EECS203A Exam #2 June 8, 2021

Name:

I.D.:

This is a 120 minute closed book exam. You are allowed one 8.5 by 11 inch sheet with notes. Calculators are not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

Question 12:

Question 13:

Question 14:

TOTAL:

Note: The Euclidean distance $E(v, w)$ between a vector $v = (v_x\ v_y\ v_z)$ and a vector $w = (w_x\ w_y\ w_z)$ is $E(v, w) = \sqrt{(v_x - w_x)^2 + (v_y - w_y)^2 + (v_z - w_z)^2}$.

Question 1 (4 points) Let $I_1(\lambda)$ and $I_2(\lambda)$ be two functions over the visible wavelengths.

a) What does it mean to say $I_1(\lambda)$ and $I_2(\lambda)$ are isomers?

b) What does it mean to say $I_1(\lambda)$ and $I_2(\lambda)$ are metamers?

Question 2 (4 points) A display device has b bits per pixel to represent gray level. A halftoned image uses an $n \times n$ grid of display device pixels to represent one halftoned pixel. How many different gray levels are possible for each halftoned pixel?

Question 3 (4 points) A filter transforms an input image $f(x, y)$ to an output image $g(x, y)$ according to $g(x, y) = \frac{1}{g'(x,y)}$ where

$$g'(x, y) = \sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} (f(i, j))^{-1}$$

Describe in words the effect of this filter on an input image.

Question 4 (6 points) A triangular region in image $f(x, y)$ with vertices $(x, y) = \{(0,0), (2,0), (2,1)\}$ appears distorted in image $f'(x', y')$ with corresponding vertices $(x', y') = \{(1,2), (2,0), (2,4)\}$. Determine the functions $x'(x, y)$ and $y'(x, y)$ using a bilinear model without the cross xy term for the distortion.

Question 5 (6 points) Let $f(x, y)$ be the 4×4 digital image

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f(0,0) f(0,1) f(0,2) f(0,3)        6   6   6   6
f(1,0) f(1,1) f(1,2) f(1,3)   =    4   4   4   4
f(2,0) f(2,1) f(2,2) f(2,3)        6   6   6   6
f(3,0) f(3,1) f(3,2) f(3,3)        4   4   4   4
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Find the DFT $F(u, v)$ of $f(x, y)$ for $u = 0, 1, 2, 3$ and $v = 0, 1, 2, 3$. Simplify your answer.

Question 6 (6 points) The digital image $g(x, y)$ is a degraded version of an ideal digital image $f(x, y)$ with

$$g(x, y) = h(x, y) * f(x, y) + n(x, y)$$

a) What must be known to apply an inverse filter to $g(x, y)$?

b) What image $f'(x, y)$ is the result of applying an inverse filter to $g(x, y)$?

c) What are the most general conditions on $h(x, y)$ and $n(x, y)$ for which $f'(x, y) = f(x, y)$?

Question 7 (8 points) We are given three display guns with spectral distributions $D_R(\lambda)$, $D_G(\lambda)$, and $D_B(\lambda)$.

a) Find the chromaticity coordinates (x, y) for each of the three display guns.

b) Draw a labeled chromaticity diagram (x, y) that shows the gamut of chromaticities that can be generated by adding nonnegative amounts of the three display guns.

c) Let $I(\lambda)$ be an arbitrary nonnegative function of wavelength. In general, can we find constants r, g, b so that $I(\lambda) = rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$? Explain your answer.

d) Let $I(\lambda)$ be an arbitrary nonnegative function of wavelength. Find constants r, g, b so that $rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$ looks the same as $I(\lambda)$ to a human observer.

Question 8 (8 points) A camera is viewing a flat surface that fills the entire image. The generated digital image $D(x, y)$ is given by

$$D(x, y) = (R(x, y)e + N(x, y))A$$

where e is the constant illumination incident on the viewed surface and $R(x, y)$ is the reflectance of the surface at pixel (x, y). Let $R(x, y) = \overline{r} + r(x, y)$ where $\overline{r}$ is the constant mean reflectance of the surface and $r(x, y)$ describes the variation in the reflectance of the surface. $r(x, y)$ has zero-mean and a variance of σ_r^2 over the pixels (x, y) in the image. $N(x, y)$ is zero-mean noise with variance σ_N^2. A is the constant gain of the system. Assume that the noise and $r(x, y)$ are independent.

a) What is the mean of $D(x, y)$ over the image.

b) What is the total variance of $D(x, y)$ over the image?

c) What fraction of the total variance of $D(x, y)$ is due to the variation in the reflectance of the viewed surface?

Question 9 (8 points) A two-dimensional function $I(x, y)$ is defined for continuous variables x and y over the space $0 \leq x \leq 99,\ 0 \leq y \leq 99$ by

$$\begin{aligned} I(x, y) &= 10 \quad \text{for } x \leq E \\ &= 0 \quad \text{for } x > E \end{aligned}$$

where E is a real number. A 100×100 digital image $D(m, n)$ is generated from $I(x, y)$ according to $D(m, n) = I(m, n)$ for all pairs of integers (m, n) in $m = 0, 1, \ldots, 99$ and $n = 0, 1, \ldots, 99$.

a) What is this process for generating $D(m, n)$ from $I(x, y)$ called?

b) If the mean value of the pixels in $D(m, n)$ is 6.0, what are the possible values of E?

c) Can we uniquely recover $I(x, y)$ from $D(m, n)$? Explain.

d) Will the use of an antialiasing filter improve the appearance of jagged edges in $D(m, n)$? If you answer No, then explain how the use of an antialiasing filter will affect the appearance of the digital image.

Question 10 (8 points) We have 5 pixels in a region of a color image with values

pixel	R	G	B
1	90	60	90
2	90	90	30
3	60	60	60
4	60	60	90
5	30	30	30

a) Use the method given in class to estimate the color covariance matrix Σ using these 5 pixels.

b) Would you expect to observe these R,G,B values for 5 pixels in the region of a single material in a color image? Explain.

Question 11 (8 points) The pixels $z = (z_r\ z_g\ z_b)^T$ in a color image have a mean vector $\mu = (\mu_R\ \mu_G\ \mu_B)^T = (50\ 100\ 200)^T$ and a covariance matrix $\Sigma = E[(z-\mu)(z-\mu)^T] = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix}$.

a) If the distribution of color pixel values in this image is multivariate Gaussian, then find the probability density function $p(z)$. Simplify.

b) Find the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of Σ where $\lambda_1 > \lambda_2 > \lambda_3$.

c) Find the unit length eigenvectors e_1, e_2, e_3 that correspond to $\lambda_1, \lambda_2, \lambda_3$ respectively.

d) Find the matrix A so that the operation Az implements the principal components transform.

Question 12 (10 points) The pixels $z = (z_r\ z_g\ z_b)^T$ in a color image region have a mean $\mu = (\mu_R\ \mu_G\ \mu_B)^T = (50\ 100\ 200)^T$ and a covariance matrix $\Sigma = E[(z-\mu)(z-\mu)^T] = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix}$.

a) Find the set of color pixel vectors $(R\ G\ B)$ that have a Euclidean distance of 10 from the point $(50\ 100\ 200)^T$ in color space.

b) Use μ and Σ to find the set of color pixel vectors (R, G, B) that have a Mahalanobis distance of 10 from the color pixel distribution for the region.

c) Consider the set of color pixel vectors (R, G, B) that have a Euclidean distance of 10 from the point $\mu = (50\ 100\ 200)^T$ in color space. Find the color pixel vector(s) (R, G, B) in this set with the smallest Mahalanobis distance from the color pixel distribution for the region.

d) Consider the set of color pixel vectors (R, G, B) that have a Euclidean distance of 10 from the point $\mu = (50\ 100\ 200)^T$ in color space. Find the color pixel vector(s) (R, G, B) in this set with the largest Mahalanobis distance from the color pixel distribution for the region.

Question 13 (10 points) Consider a spatial domain filter that transforms an input image $f(x, y)$ with N rows and N columns where N is divisible by 4 into an output image $g(x, y)$ according to

$$g(x, y) = \frac{1}{6}\left[f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1)\ \right] + \frac{1}{3}f(x, y)$$

Let $G(u, v)$ be the DFT of $g(x, y)$ and let $F(u, v)$ be the DFT of $f(x, y)$.

a) For this filter, find $H(u, v)$ so that $G(u, v) = H(u, v)F(u, v)$. Simplify your answer.

b) Find $|H(u, v)|$

c) Plot $|H(u, 0)|$ for the range $u = 0, 1, \ldots, N - 1$.

d) What is the maximum value of $|H(u,v)|$? For what value(s) of (u,v) over $u = 0, 1, \ldots, N-1$ and $v = 0, 1, \ldots, N-1$ does $|H(u,v)|$ take on this maximum value?

e) What is the minimum value of $|H(u,v)|$? For what value(s) of (u,v) over $u = 0, 1, \ldots, N-1$ and $v = 0, 1, \ldots, N-1$ does $|H(u,v)|$ take on this minimum value?

Question 14 (10 points) An input digital image $f(x, y)$ consists of only a bright rectangle of constant brightness on a dark background of constant brightness. We process $f(x, y)$ with a linear space-invariant filter defined by a 3×3 mask where the nine mask coefficients add up to one. The central part of the output image is given below. Find the $f(x, y)$ pixel values corresponding to the 11×11 region shown below and find the 3×3 mask.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	4	4	4	4	4	4	0	0
0	0	4	12	16	16	16	16	12	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	12	16	16	16	16	12	4	0
0	0	0	4	4	4	4	4	4	0	0
0	0	0	0	0	0	0	0	0	0	0