EECS203A Exam #1 April 29, 2021

Name:

I.D.:

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

TOTAL:

Question 1 (5 points) Consider a 2-D spatial filter $h(x, y)$ defined by the 3×3 mask

```
0.01   0.10   0.01
0.10   0.56   0.10
0.01   0.10   0.01
```

a) Is $h(x, y)$ best described as a lowpass or highpass filter? Explain.

b) What is the output image if $h(x, y)$ is applied to the constant input image $I(x, y) = 100$?

Question 2 (6 points) Suppose that we generate an output image $g(x, y)$ from an input image $f(x, y)$ according to

$$g(x, y) = 5f(x, y) - f(x + 1, y) - f(x - 1, y) - f(x, y + 1) - f(x, y - 1)$$

a) Find a mask that implements this operation.

b) Describe the appearance of the filtered image $g(x, y)$ compared to the input image $f(x, y)$.

Question 3 (8 points) Consider a 2-D spatial filter $h(x, y)$ that has the frequency response

$$\begin{aligned} H(u,v) &= 0 \text{ if } D(u,v) \leq D_0 \\ &= 1 \text{ if } D(u,v) > D_0 \end{aligned}$$

where $D(u, v) = \sqrt{u^2 + v^2}$.

a) What is this filter called?

b) Will ringing effects in an image filtered with $h(x, y)$ become more prominent as D_0 increases? Explain.

c) Will a filtered image $f(x, y) * h(x, y)$ have a larger fraction of the total power in the input image $f(x, y)$ for small D_0 or for large D_0? Explain.

Question 4 (8 points) Let H be an operator that maps an input image $I(x, y)$ to an output image $O(x, y)$ according to

$$O(x, y) = c[I(x, y)]^A$$

where c and A are real constants. For what values of c and A is H a linear operator?

Question 5 (8 points) Suppose that we capture a sequence of images

$$g(x, y, t_i) = f(x, y) + n(x, y, t_i)$$

where $f(x, y)$ is a noise-free image and $n(x, y, t_i)$ is a zero-mean additive noise source with variance $\sigma_n^2(x, y)$. Assume that the noise at any time is independent of the noise at any other time. Suppose that we form the image

$$h(x, y) = \frac{1}{4}(g(x, y, t_1) + g(x, y, t_2) - g(x, y, t_3) - g(x, y, t_4))$$

a) What is the expected value of $h(x, y)$?

b) What is the variance of $h(x, y)$?

Question 6 (9 points) Let $I(x, y)$ be an input digital image and let $O(x, y)$ be the output digital image obtained by processing $I(x, y)$ with a 3×3 median filter. Let N_I be the number of different gray levels that occur in $I(x, y)$ and let N_O be the number of different gray levels that occur in $O(x, y)$. For each part of this question, if you answer YES then give an example that satisifes the condition. If you answer NO then explain why not.

a) Can we have $N_I > N_O$?

b) Can we have $N_I = N_O$?

c) Can we have $N_I < N_O$?

Question 7 (10 points) Consider a 2-D spatial filter $h(x, y)$ defined by the 3×3 mask

```
-1   -2   0
-2    0   3
 0    3   1
```

Suppose that we are given a digital image $I(x, y)$ with gray levels represented using 6 bits so that pixels have the possible values 0,1,2,...,63. Let $O(x, y)$ be the output image after $h(x, y)$ is applied to $I(x, y)$. Assume pixels in $O(x, y)$ can take any integer values including negatives.

a) What is the minimum possible value MIN of a pixel in $O(x, y)$?

b) What is the maximum possible value MAX of a pixel in $O(x, y)$?

c) Find a gray-level transform $T(r)$ that maps a gray level r in $O(x, y)$ to a gray level in the 6-bit range 0,1,2,...,63 where $T(\text{MIN}) = 0$, $T(\text{MAX}) = 63$, and gray levels r in $O(x, y)$ with $\text{MIN} < r < \text{MAX}$ are mapped to the 6-bit range so that $T(r)$ is a monotonically increasing linear function of r. Note that with rounding error, $T(r)$ may deviate slightly from linear.

Question 8 (10 points) Let $f(x, y)$ be the 4×4 digital image with DFT $F(u, v)$ given by

```
F(0,0) F(0,1) F(0,2) F(0,3)         6    0    0    0
F(1,0) F(1,1) F(1,2) F(1,3)    =    0    0    0    1
F(2,0) F(2,1) F(2,2) F(2,3)         0    0    0    0
F(3,0) F(3,1) F(3,2) F(3,3)         0    1    0    0
```

Find the 4×4 digital image $f(x, y)$ for $x = 0, 1, 2, 3$ and $y = 0, 1, 2, 3$. Simplify your answer.

Question 9 (12 points) Consider an $N \times N$ digital image f with 8 gray levels from 0 to 7. Suppose that the gray level histogram for f is given by

$$h(r_k) = 2r_k + 1 \qquad\qquad r_k = 0, 1, 2, \ldots, 7$$

a) Find N.

b) Use the method described in class to determine the gray level transformation $M(r_k)$ for $r_k = 0, 1, 2, \ldots, 7$ that corresponds to histogram equalization.

c) Find the histogram $h'(r_k)$ $\quad r_k = 0, 1, 2, \ldots, 7$ for the transformed image that results after applying histogram equalization to f.

Question 10 (12 points) Consider the 4×4 digital image $f(x, y) = 6x^2y$ defined for $x = 0, 1, 2, 3, \quad y = 0, 1, 2, 3$. Let $g(x, y)$ be a 31×31 zoomed version of $f(x, y)$ defined for $x = 0, 1, \ldots, 30, \quad y = 0, 1, \ldots, 30$ as described in class.

a) Find $g(11, 17)$ using nearest neighbor interpolation.

b) Find the coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ for the pixels in $f(x, y)$ that will contribute to the bilinear interpolation value for $g(11, 17)$.

c) Find $g(11, 17)$ using bilinear interpolation. Your answer may be a floating point (non-integer) number.

Question 11 (12 points) Define the masks for two 3×3 spatial filters $h_1(x, y)$ and $h_2(x, y)$ and define the 5×5 digital image $f(x, y)$ as shown below

```
                                                 0  0  0  0  0
        0 1 0              0 0 0                 0  0  0  0  0
h1 =    0 2 0       h2 =   4 5 6          f =    0  0  1  0  0
        0 3 0              0 0 0                 0  0  0  0  0
                                                 0  0  0  0  0
```

where you may assume for filtering that $f(x, y)$ has all zeros outside the boundaries.

a) Find the 5×5 output image $g_1(x, y)$ if we apply $h_1(x, y)$ to $f(x, y)$.

b) Find the 5×5 output image $g_2(x, y)$ if we apply $h_2(x, y)$ to $g_1(x, y)$. You may assume that $g_1(x, y)$ has all zeros outside the boundaries.

c) Find a mask that when applied to an input image is equivalent to the double filtering operation of applying $h_1(x, y)$ to an input image and then applying $h_2(x, y)$ to the result.