Image Enhancement

Define a spatial domain enhancement operator by

$$g(x,y) = T[f(x,y)]$$

where $f(x,y)$ is the input image, $g(x,y)$ is the output image, and T is a function of the pixels of $f(x,y)$ in a neighborhood $N(x,y)$ of (x,y).

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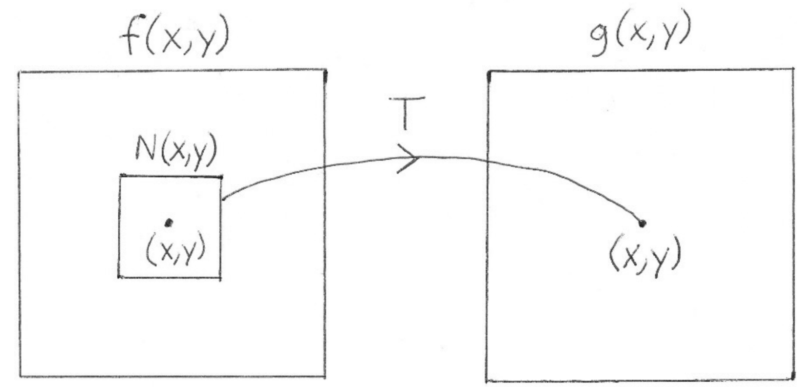

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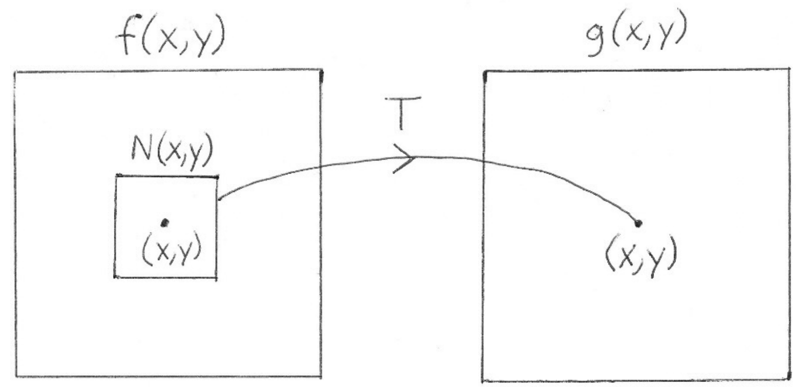

T is applied to the neighborhood of every pixel (x,y) in the input image to generate the pixel value at (x,y) in the output image.

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$$g(x,y) = T[f(x,y)]$$

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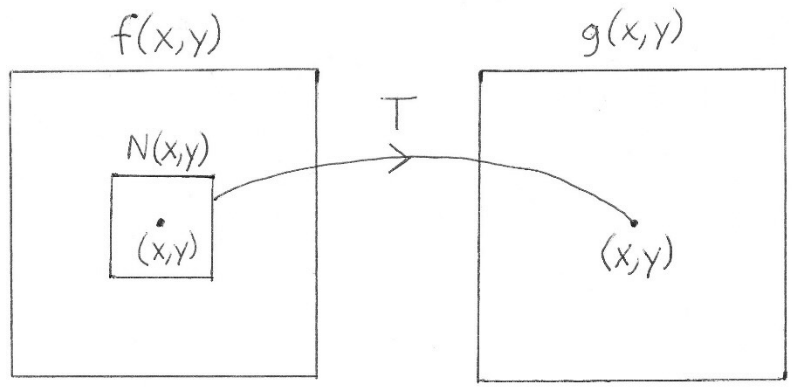

T is applied to the neighborhood of every pixel (x,y) in the input image to generate the pixel value at (x,y) in the output image.

$N(x,y)$ is usually a square or rectangle centered at (x,y).

Gray-level transformation (GLT) - $N(x,y)$ is a 1×1 neighborhood so that at any (x,y), $g(x,y)$ depends only on $f(x,y)$.

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A GLT can be represented by $s = T(r)$ where r is the gray-level at (x,y) in the input image and s is the gray-level at (x,y) in the output image.

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A GLT can be represented by $s = T(r)$ where r is the gray-level at (x,y) in the input image and s is the gray-level at (x,y) in the output image.

Contrast stretching - a GLT where dark pixels are made darker and bright pixels are made brighter

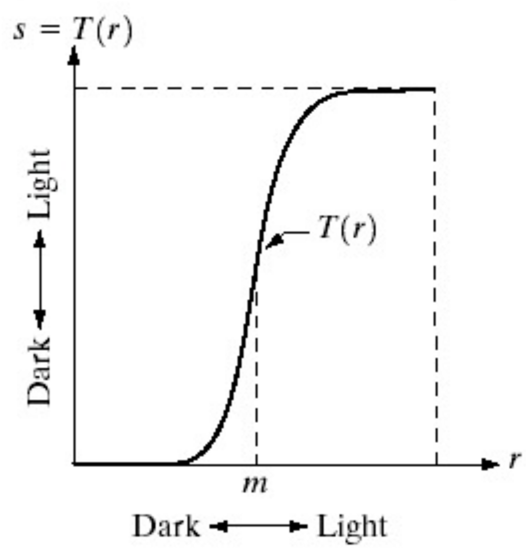
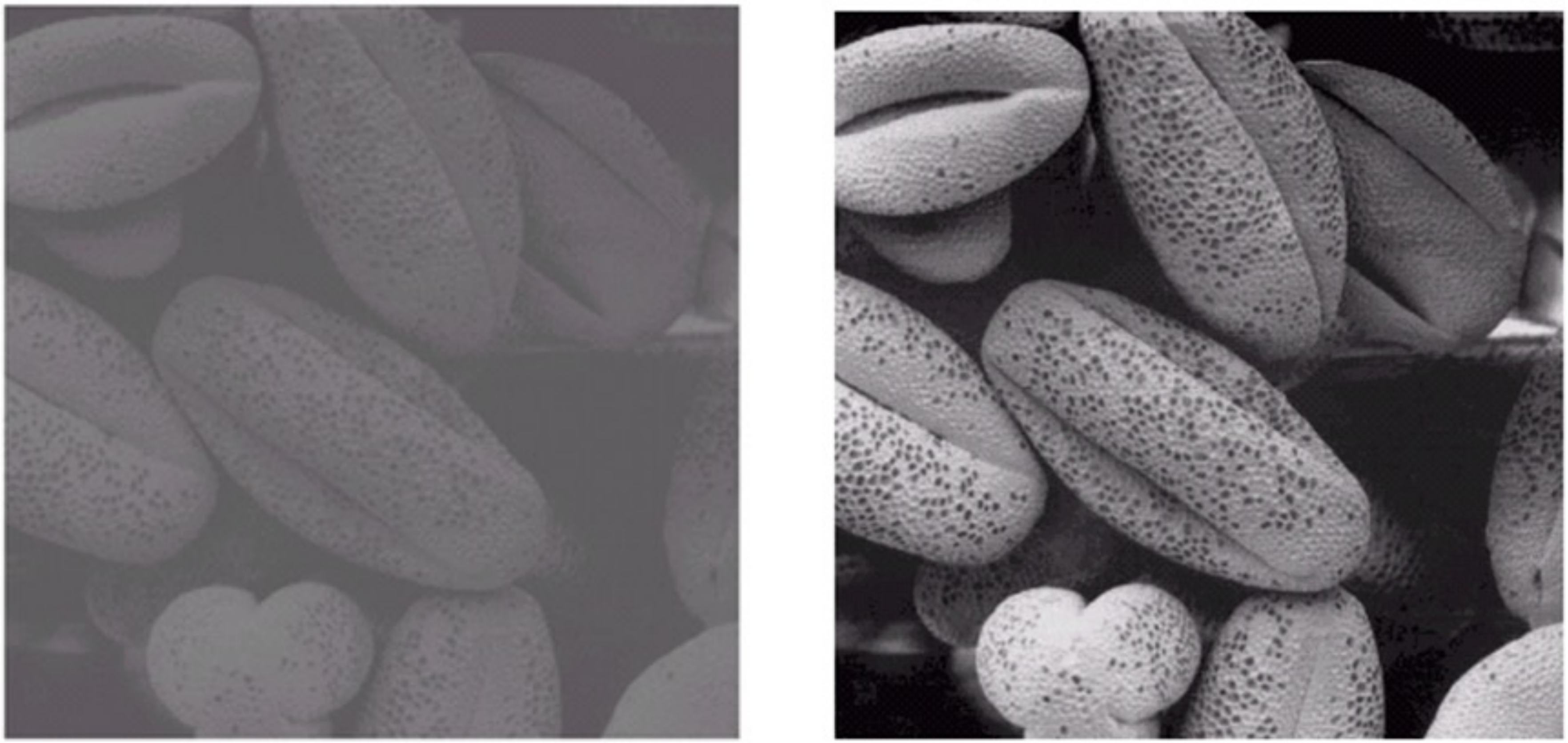


FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

FIGURE 3.10
Contrast stretching.



Thresholding – a GLT where pixels with a brightness less than a threshold are set to the minimum gray level and pixels with a brightness greater than or equal to the threshold are set to the maximum gray level

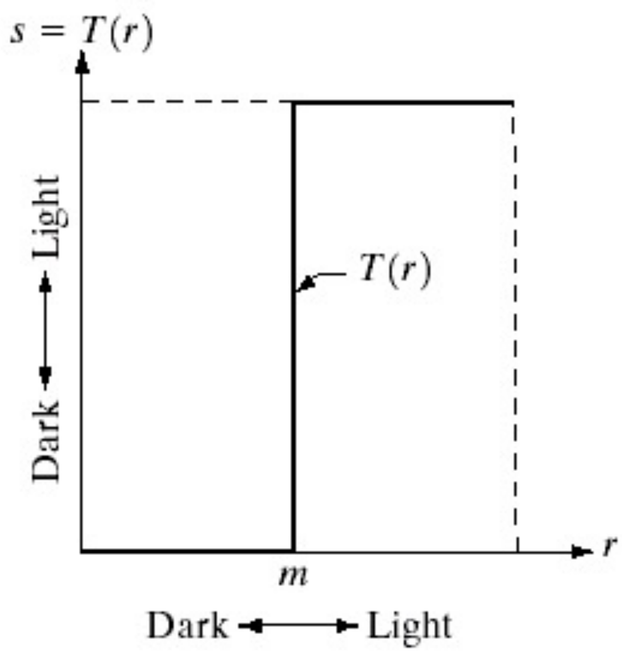


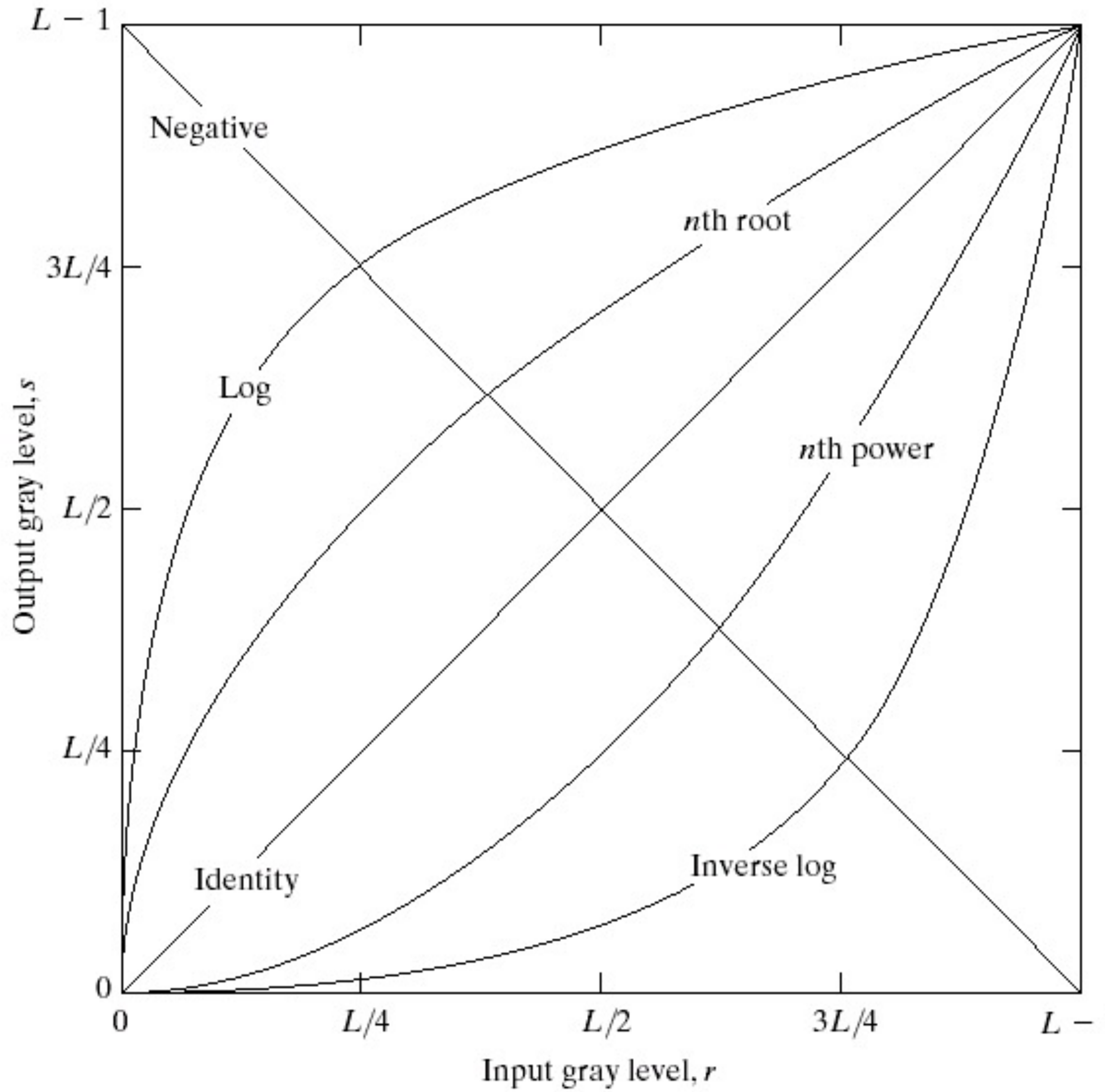
FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Thresholding

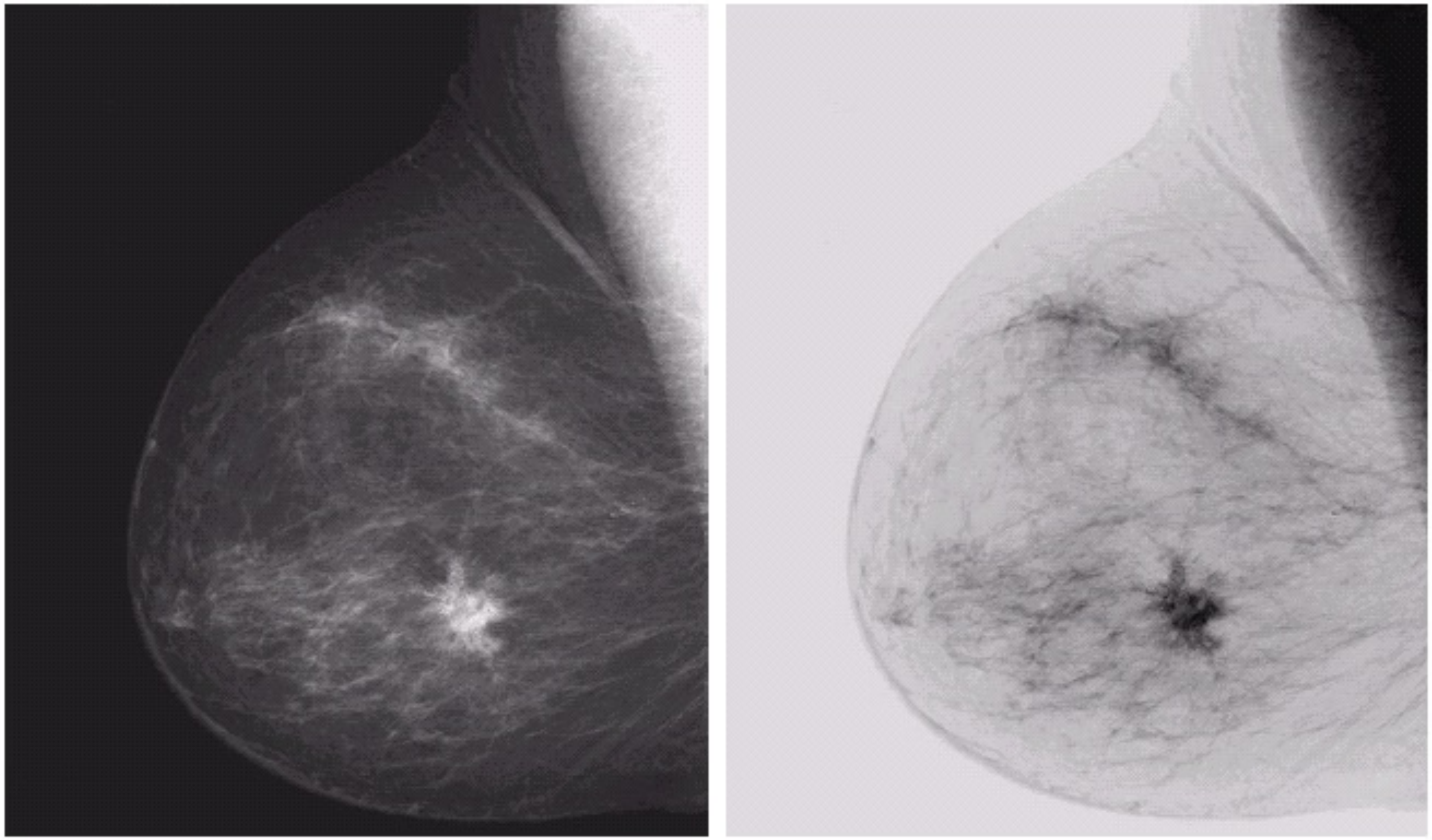
FIGURE 3.10

Negative – an image with gray levels in the range $[0, L-1]$ is transformed according to GLT $s = L-1-r$

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



Negative



a b

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

Log – a GLT defined by $s = c\log(1+r)$ for $r \geq 0$ where c is chosen so maximum r level maps to maximum s level. The log GLT provides more dynamic range for dark gray levels and less dynamic range for bright gray levels. The log GLT will brighten the image.

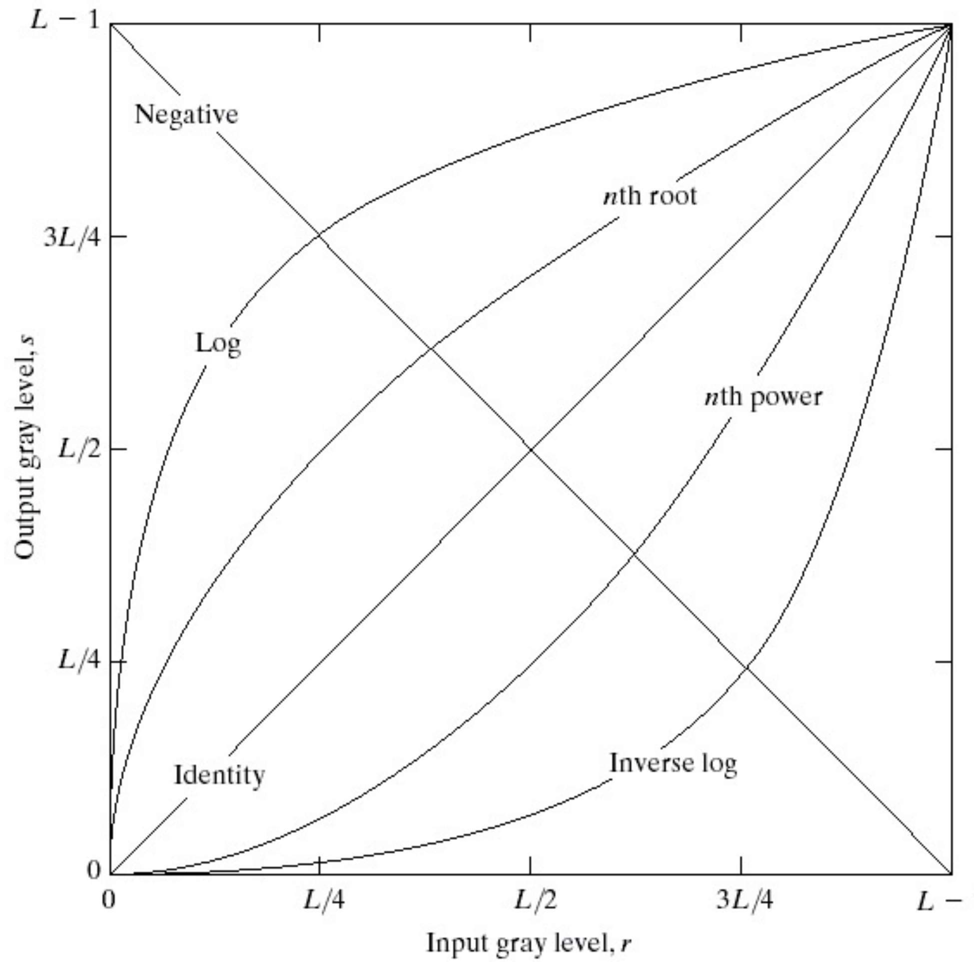


FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

Log

a b

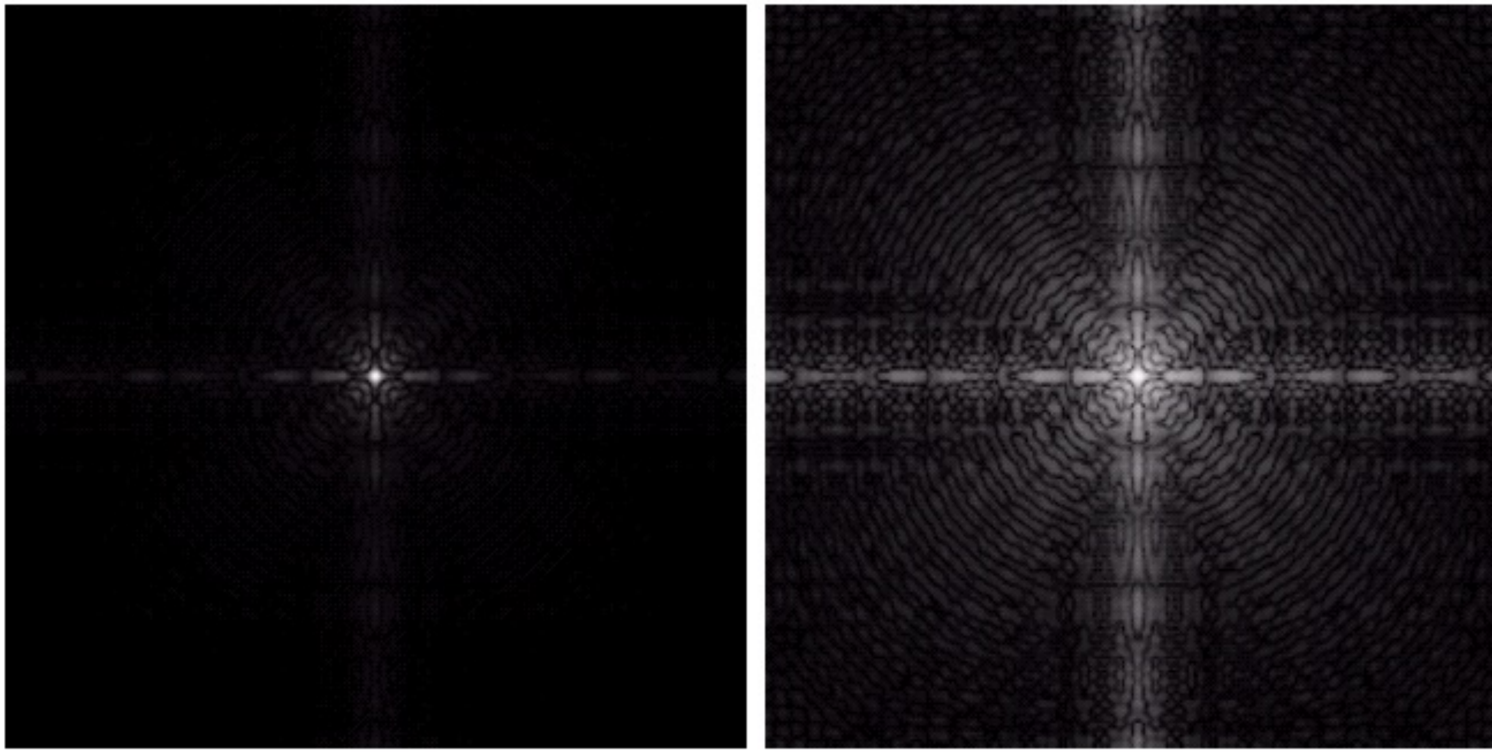


FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

Power Law – a GLT defined by $s = cr^{\gamma}$ for $c > 0$, $\gamma > 0$ where c is chosen so maximum r level maps to maximum s level.

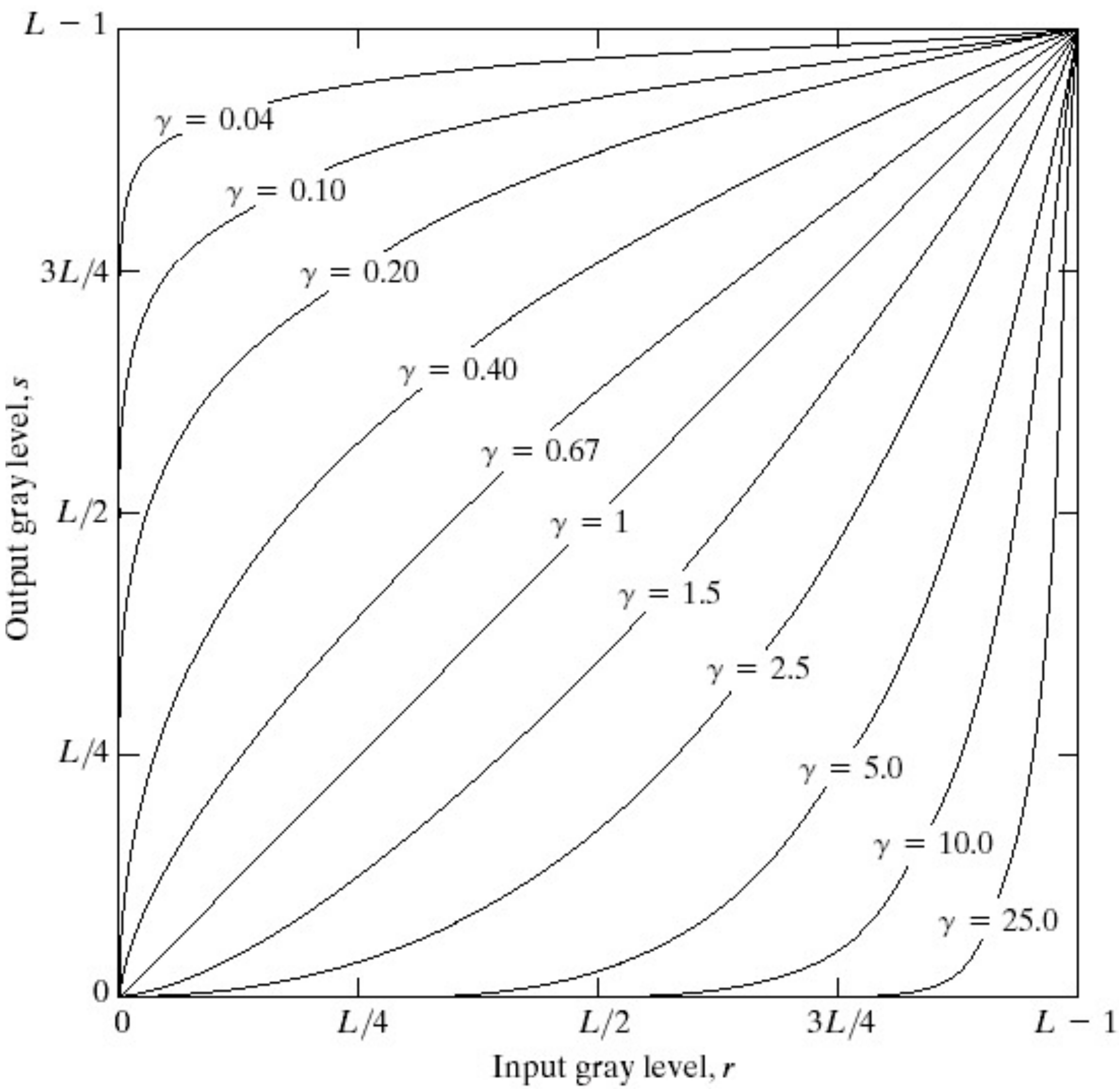


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

$\gamma < 1$ More dynamic range for dark gray levels
Less dynamic range for bright gray levels
Brightens image

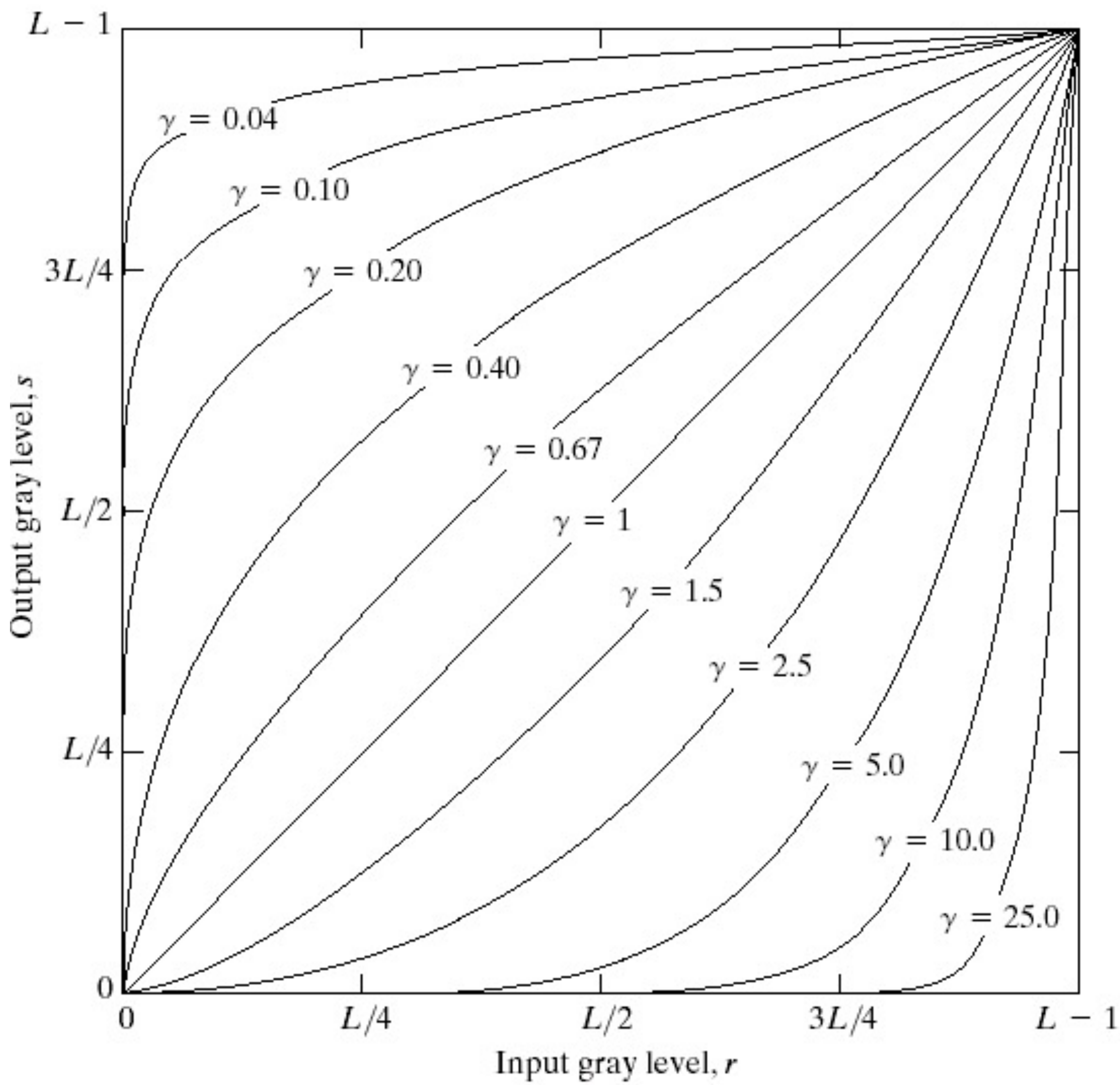


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

$\gamma = 1$ Linear scaling of input

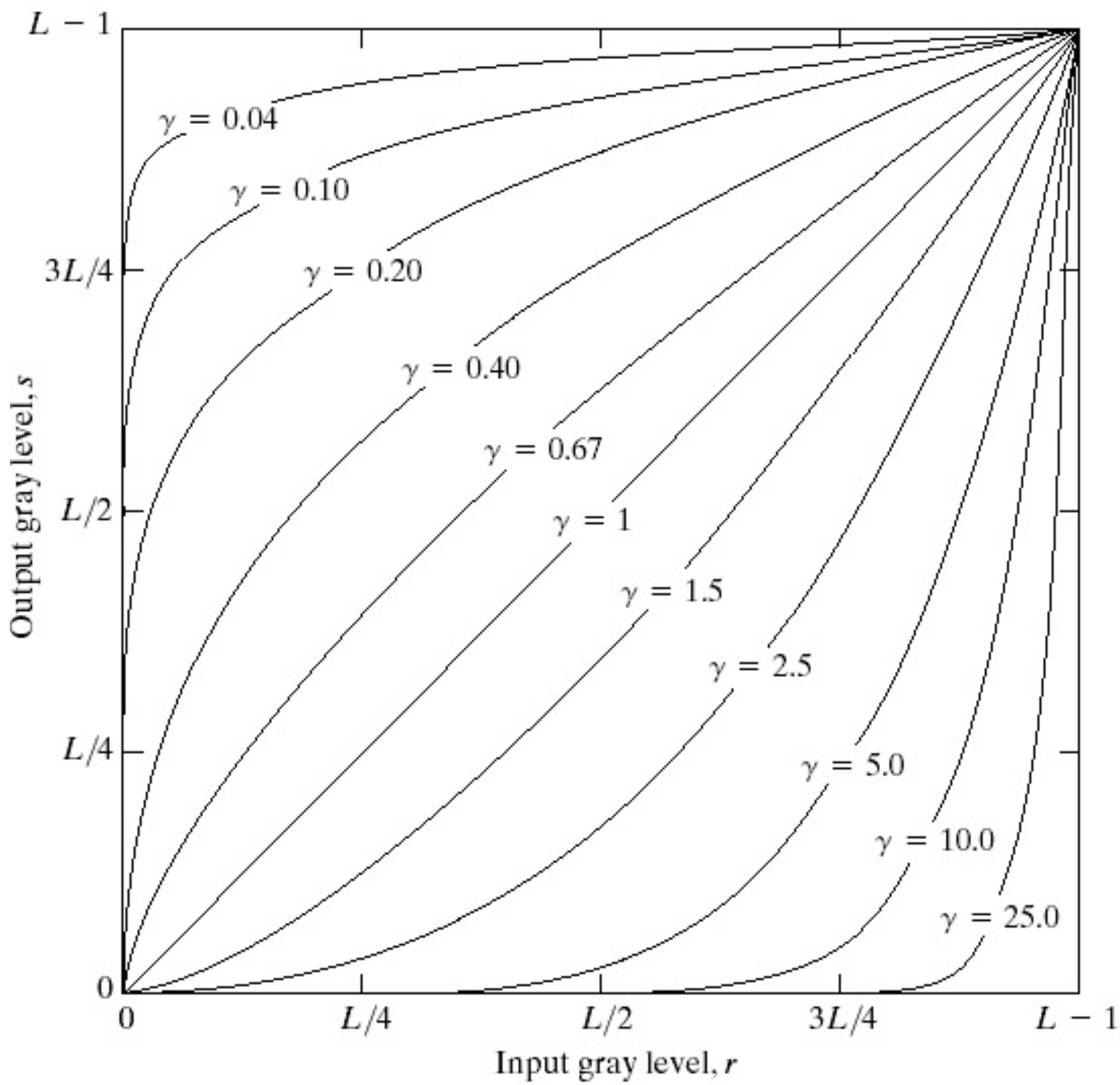


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

$\gamma > 1$ More dynamic range for bright gray levels
Less dynamic range for dark gray levels
Darkens image

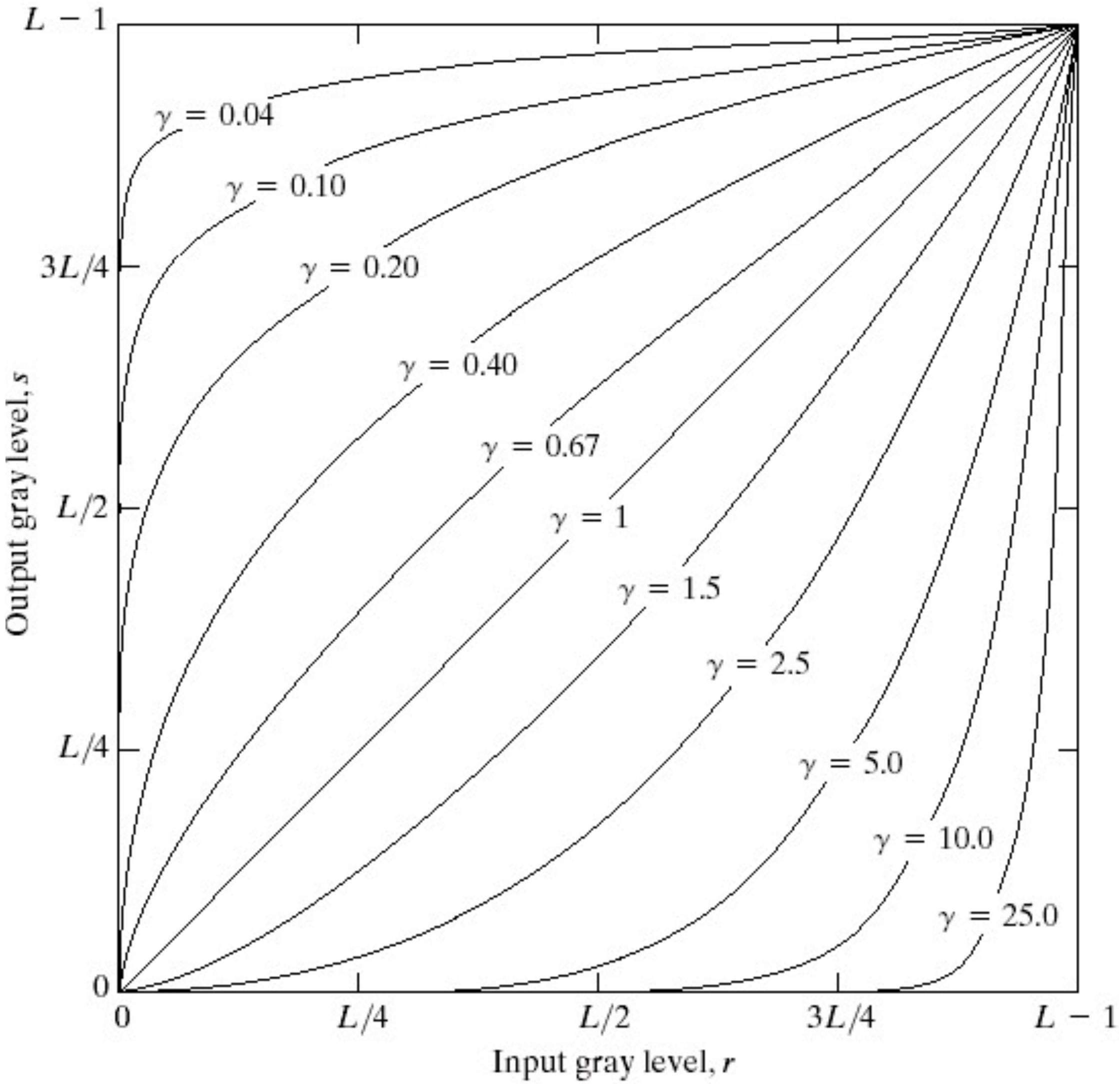
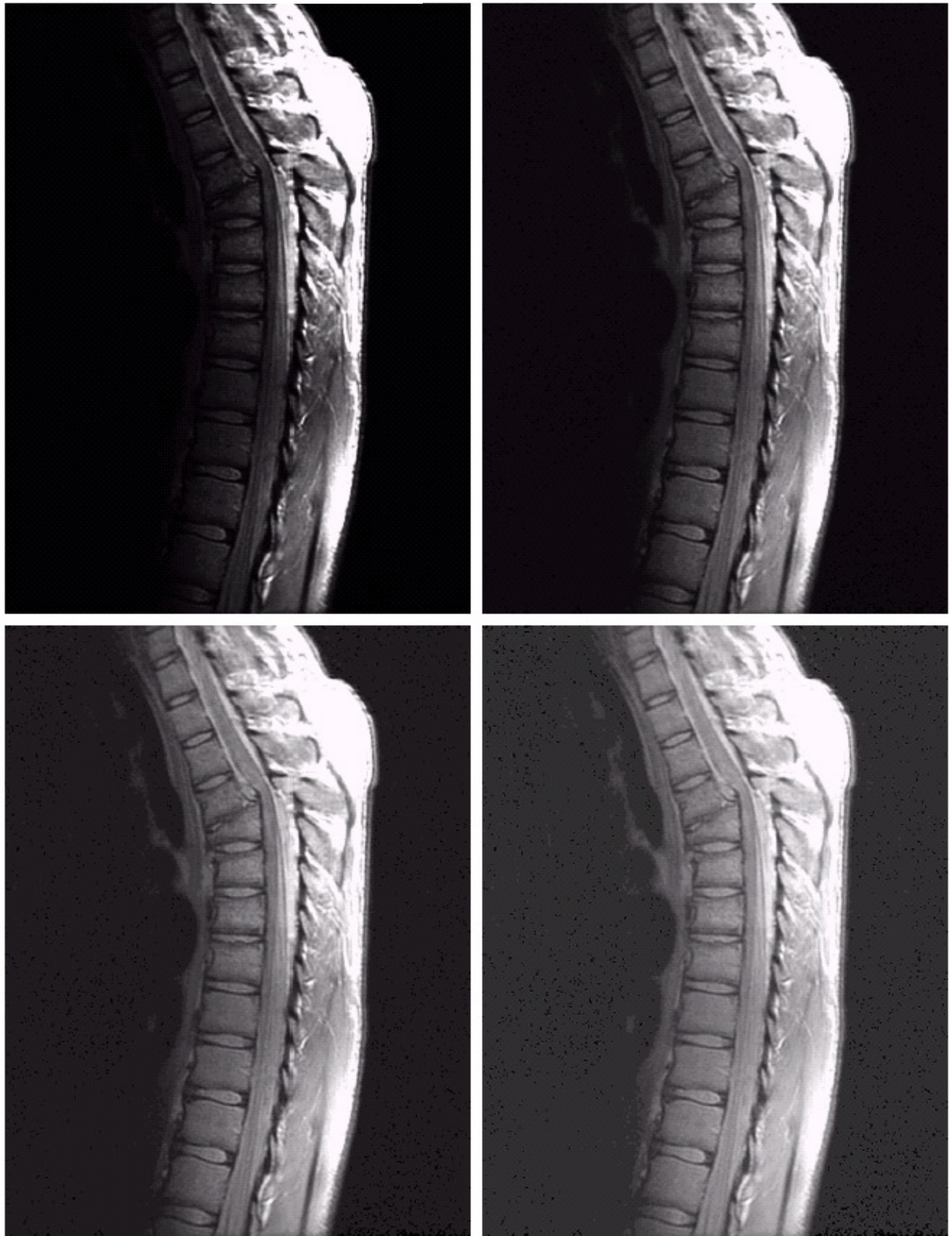


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

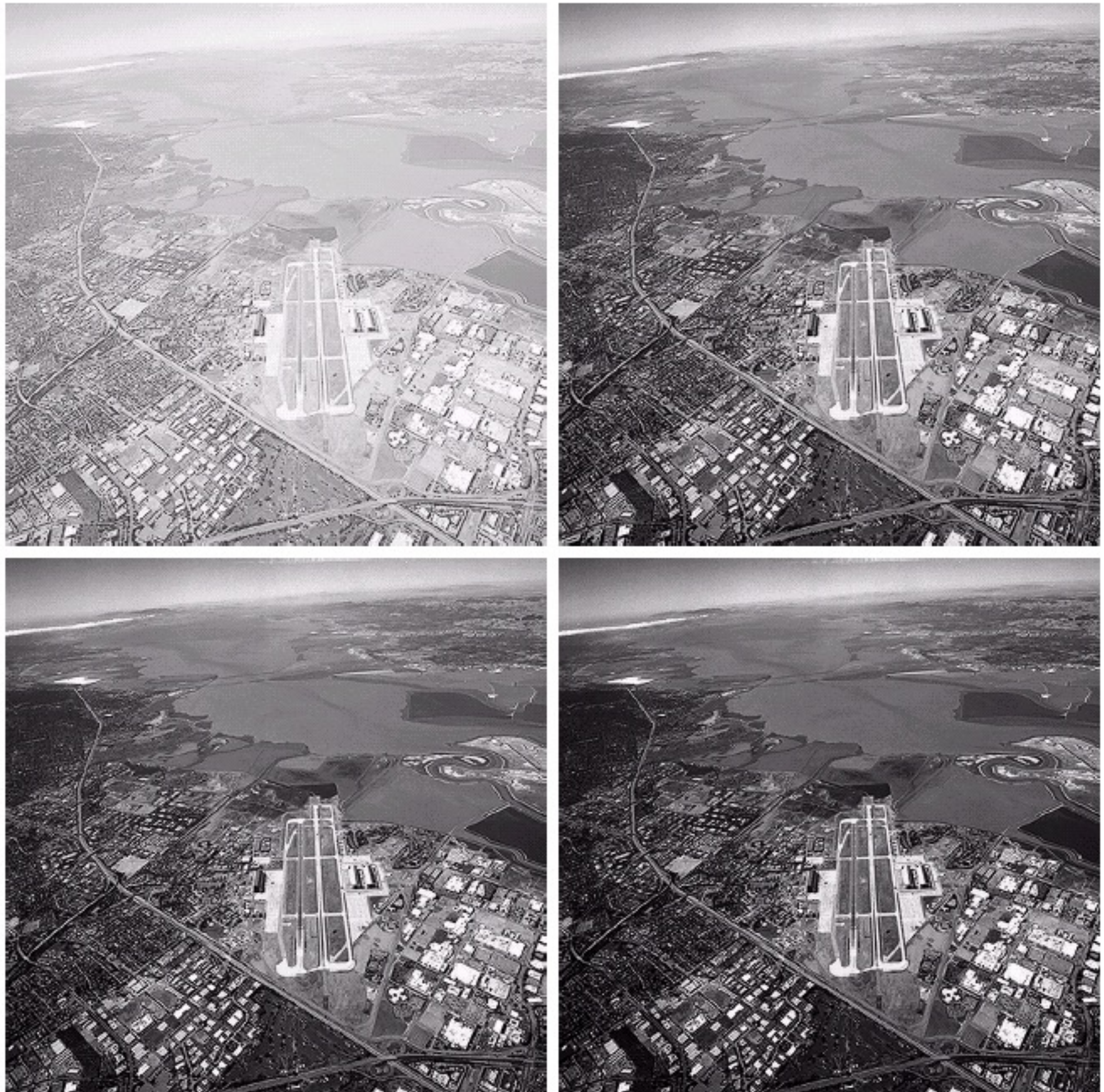
$\underline{\gamma < 1}$ Brightens image



a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

$\underline{\gamma > 1}$ Darkens image



a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Gamma Correction

Displays produce an output $O = c I^{\gamma}$ for input I with $1.8 \le \gamma \le 2.5$. Since $\gamma > 1$, the displayed image is too dark.

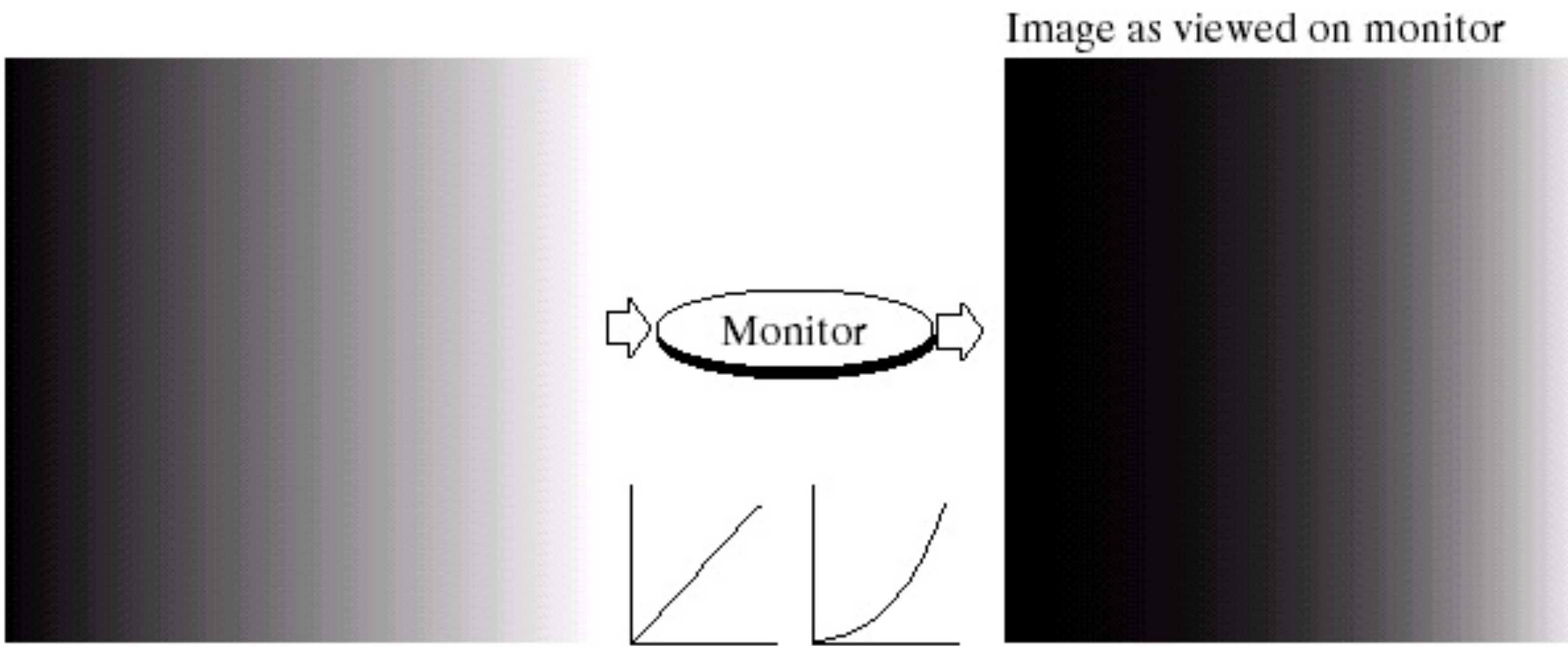


FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.

Gamma correction – if display output is $O = I^{\gamma}$, then apply GLT $s = I^{1/\gamma}$ before display so that $O = (I^{1/\gamma})^{\gamma} = I$

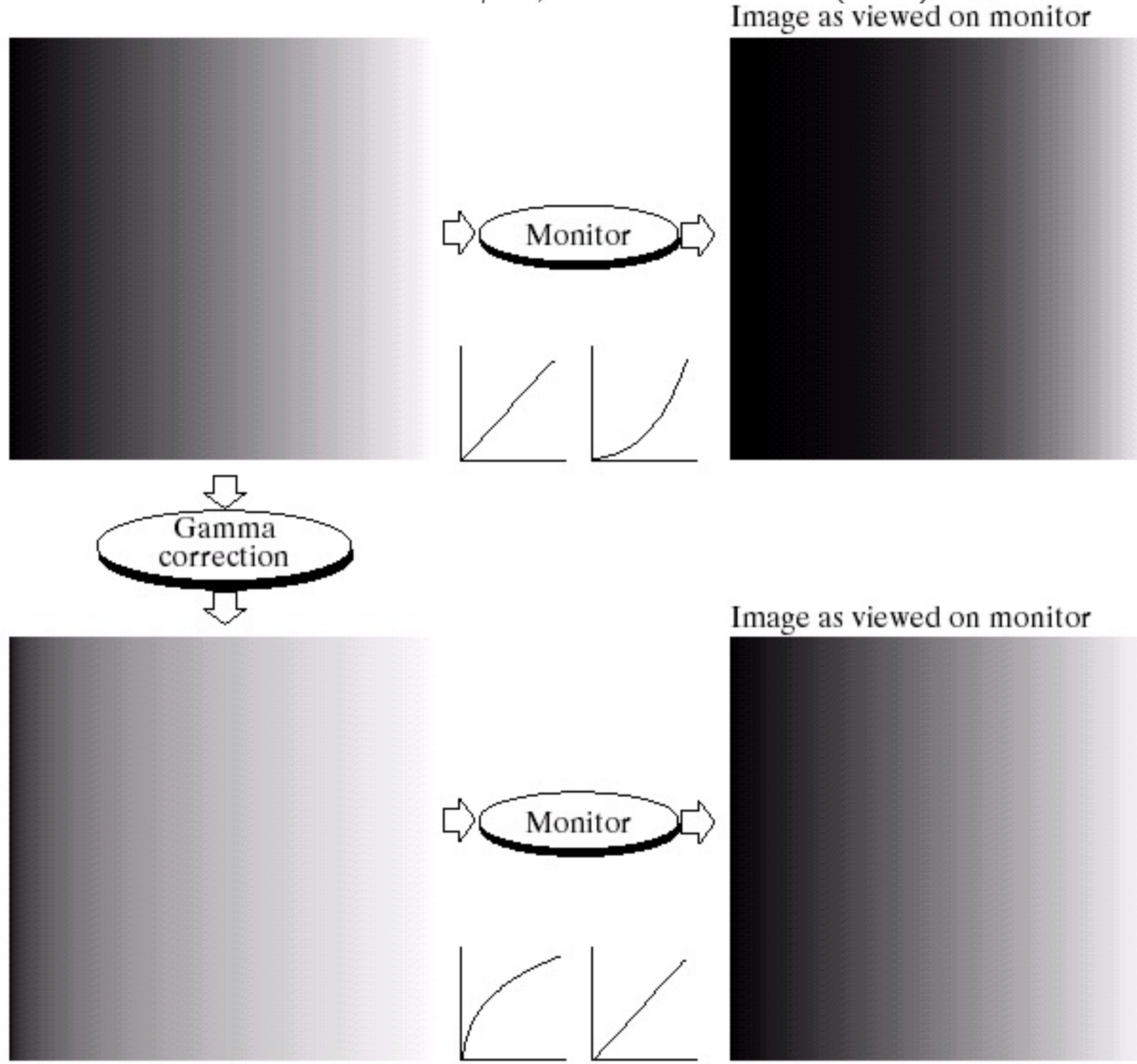
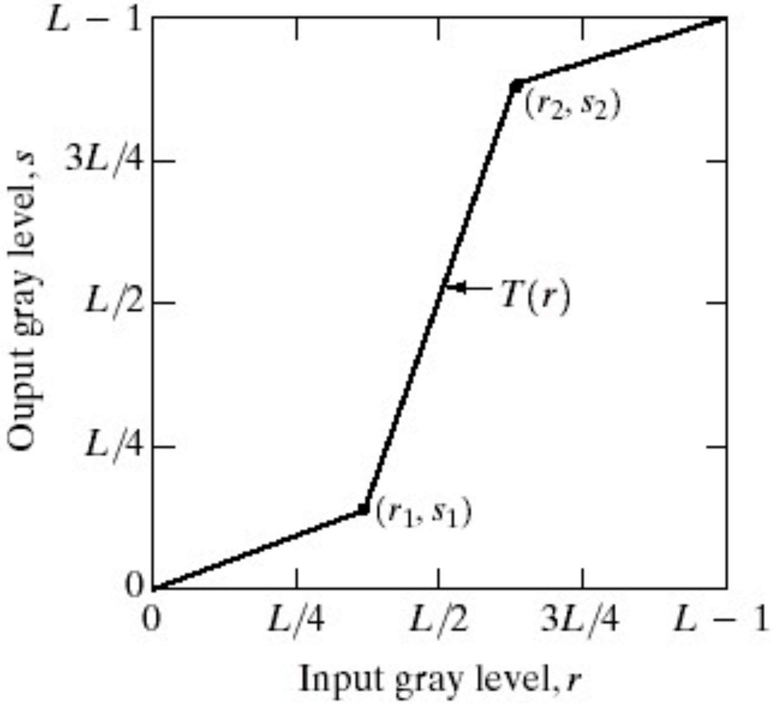


FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

Piecewise Linear Contrast Stretching

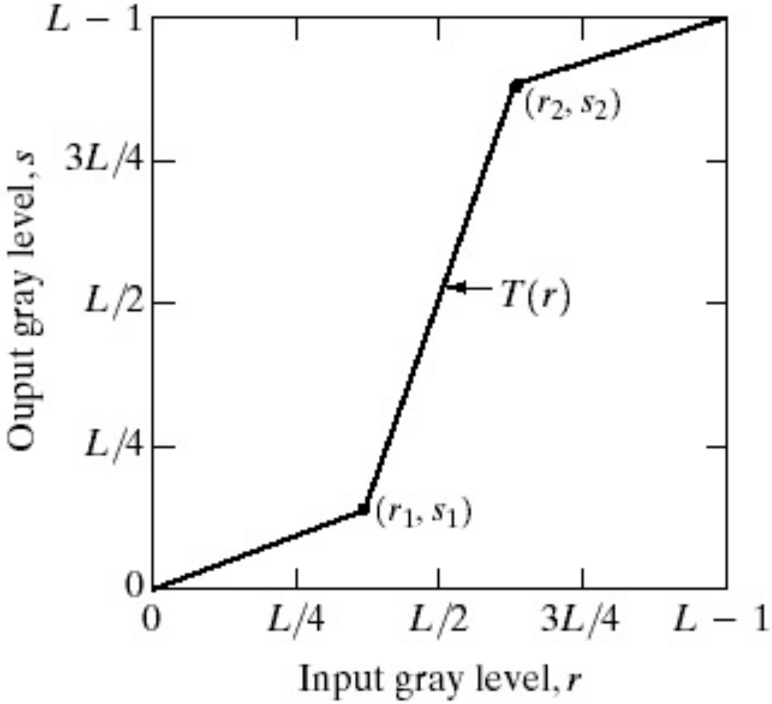
FIGURE 3.10 Contrast stretching.



identity GLT $\quad r_1 = s_1, \; r_2 = s_2$

Piecewise Linear Contrast Stretching

FIGURE 3.10 Contrast stretching.

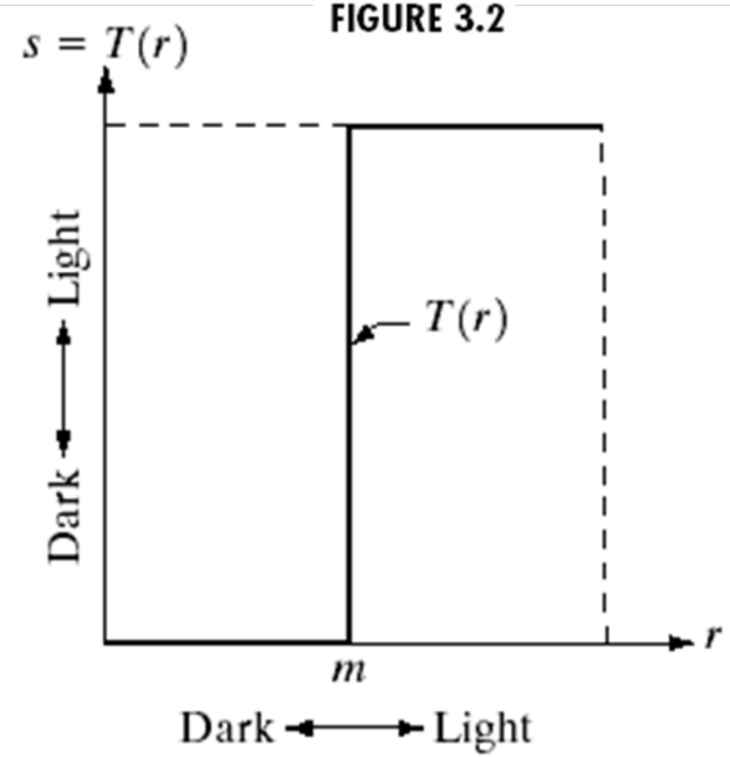


identity GLT $r_1 = s_1$, $r_2 = s_2$

threshold GLT

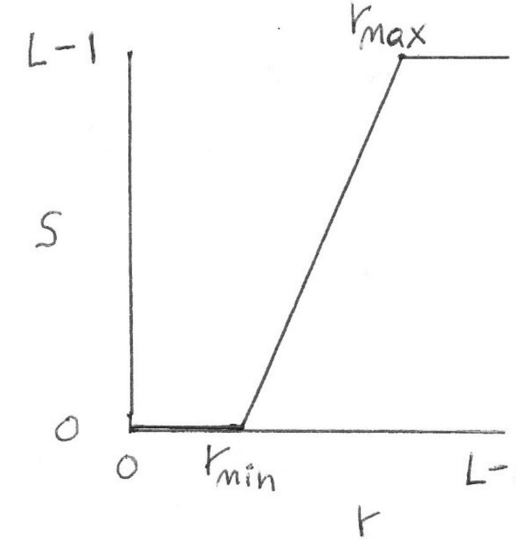
$r_1 = r_2$, $s_1 = 0$, $s_2 = L-1$

FIGURE 3.2



If input image has minimum gray level r_{min} and maximum gray level r_{max}, then can set

$(r_1, s_1) = (r_{min}, 0) \quad (r_2, s_2) = (r_{max}, L-1)$



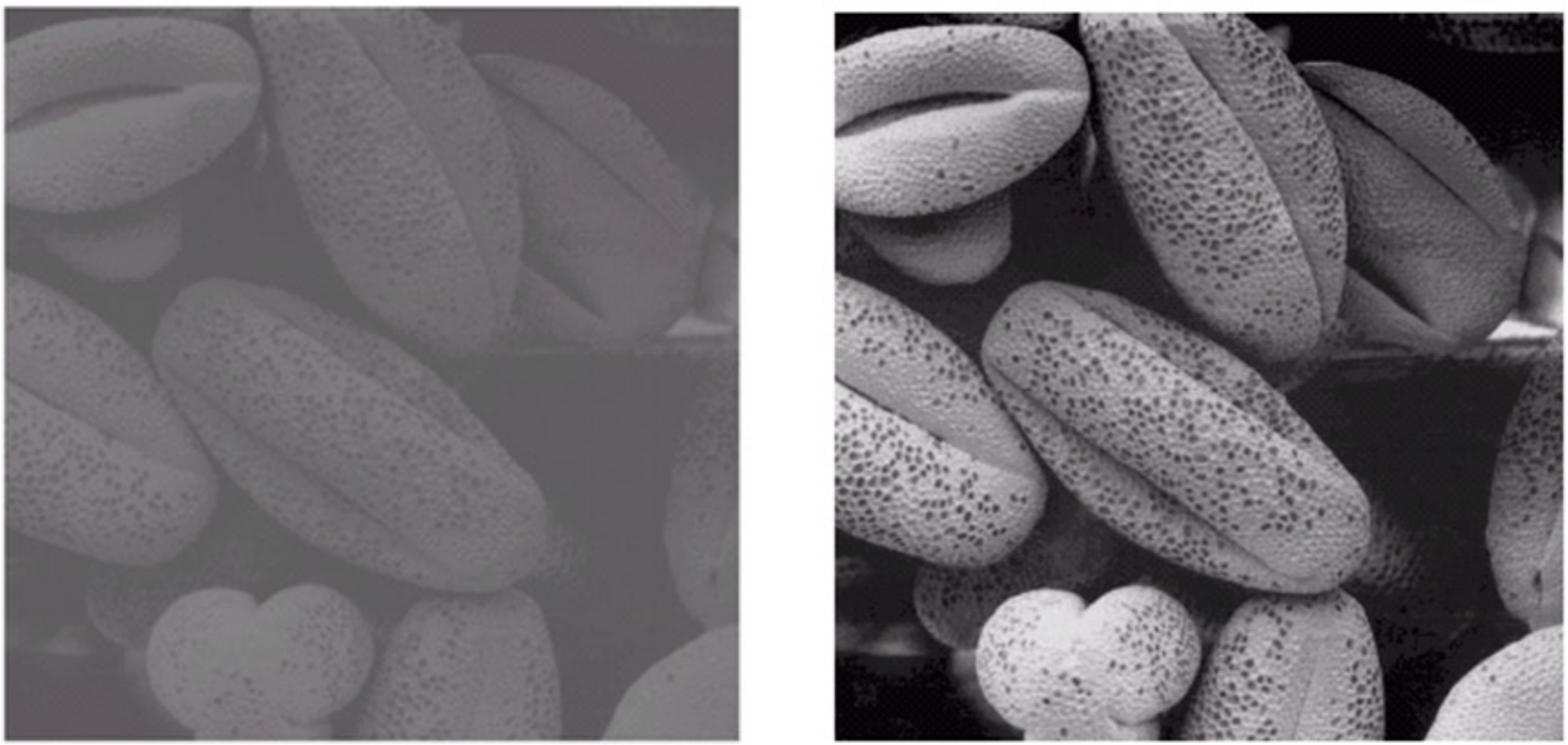
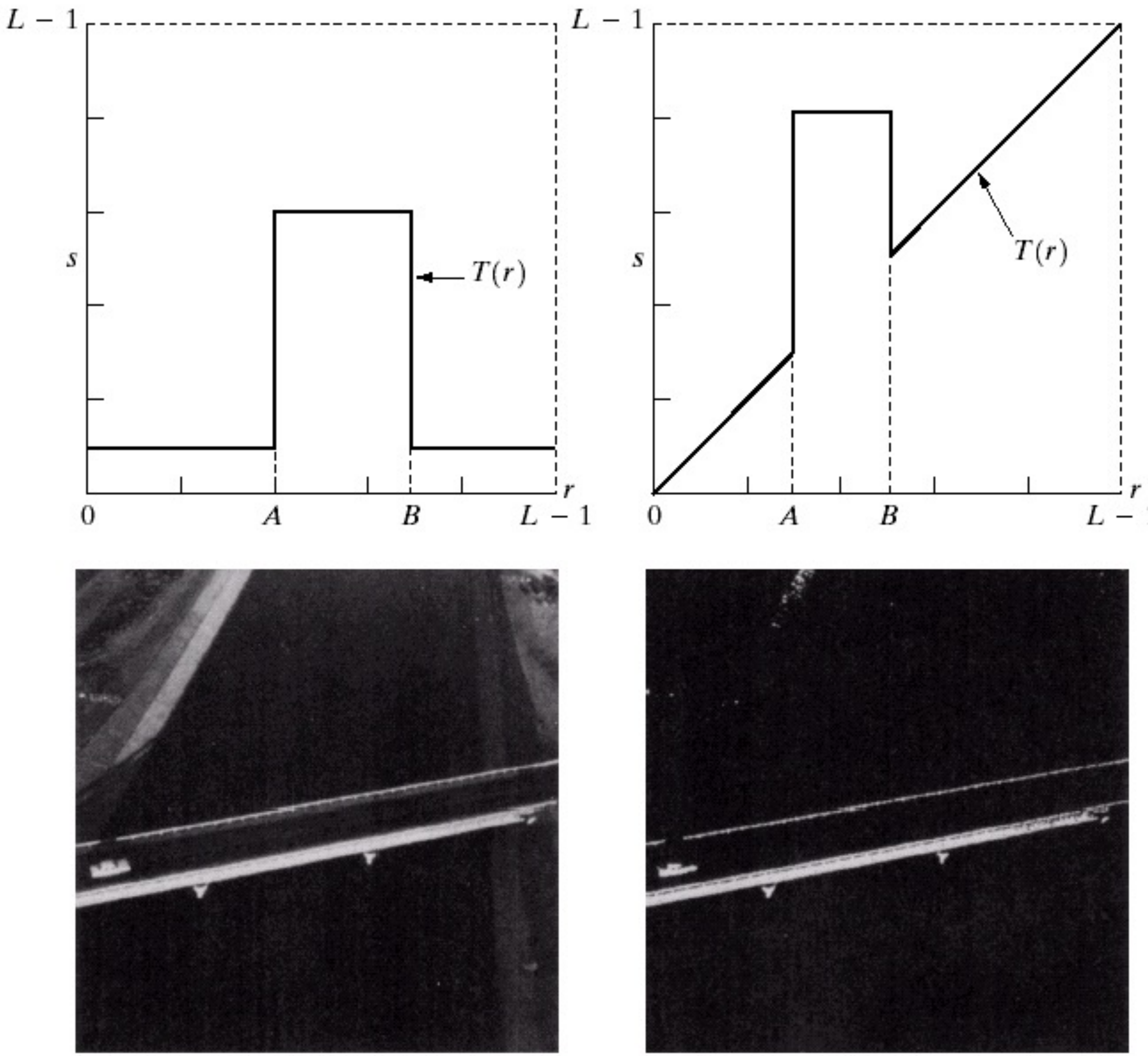


FIGURE 3.10
Contrast stretching.

Piecewise Linear Gray-Level Slicing – a GLT that enhances a specific range of gray levels



a b
c d

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Histogram Processing

The histogram of a digital image is the discrete function $h(r_k) = n_k$ where r_k is the kth gray level and n_k is the number of pixels in the image with gray level r_k

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A normalized histogram is defined by $p(r_k) = \frac{n_k}{n}$ where n is the total number of pixels in the image.

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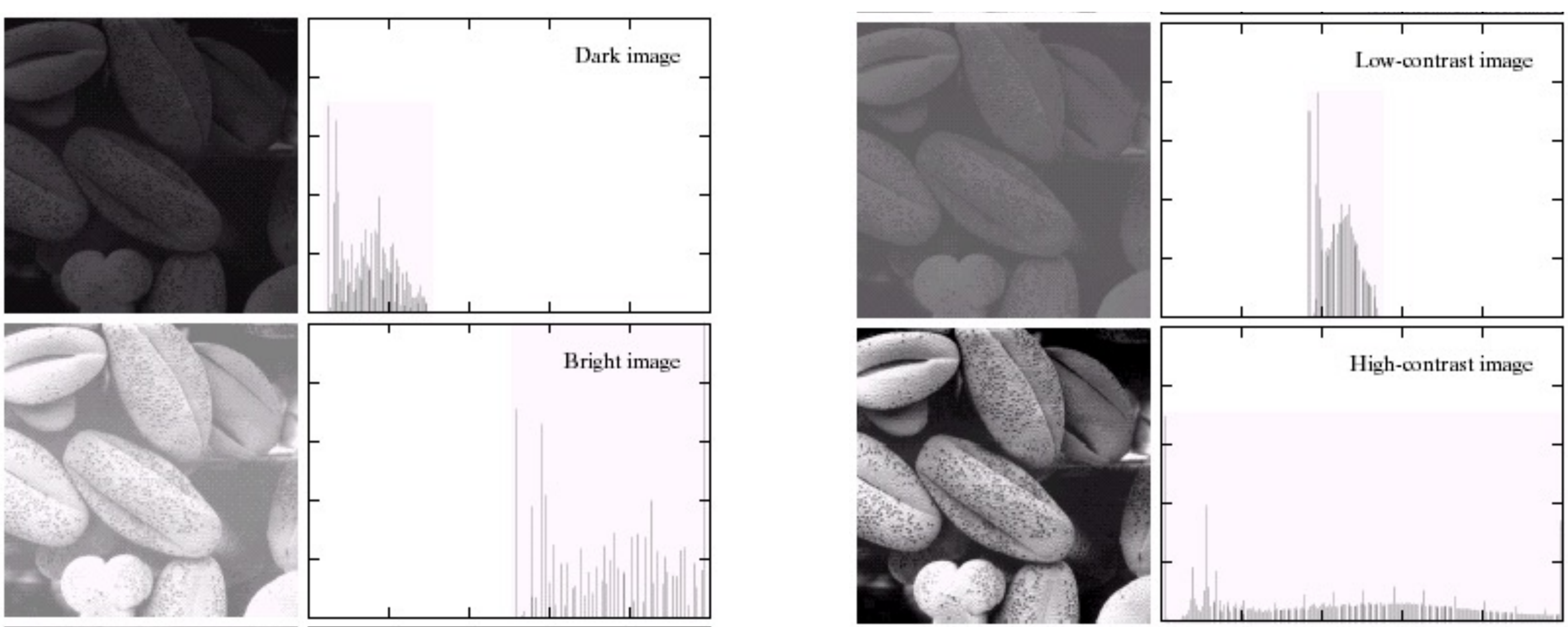

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Histogram Matching

Let $p_r(r_k)$ be the histogram for an input image

Let $p_z(z_k)$ be the desired histogram for the output image

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Let $p_r(r_k)$ be the histogram for an input image

Let $p_z(z_k)$ be the desired histogram for the output image

We want a monotonically nondecreasing GLTM that maps each r_k to a z_k so that the histogram of the transformed input image is as close as possible to the desired histogram.

Histogram Matching

Let $p_r(r_k)$ be the histogram for an input image

Let $p_z(z_k)$ be the desired histogram for the output image

We want a monotonically nondecreasing GLT M that maps each r_k to a z_k so that the histogram of the transformed input image is as close as possible to the desired histogram.

Monotonically nondecreasing means

if $r_k > r_i$ then $M[r_k] \geq M[r_i]$

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Define cumulative distributions

$$T(r_k) = \sum_{j=0}^{k} p_r(r_j) \qquad G(z_k) = \sum_{i=0}^{k} p_z(z_i)$$

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if $r_k > r_i$ then $M[r_k] \geq M[r_i]$

Define cumulative distributions

$$T(r_k) = \sum_{j=0}^{k} p_r(r_j) \qquad G(z_k) = \sum_{i=0}^{k} p_z(z_i)$$

Map each r_k to z_k so that $T(r_k)$ is as close as possible to $G(z_k)$

Ex

Input

r_k	p_r	$T(r_k)$
0	10	10
1	10	20
2	3	23
3	2	25
4	0	25

Desired

z_k	p_z	$G(z_k)$
0	5	5
1	5	10
2	5	15
3	5	20
4	5	25

Ex

Input

r_k	p_r	$T(r_k)$
0	10	10
1	10	20
2	3	23
3	2	25
4	0	25

Desired

z_k	p_z	$G(z_k)$
0	5	5
1	5	10
2	5	15
3	5	20
4	5	25

Map each r_k to z_k so that $T(r_k)$ is as close as possible to $G(z_k)$

r_k	$M(r_k)$
0	1
1	3
2	4
3	4
4	4

What is the histogram of the output image $O(r_k)$?

Input

r_k	P_r	$T(r_k)$
0	10	10
1	10	20
2	3	23
3	2	25
4	0	25

r_k	$M(r_k)$
0	1
1	3
2	4
3	4
4	4

What is the histogram of the output image $O(r_k)$?

Input

r_k	p_r	$T(r_k)$
0	10	10
1	10	20
2	3	23
3	2	25
4	0	25

r_k	$M(r_k)$
0	1
1	3
2	4
3	4
4	4

10 pixels of gray level 0 map to gray level 1

10 pixels of gray level 1 map to gray level 3

3 pixels of gray level 2 map to gray level 4

2 pixels of gray level 3 map to gray level 4

What is the histogram of the output image $O(r_k)$?

Input

r_k	P_r	$T(r_k)$
0	10	10
1	10	20
2	3	23
3	2	25
4	0	25

r_k	$M(r_k)$
0	1
1	3
2	4
3	4
4	4

r_k	$O(r_k)$
0	0
1	10
2	0
3	10
4	5

10 pixels of gray level 0 map to gray level 1

10 pixels of gray level 1 map to gray level 3

3 pixels of gray level 2 map to gray level 4

2 pixels of gray level 3 map to gray level 4

Histogram Equalization refers to histogram matching when the desired histogram is uniform.

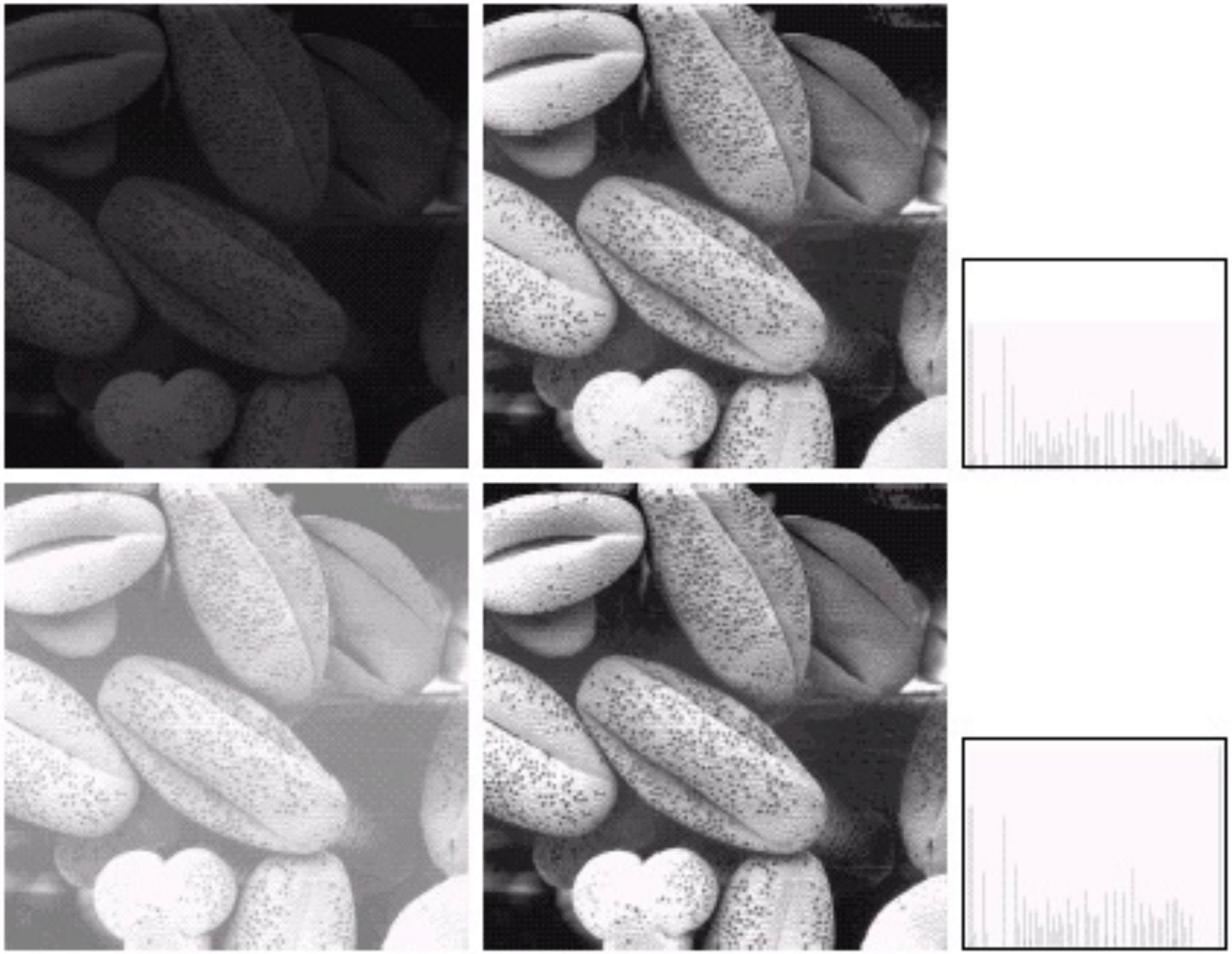


FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

Histogram Equalization

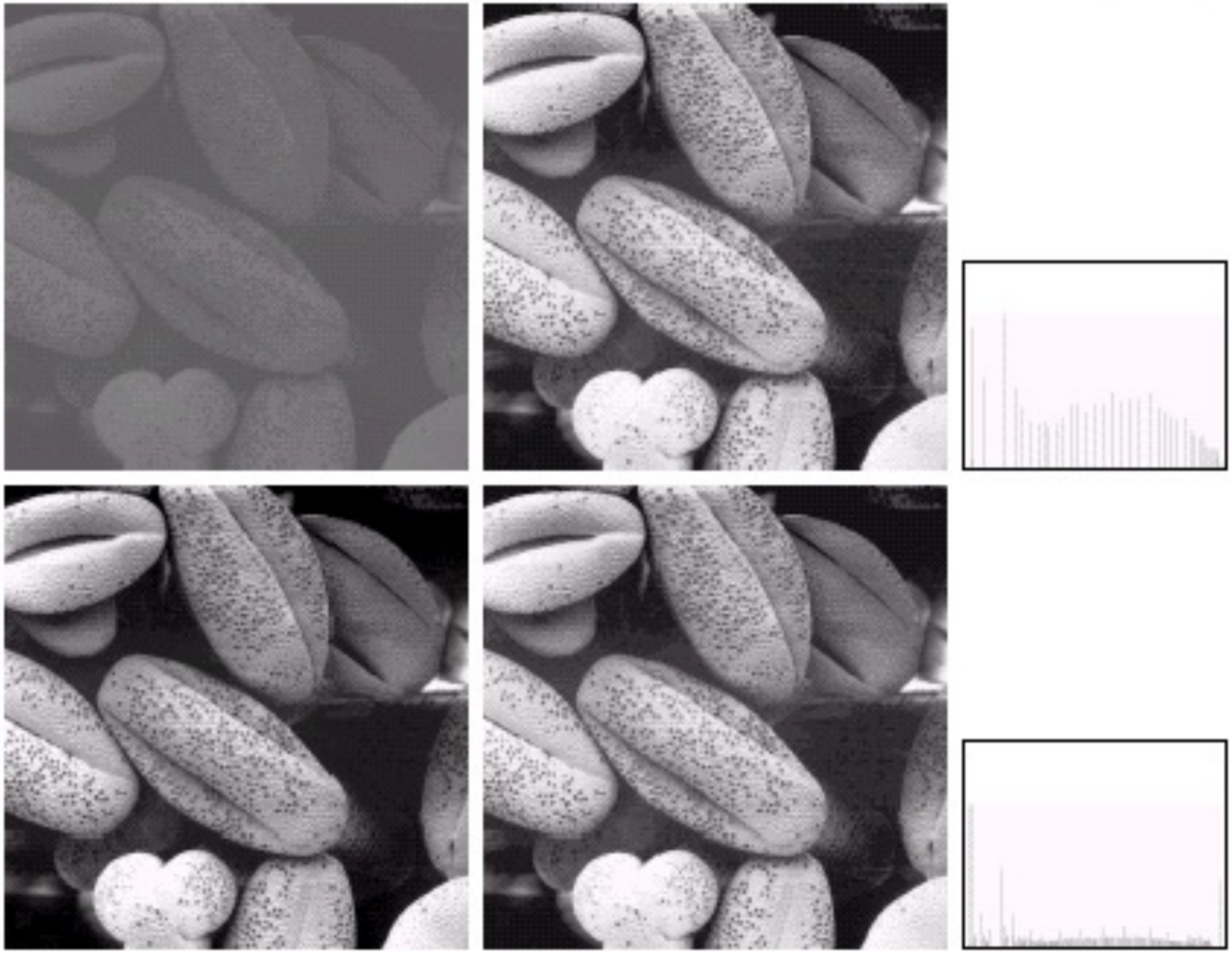


FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

Histogram Equalization

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).

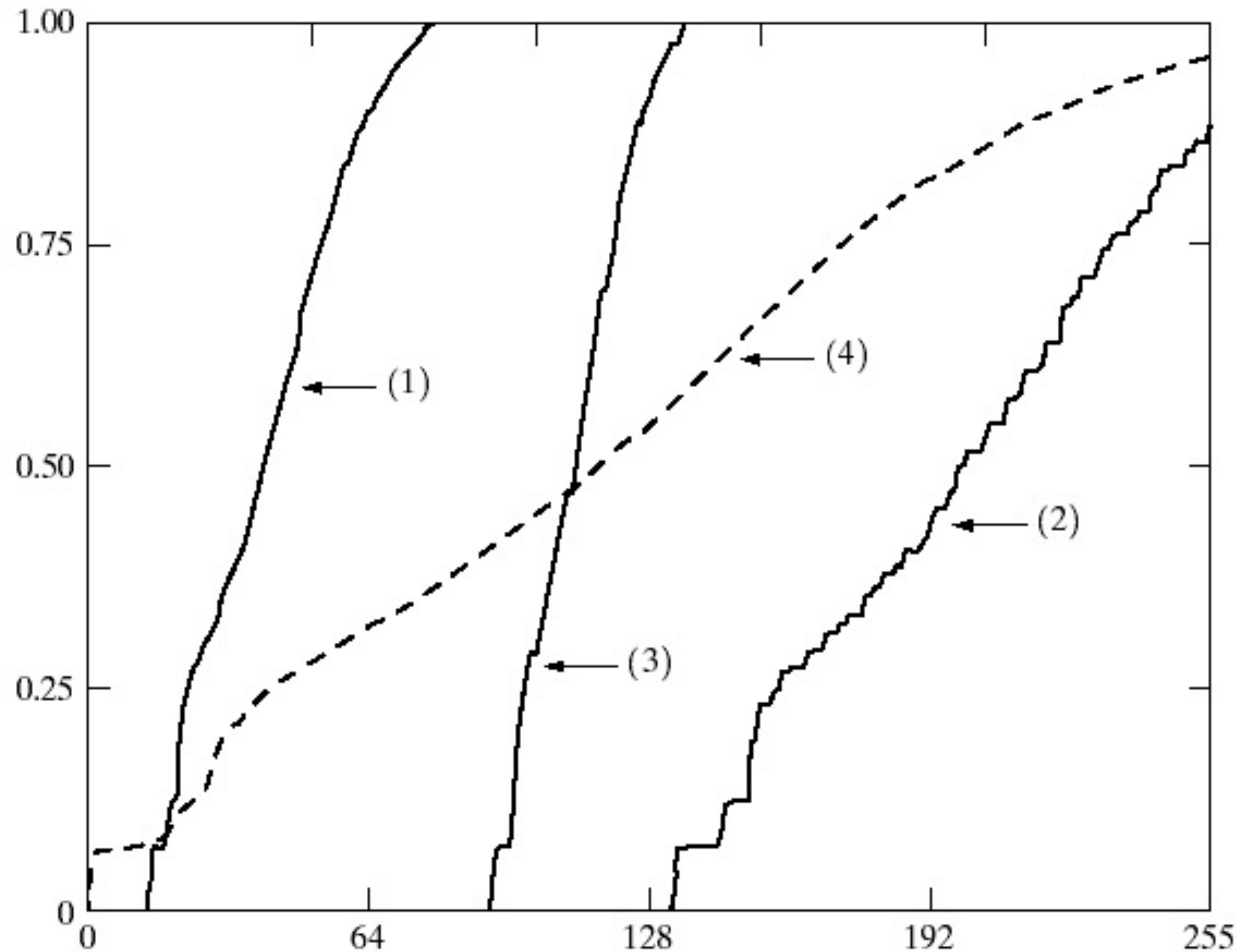
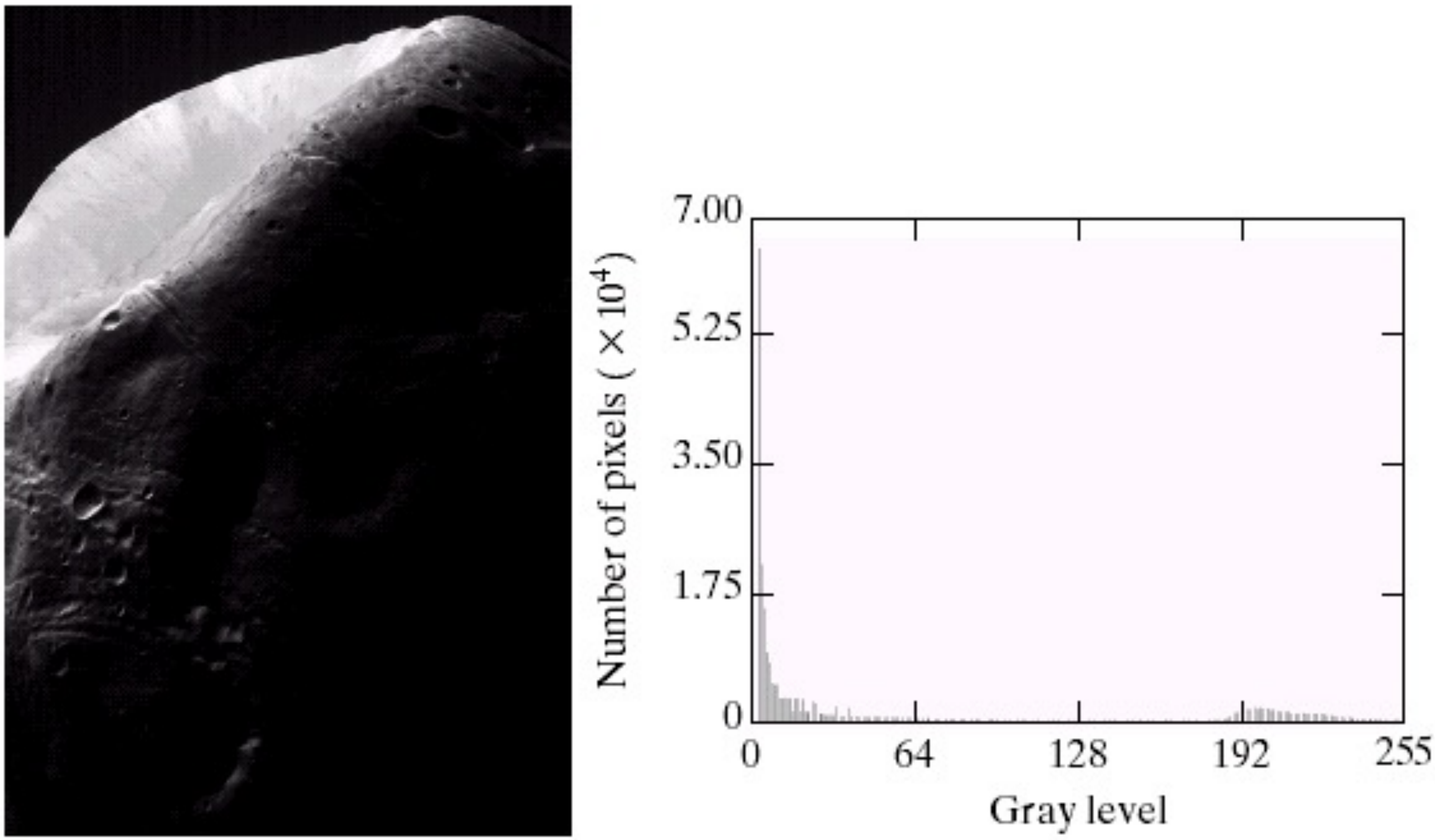


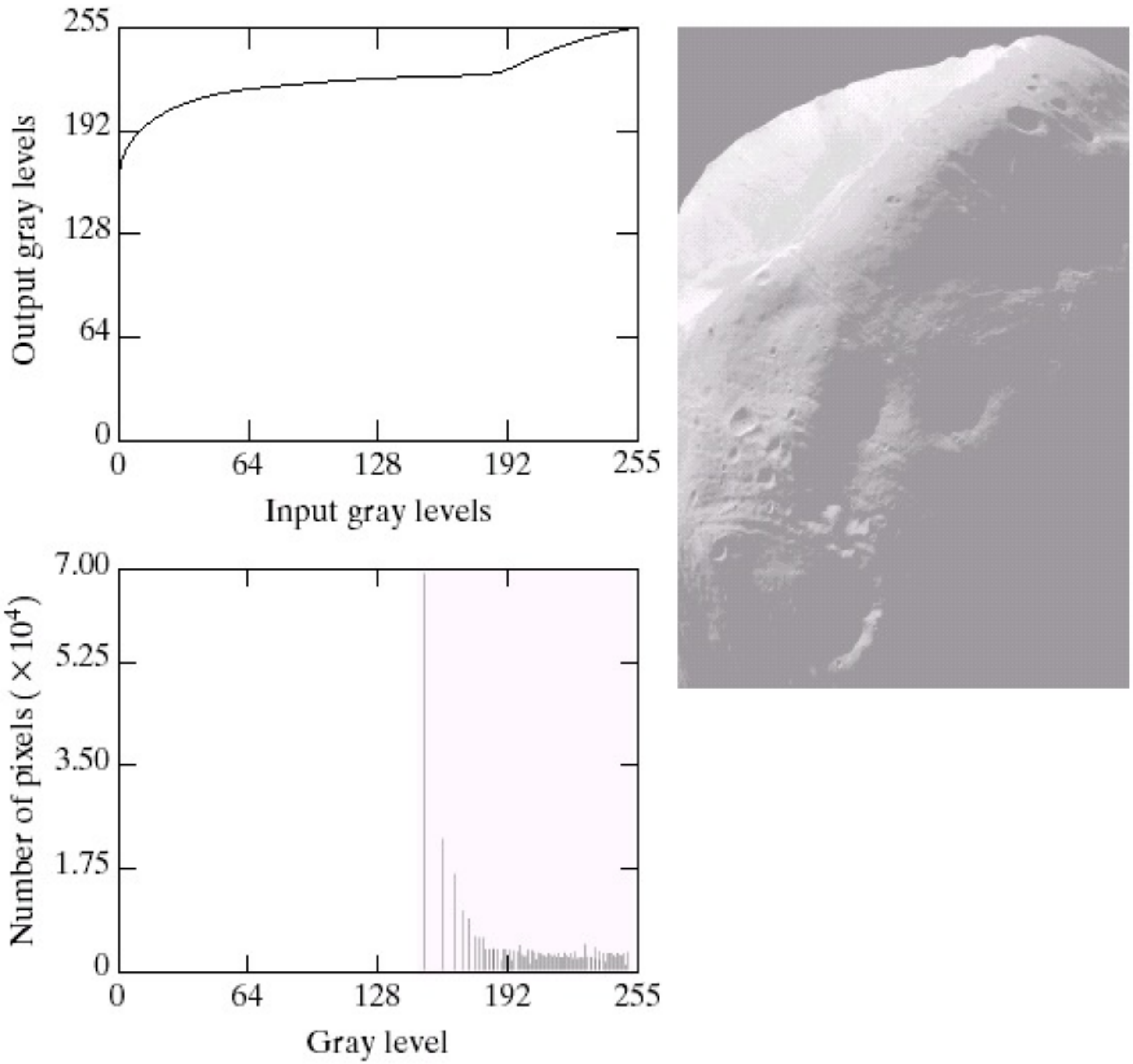
Image with Large Number of Dark Pixels



a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Histogram Equalization

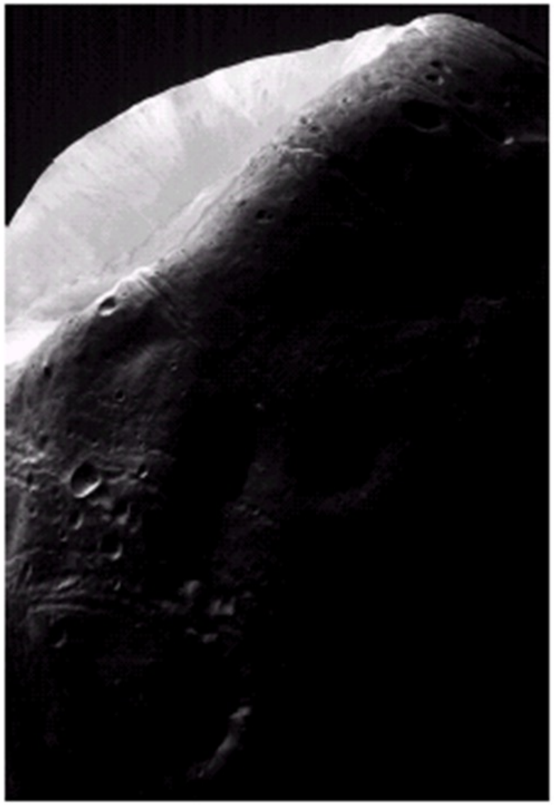


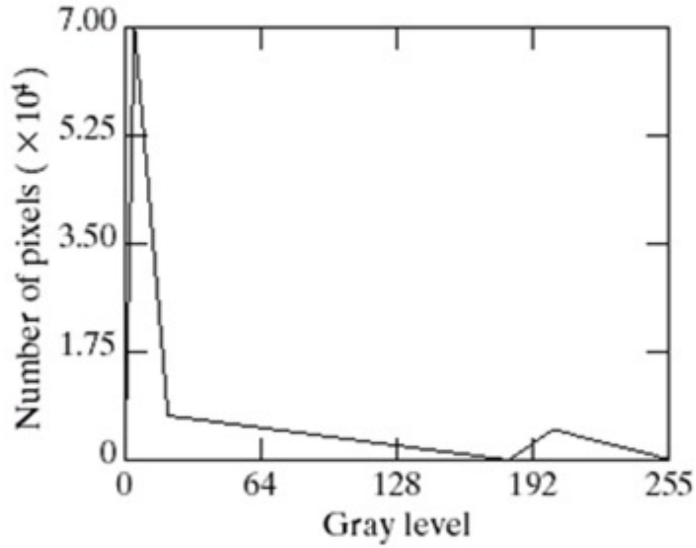
a b
c

FIGURE 3.21
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

Custom Desired Histogram

FIGURE 3.22





Global histogram – a histogram computed using all of the pixels in an image

Global histogram – a histogram computed using all of the pixels in an image

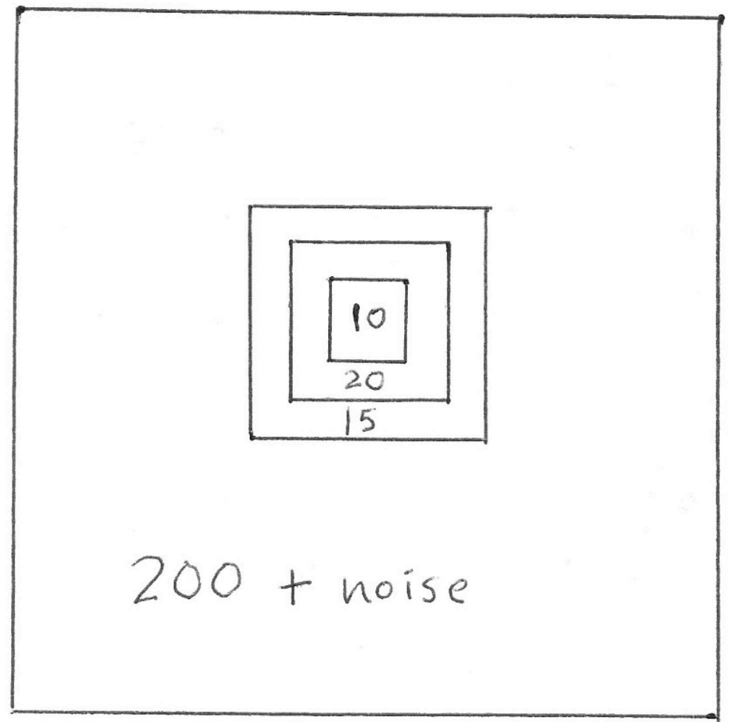
Local histogram – a histogram computed using the pixels in a small region of an image

Global histogram – a histogram computed using all of the pixels in an image

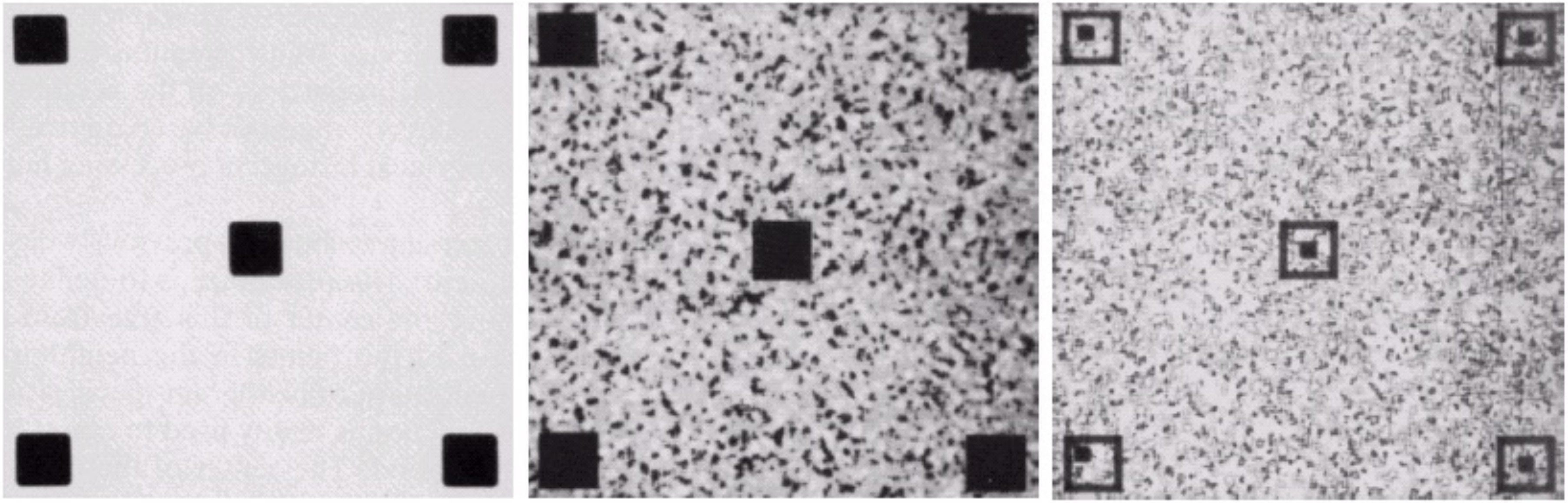
Local histogram – a histogram computed using the pixels in a small region of an image

Local histogram equalization – for each pixel location (x,y) compute a histogram equalization transform T_{xy} using an $n \times n$ region centered at (x,y). Transform the gray level at (x,y) using T_{xy}.

Ex

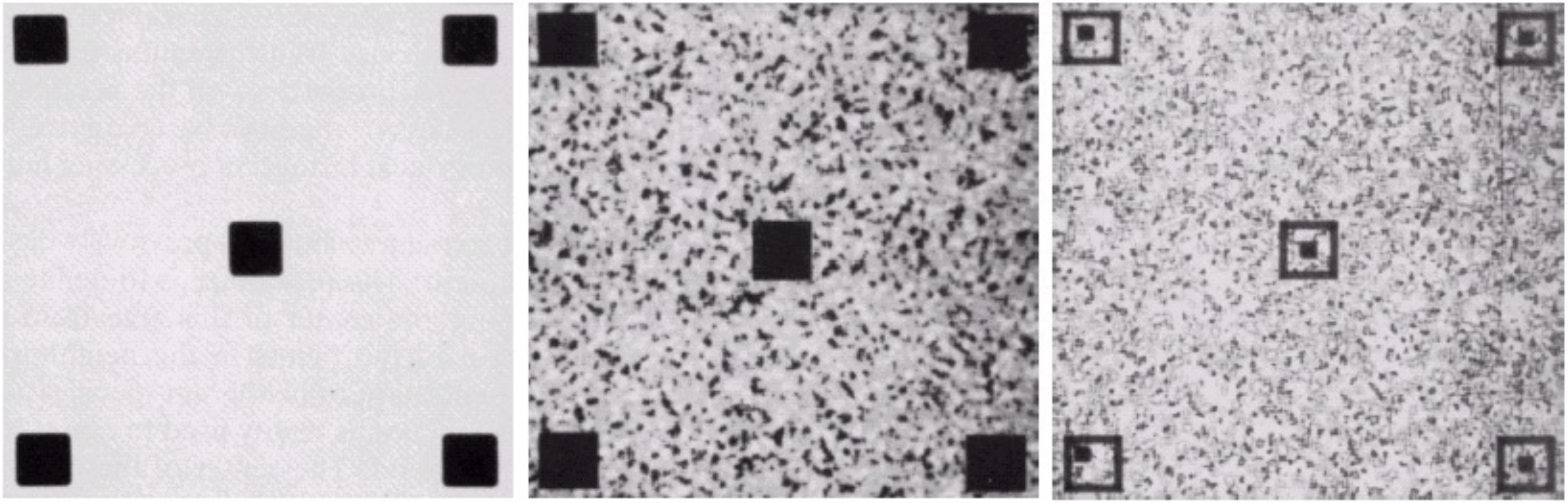


In the original image or when using global histogram equalization, the image will appear as a dark square on a bright background. Using local histogram equalization, the internal structure of the square will appear



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

Using local histogram equalization, a gray level in the input image can map to more than one gray level in the output image. A pixel P_1 that has a smaller gray level than pixel P_2 in the input image can have a larger gray than pixel P_2 in the output image.

Noise Reduction

Let $g(x,y,t) = f(x,y) + n(x,y,t)$

$g(x,y,t)$ = noisy image of static scene at time t

$f(x,y)$ = noise-free image of scene

$n(x,y,t)$ = zero-mean additive noise at time t with variance $\sigma_n^2(x,y)$

Assume that $n(x,y,t_1)$ is independent of $n(x,y,t_2)$ for $t_1 \neq t_2$

Average of images taken at $t_1, t_2, \cdots, t_K$ is

$$\bar{g}(x,y) = \frac{1}{K} \sum_{i=1}^{K} g(x,y,t_i) = \frac{1}{K} \sum_{i=1}^{K} \left[f(x,y) + n(x,y,t_i) \right]$$

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Find mean and variance of $\bar{g}(x,y)$ at pixel (x,y)

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Find mean and variance of $\bar{g}(x,y)$ at pixel (x,y)

mean $E\left[\bar{g}(x,y)\right] = E\left[\frac{1}{K}\left(K f(x,y) + n(x,y,t_1) + \cdots + n(x,y,t_K)\right)\right]$

$$= f(x,y) + \frac{1}{K} E\left[n(x,y,t_1) + \cdots + n(x,y,t_K)\right] = f(x,y)$$

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$$= f(x,y) + \frac{1}{K} E\left[n(x,y,t_1) + \cdots + n(x,y,t_K)\right] = f(x,y)$$

variance $\quad VAR\left[\bar{g}(x,y)\right] = VAR\left[\frac{1}{K}\left(K f(x,y)\right) + \frac{1}{K}\left(n(x,y,t_1) + \cdots + n(x,y,t_K)\right)\right]$

$$= \frac{1}{K^2} VAR\left[n(x,y,t_1) + \cdots + n(x,y,t_K)\right] = \frac{1}{K} \sigma_n^2(x,y)$$

Average of images taken at $t_1, t_2, \cdots, t_K$ is

$$\bar{g}(x,y) = \frac{1}{K} \sum_{i=1}^{K} g(x,y,t_i) = \frac{1}{K} \sum_{i=1}^{K} \left[f(x,y) + n(x,y,t_i) \right]$$

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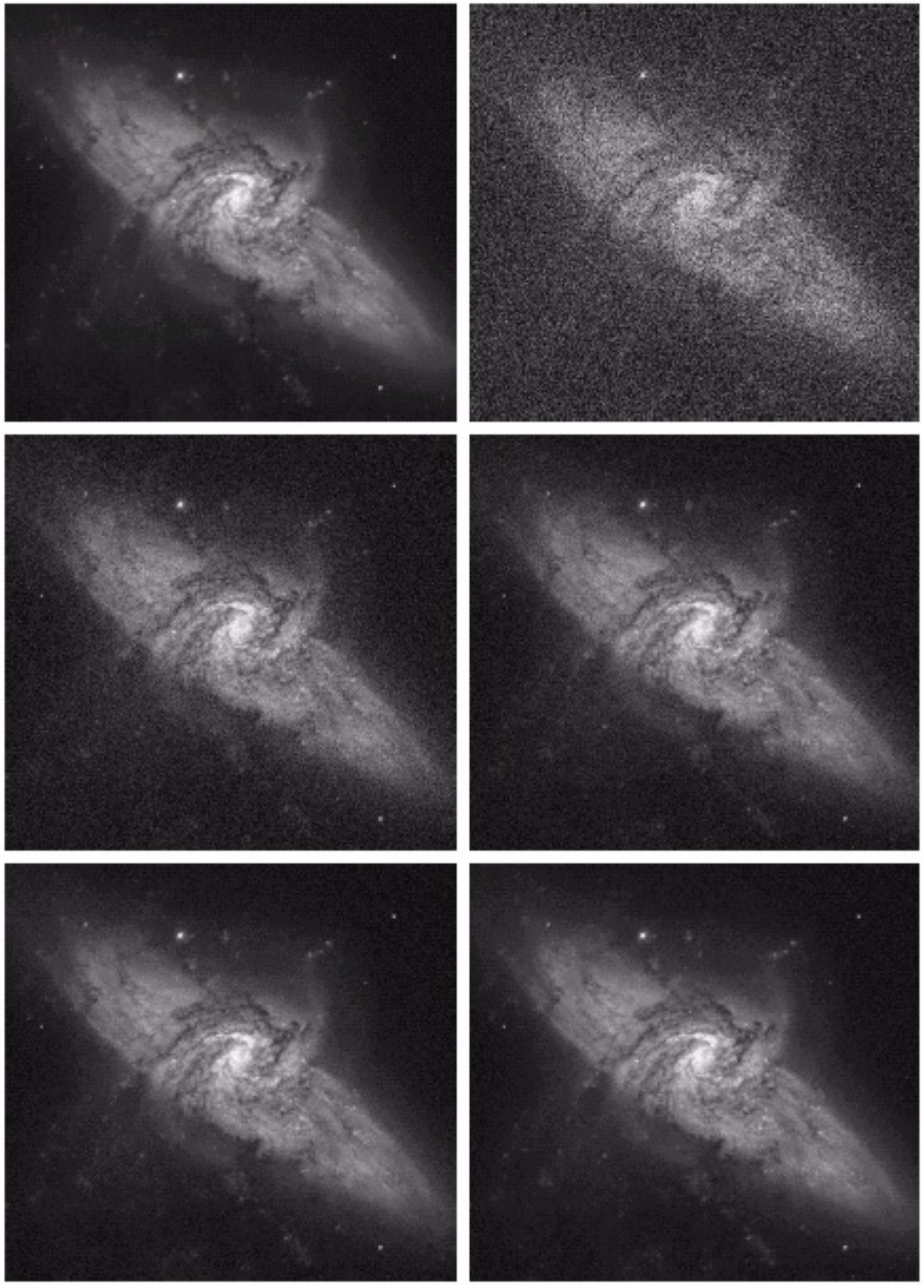
$$= \frac{1}{K^2} VAR\left[n(x,y,t_1) + \cdots + n(x,y,t_K)\right] = \frac{1}{K} \sigma_n^2(x,y)$$

If X and Y are independent random variables,

$$VAR[aX + bY] = a^2 VAR[X] + b^2 VAR[Y]$$



standard deviation $\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}} \sigma_{n}(x,y)$

a	b
c	d
e	f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

As images are averaged, the standard deviation of the noise decreases by $\frac{1}{\sqrt{K}}$