Color Images

A color image $f(x,y)$ has 3 values at each pixel

$$f(x,y) = [R(x,y), G(x,y), B(x,y)]$$

where $R(x,y)$, $G(x,y)$, $B(x,y)$ specifies the color at (x,y).

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If each of the red, green, and blue bands has 8 bits/pixel, then $f(x,y)$ has 24 bits/pixel ($>$16 million colors)

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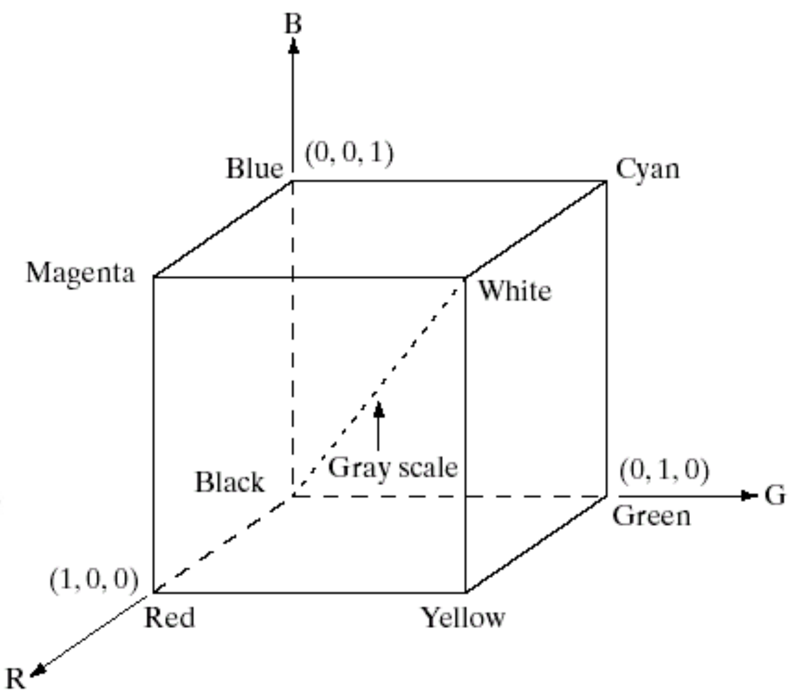

FIGURE 6.7 Schematic of the RGB color cube. Points along the main diagonal have gray values, from black at the origin to white at point (1, 1, 1).

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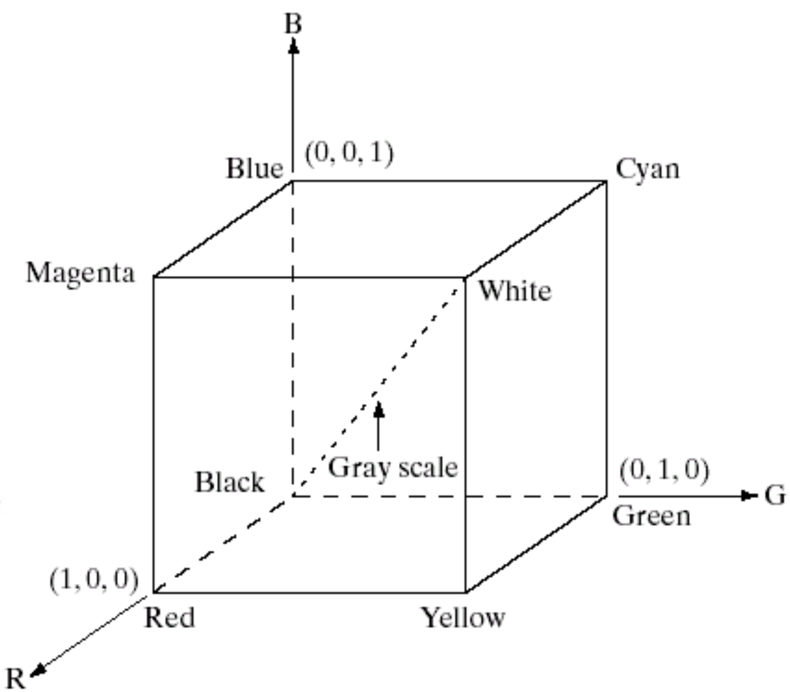


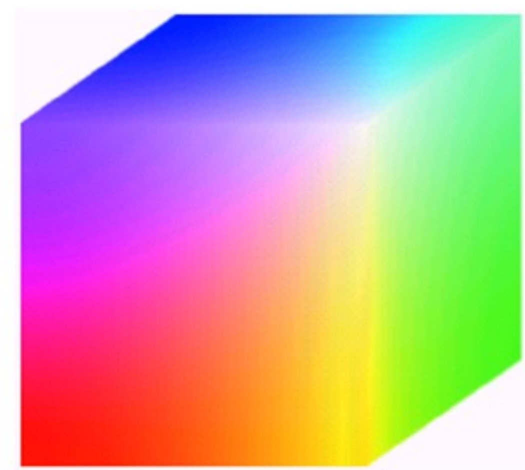
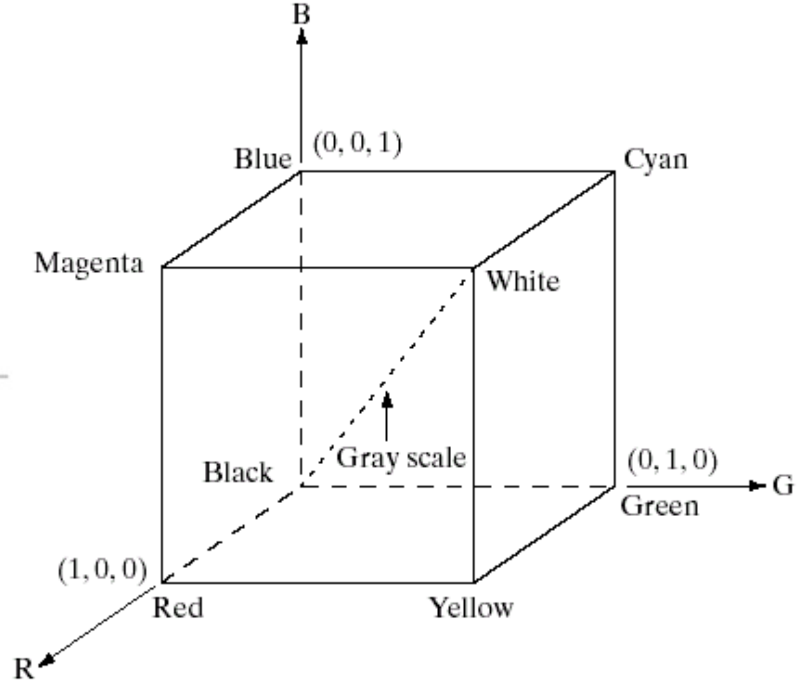


FIGURE 6.8 RGB 24-bit color cube.



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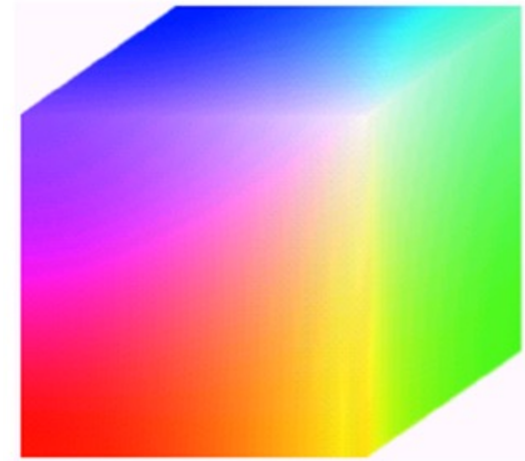
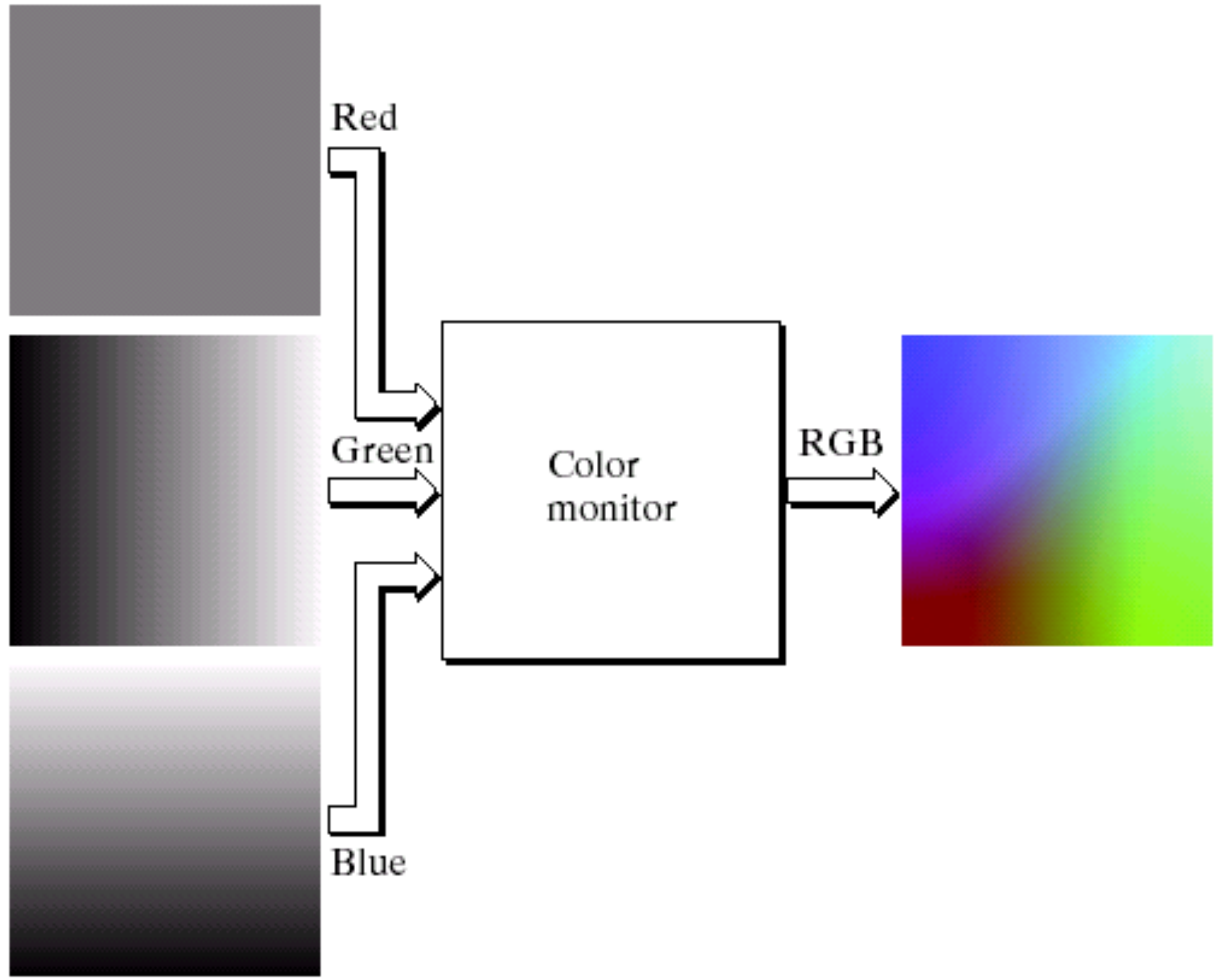


FIGURE 6.8 RGB 24-bit color cube.

($R = 0$) ($G = 0$) ($B = 0$)

Each individual band of a color image, e.g. $R(x,y)$, can be displayed as a gray level image.



a
b

FIGURE 6.9
(a) Generating the RGB image of the cross-sectional color plane $(127, G, B)$.
(b) The three hidden surface planes in the color cube of Fig. 6.8.

Ex Limit R,G,B to the 6 values 0, 51, 102, 153, 204, 255

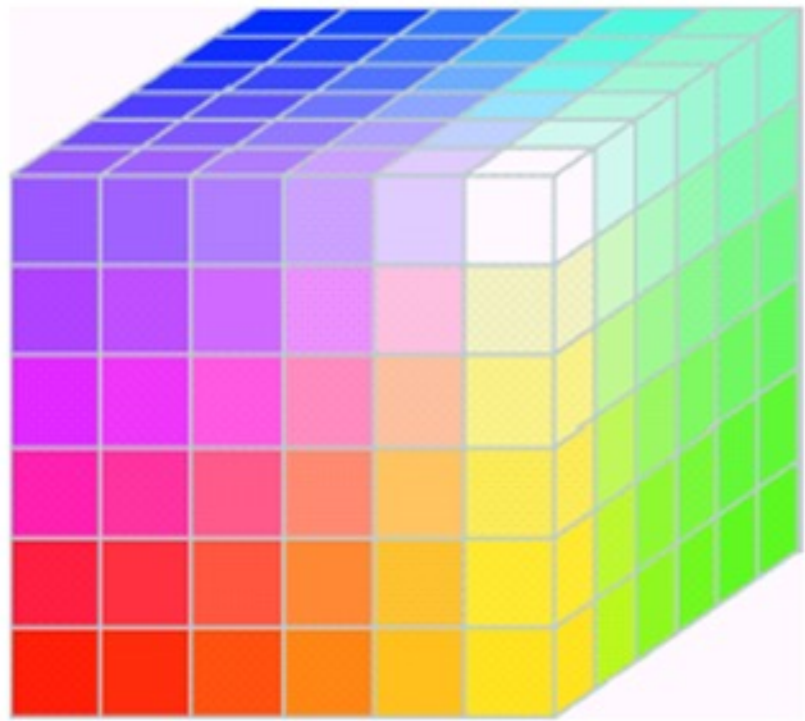


FIGURE 6.11

Ex Limit R,G,B to the 6 values 0,51,102,153,204,255

blue

255 204 153 102 51 0

R=255 R=204 R=153

R=102 R=51 R=0

green

255
204
153
102
51
0

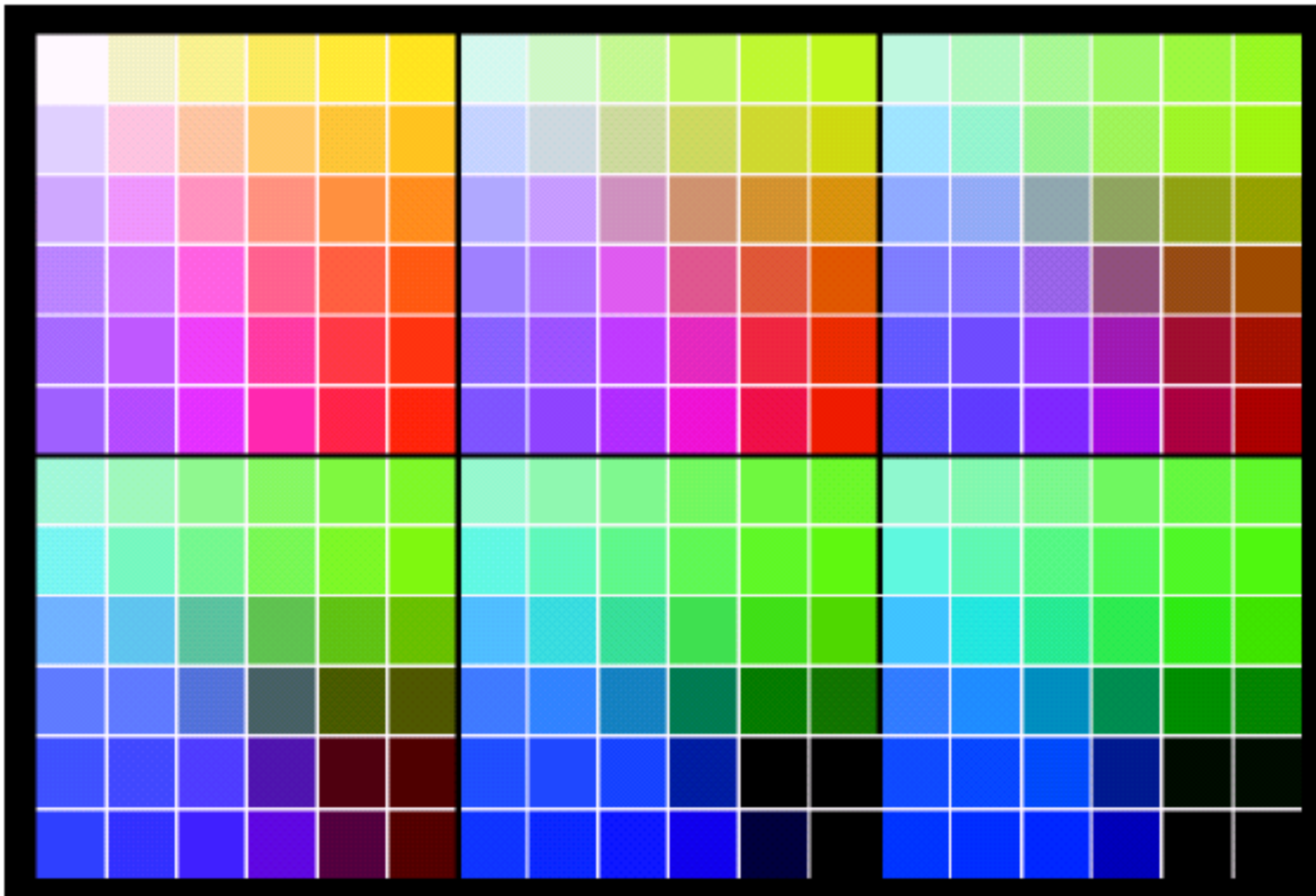


FIGURE 6.10

Ex Limit R,G,B to the 6 values 0, 51, 102, 153, 204, 255

blue

255 204 153 102 51 0

R = 255 R = 204 R = 153

R = 102 R = 51 R = 0

green

255
204
153
102
51
0

Where are the gray colors?

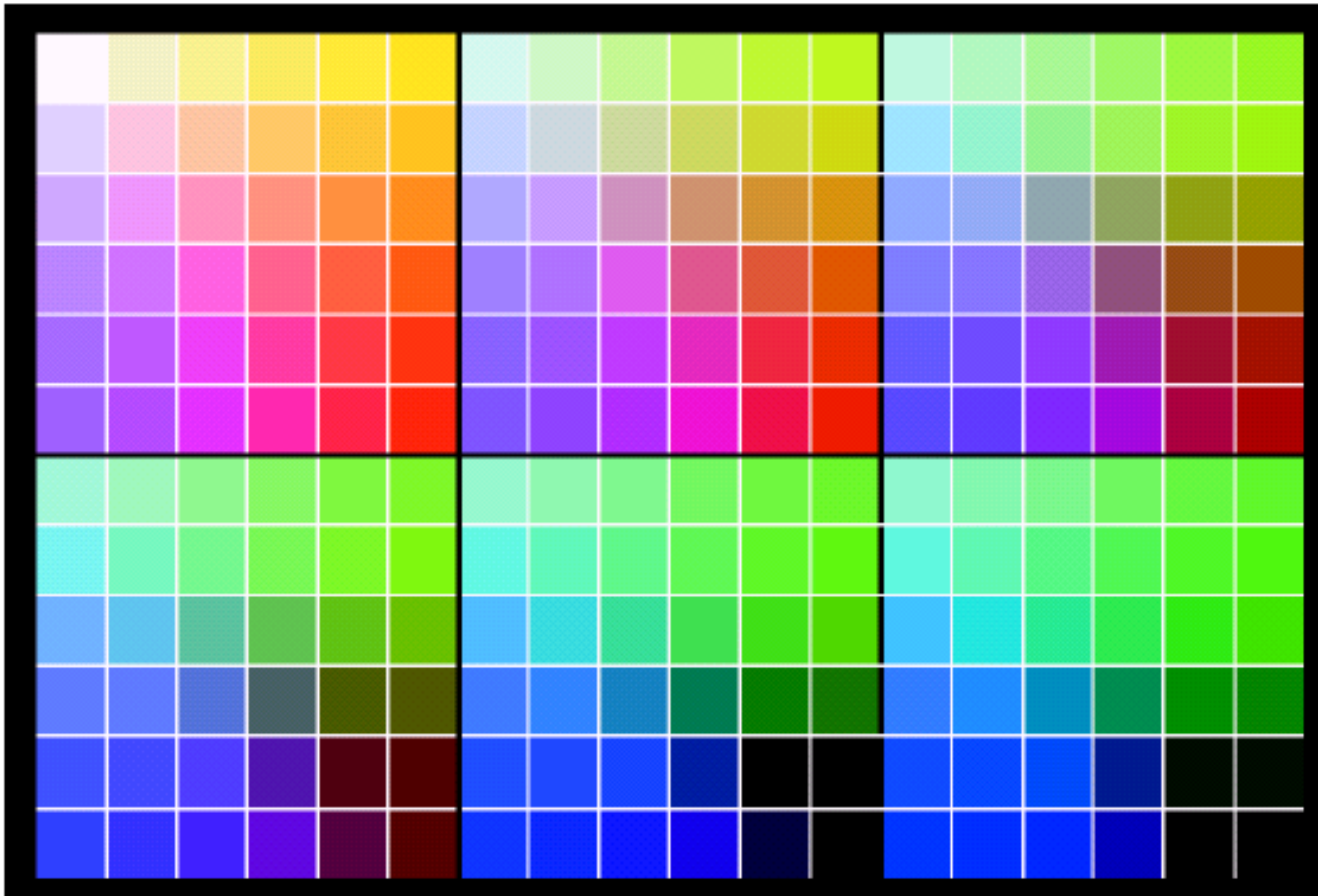


FIGURE 6.10

Human Color Vision

The human retina has 3 kinds of color receptors called cones. Each cone type has a certain photopigment with a characteristic spectral response.

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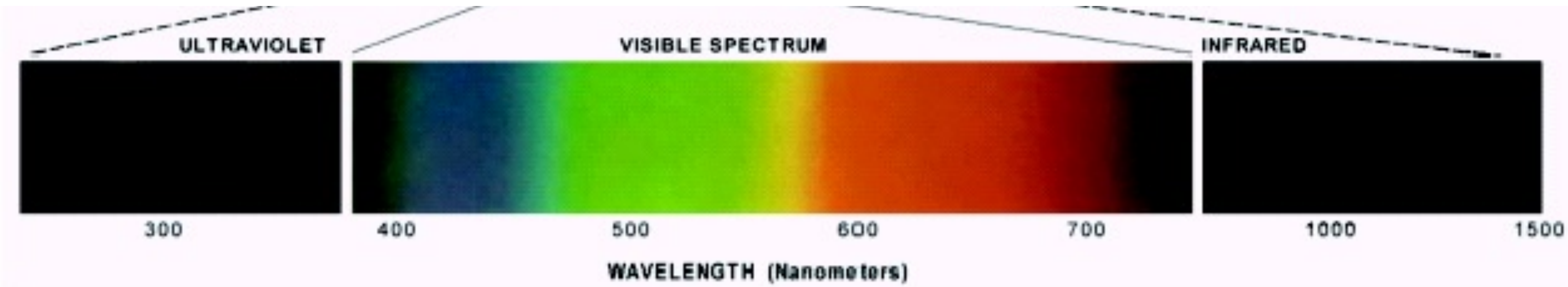

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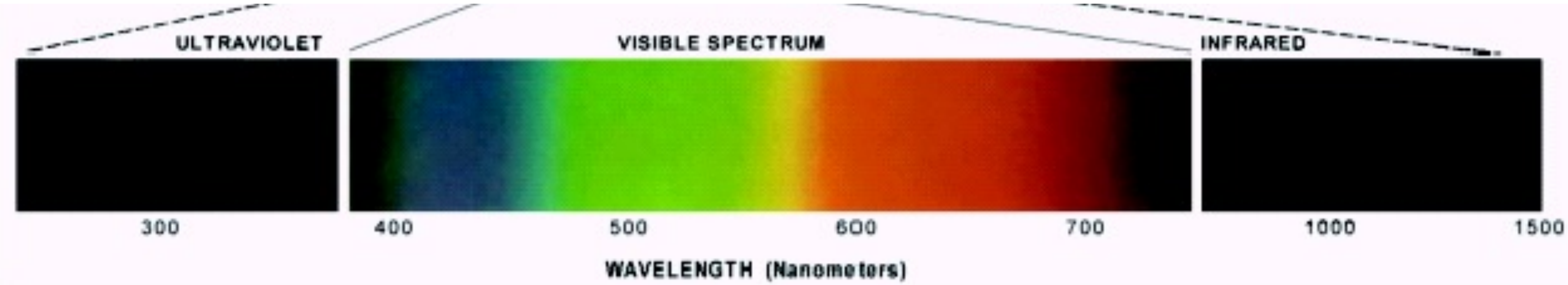

At a retina location (x,y), the light stimulus $I(x,y,\lambda)$ is encoded as 3 numbers R_L, R_M, R_S

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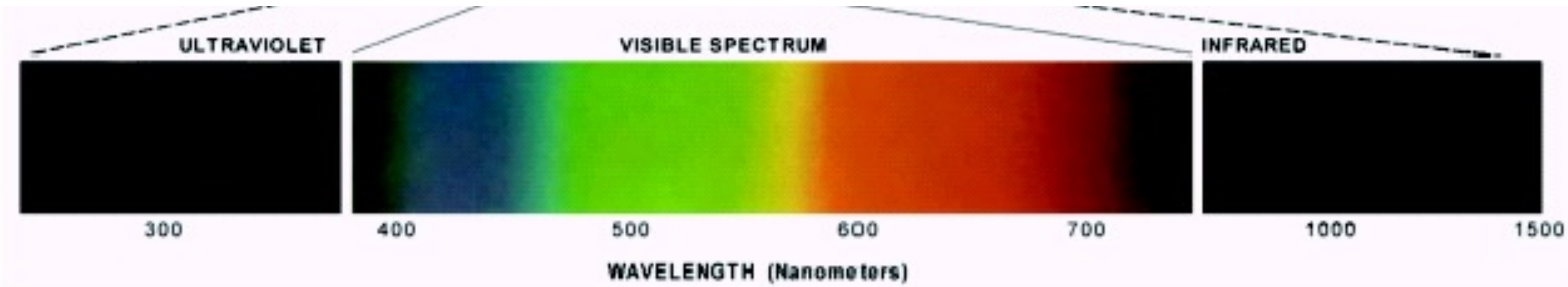

At a retina location (x,y), the light stimulus $I(x,y,\lambda)$ is encoded as 3 numbers R_L, R_M, R_S

$$R_L = \int I(\lambda) L(\lambda) d\lambda \qquad R_M = \int I(\lambda) M(\lambda) d\lambda \qquad R_S = \int I(\lambda) S(\lambda) d\lambda$$

$L(\lambda)$ = response of long wavelength cones

$M(\lambda)$ = response of middle wavelength cones

$S(\lambda)$ = response of short wavelength cones

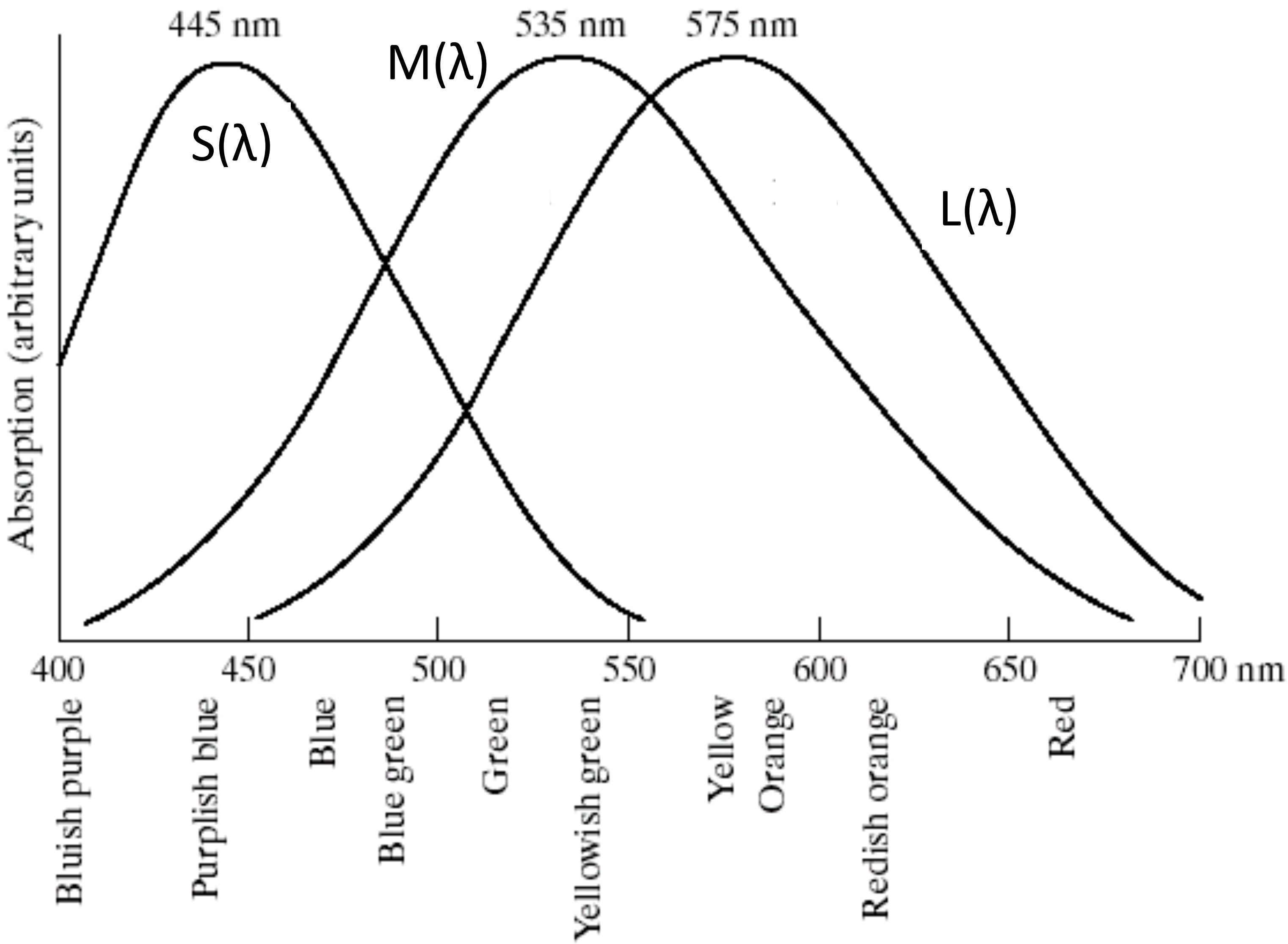

FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.

Stimuli $I_1(\lambda)$ and $I_2(\lambda)$ will appear identical if they give the same values R_L, R_M, R_S.

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If $I_1(\lambda) \neq I_2(\lambda)$ but $I_1(\lambda)$ and $I_2(\lambda)$ give the same R_L, R_M, R_S then $I_1(\lambda)$ and $I_2(\lambda)$ are metamers.

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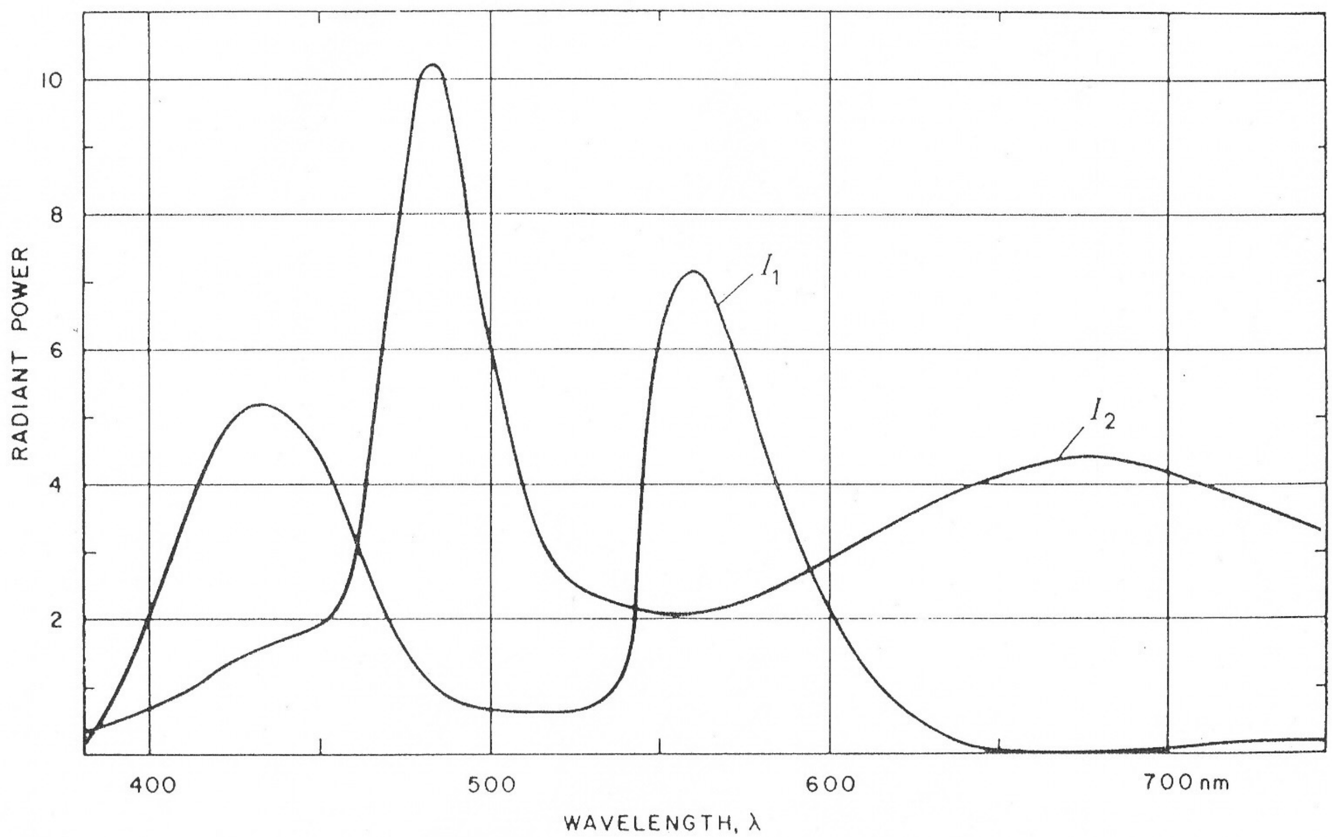

If $I_1(\lambda)$ and $I_2(\lambda)$ appear identical (isomeric or metameric match), we write $I_1(\lambda) \stackrel{M}{=} I_2(\lambda)$

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Human vision is assumed to be trichromatic meaning

1) Any color can be matched by one additive combination of 3 fixed independent primaries

2) Color processing is linear

Color Matching Experiments

Consider the monochromatic primaries $R'(\lambda_1), G'(\lambda_2), B'(\lambda_3)$ where $\lambda_1 = 700\text{nm}$, $\lambda_2 = 546.1\text{ nm}$, $\lambda_3 = 435.8\text{ nm}$

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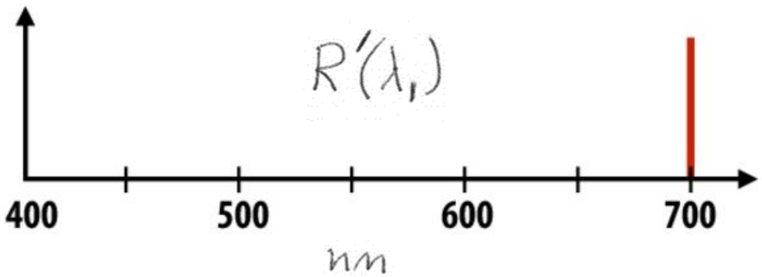

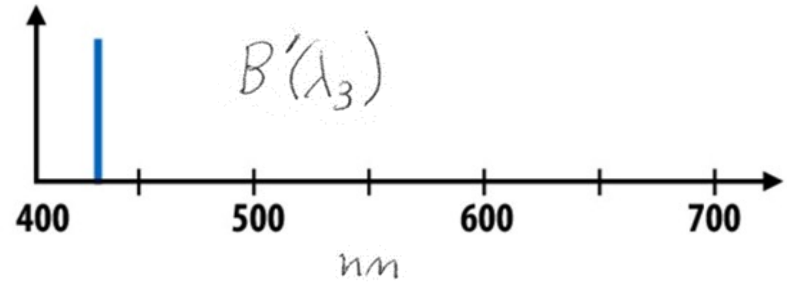

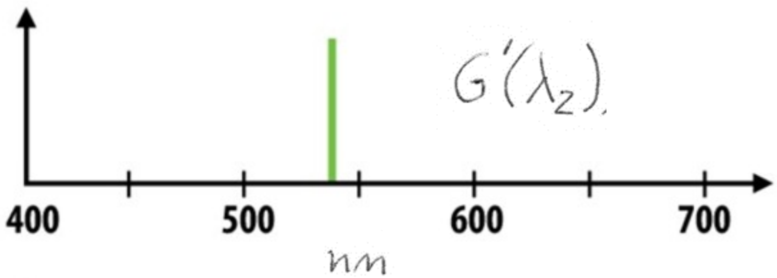

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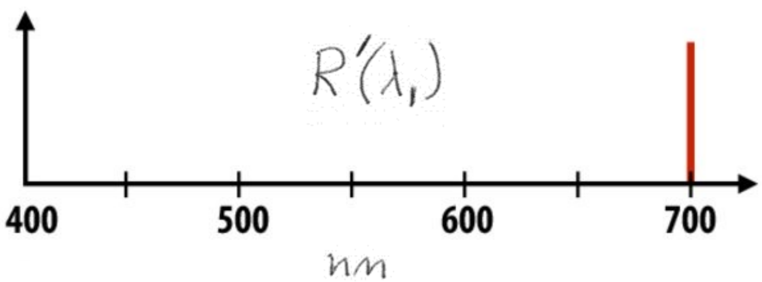

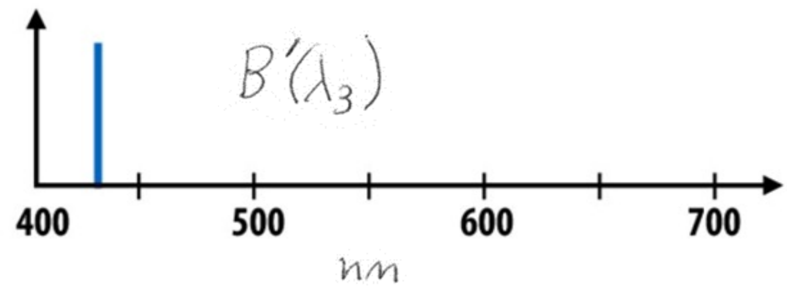

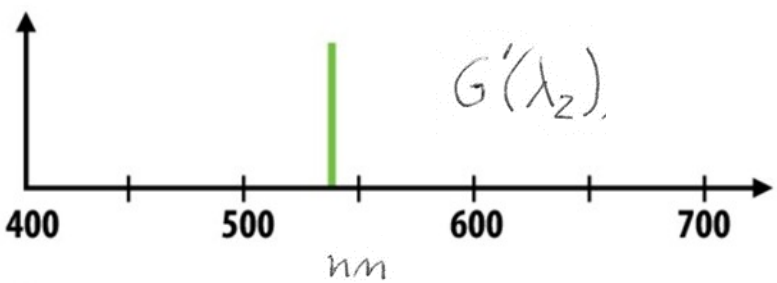

Any unit energy monochromatic test stimulus $Q(\lambda)$ can be matched using

$$Q(\lambda) \overset{M}{=} x'R'(\lambda_1) + y'G'(\lambda_2) + z'B'(\lambda_3)$$

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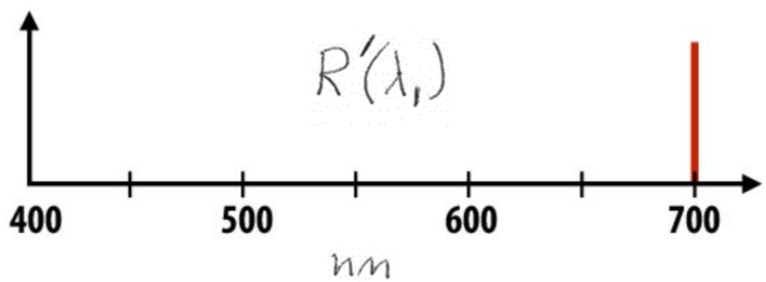
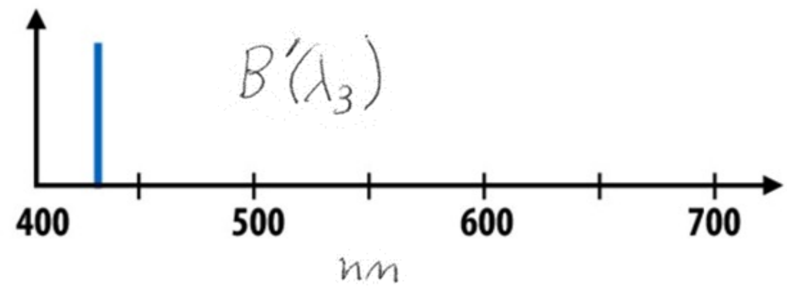
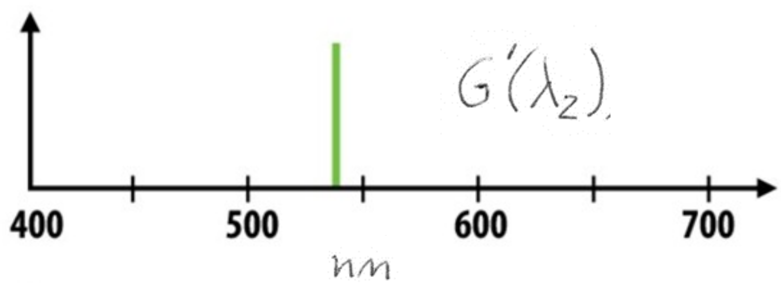


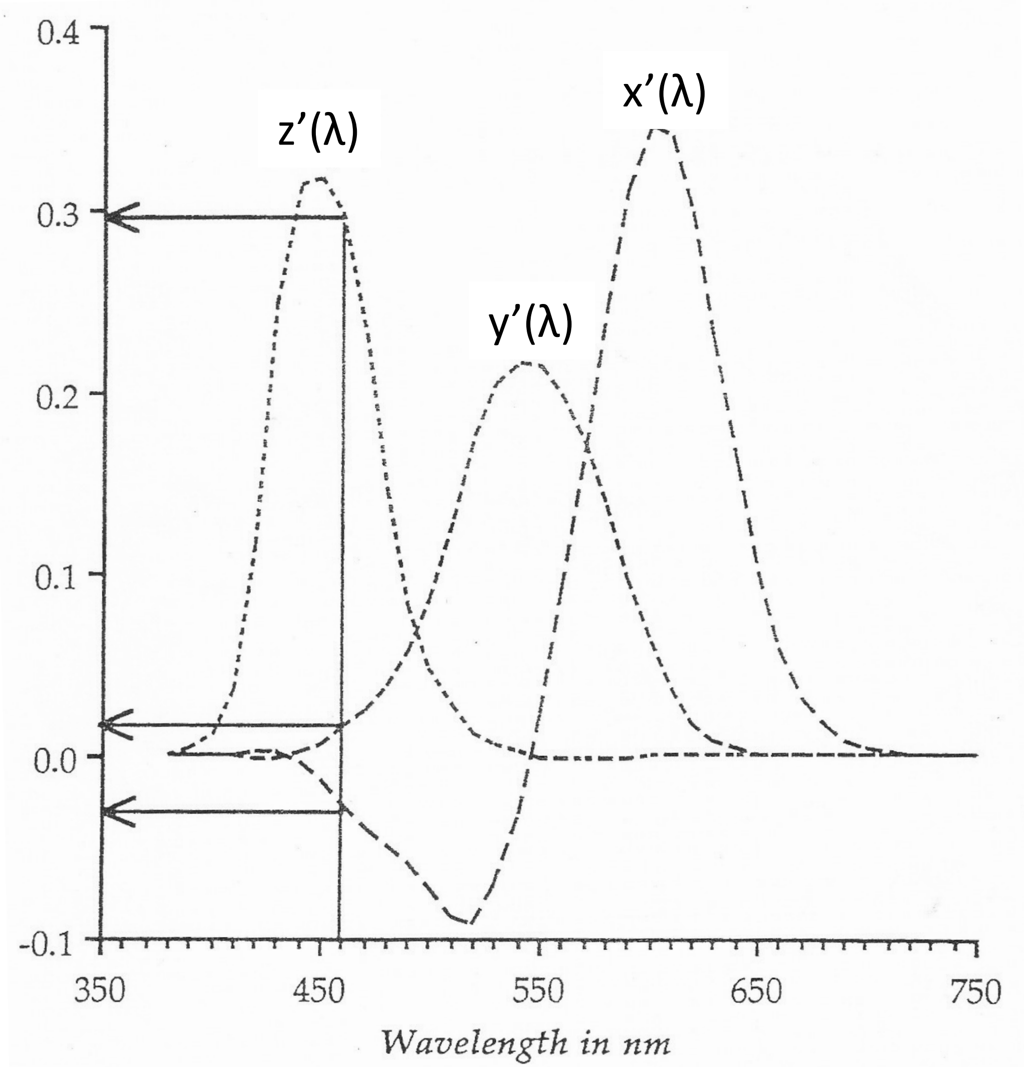




Any unit energy monochromatic test stimulus $Q(\lambda)$ can be matched using

$$Q(\lambda) \stackrel{M}{=} x'R'(\lambda_1) + y'G'(\lambda_2) + z'B'(\lambda_3)$$

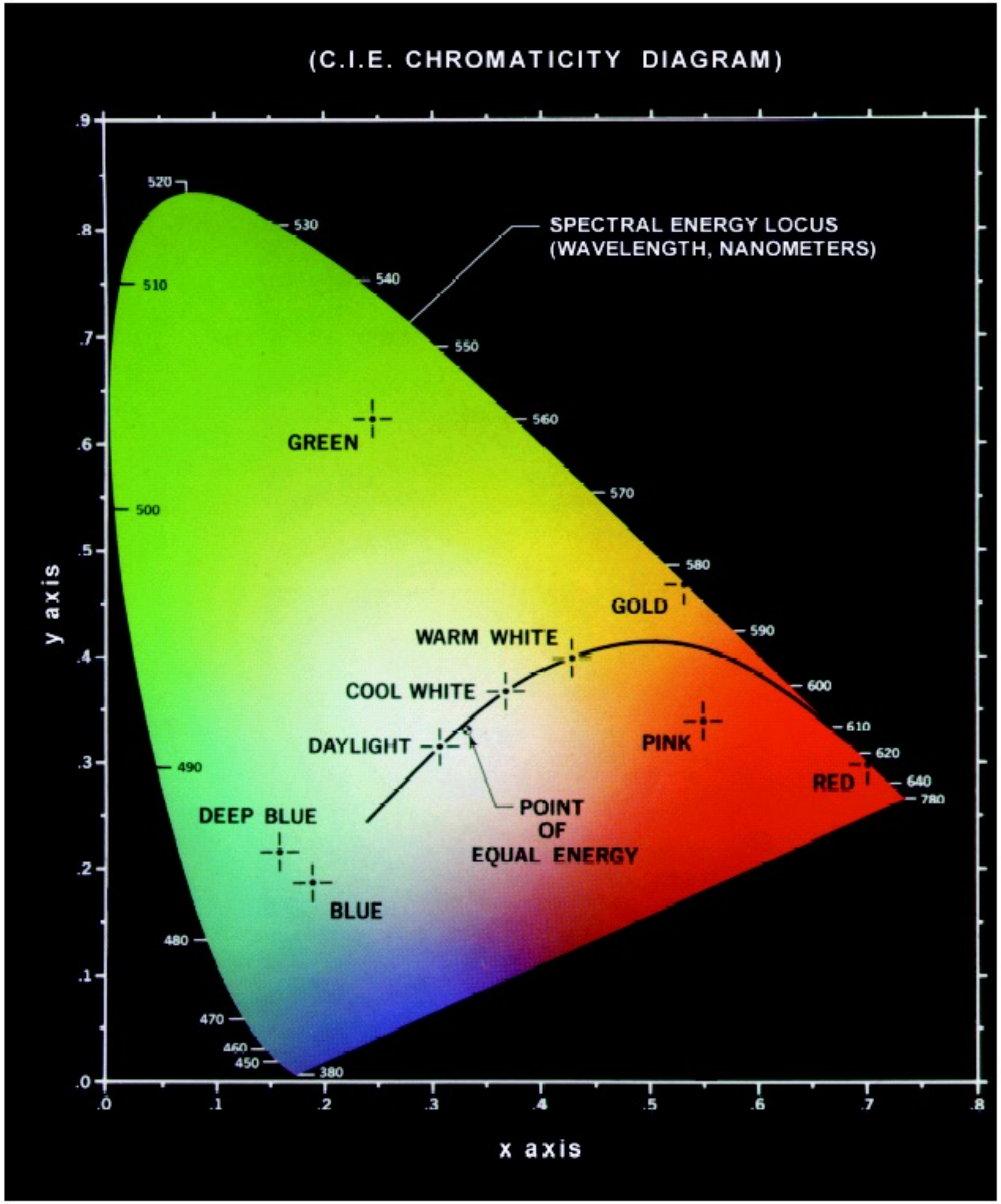
As λ varies, we get functions $x'(\lambda), y'(\lambda), z'(\lambda)$
called color matching functions for these primaries.



Example: Match $Q(\lambda)$ if $\lambda = 460\,nm \longrightarrow x' = -0.026 \quad y' = 0.015 \quad z' = 0.298$

$$Q(460) \stackrel{M}{=} -0.026\,R'(\lambda_1) + 0.015\,G'(\lambda_2) + 0.298\,B'(\lambda_3)$$

FIGURE 6.5
Chromaticity diagram. (Courtesy of the General Electric Co., Lamp Business Division.)



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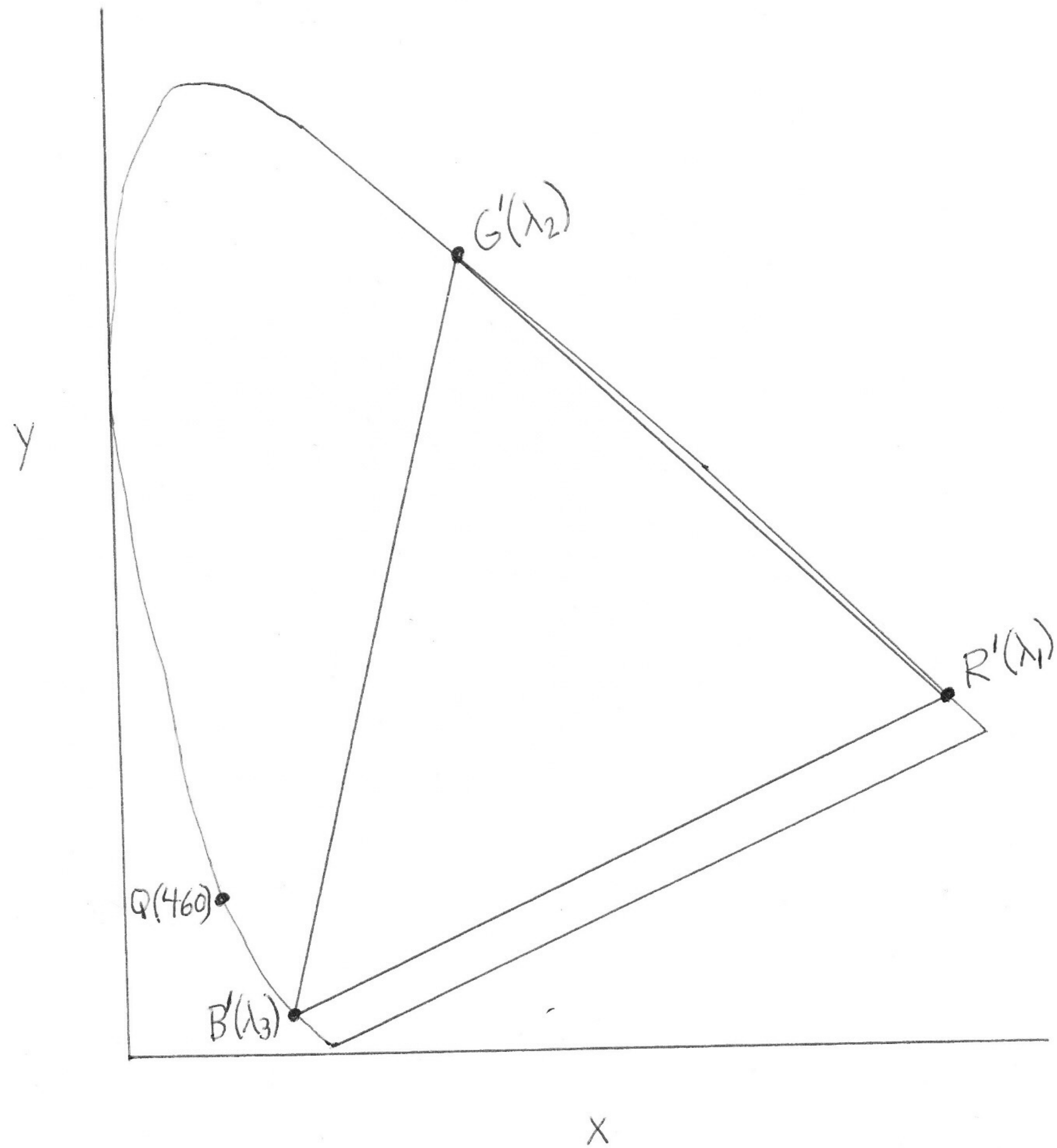
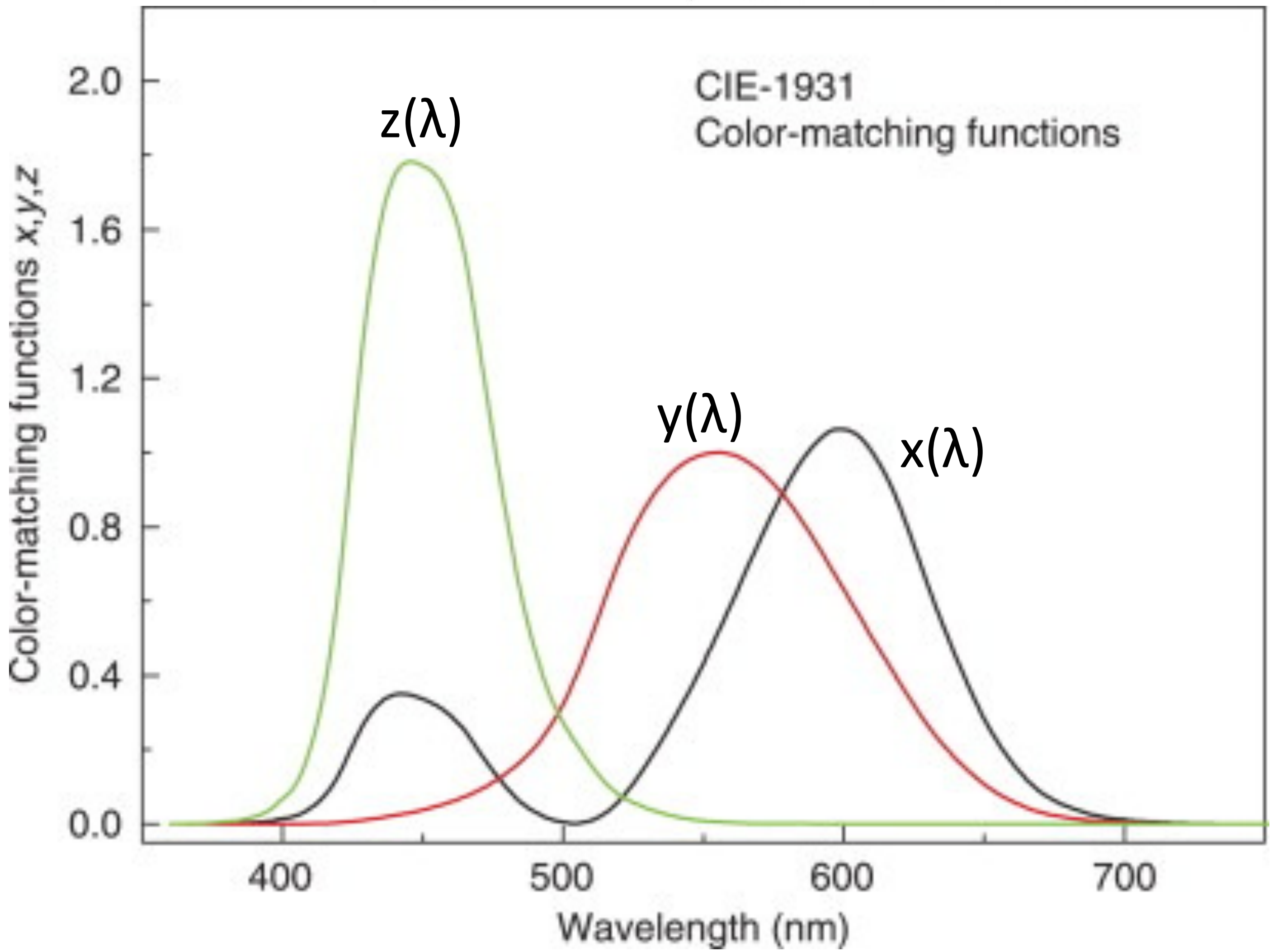


To avoid negative values of color matching functions,
CIE (Commission Internationale de l'Eclairage)
in 1931 defined all positive color matching functions
$x(\lambda), y(\lambda), z(\lambda)$ for hypothetical primaries $R(\lambda_1), G(\lambda_2), B(\lambda_3)$

To avoid negative values of color matching functions, CIE (Commission Internationale de l'Eclairage) in 1931 defined all positive color matching functions $x(\lambda), y(\lambda), z(\lambda)$ for hypothetical primaries $R(\lambda_1), G(\lambda_2), B(\lambda_3)$



Let $I(\lambda)$ be an arbitrary (not monochromatic) light

$$I(\lambda) \overset{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

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$$I(\lambda) \stackrel{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

where $X_I = \int x(\lambda) I(\lambda) d\lambda \quad Y_I = \int y(\lambda) I(\lambda) d\lambda$

$$Z_I = \int z(\lambda) I(\lambda) d\lambda$$

Let $I(\lambda)$ be an arbitrary (not monochromatic) light

$$I(\lambda) \overset{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

where $X_I = \int x(\lambda) I(\lambda) d\lambda$ $\quad Y_I = \int y(\lambda) I(\lambda) d\lambda$

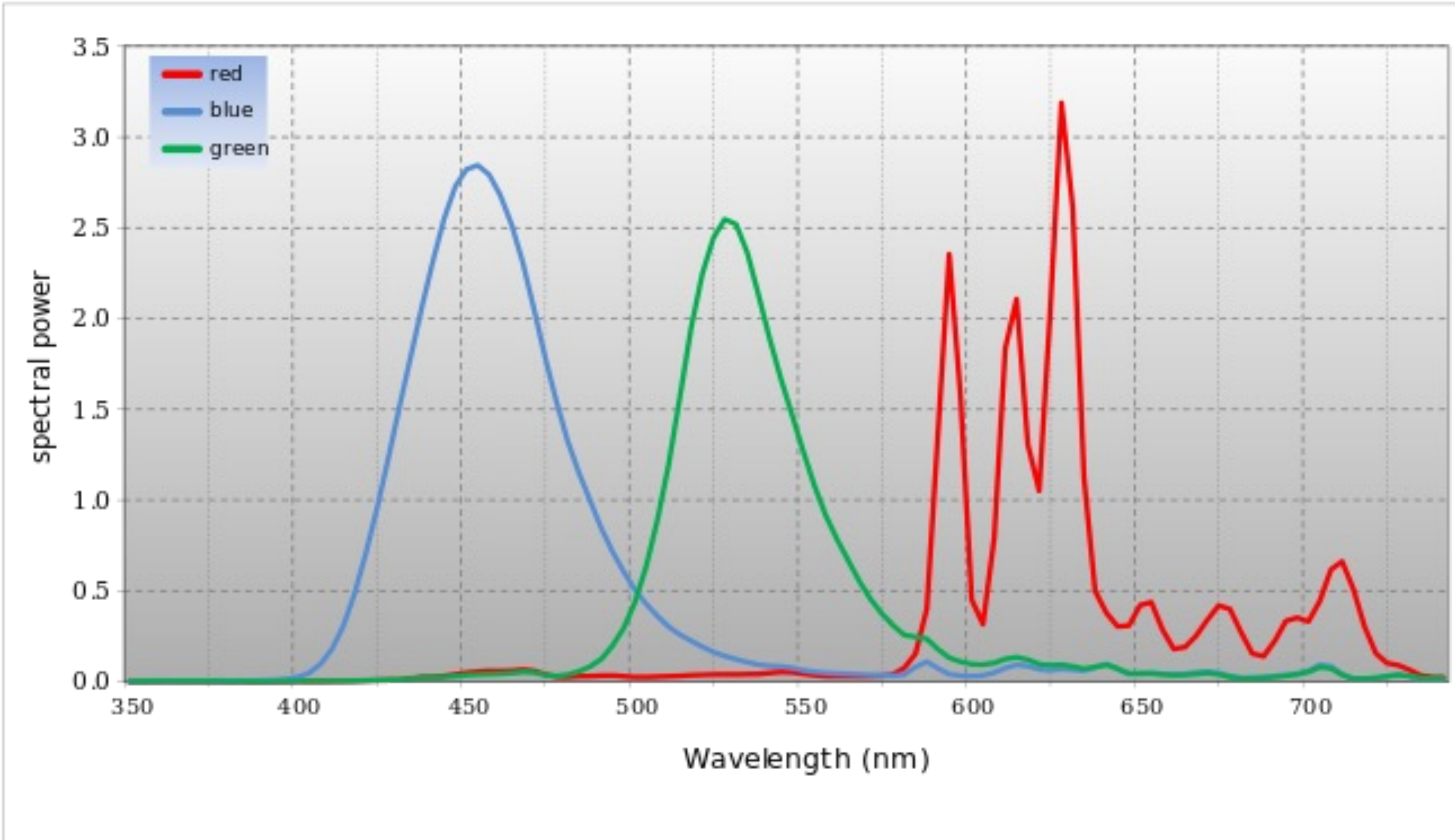
$$Z_I = \int z(\lambda) I(\lambda) d\lambda$$

Problem: Given $I(\lambda)$, find r, g, b so that

$$I(\lambda) \overset{M}{=} r D_R(\lambda) + g D_G(\lambda) + b D_B(\lambda)$$

where $D_R(\lambda), D_G(\lambda), D_B(\lambda)$ are the Known emitted energy distributions for the display guns for a color monitor

$D_R(\lambda)$, $D_G(\lambda)$,$D_B(\lambda)$ for Plasma Display



Represent $D_R(\lambda), D_G(\lambda), D_B(\lambda)$ using X, Y, Z system

$$D_R(\lambda) \overset{M}{=} X_R R(\lambda_1) + Y_R G(\lambda_2) + Z_R B(\lambda_3)$$

$$D_G(\lambda) \overset{M}{=} X_G R(\lambda_1) + Y_G G(\lambda_2) + Z_G B(\lambda_3)$$

$$D_B(\lambda) \overset{M}{=} X_B R(\lambda_1) + Y_B G(\lambda_2) + Z_B B(\lambda_3)$$

Represent $D_R(\lambda), D_G(\lambda), D_B(\lambda)$ using X, Y, Z system

$$D_R(\lambda) \stackrel{M}{=} X_R R(\lambda_1) + Y_R G(\lambda_2) + Z_R B(\lambda_3)$$

$$D_G(\lambda) \stackrel{M}{=} X_G R(\lambda_1) + Y_G G(\lambda_2) + Z_G B(\lambda_3)$$

$$D_B(\lambda) \stackrel{M}{=} X_B R(\lambda_1) + Y_B G(\lambda_2) + Z_B B(\lambda_3)$$

$$X_R = \int x(\lambda) D_R(\lambda) d\lambda \qquad Y_R = \int y(\lambda) D_R(\lambda) d\lambda \qquad Z_R = \int z(\lambda) D_R(\lambda) d\lambda$$

$$X_G = \int x(\lambda) D_G(\lambda) d\lambda \qquad Y_G = \int y(\lambda) D_G(\lambda) d\lambda \qquad Z_G = \int z(\lambda) D_G(\lambda) d\lambda$$

$$X_B = \int x(\lambda) D_B(\lambda) d\lambda \qquad Y_B = \int y(\lambda) D_B(\lambda) d\lambda \qquad Z_B = \int z(\lambda) D_B(\lambda) d\lambda$$

Represent $D_R(\lambda), D_G(\lambda), D_B(\lambda)$ using X, Y, Z system

$$D_R(\lambda) \overset{M}{=} X_R R(\lambda_1) + Y_R G(\lambda_2) + Z_R B(\lambda_3)$$

$$D_G(\lambda) \overset{M}{=} X_G R(\lambda_1) + Y_G G(\lambda_2) + Z_G B(\lambda_3)$$

$$D_B(\lambda) \overset{M}{=} X_B R(\lambda_1) + Y_B G(\lambda_2) + Z_B B(\lambda_3)$$

$$X_R = \int x(\lambda) D_R(\lambda)\, d\lambda \qquad Y_R = \int y(\lambda) D_R(\lambda)\, d\lambda \qquad Z_R = \int z(\lambda) D_R(\lambda)\, d\lambda$$

$$X_G = \int x(\lambda) D_G(\lambda)\, d\lambda \qquad Y_G = \int y(\lambda) D_G(\lambda)\, d\lambda \qquad Z_G = \int z(\lambda) D_G(\lambda)\, d\lambda$$

$$X_B = \int x(\lambda) D_B(\lambda)\, d\lambda \qquad Y_B = \int y(\lambda) D_B(\lambda)\, d\lambda \qquad Z_B = \int z(\lambda) D_B(\lambda)\, d\lambda$$

$$\begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix} \overset{M}{=} \underbrace{\begin{bmatrix} X_R & Y_R & Z_R \\ X_G & Y_G & Z_G \\ X_B & Y_B & Z_B \end{bmatrix}}_{T} \begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix}$$

Represent $D_R(\lambda), D_G(\lambda), D_B(\lambda)$ using X, Y, Z system

$$D_R(\lambda) \stackrel{M}{=} X_R R(\lambda_1) + Y_R G(\lambda_2) + Z_R B(\lambda_3)$$
$$D_G(\lambda) \stackrel{M}{=} X_G R(\lambda_1) + Y_G G(\lambda_2) + Z_G B(\lambda_3)$$
$$D_B(\lambda) \stackrel{M}{=} X_B R(\lambda_1) + Y_B G(\lambda_2) + Z_B B(\lambda_3)$$

$$X_R = \int x(\lambda) D_R(\lambda)\, d\lambda \quad Y_R = \int y(\lambda) D_R(\lambda)\, d\lambda \quad Z_R = \int z(\lambda) D_R(\lambda)\, d\lambda$$
$$X_G = \int x(\lambda) D_G(\lambda)\, d\lambda \quad Y_G = \int y(\lambda) D_G(\lambda)\, d\lambda \quad Z_G = \int z(\lambda) D_G(\lambda)\, d\lambda$$
$$X_B = \int x(\lambda) D_B(\lambda)\, d\lambda \quad Y_B = \int y(\lambda) D_B(\lambda)\, d\lambda \quad Z_B = \int z(\lambda) D_B(\lambda)\, d\lambda$$

$$\underset{\sim}{\begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}} \stackrel{M}{=} \underbrace{\begin{bmatrix} X_R & Y_R & Z_R \\ X_G & Y_G & Z_G \\ X_B & Y_B & Z_B \end{bmatrix}}_{T} \begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix}$$

T matrix depends on monitor and properties of the human vision system $(x(\lambda), y(\lambda), z(\lambda))$

$$\begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix} \stackrel{M}{=} T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

$$\begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix} \overset{M}{=} T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

$$I(\lambda) \overset{M}{=} \begin{bmatrix} X_I & Y_I & Z_I \end{bmatrix} \begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix}$$

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$$\begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix} \overset{M}{=} T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

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$$\overset{M}{=} \begin{bmatrix} X_I & Y_I & Z_I \end{bmatrix} T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

$$\begin{bmatrix} r & g & b \end{bmatrix} = \begin{bmatrix} X_I & Y_I & Z_I \end{bmatrix} T^{-1}$$

Relationship between $x(\lambda), y(\lambda), z(\lambda)$ and $S(\lambda), M(\lambda), L(\lambda)$

In the X, Y, Z system we represent an input $I(\lambda)$ using the primaries $R(\lambda_1), G(\lambda_2), B(\lambda_3)$ as

$$I(\lambda) \overset{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

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In the X,Y,Z system we represent an input $I(\lambda)$ using the primaries $R(\lambda_1), G(\lambda_2), B(\lambda_3)$ as

$$I(\lambda) \overset{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

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$$R_L = \int I(\lambda) L(\lambda) d\lambda \qquad R_M = \int I(\lambda) M(\lambda) d\lambda \qquad R_S = \int I(\lambda) S(\lambda) d\lambda$$

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$$I(\lambda) \overset{M}{=} X_I R(\lambda_1) + Y_I G(\lambda_2) + Z_I B(\lambda_3)$$

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$$I(\lambda) \stackrel{M}{=} \hat{x}' \hat{R}'(\lambda) + \hat{y}' \hat{G}'(\lambda) + \hat{z}' \hat{B}'(\lambda)$$

$$\hat{x}' = a_{11} X_I + a_{12} Y_I + a_{13} Z_I$$
$$\hat{y}' = a_{21} X_I + a_{22} Y_I + a_{23} Z_I$$
$$\hat{z}' = a_{31} X_I + a_{32} Y_I + a_{33} Z_I$$

$$\hat{X}' = a_{11}\int x(\lambda)\,I(\lambda)\,d\lambda + a_{12}\int y(\lambda)\,I(\lambda)\,d\lambda + a_{13}\int z(\lambda)\,I(\lambda)\,d\lambda$$

$$= \int \underbrace{\left(a_{11}x(\lambda) + a_{12}y(\lambda) + a_{13}z(\lambda)\right)}_{\text{new color matching function}} I(\lambda)\,d\lambda$$

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$$\hat{y}' = a_{21} \int x(\lambda) I(\lambda) d\lambda + a_{22} \int y(\lambda) I(\lambda) d\lambda + a_{23} \int z(\lambda) I(\lambda) d\lambda$$

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$$\hat{Y}' = a_{21}\int x(\lambda) I(\lambda)\,d\lambda + a_{22}\int y(\lambda) I(\lambda)\,d\lambda + a_{23}\int z(\lambda) I(\lambda)\,d\lambda$$
$$= \int \underbrace{\left(a_{21}x(\lambda) + a_{22}y(\lambda) + a_{23}z(\lambda)\right)}_{\text{new color matching function}} I(\lambda)\,d\lambda$$

$$\hat{Z}' = a_{31}\int x(\lambda) I(\lambda)\,d\lambda + a_{32}\int y(\lambda) I(\lambda)\,d\lambda + a_{33}\int z(\lambda) I(\lambda)\,d\lambda$$
$$= \int \underbrace{\left(a_{31}x(\lambda) + a_{32}y(\lambda) + a_{33}z(\lambda)\right)}_{\text{new color matching function}} I(\lambda)\,d\lambda$$

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For different primaries, new color matching functions are a linear transformation of $x(\lambda), y(\lambda), z(\lambda)$

$$\hat{x}' = a_{11}\int x(\lambda)I(\lambda)d\lambda + a_{12}\int y(\lambda)I(\lambda)d\lambda + a_{13}\int z(\lambda)I(\lambda)d\lambda$$

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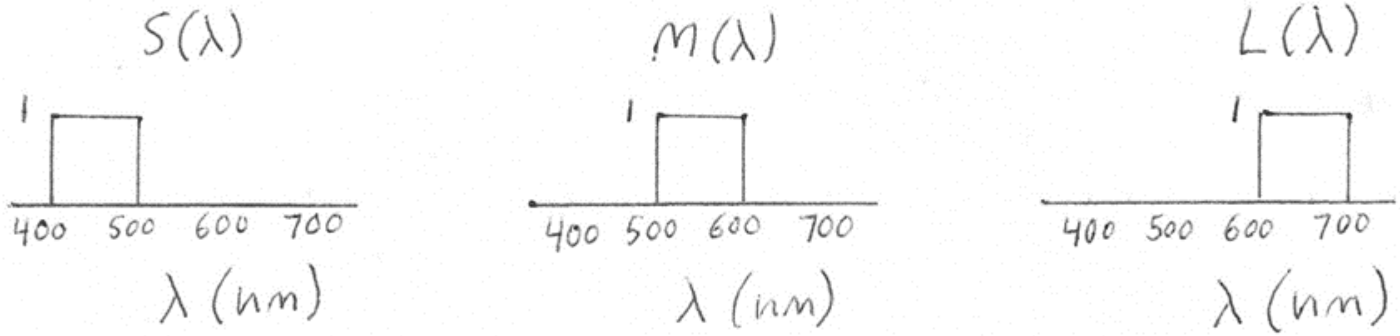
$$\hat{z}' = a_{31}\int x(\lambda)I(\lambda)d\lambda + a_{32}\int y(\lambda)I(\lambda)d\lambda + a_{33}\int z(\lambda)I(\lambda)d\lambda$$

$$= \int \underbrace{(a_{31}x(\lambda) + a_{32}y(\lambda) + a_{33}z(\lambda))}_{\text{new color matching function}} I(\lambda)d\lambda$$

For different primaries, new color matching functions are a linear transformation of $x(\lambda), y(\lambda), z(\lambda)$

$x(\lambda), y(\lambda), z(\lambda)$ are a linear transformation of $L(\lambda), M(\lambda), S(\lambda)$ so that the color matching experiments and physiological data agree.

Ex Suppose the human vision system has the idealized response functions



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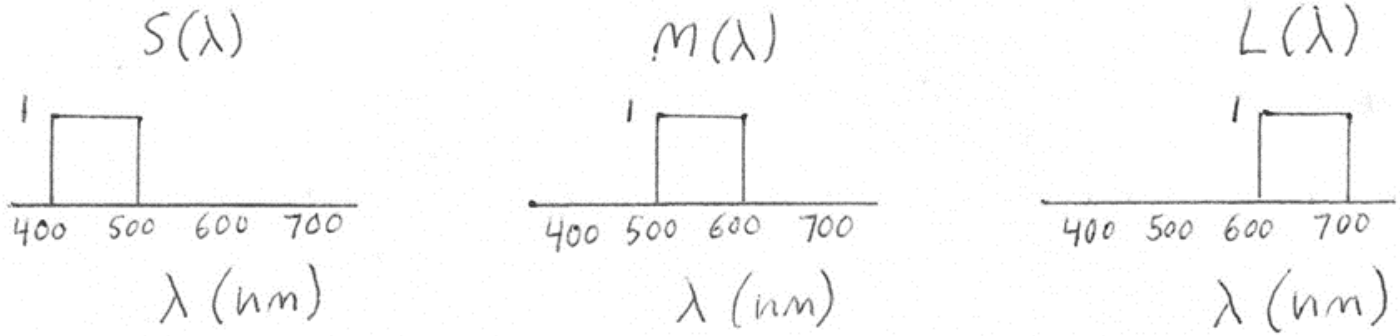


so that any stimuli $I(\lambda)$ having the same R_S, R_M, R_L

$$R_S = \int I(\lambda) S(\lambda)\, d\lambda \quad R_M = \int I(\lambda) M(\lambda)\, d\lambda \quad R_L = \int I(\lambda) L(\lambda)\, d\lambda$$

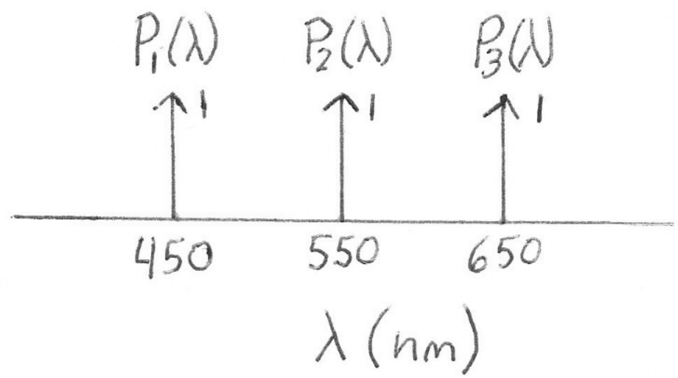
will appear the same.

Consider 3 primaries

$P_1(\lambda)$ = monochromatic unit energy light at 450 nm

$P_2(\lambda)$ = monochromatic unit energy light at 550 nm

$P_3(\lambda)$ = monochromatic unit energy light at 650 nm



a) Find the color matching functions $x'(\lambda), y'(\lambda), z'(\lambda)$ corresponding to $P_1(\lambda), P_2(\lambda), P_3(\lambda)$ respectively that will be measured for these primaries.

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Let $Q(\lambda_0)$ = monochromatic unit energy light at λ_0 nm

$$Q(\lambda_0) \overset{M}{=} x'(\lambda_0) P_1(\lambda) + y'(\lambda_0) P_2(\lambda) + z'(\lambda_0) P_3(\lambda)$$

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Let $Q(\lambda_0)$ = monochromatic unit energy light at λ_0 nm

$$Q(\lambda_0) \overset{M}{=} x'(\lambda_0)P_1(\lambda) + y'(\lambda_0)P_2(\lambda) + z'(\lambda_0)P_3(\lambda)$$

For a match

$$(1) \int Q(\lambda_0)S(\lambda)d\lambda = \int [x'(\lambda_0)P_1(\lambda) + y'(\lambda_0)P_2(\lambda) + z'(\lambda_0)P_3(\lambda)]S(\lambda)d\lambda$$

$$(2) \int Q(\lambda_0)M(\lambda)d\lambda = \int [x'(\lambda_0)P_1(\lambda) + y'(\lambda_0)P_2(\lambda) + z'(\lambda_0)P_3(\lambda)]M(\lambda)d\lambda$$

$$(3) \int Q(\lambda_0)L(\lambda)d\lambda = \int [x'(\lambda_0)P_1(\lambda) + y'(\lambda_0)P_2(\lambda) + z'(\lambda_0)P_3(\lambda)]L(\lambda)d\lambda$$

$$(1) \int Q(\lambda_0) S(\lambda) d\lambda = \int x'(\lambda_0) P_1(\lambda) S(\lambda) d\lambda$$

$$(2) \int Q(\lambda_0) M(\lambda) d\lambda = \int y'(\lambda_0) P_2(\lambda) M(\lambda) d\lambda$$

$$(3) \int Q(\lambda_0) L(\lambda) d\lambda = \int z'(\lambda_0) P_3(\lambda) L(\lambda) d\lambda$$

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(3) $\int Q(\lambda_0) L(\lambda) d\lambda = \int z'(\lambda_0) P_3(\lambda) L(\lambda) d\lambda$

If $400\,nm < \lambda_0 < 500\,nm$ then

(1) $1 = \int x'(\lambda_0) P_1(\lambda) S(\lambda) d\lambda \longrightarrow x'(\lambda_0) = 1$

(2) $0 = \int y'(\lambda_0) P_2(\lambda) M(\lambda) d\lambda \longrightarrow y'(\lambda_0) = 0$

(3) $0 = \int z'(\lambda_0) P_3(\lambda) L(\lambda) d\lambda \longrightarrow z'(\lambda_0) = 0$

If $500\,nm < \lambda_0 < 600\,nm$ then

(1) $$0 = \int x'(\lambda_0) P_1(\lambda) S(\lambda) d\lambda \longrightarrow x'(\lambda_0) = 0$$

(2) $$1 = \int y'(\lambda_0) P_2(\lambda) M(\lambda) d\lambda \longrightarrow y'(\lambda_0) = 1$$

(3) $$0 = \int z'(\lambda_0) P_3(\lambda) L(\lambda) d\lambda \longrightarrow z'(\lambda_0) = 0$$

If $500\,nm < \lambda_o < 600\,nm$ then

(1) $$0 = \int x'(\lambda_o) P_1(\lambda) S(\lambda)\, d\lambda \longrightarrow x'(\lambda_o) = 0$$

(2) $$1 = \int y'(\lambda_o) P_2(\lambda) M(\lambda)\, d\lambda \longrightarrow y'(\lambda_o) = 1$$

(3) $$0 = \int z'(\lambda_o) P_3(\lambda) L(\lambda)\, d\lambda \longrightarrow z'(\lambda_o) = 0$$

If $600\,nm < \lambda_o < 700\,nm$ then

(1) $$0 = \int x'(\lambda_o) P_1(\lambda) S(\lambda)\, d\lambda \longrightarrow x'(\lambda_o) = 0$$

(2) $$0 = \int y'(\lambda_o) P_2(\lambda) M(\lambda)\, d\lambda \longrightarrow y'(\lambda_o) = 0$$

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If 500 nm < λ_o < 600 nm then

$$(1) \qquad 0 = \int x'(\lambda_o) P_1(\lambda) S(\lambda) d\lambda \longrightarrow x'(\lambda_o) = 0$$

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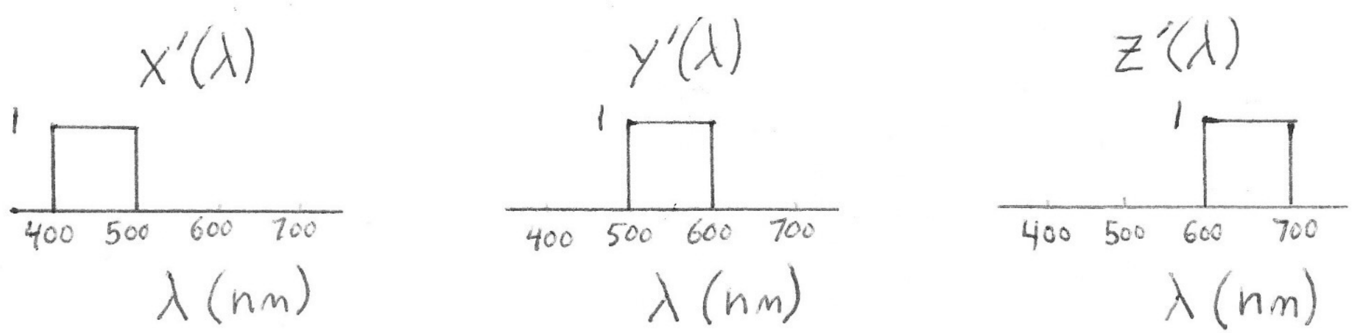
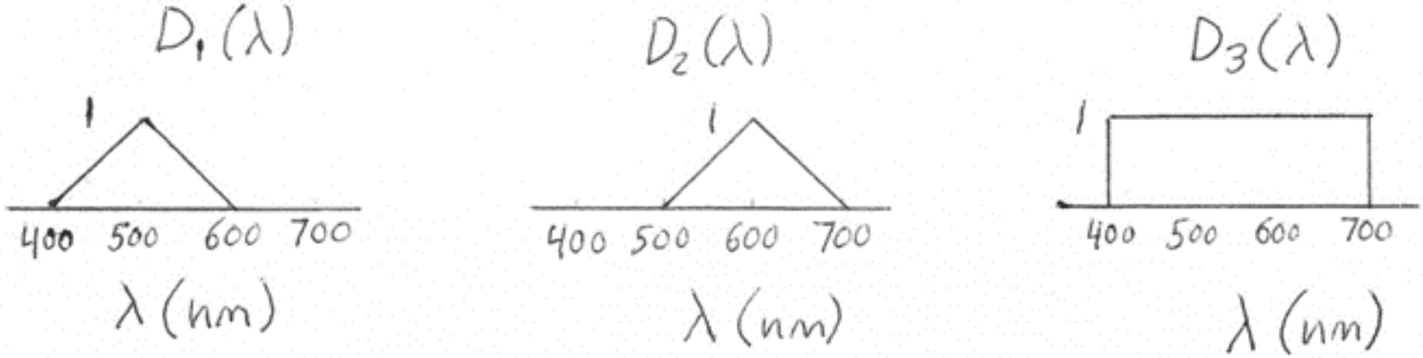


b) Consider display guns $D_1(\lambda)$, $D_2(\lambda)$, $D_3(\lambda)$ with spectral distributions



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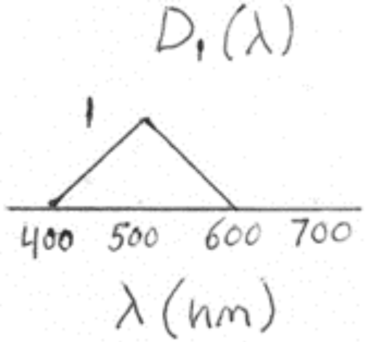
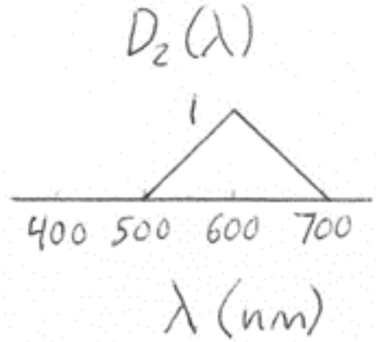
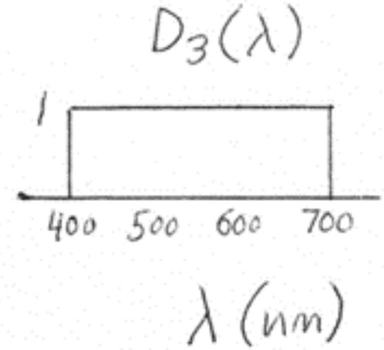




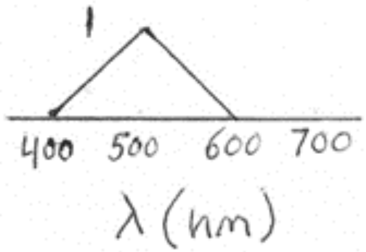


Find a 3x3 matrix C such that

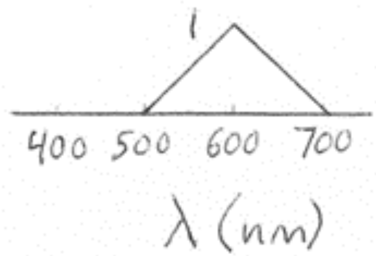
$$[D_1(\lambda)\ D_2(\lambda)\ D_3(\lambda)] \stackrel{M}{=} [P_1(\lambda)\ P_2(\lambda)\ P_3(\lambda)]\, C$$

b) Consider display guns $D_1(\lambda)$, $D_2(\lambda)$, $D_3(\lambda)$ with spectral distributions

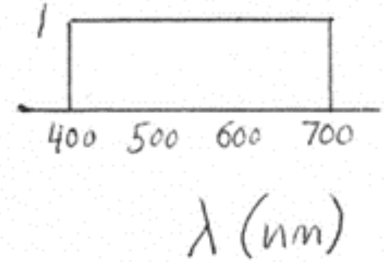
$D_1(\lambda)$



$D_2(\lambda)$



$D_3(\lambda)$



Find a 3x3 matrix C such that

$$[D_1(\lambda)\ D_2(\lambda)\ D_3(\lambda)] \overset{M}{=} [P_1(\lambda)\ P_2(\lambda)\ P_3(\lambda)]\, C$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$D_1(\lambda) \overset{M}{=} c_{11} P_1(\lambda) + c_{21} P_2(\lambda) + c_{31} P_3(\lambda)$$

$$D_2(\lambda) \overset{M}{=} c_{12} P_1(\lambda) + c_{22} P_2(\lambda) + c_{32} P_3(\lambda)$$

$$D_3(\lambda) \overset{M}{=} c_{13} P_1(\lambda) + c_{23} P_2(\lambda) + c_{33} P_3(\lambda)$$

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$$D_3(\lambda) \overset{M}{=} c_{13} P_1(\lambda) + c_{23} P_2(\lambda) + c_{33} P_3(\lambda)$$

$$c_{11} = \int D_1(\lambda)\, x'(\lambda)\, d\lambda = 50 \qquad c_{21} = \int D_1(\lambda)\, y'(\lambda)\, d\lambda = 50 \qquad c_{31} = \int D_1(\lambda)\, z'(\lambda)\, d\lambda = 0$$

$$D_1(\lambda) \stackrel{M}{=} c_{11} P_1(\lambda) + c_{21} P_2(\lambda) + c_{31} P_3(\lambda)$$

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$$c_{12} = \int D_2(\lambda)\, x'(\lambda)\, d\lambda = 0 \qquad c_{22} = \int D_2(\lambda)\, y'(\lambda)\, d\lambda = 50 \qquad c_{32} = \int D_2(\lambda)\, z'(\lambda)\, d\lambda = 50$$

$$c_{13} = \int D_3(\lambda)\, x'(\lambda)\, d\lambda = 100 \qquad c_{23} = \int D_3(\lambda)\, y'(\lambda)\, d\lambda = 100 \qquad c_{33} = \int D_3(\lambda)\, z'(\lambda)\, d\lambda = 100$$

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$$c_{12} = \int D_2(\lambda) x'(\lambda) d\lambda = 0 \qquad c_{22} = \int D_2(\lambda) y'(\lambda) d\lambda = 50 \qquad c_{32} = \int D_2(\lambda) z'(\lambda) d\lambda = 50$$

$$c_{13} = \int D_3(\lambda) x'(\lambda) d\lambda = 100 \qquad c_{23} = \int D_3(\lambda) y'(\lambda) d\lambda = 100 \qquad c_{33} = \int D_3(\lambda) z'(\lambda) d\lambda = 100$$

$$C = \begin{bmatrix} 50 & 0 & 100 \\ 50 & 50 & 100 \\ 0 & 50 & 100 \end{bmatrix}$$

c) For an arbitrary $I(\lambda)$, find a_1, a_2, a_3 so that

$$I(\lambda) \stackrel{M}{=} a_1 D_1(\lambda) + a_2 D_2(\lambda) + a_3 D_3(\lambda)$$

c) For an arbitrary $I(\lambda)$, find a_1, a_2, a_3 so that

$$I(\lambda) \stackrel{M}{=} a_1 D_1(\lambda) + a_2 D_2(\lambda) + a_3 D_3(\lambda)$$

$$I(\lambda) \stackrel{M}{=} \begin{bmatrix} P_1(\lambda) & P_2(\lambda) & P_3(\lambda) \end{bmatrix} \begin{bmatrix} X_I & Y_I & Z_I \end{bmatrix}^T$$

$$X_I = \int I(\lambda) x'(\lambda) d\lambda \quad Y_I = \int I(\lambda) y'(\lambda) d\lambda \quad Z_I = \int I(\lambda) z'(\lambda) d\lambda$$

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$$X_I = \int I(\lambda) x'(\lambda) d\lambda \quad Y_I = \int I(\lambda) y'(\lambda) d\lambda \quad Z_I = \int I(\lambda) z'(\lambda) d\lambda$$

From b), $\begin{bmatrix} P_1(\lambda) & P_2(\lambda) & P_3(\lambda) \end{bmatrix} \overset{M}{=} \begin{bmatrix} D_1(\lambda) & D_2(\lambda) & D_3(\lambda) \end{bmatrix} C^{-1}$

c) For an arbitrary $I(\lambda)$, find a_1, a_2, a_3 so that

$$I(\lambda) \overset{M}{=} a_1 D_1(\lambda) + a_2 D_2(\lambda) + a_3 D_3(\lambda)$$

$$I(\lambda) \overset{M}{=} [P_1(\lambda) \ P_2(\lambda) \ P_3(\lambda)] [X_I \ Y_I \ Z_I]^T$$

$$X_I = \int I(\lambda) x'(\lambda) d\lambda \quad Y_I = \int I(\lambda) y'(\lambda) d\lambda \quad Z_I = \int I(\lambda) z'(\lambda) d\lambda$$

From b), $[P_1(\lambda) \ P_2(\lambda) \ P_3(\lambda)] \overset{M}{=} [D_1(\lambda) \ D_2(\lambda) \ D_3(\lambda)] C^{-1}$

$$I(\lambda) \overset{M}{=} [D_1(\lambda) \ D_2(\lambda) \ D_3(\lambda)] C^{-1} [X_I \ Y_I \ Z_I]^T$$

c) For an arbitrary $I(\lambda)$, find a_1, a_2, a_3 so that

$$I(\lambda) \overset{M}{=} a_1 D_1(\lambda) + a_2 D_2(\lambda) + a_3 D_3(\lambda)$$

$$I(\lambda) \overset{M}{=} \left[P_1(\lambda) \;\; P_2(\lambda) \;\; P_3(\lambda)\right] \left[X_I \;\; Y_I \;\; Z_I\right]^T$$

$$X_I = \int I(\lambda) x'(\lambda) d\lambda \quad Y_I = \int I(\lambda) y'(\lambda) d\lambda \quad Z_I = \int I(\lambda) z'(\lambda) d\lambda$$

From b), $\left[P_1(\lambda) \;\; P_2(\lambda) \;\; P_3(\lambda)\right] \overset{M}{=} \left[D_1(\lambda) \;\; D_2(\lambda) \;\; D_3(\lambda)\right] C^{-1}$

$$I(\lambda) \overset{M}{=} \left[D_1(\lambda) \;\; D_2(\lambda) \;\; D_3(\lambda)\right] C^{-1} \left[X_I \;\; Y_I \;\; Z_I\right]^T$$

$$\left[a_1 \;\; a_2 \;\; a_3\right]^T = C^{-1} \left[X_I \;\; Y_I \;\; Z_I\right]^T$$

c) For an arbitrary $I(\lambda)$, find a_1, a_2, a_3 so that

$$I(\lambda) \overset{M}{=} a_1 D_1(\lambda) + a_2 D_2(\lambda) + a_3 D_3(\lambda)$$

$$I(\lambda) \overset{M}{=} \left[P_1(\lambda) \; P_2(\lambda) \; P_3(\lambda) \right] \left[X_I \; Y_I \; Z_I \right]^T$$

$$X_I = \int I(\lambda) x'(\lambda) d\lambda \quad Y_I = \int I(\lambda) y'(\lambda) d\lambda \quad Z_I = \int I(\lambda) z'(\lambda) d\lambda$$

From b),
$$\left[P_1(\lambda) \; P_2(\lambda) \; P_3(\lambda) \right] \overset{M}{=} \left[D_1(\lambda) \; D_2(\lambda) \; D_3(\lambda) \right] C^{-1}$$

$$I(\lambda) \overset{M}{=} \left[D_1(\lambda) \; D_2(\lambda) \; D_3(\lambda) \right] C^{-1} \left[X_I \; Y_I \; Z_I \right]^T$$

$$\left[a_1 \; a_2 \; a_3 \right]^T = C^{-1} \left[X_I \; Y_I \; Z_I \right]^T$$

$$C^{-1} = \frac{1}{100} \begin{bmatrix} 0 & 2 & -2 \\ -2 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$