EECS 203A: HW #2 Solution, Spring 2022

1.a)

Input			Desired			Transform	
r_k	h(r_k)	T(r_k)	r_k	d(r_k)	G(r_k)	r_k	M(r_k)
0	0	0	0	30	30	0	0
1	3	3	1	27	57	1	0
2	6	9	2	24	81	2	0
3	9	18	3	21	102	3	0
4	12	30	4	18	120	4	0
5	15	45	5	15	135	5	1
6	18	63	6	12	147	6	1
7	21	84	7	9	156	7	2
8	24	108	8	6	162	8	3
9	27	135	9	3	165	9	5
10	30	165	10	0	165	10	10

Choose M for each r_k so that $T(r_k)$ is as close as possible to $G(r_k)$. Note: M(10) can also be 9.

b)

r_k	0	1	2	3	4	5	6	7	8	9	10
o(r_k)	30	33	21	24	0	27	0	0	0	0	30

2.a)

$$h(x,y) = \frac{1}{N}[-f(x,y) - n(x,y,t_1) + f(x,y) + n(x,y,t_2) + \ldots - f(x,y) - n(x,y,t_{N-1}) + f(x,y) + n(x,y,t_N)]$$

$$E[h(x,y)] = 0$$

b) $\mathrm{VAR}[h(x,y)] = \mathrm{VAR}\left[\frac{1}{N}(-n(x,y,t_1) + n(x,y,t_2) + \ldots + n(x,y,t_N))\right]$

$$= \tfrac{1}{N^2}\mathrm{VAR}[-n(x,y,t_1) + n(x,y,t_2) + \ldots + n(x,y,t_N)]$$

and since noise is independent

$$\mathrm{VAR}[h(x,y)] = \tfrac{1}{N^2}(N\sigma_n^2(x,y)) = \sigma_n^2(x,y)/N$$