(11) \qquad s_i(a_j) = I; \qquad i \neq j; \qquad s_i(a_i) = -a_i$$

(assuming the convention $-(-1) = +1$). Thus all non-assimilatory phonological rules may be considered as a finite compositions of the s-mappings defined in (11).

Returning to the case of coefficient assimilation (or dissimilation) by means of variables, the manner in which variables apply within the present framework may now be studied. Although without direct theoretical consequence to the goals of this study, it is useful to examine the fashion in which the theory outlined above accomodates the use of alpha variables to indicate similarity or dissimilarity of coefficients in the same segment. Suppose one is given a fully specified matrix Ma_i as defined in (7), together with the stipulation that the coefficients of two features, say F_i and F_j are to be equal. In this case, the specification mapping g_i and g_j which supply the coefficients for F_i

and F_j, respectively, must be specified as providing the same values. The easiest manner to indicate this equivalence in the notation hitherto employed is to equate the index numbers of the appropriate specification mappings. In the particular example just mentioned, the matrix Ma_i would be specified as follows:

(12) $$g_n {}^\circ \cdots {}^\circ g_{i=j} {}^\circ g_{j=i} {}^\circ \cdots {}^\circ g_1$$

Assimilation of the coefficients of more than two features in the same matrix may be handled by extending the number of similarly indexed mappings, while dissimilation may be handled by requiring the appropriate specification mappings to assume opposite values.

Given a fully specified matrix Mb_i, defined as:

(13)
$$\begin{vmatrix} b_1 \ F_1 \\ \cdot \\ \cdot \\ \cdot \\ b_n \ F_n \end{vmatrix}$$

and given the totally unspecified matrix $M_\emptyset$ as defined in (6), one may define an assimilation mapping which will duplicate the coefficients of Mb_i onto the respective features of $M_\emptyset$. Such a mapping may be called a b_i-dominated specification mapping, and is defined as:

(14) $$g_{bi}(b_j, \langle +1, -1 \rangle_j) = I, \quad i \neq j; \quad g_{bi}(b_i, \langle +1, -1 \rangle_i) = b_i$$

These functions may be combined by the usual action of composition to yield a duplicate of the matrix Mb_i, as illustrated in (9).

We must now consider those instances where, given a fully specified matrix Mb_i, only one of its feature coefficients is to be duplicated in another partially specified matrix. This would represent a typical case of neighborhood assimilation. Let us assume a given partially specified matrix Ma_{i-j} whose jth feature coefficient is unspecified:

(15)
$$\begin{vmatrix} a_1 \ F_1 \\ \cdot \\ a_{j-1} \ F_{j-1} \\ \langle +1 \ , \ -1 \rangle_j \ F_j \\ \cdot \\ a_n \ F_n \end{vmatrix}$$

Such a matrix is equivalent to a segment whose jth distinctive feature is represented by an alpha variable. Since the coefficient of F_j in Ma_{i-j} may take either of the two values $+1$ or -1, two possible fully specified matrices Mb_i and $M*b_i$ must actually be taken into consideration:

(16) $$Mb_i = \begin{vmatrix} b_1 \ F_1 \\ \cdot \\ \cdot \\ b_j \ F_j \\ \cdot \\ b_n \ F_n \end{vmatrix} \qquad M*b_i = \begin{vmatrix} b_1 \ F_1 \\ \cdot \\ \cdot \\ -b_j \ F_j \\ \cdot \\ -b_n \ F_n \end{vmatrix}$$

The two matrices Mb_i and $M*b_i$ differ only in the coefficient of F_j, and thus represent the two possibilities allowed by the alpha variable present in Ma_{i-j}. Since the two matrices Mb_i and $M*b_i$ are completely specified, while the matrix Ma_{i-j} remains to be specified for the feature F_j, one may use the concept of b_i-dominated specification mappings defined in (14) to complete the specification of Ma_{i-j}. The latter matrix, it is recalled, is presently of the form:

(17) $$g_n {}^\circ \cdots {}^\circ g_{j+1} {}^\circ g_{j-1} {}^\circ \cdots {}^\circ g_1$$

Therefore, only one b_i-dominated specification mapping will be required to fully specify it. Such a completion mapping would be of the form g_{bj}, defined as:

(18) $$g_{bj}(a_i, b_i) = I, \quad i \neq j; \quad g_{bj}(a_j, b_j) = b_j$$

By composing the mapping g_{bj} with the incompletely specified matrix Ma_{i-j} defined in (17), a fully specified matrix results, whose jth feature coefficient matches that of Mb_i or $M*b_i$, depending on which of the latter two segments is actually present. Assimilation of several unspecified features may be similarly handled by the appropriate b_i-dominated specification mappings. The so-called "alpha-switching" rules, which call for dissimilation of the coefficients of the same feature, may be handled by a b_i-dominated dissimilation mapping:

(19) $$g*_{bj}(a_i, b_i) = I, \quad i \neq j; \quad g*_{bj}(a_j, b_j) = -b_j$$

The process described above corresponds to assimilatory redundancy rules, of the type illustrated in (3). One must now consider the case where, given two *fully* specified matrices Ma_i and Mb_i, one wishes to assimilate the coefficient of a feature in Ma_i, say F_j, to the coefficient of the same feature

in Mb_i. This represents the true phonological change rules. In the case where the coefficient of F_j in Ma_i is to be represented by a variable, one must again consider two matrices Mb_i and M^*b_i as defined in (16). Since the matrix Ma_i is already fully specified, the notion of b_i-dominated specification mappings does not apply. One must consider instead the existence of a b_i-dominated switching map s_{bj}, defined as follows:

$$(20) \qquad\qquad s_{bj}(a_i) = I, \qquad i \neq j; \qquad s_{bj}(a_j) = b_j$$

Similarly, a dissimilatory switching map s^*_{bj} may be defined:

$$(21) \qquad\qquad s^*_{bj}(a_i) = I, \qquad i \neq j; \qquad s^*_{bj}(a_j) = -b_j$$

It may be seen, therefore, that there exists a natural method of establishing a canonical, one-to-one mapping which will duplicate the coefficients of a given fully specified matrix Mb_i in the appropriate spot in any other matrix. It is seen, furthermore, that the assimilation of feature coefficients to adjacent segments may be considered either as a phonological change, effected by s-mappings, or as the filling in of an incompletely specified matrix by means of coefficients drawn from another segment, effected by specification mappings.

At this stage, it may already be seen why the use of variables to unite the coefficients of the same distinctive feature may be formally regarded as a simplification. Between the coefficients of the distinctive features of any two segments, there exists exactly one set of canonical mappings, which establish a one-to-one relationship between the features involved. Therefore, if the binary coefficients of a feature in a given segment are replaced by a variable, these canonical mappings will pair off the variable with the corresponding coefficients of the two matrices entailed by the binary system. Similarly, if the variable represents an n-ary coefficient, for any value of n, then n matrices will all be drawn together in one expression.

The situation is fundamentally different, however, when one has to consider the case of assimilation or dissimilation of coefficients of *different* features on either side of the equation, as in (4) and (5). Let us assume, as in the preceding paragraphs, an incompletely specified matrix Ma_{i-j} whose jth feature coefficient is unspecified. Suppose we wish to specify the coefficient of F_j in Ma_{i-j} by assimilation to the coefficient of F_i in another segment Mb_i, where $i \neq j$. Clearly, the simple b_i-dominated specification mappings will not work, since these mappings only apply in cases of assimilation to the coefficients of the *same* feature. One must define instead of the one-to-one mappings usable in the case of assimilation of the same feature, an entire set of n-to-one mappings which equate the coefficient of a given distinctive feature in one

matrix to every possible coefficient of another matrix. Such a function might be denoted as $G_{i/j}$, and is defined as follows:

$$(22) \qquad G_{i/j}(a_p, b_q) = I, \qquad j \neq p, \qquad i \neq q; \qquad G_{i/j}(a_j, b_i) = b_i$$

The case where both Ma_i and Mb_i are fully specified would be handled by a similar function, this time a coefficient switching function $S_{i/j}$, similarly defined:

$$(23) \qquad S_{i/j}(a_p, b_q) = I, \qquad j \neq p, \qquad i \neq q; \qquad S_{i/j}(a_j, b_i) = b_i$$

There is thus seen to be no method of establishing a natural one-to-one mapping between Mb_i and Ma_i which will relate the features in each matrix in such a way as to allow for assimilation of coefficients of different features in different segments. This is so since in the case of assimilation of coefficients of the same feature, there exists a natural pairing of the feature coefficients on both sides of the equation. In the case of assimilation of coefficients of different features of the equation, one must separately ennumerate the individual features of both matrices under consideration. Instead of a one-to-one relation between the two matrices, one is forced to consider a total of $n \times n$ individual mappings, of which only n are the natural assimilation mappings defined in (14) and (20). It is thus seen that the extension of alpha variable assimilation to cases where the coefficients of two different features must be considered may only be effected at the cost of an exponential increase in formal complexity. This may be illustrated by considering a more specific example. Suppose we are working within a system employing 4 distinctive features, and suppose furthermore that we are given an incompletely specified matrix, say Ma_{i-2} where the coefficient of F_2 is unspecified; i.e. is represented by a variable. Moreover, assume that we are given two fully specified matrices Mb_i and M^*b_i as defined above, from which the coefficient of F_2 in Ma_{i-2} is to be chosen. Remaining within a system which allows only for assimilation of the coefficients of the same features, there is only one possible set of mappings which may be used to furnish a coefficient for F_2 in Ma_{i-2}; namely, g_{b2} and g^*_{b2}. This may be illustrated as follows:

$$(24) \qquad \begin{vmatrix} b_1 & F_1 \\ b_2 & F_2 \\ b_3 & F_3 \\ b_4 & F_4 \end{vmatrix} \xrightarrow{\ g_{ba}\ } \begin{vmatrix} a_1 & F_1 \\ \langle +1, -1 \rangle_2 & F_2 \\ a_3 & F_4 \\ a_4 & F_4 \end{vmatrix} \xleftarrow{\ g^*_{ba}\ } \begin{vmatrix} b_1 & F_1 \\ -b_2 & F_2 \\ b_3 & F_3 \\ b_4 & F_4 \end{vmatrix}$$

$$\qquad\qquad Mb_i \qquad\qquad\qquad\qquad Ma_{i-2} \qquad\qquad\qquad M^*b_i$$

Here there is only one choice to be made, that between g_{b2} and g^*_{b2}; in practice, this is determined by which segment is actually present. Suppose, however, that we wish to assimilate the coefficient of F_2 in Ma_{i-2} to the coefficient of, say, F_3 in Mb_i or M^*b_i. In order to do this, we must transfer to the system which, instead of providing for four mappings between Ma_{i-2} and another matrix, provide for $4 \times 4 = 16$ mappings. Thus, in the specific case under consideration, there are a total of eight mappings which could potentially supply the coefficient for F_2 in Ma_{i-2}. This may be shown as:

$$(25) \quad \begin{vmatrix} b_1 & F_1 & G_{1/a} \\ b_2 & F_2 & G_{2/a} \\ b_3 & F_3 & G_{3/a} \\ b_4 & F_4 & G_{4/a} \end{vmatrix} \rightarrow \begin{vmatrix} a_1 & F_1 \\ \langle +1, -1 \rangle_2 & F_2 \\ a_3 & F_3 \\ a_4 & F_4 \end{vmatrix} \leftarrow \begin{vmatrix} G^*_{1/a} & b_1 & F_1 \\ G^*_{2/a} & b_2 & F_2 \\ G^*_{3/a} & -b_3 & F_3 \\ G^*_{4/a} & b_4 & F_4 \end{vmatrix}$$
$$\qquad\qquad Mb_i \qquad\qquad\qquad Ma_{i-2} \qquad\qquad\qquad M^*b_i$$

Here again, the choice between Mb_i and M^*b_i will be determined by the actual linguistic context. However, once one of the latter two matrices has been selected, the choice must still be made among the four potentially operable functions. This choice is not determined by context, but calls for a purely formal means of arriving at a choice; i.e. an additional decision function. Thus it may be seen that using variables to specify different features in different segments is not only formally more complex than when the same feature is assimilated, but it requires an additional and more complicated mathematical system to even be made possible. Thus the "simplicity metric" employed in generative phonology cannot be equally applied to the three fundamentally different uses of variables outlined earlier. The use of variables within the same segment cannot be formally evaluated at all, but rather depends on language-specific data. Variables specifying the same feature in different segments may be represented by a simple and straight-forward mathematical model which highlights the inherent power of simplification contained in such a device. Finally, using variables to specify different features in different segments entails an entirely different system, which may not be evaluated in the same manner as the preceding one, but which is in any event formally more complex.

As an epilogue to the study of the formal properties of variables, it is interesting to consider some of the observations made by Newton (1972). Newton, spurred by the claims of King (1972b) and others that "drag chains" represent a simplification over the individual rules involved, set out to investigate the formal degree of simplicity of *all* possible types of covariant sound shifts. Using for his examples an idealized square vocalic system consisting of i, e, o and u, Newton isolated six different possibilities for covariant shifts: 1. *parallel rules*; e.g. $e > i, o > u$; 2. *counterparallel rules*; e.g. $i > e, o > u$; 3. *Flip-flop rules*; e.g. $i > e, e > i$; 4. *Converging rules*; e.g. $e > i, u > i$; 5. *diverging rules*; e.g. $i > e, i > u$; 6. *chain rules*; e.g. $i > e, e > o$. All these

pairs of rules (except parallel rules, which may be collapsed by "feature suppression") may be collapsed by the use of variables, and with some rather surprising formal results. Newton notes (p. 43): "If the direction taken by the second of a sequence of two rules were determined by the simplicity of the composite representation allowed by alpha notation, parallel, counterparallel and flip-flop rules would as a group be commonest, and converging, diverging and chain rules tie for last place. For example, if we had $i > e, \ldots$ we would expect this to trigger, in order of probability (a) $u > o$ or $o > u$, (b) $e > i$, (c) either $u > i, e > o$ or $i > u$." This statement, however, appears rather strange in view of the specific situations described. Given the change $i > e$, the parallel change $u > o$ would not seem particularly surprising.[4] Also, the changes $e > o$ ("push-chain") and $u > i$ ("drag-chain") find their analogues among the history of many languages. The flip-flop shift of $e > i$ would be unusual under ordinary circumstances, but such examples have evidently occurred in some languages.[5] However, the changes $o > u$ ur $i > u$ would be, under normal conditions, most unlikely. The reason for these rather disparate results seems to be that Newton, like most other investigators, has counted the use of variables equally regardless of the contexts in which they occur. In the case of the six possible pairs of covariant shifts, however, variables are used in several different ways. As noted above, parallel rules do not require the use of variables in their formulation, but rather employ feature suppression (in the case of $i > e, u > o$, the feature *front* or *back*), and thus represent formally valid, although relatively uncommon form of simplification. Diverging and converging pairs of rules make use of variables relating two features in the same segment; therefore, as shown above, they should not be formally evaluated in the same manner as an ordinary assimilation rule using variables; in fact, they should not be formally evaluated at all. There is no reason to suppose, for example, that the emergence of converging or diverging rules represents a natural or predictable evolution; those relatively few attested cases of the converging or diverging of sounds all seem to have resulted from fortuitous historical accidents rather than from the principled interaction of a pair of rules. Counterparallel and chain rules both involve variables representing different features in different segments, and hence cannot be evaluated by a metric which simply counts variables less than plus or minus values. If chain-type rule pairs are more common than counterparallel pairs, this should be attributed to the particular paradigmatic configuration presupposed by these two situations, since formally both types are equivalently constructed. This is analogous to the fact that, while certain features, such as nasality, voi-

cing and sometimes even occlusion are often assimilated, cases in which such features as vocalicity, consonanticity, etc. are assimilated are relatively rare.

Of the six types of rule pairs isolated by Newton, only flip-flop rules involve the use of variables to specify the same feature in different segments. In this sense, then, flip-flop rules represent a formal simplification of their constituent rules, and again their relative scarcity should be regarded as a result of the situation they define: the switching of two sounds is simply not a normal form of sound change.[6]

In conclusion, it is proposed that linguistics take a further look at the employment of variables in phonological rules, to determine where they may be legitimately considered as a true formal simplification, and when they serve merely as a convenient descriptive device. The use of variables in phonological descriptions can serve as a tool of not only descriptive but also even explanatory power, but when used incautiously, variables may also admit into linguistics an uncontrollable array of specious and meaningless formulations.

References

Anttila, R. 1972: An Introduction to Historical and Comparative Linguistics. New York.

Chomsky, N. 1957: Syntactic Structures. The Hague.

Chomsky, N. and Halle, M. 1968: The Sound Pattern of English. New York.

Contreras, H. 1969: Simplicity, Descriptive Adequacy, and Binary Features. In: Language 45.1—8.

Cressey, W. W. 1970: A Note on Specious Simplifications and the Theory of Markedness. In: Papers in Linguistics 2.227—37.

Hall, R. A., Jr. 1970: Some Recent Developments in American Linguistics. In: Neuphilologische Mitteilungen 70.192—227.

Halle, M. 1962: A Descriptive Convention for Treating Assimilation and Dissimilation. In: Quarterly Progress Report No. 63, M. I. T. Research Laboratory of Electronics, 295—6.

Harms, R. T. 1968: Introduction to Phonological Theory. Englewood Cliffs, N. J.

Harris, J. 1970: Sequences of Vowels in Spanish. In: Linguistic Inquiry. 1.129—34.

King, R. D. 1969a: Historical Linguistics and Generative Grammar. Englewood Cliffs, N. J.

—   1969b: Push chains and drag chains. In: Glossa 3.3—21.

Kiparsky, P. 1965: Phonological Change. (Unpublished Rh. D. dissertation, M. I. T.)

—   1968: Linguistic Universals and Linguistic Change. In: Universals in Linguistic Theory, ed. Bach & Harms, 170—202.

—   1971: Historical Linguistics. In: A Survey of Linguistic Science, ed. Dingwall, 576—649.

Lightner, Th. M. 1963: A Note on the Formulation of Phonological Rules. In: Quarterly Progress Report No. 68, M. I. T. Research Laboratory of Electronics, 187—89.

Newton, B. E. 1972: Are Drag Chains Really Simple? In: Linguistische Berichte 19.38—44.

Stanley, R. 1967: Redundancy Rules in Phonology. In: Language 43.393—436.

Wheeler, M. W. 1972: Distinctive Features and Natural Classes in Phonological Theory. In: Journal of Linguistics 8.87—102.

Zwicky, A. M. 1970: Greek-Letter Variables and the Sanskrit *ruki* Class. In: Linguistic Inquiry 1.549—55.

[6] Switching, that is, in the sense of the two sounds crossing over phonetically at some point. There is nothing unusual for a given sound *a* to shift to *b*, while *b* shifts to another sound *c*, which later becomes *a*. In such a case of discrete processes, however, it is not legitimate to collapse them into a single rule, even though the end result may suggest a single process.