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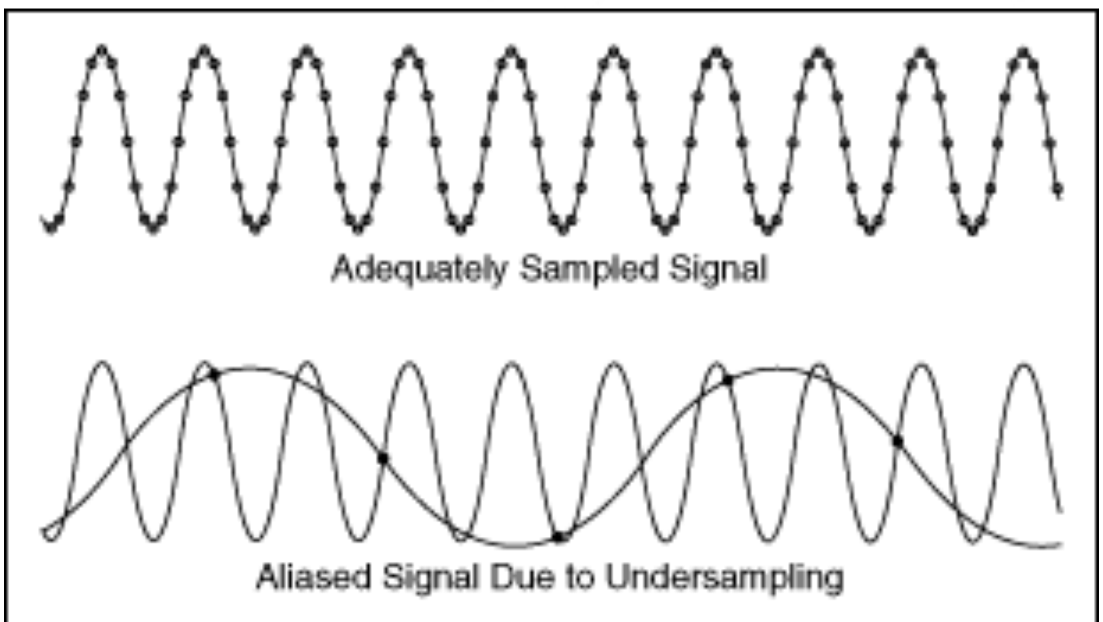
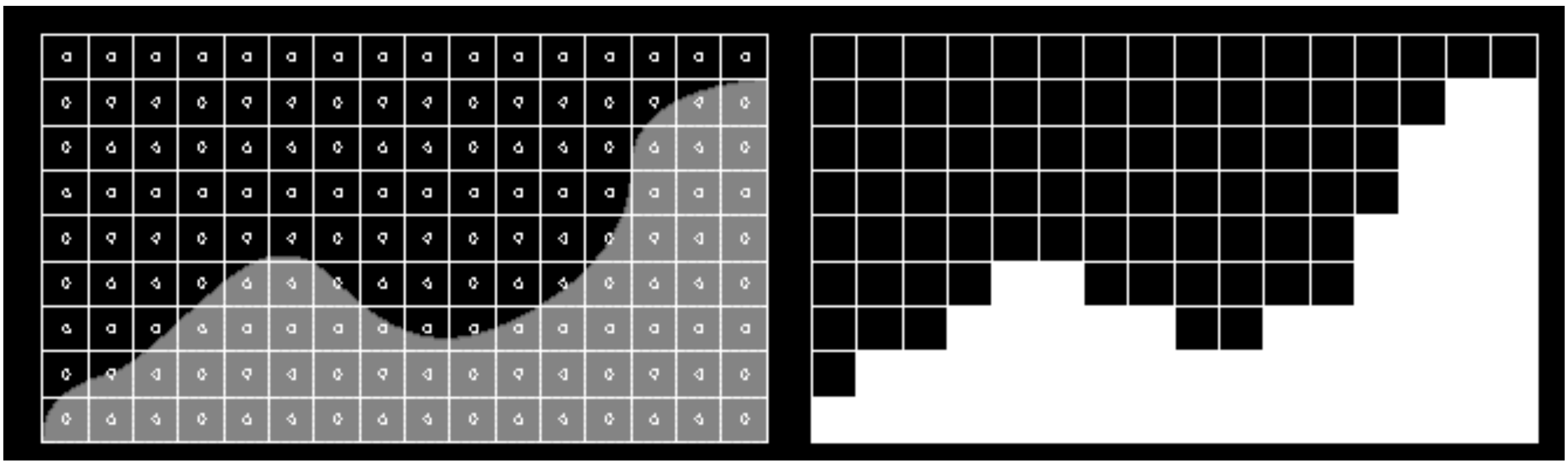
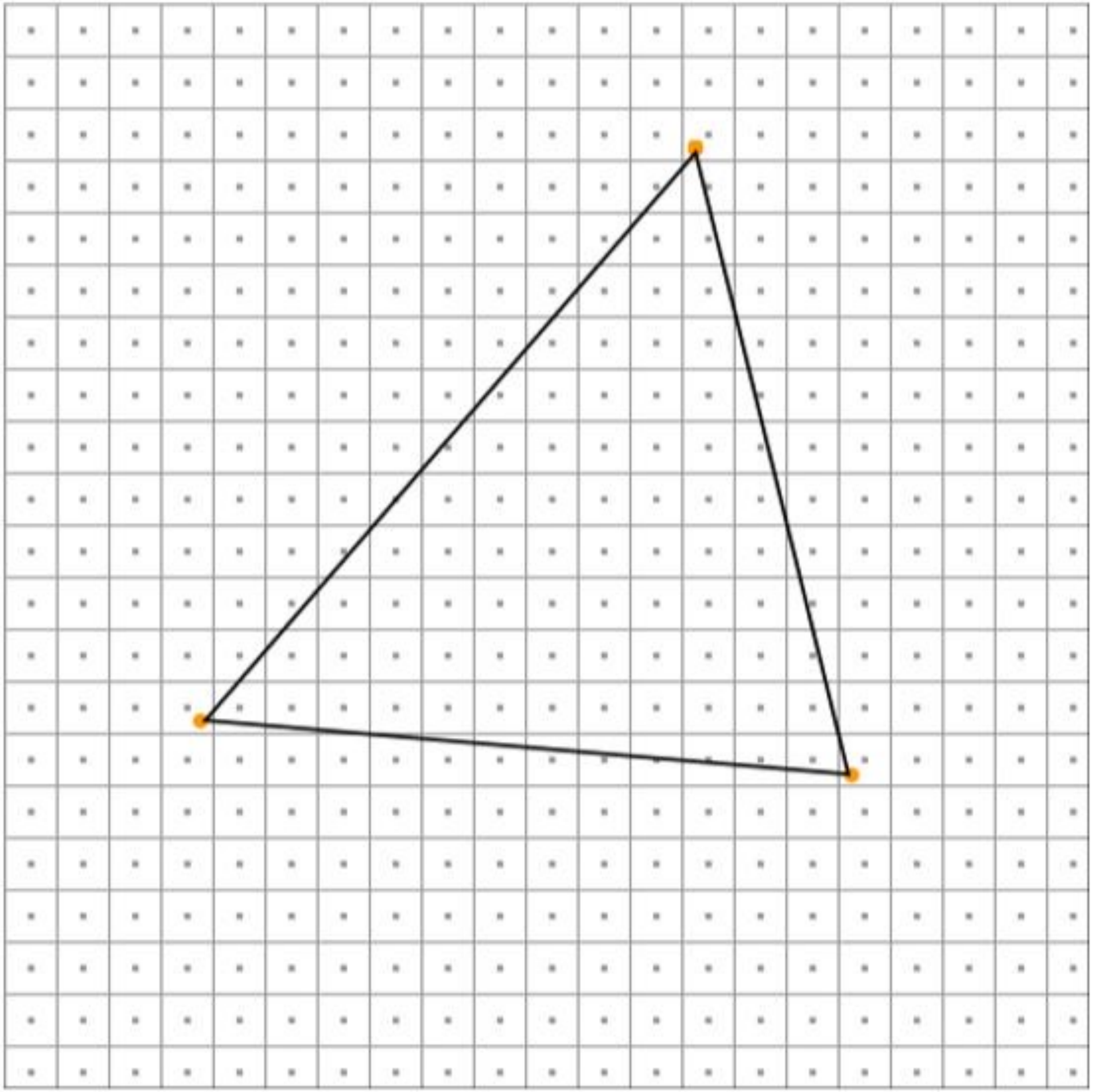


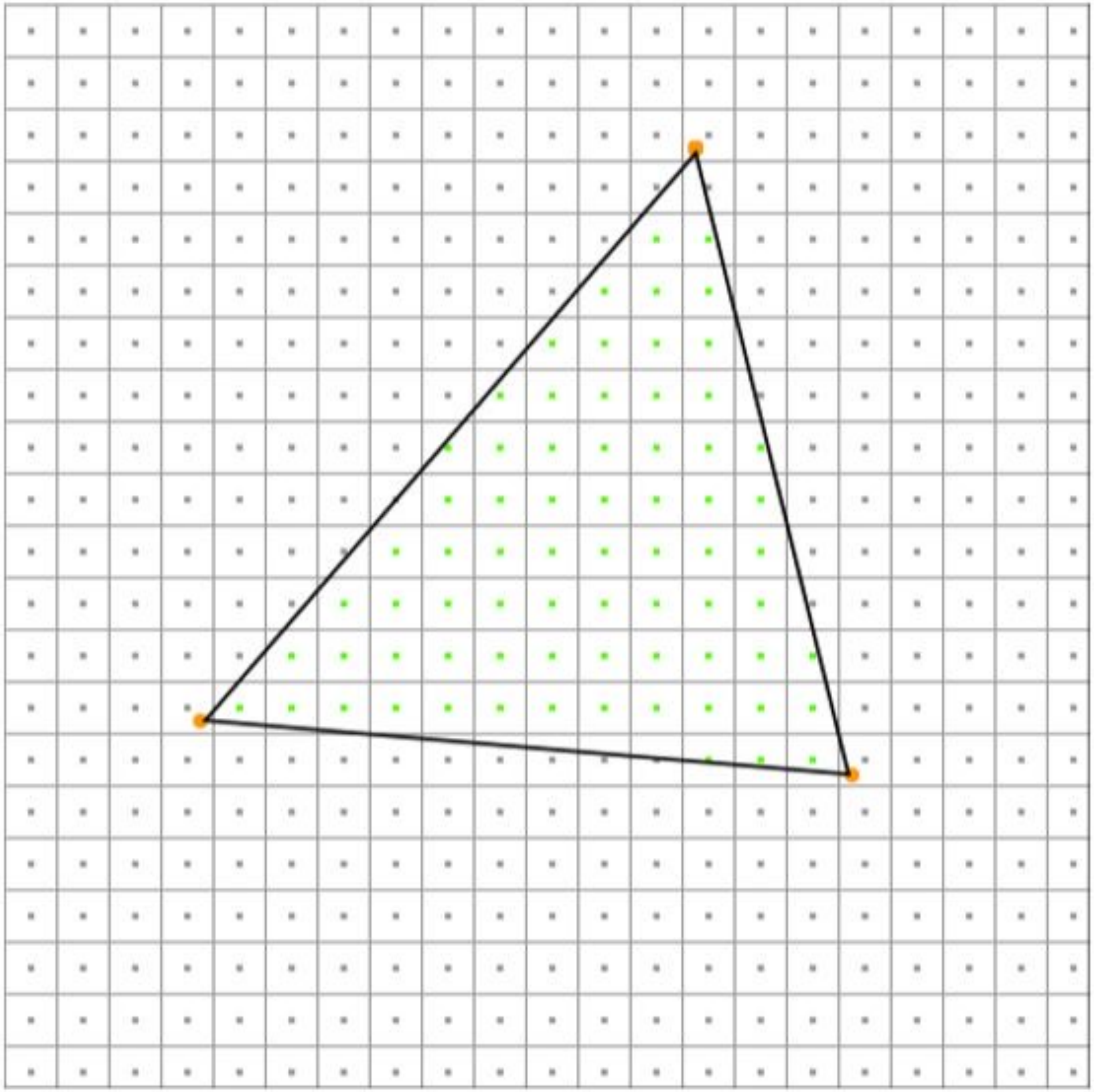
In images, aliasing is observed as

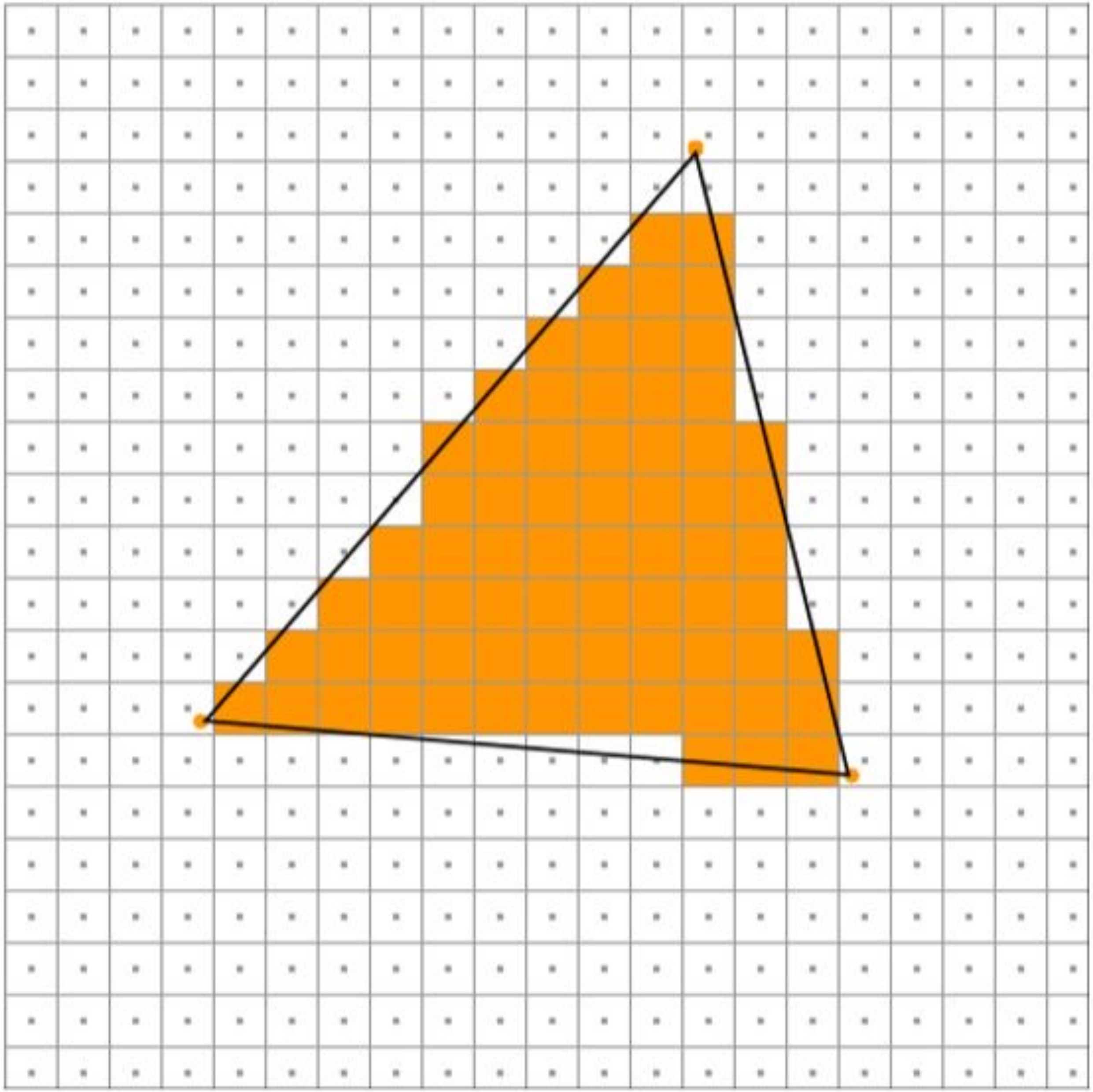
1) jagged edges
2) incorrectly rendered fine detail
3) loss of small objects

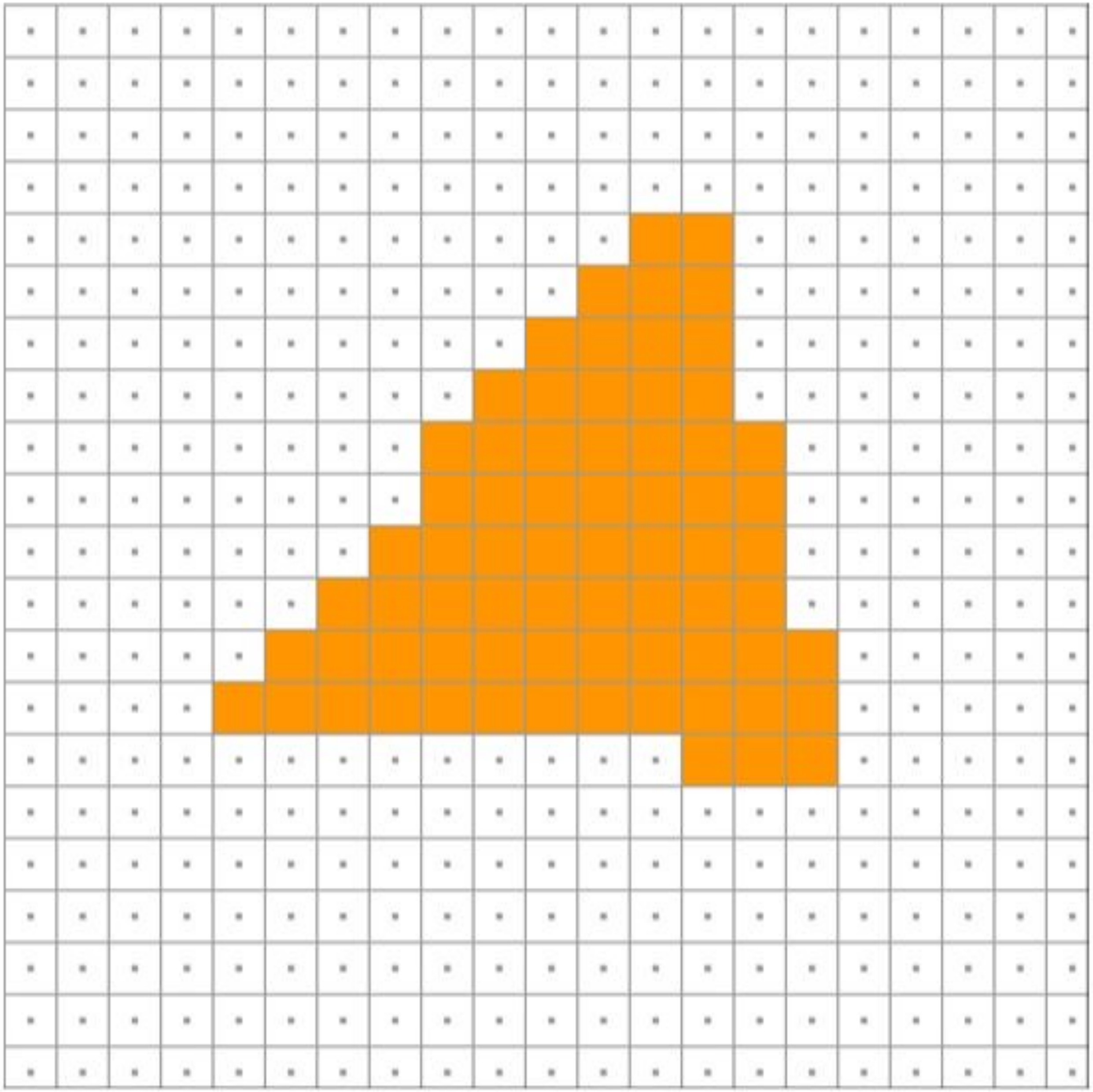
Jagged Edges



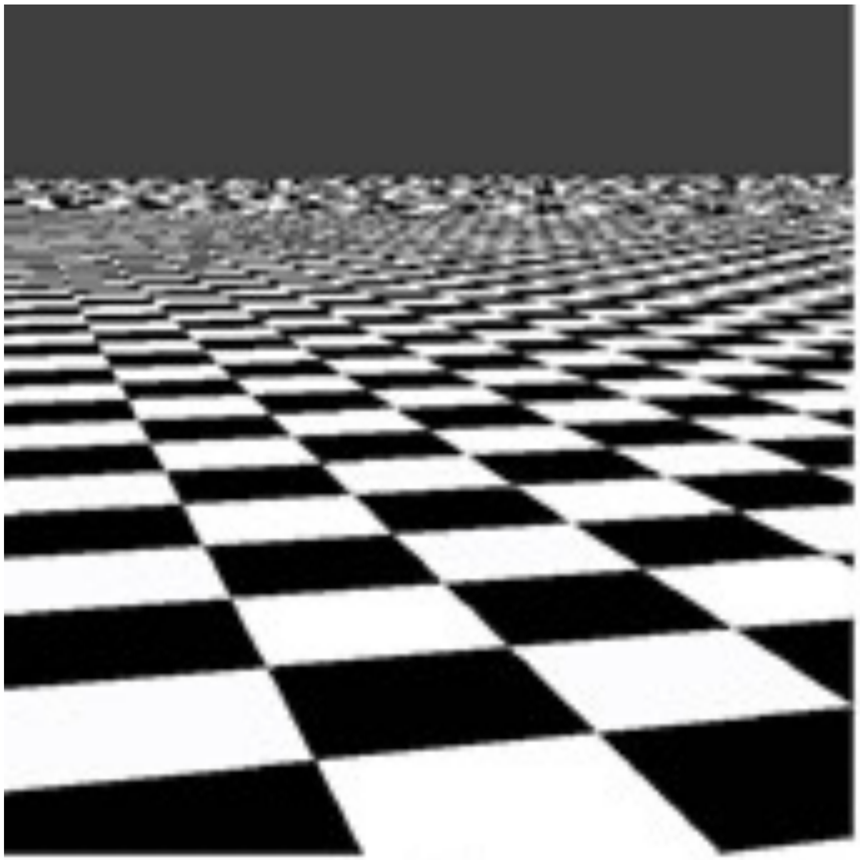


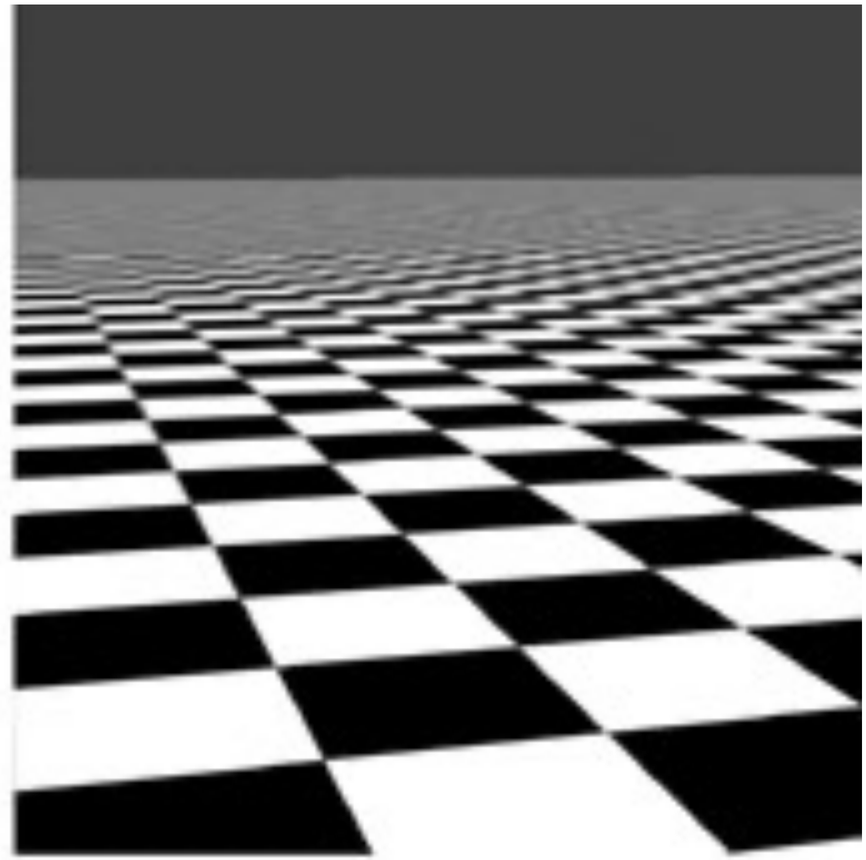




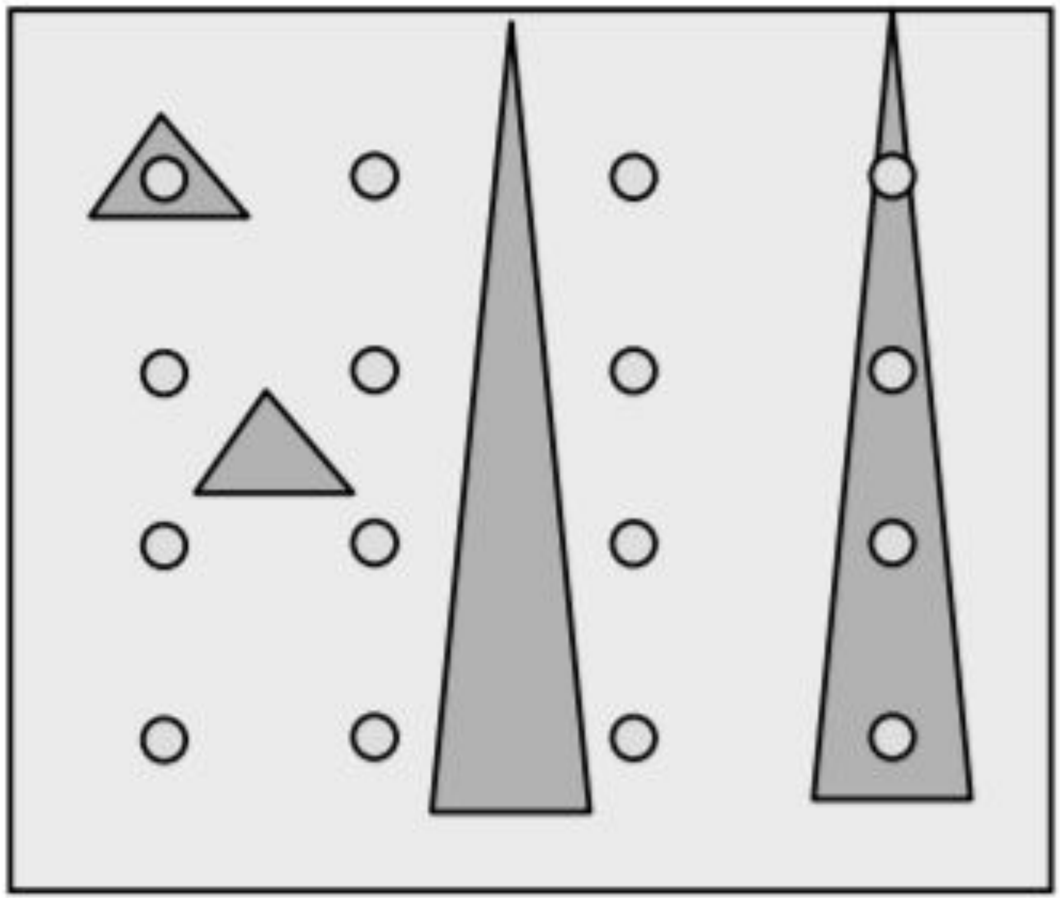


Incorrectly Rendered Fine Detail





Loss of Small Objects



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Antialiasing Integral $$s(i,j) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x+i, y+j)\, f(x,y)\, dx\, dy$$

$s(i,j)$ = spatially discrete image

$I(x,y)$ = spatially continuous image

$f(x,y)$ = antialiasing lowpass filter

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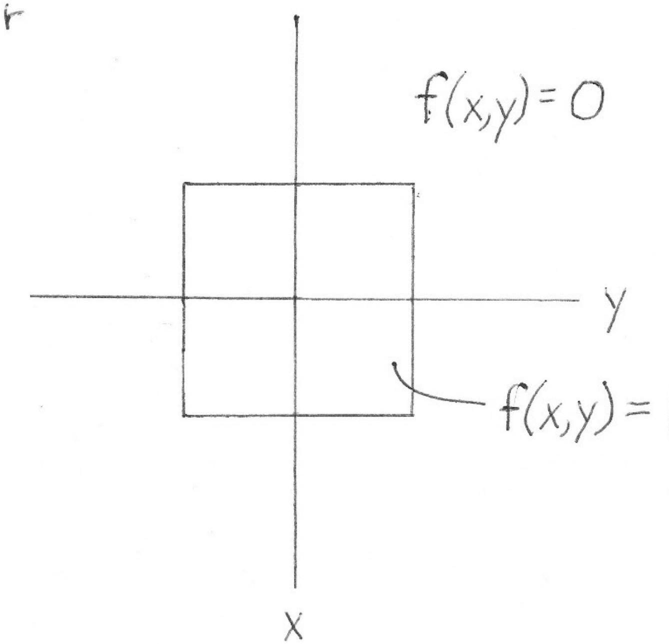
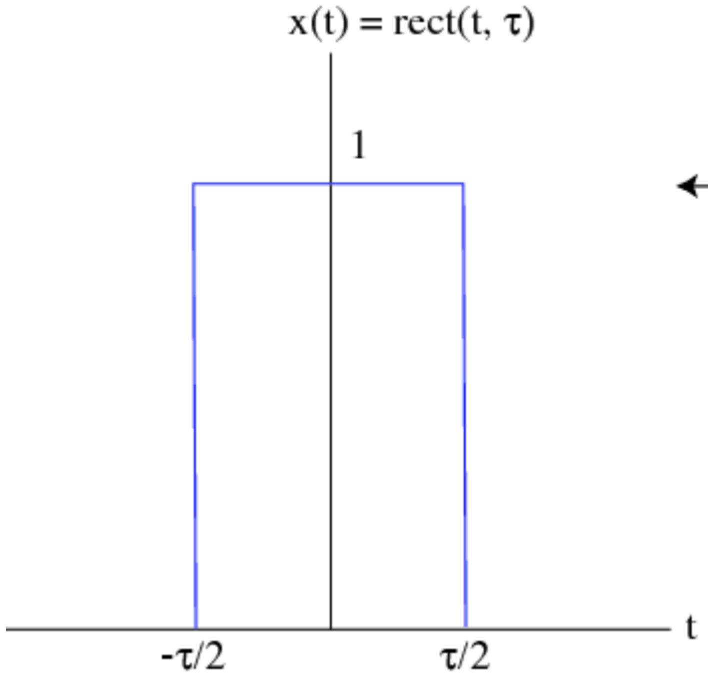
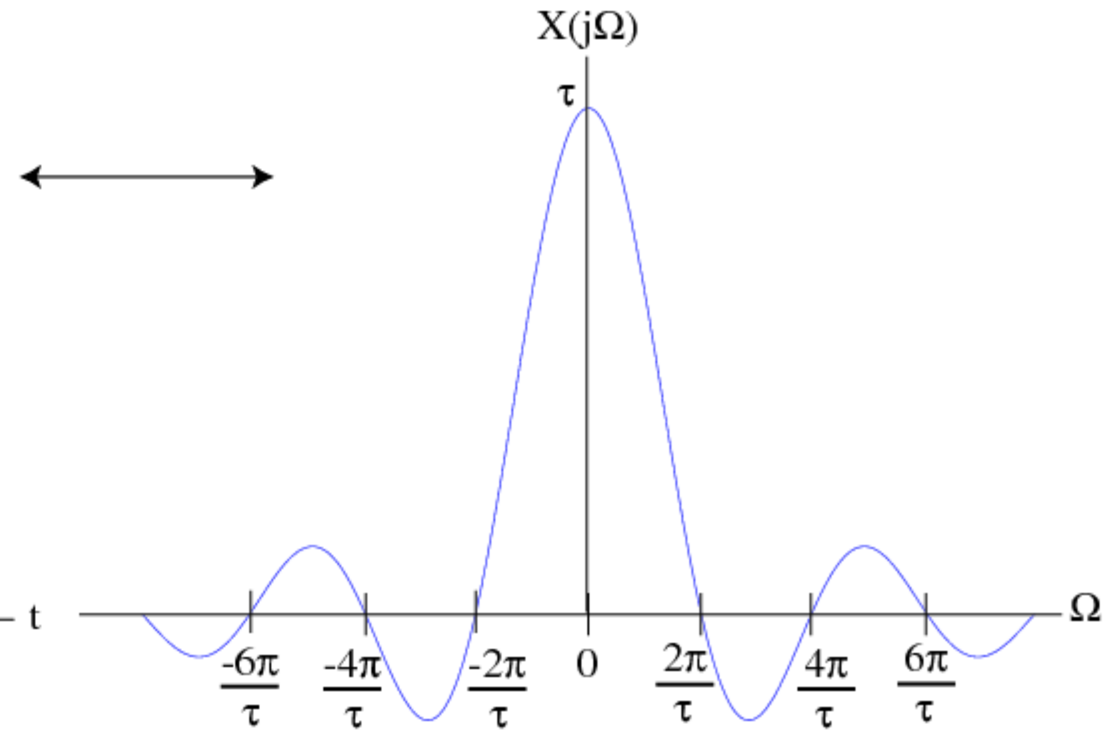


Simple $f(x,y)$ is a box function the size of a pixel.

Box Function in Space



Sinc in Frequency



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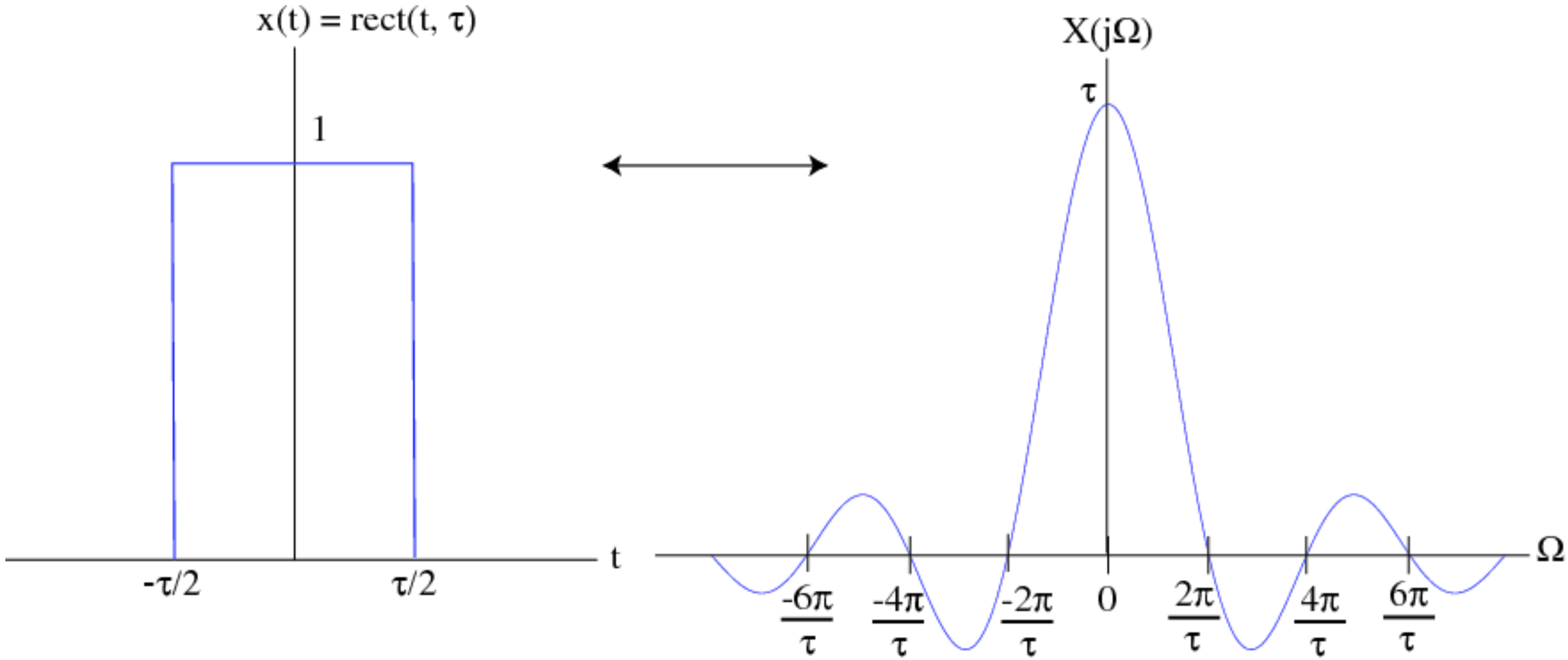
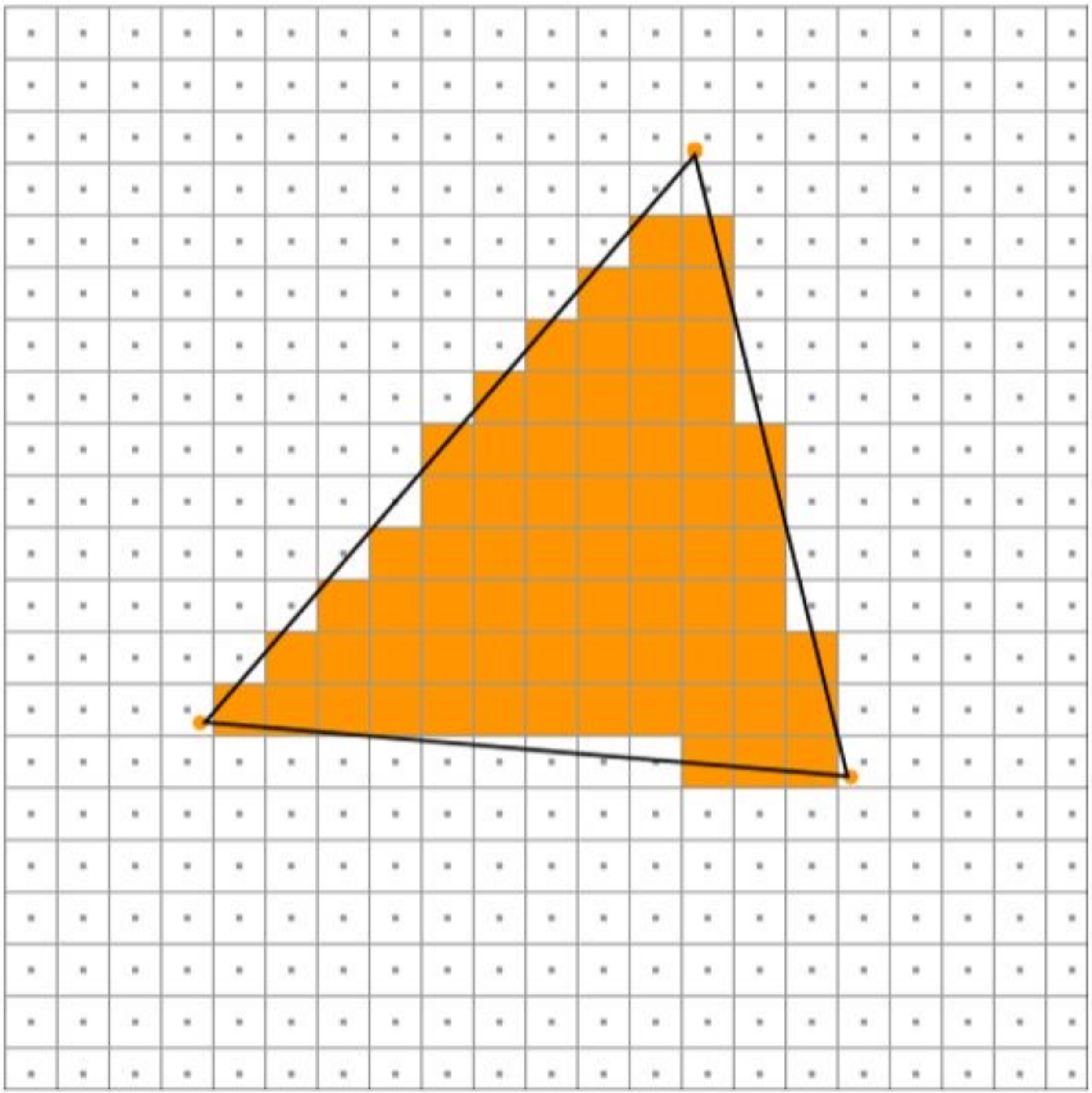
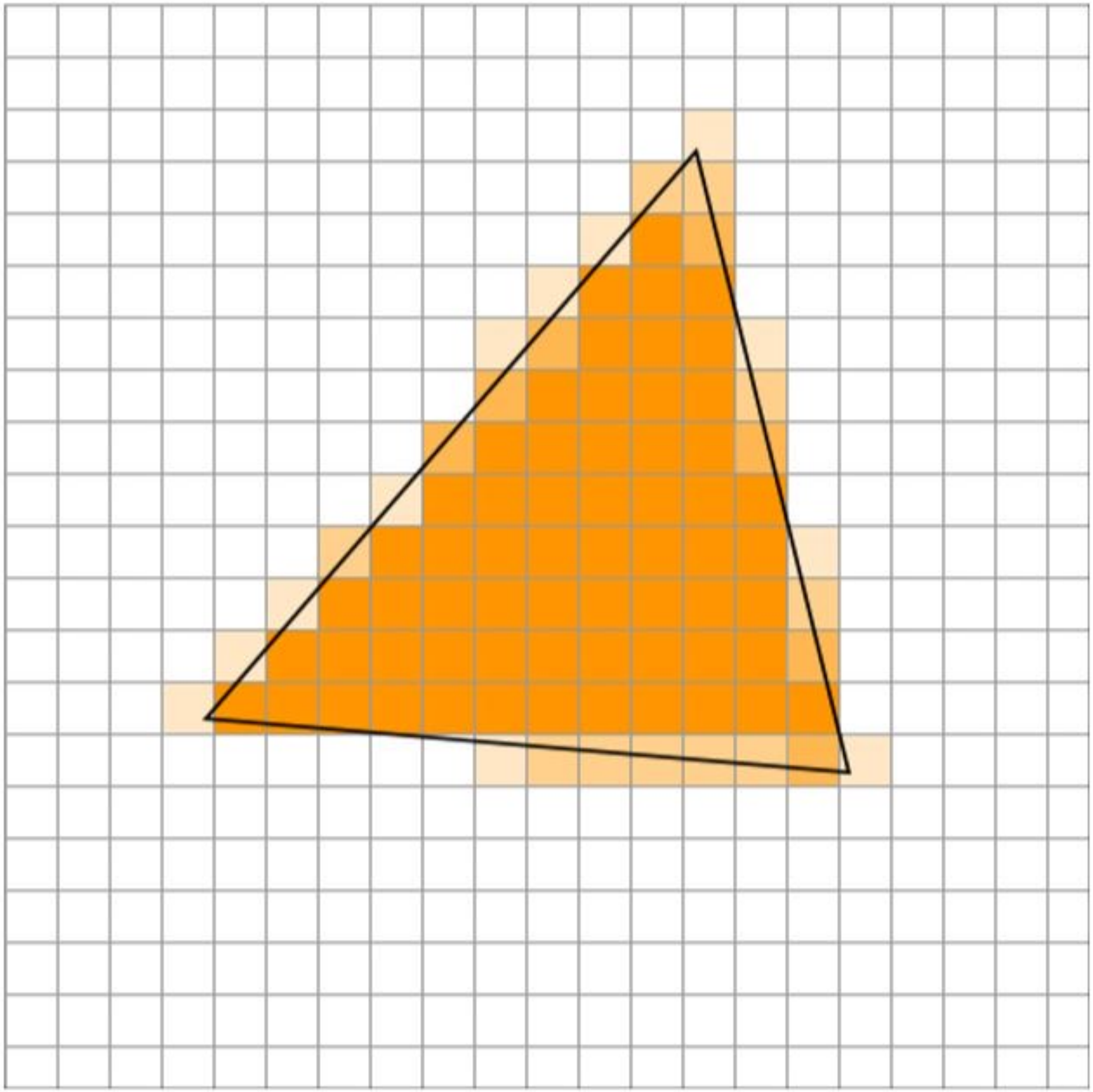
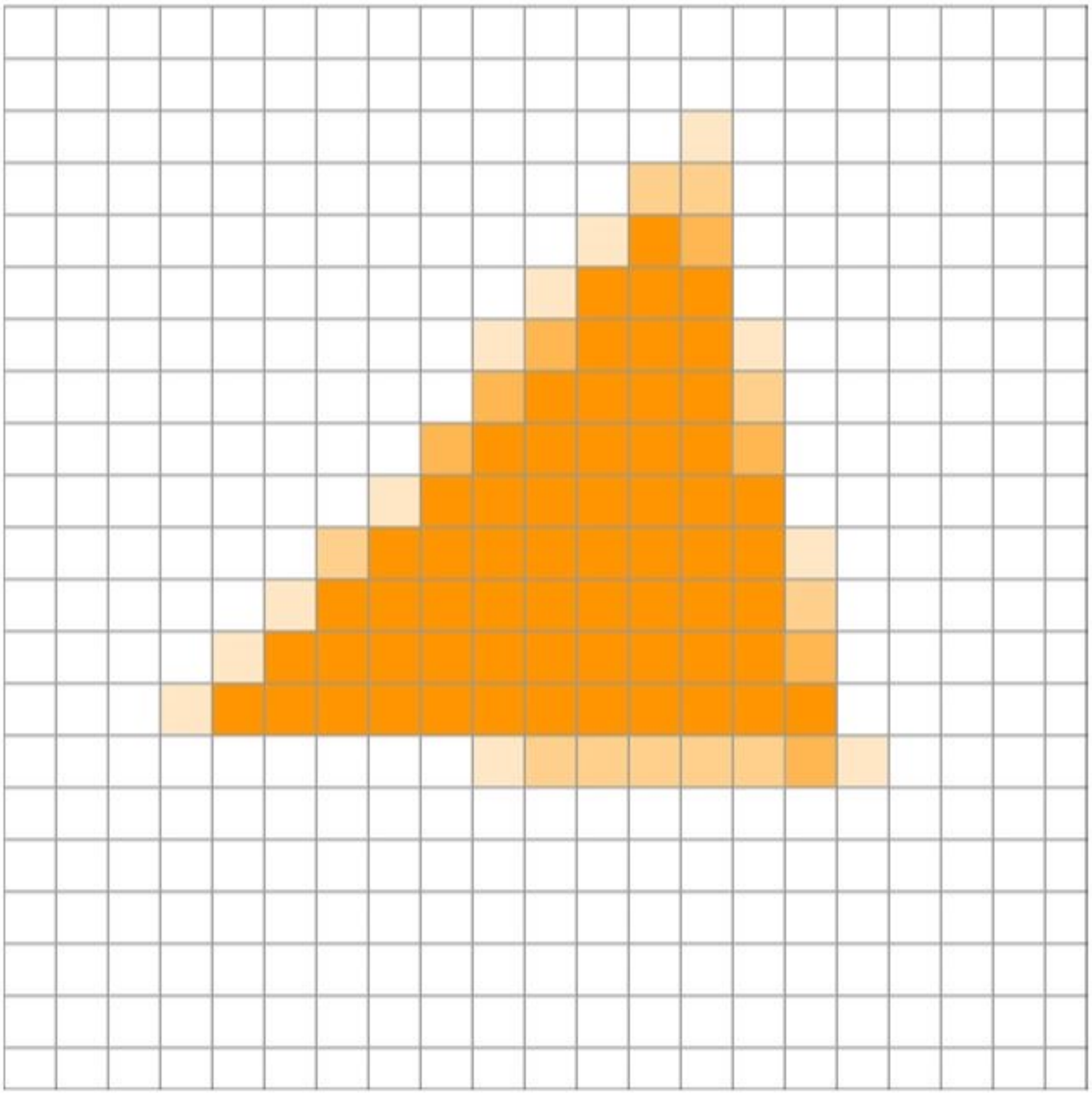
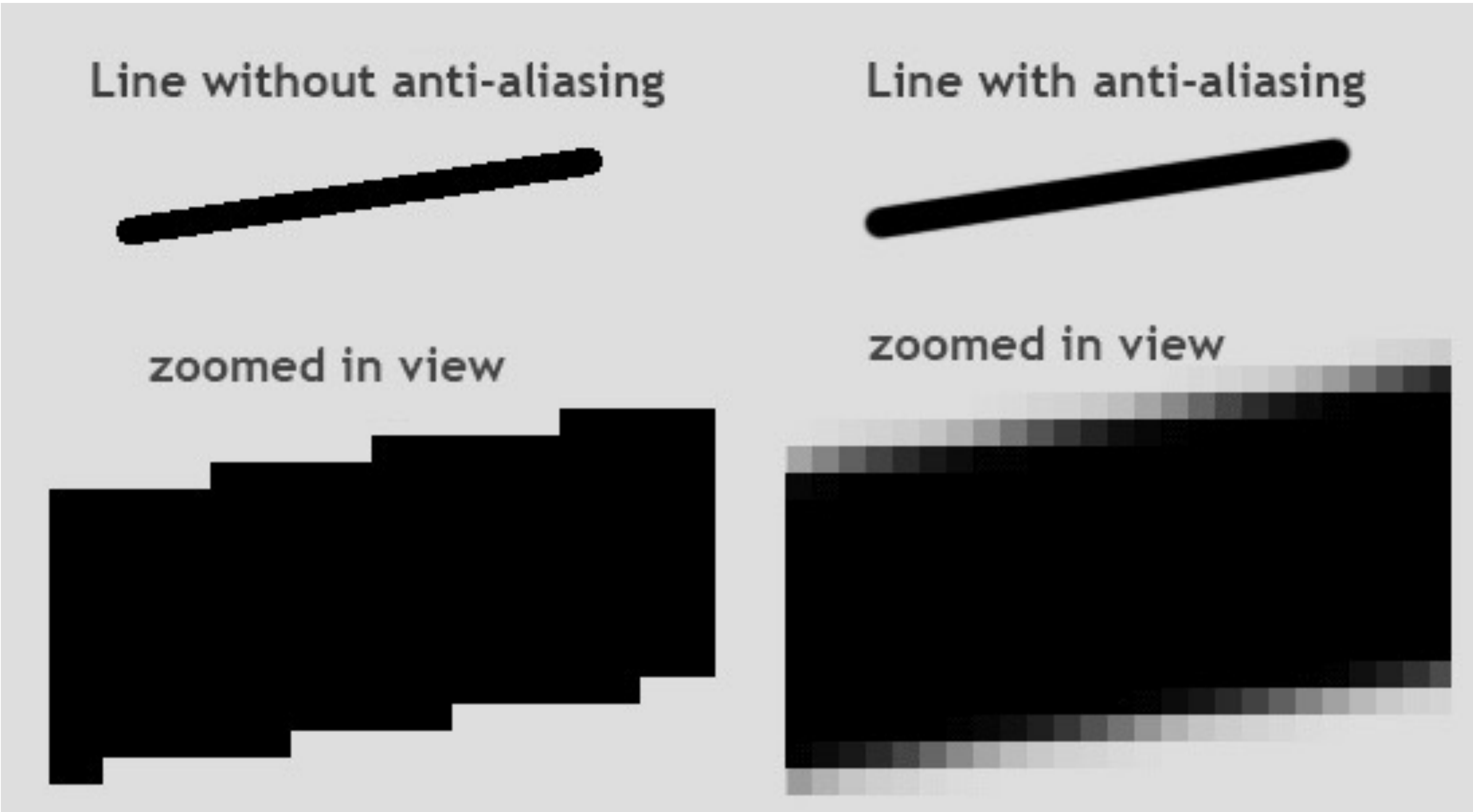


Filter passes high frequencies, does not completely eliminate aliasing.

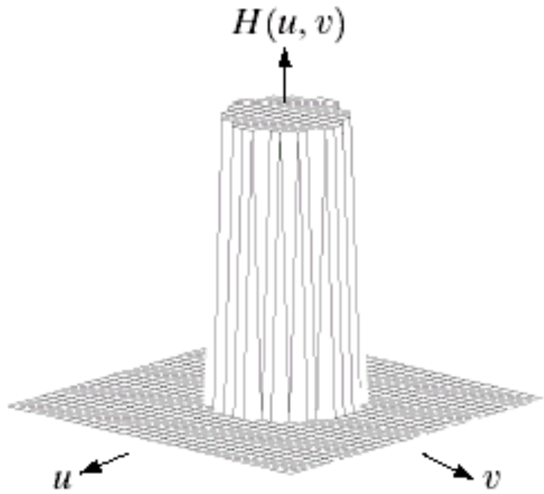


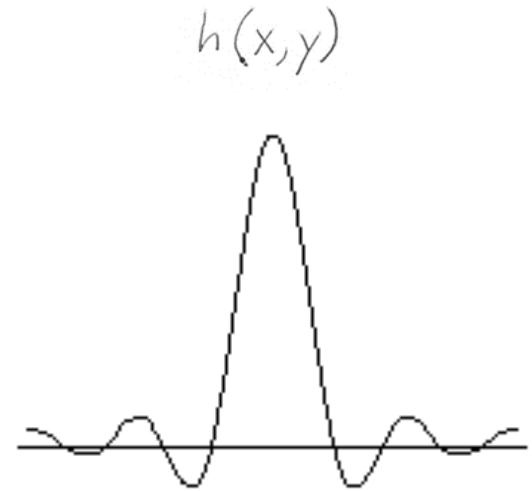





Line without anti-aliasing
Line with anti-aliasing
zoomed in view
zoomed in view

Ideal lowpass filter removes all frequencies that alias.





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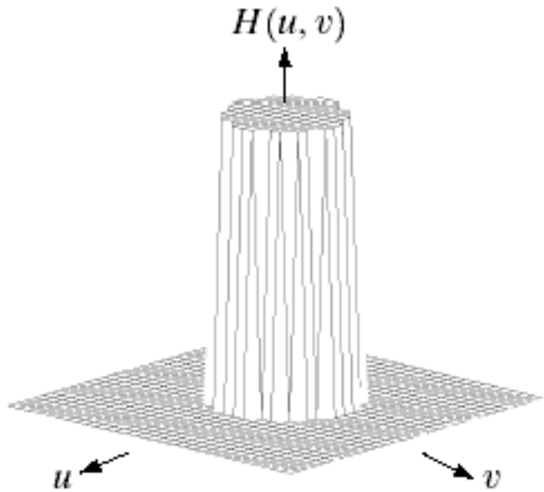
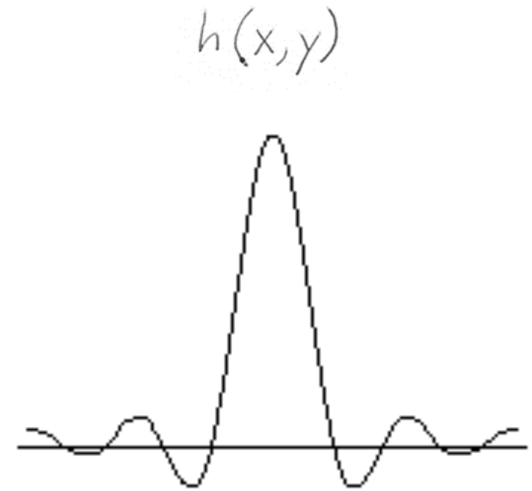
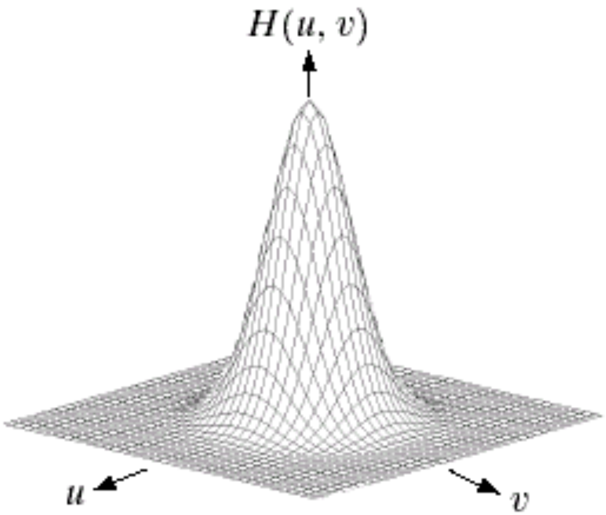


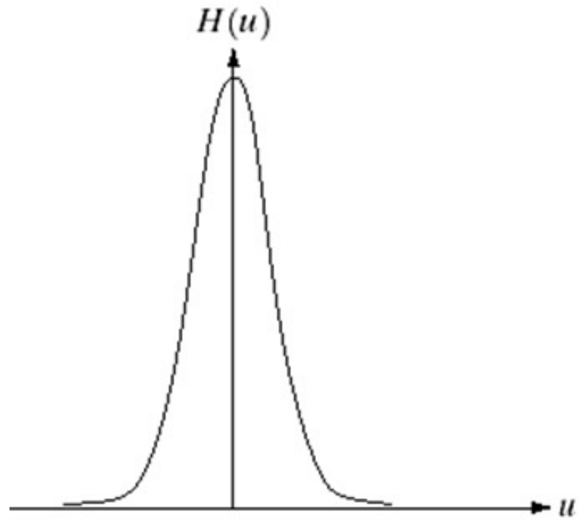


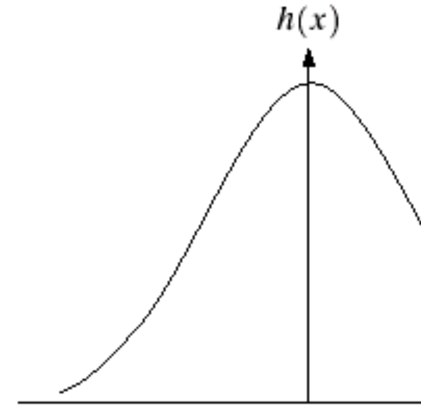
Problems with ideal lowpass filter

- large spatial extent
- ringing
- negative gray levels in filtered image

Gaussian lowpass filter is often used for antialiasing







$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

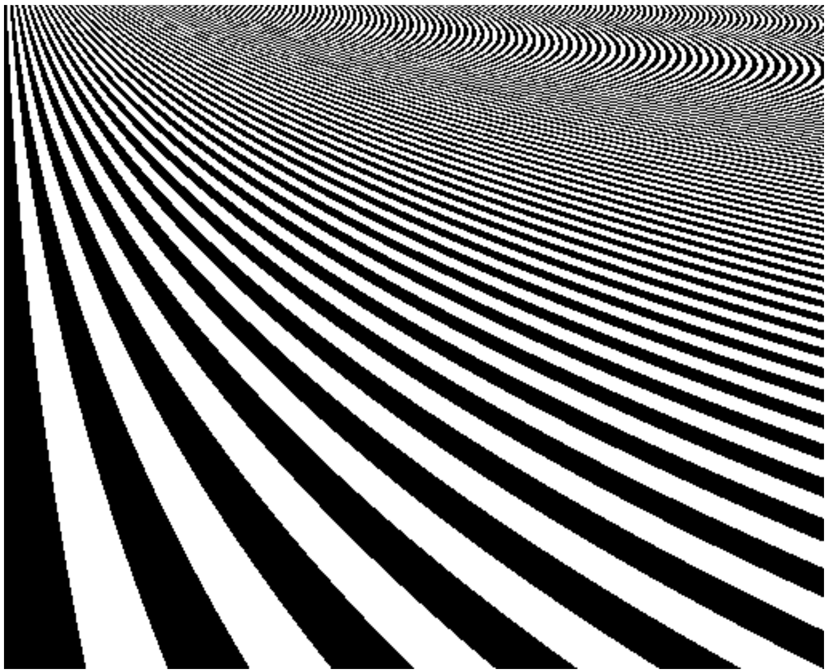
$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Gaussian has

smooth, all positive, nearly finite spatial extent

smooth, nearly finite extent frequency response

Sampled Input Image



jagged edges

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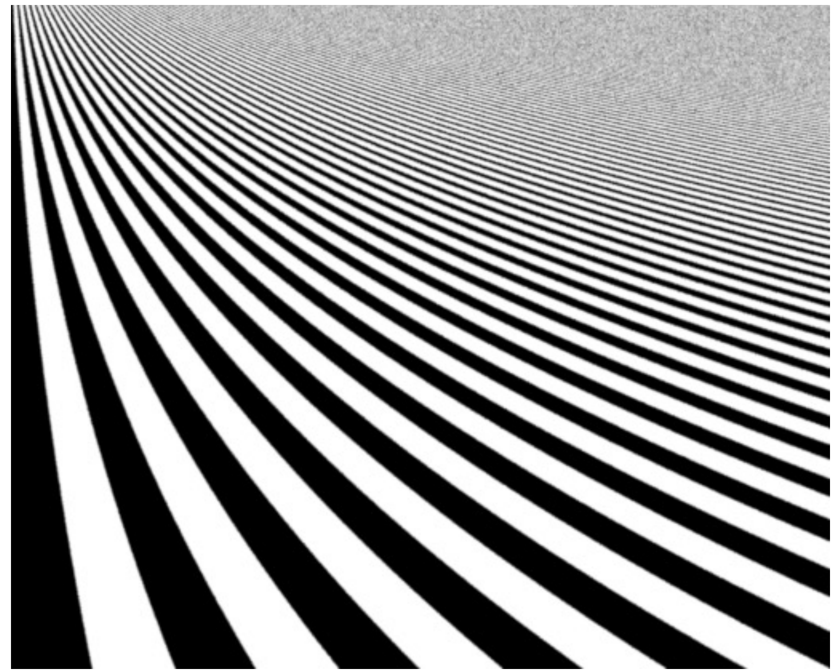
Gaussian Antialiasing



Stochastic Sampling

Idea – sample $I(x,y)$ at nonuniformly spaced locations rather than systematically at pixel centers

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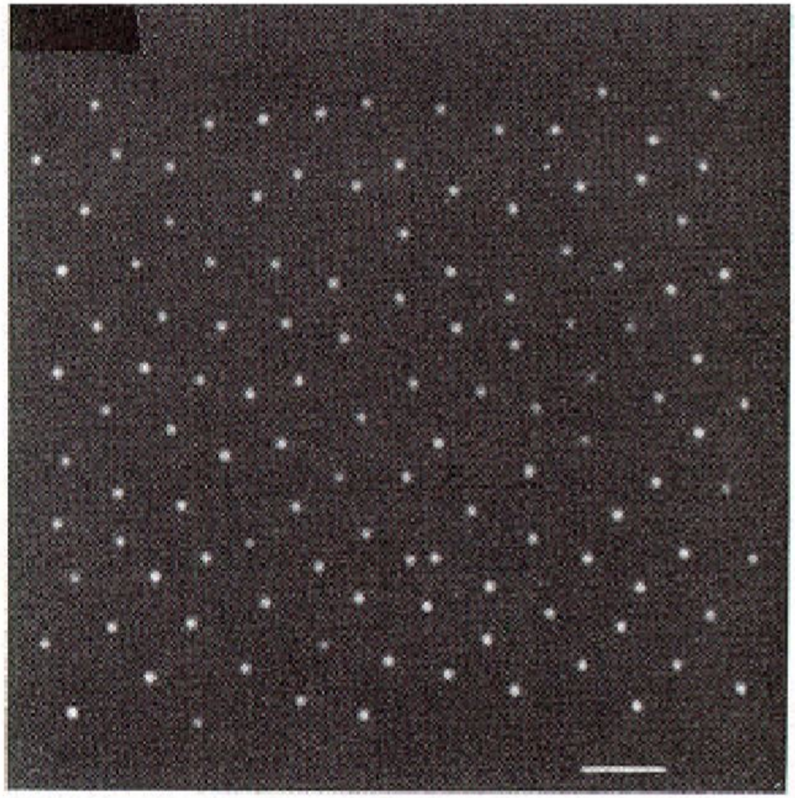

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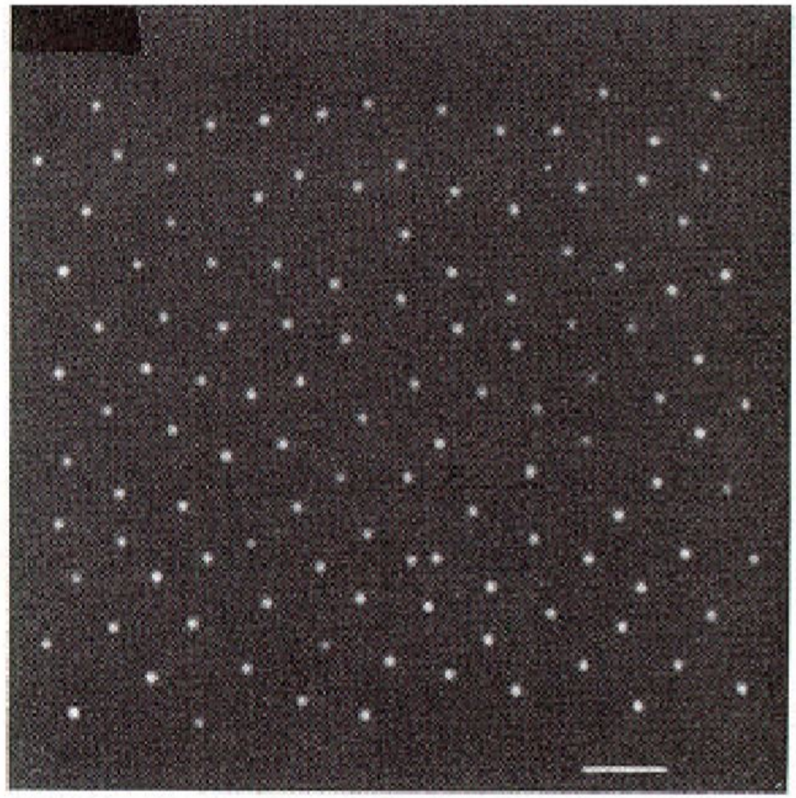

Result – frequencies beyond the Nyquist rate will appear as white noise after sampling rather than aliasing as lower frequencies

Continuous 1-D signal $I(x)$

Continuous 1-D uniform sampling function $T(x) = \sum_{i=-\infty}^{\infty} \delta(x - Ki)$

K = sampling period $\quad \omega_s = \frac{2\pi}{K}$ = sampling frequency

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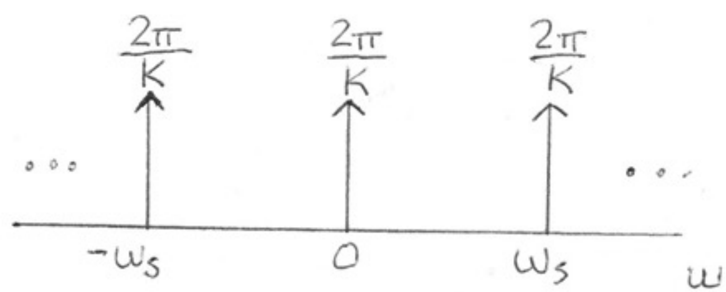

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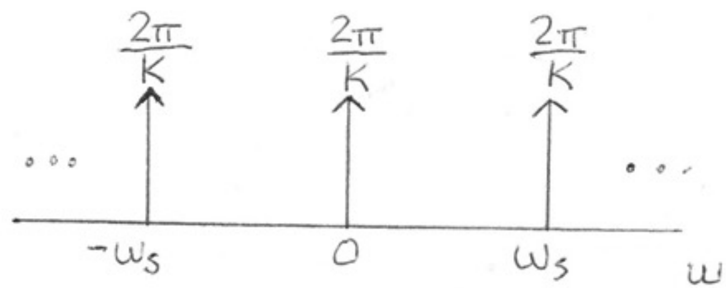

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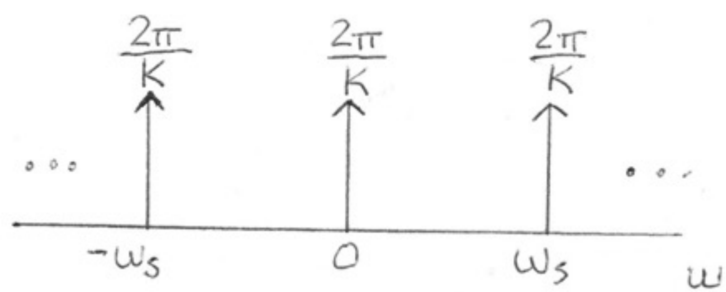

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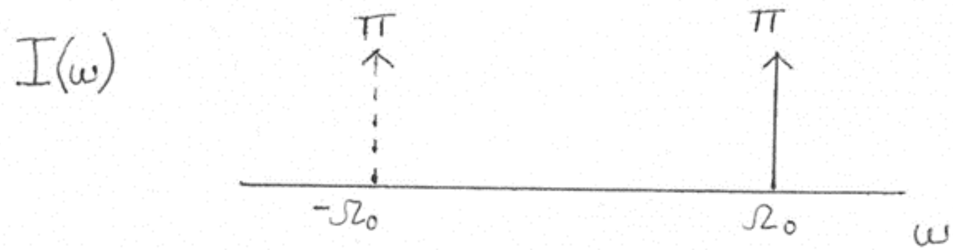

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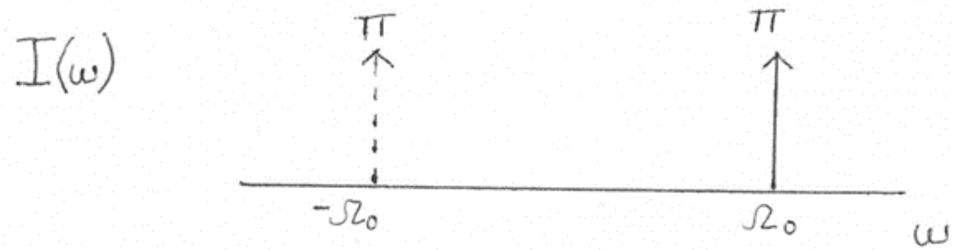

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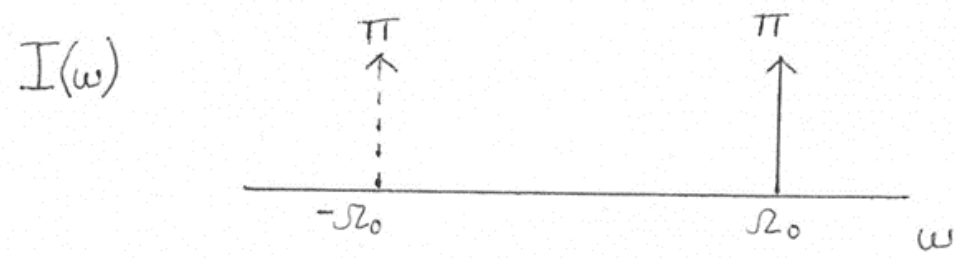

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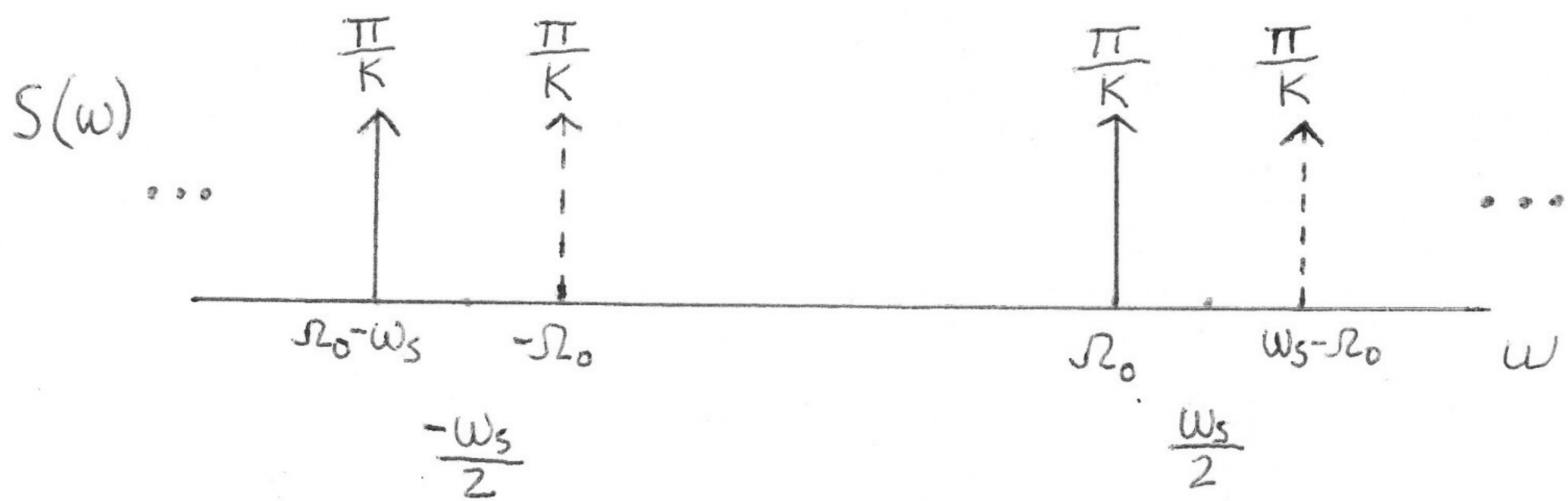

No Aliasing ($\omega_s > 2\Omega_0$)

$$\omega_s > 2\Omega_0 \longrightarrow \frac{\omega_s}{2} > \Omega_0$$

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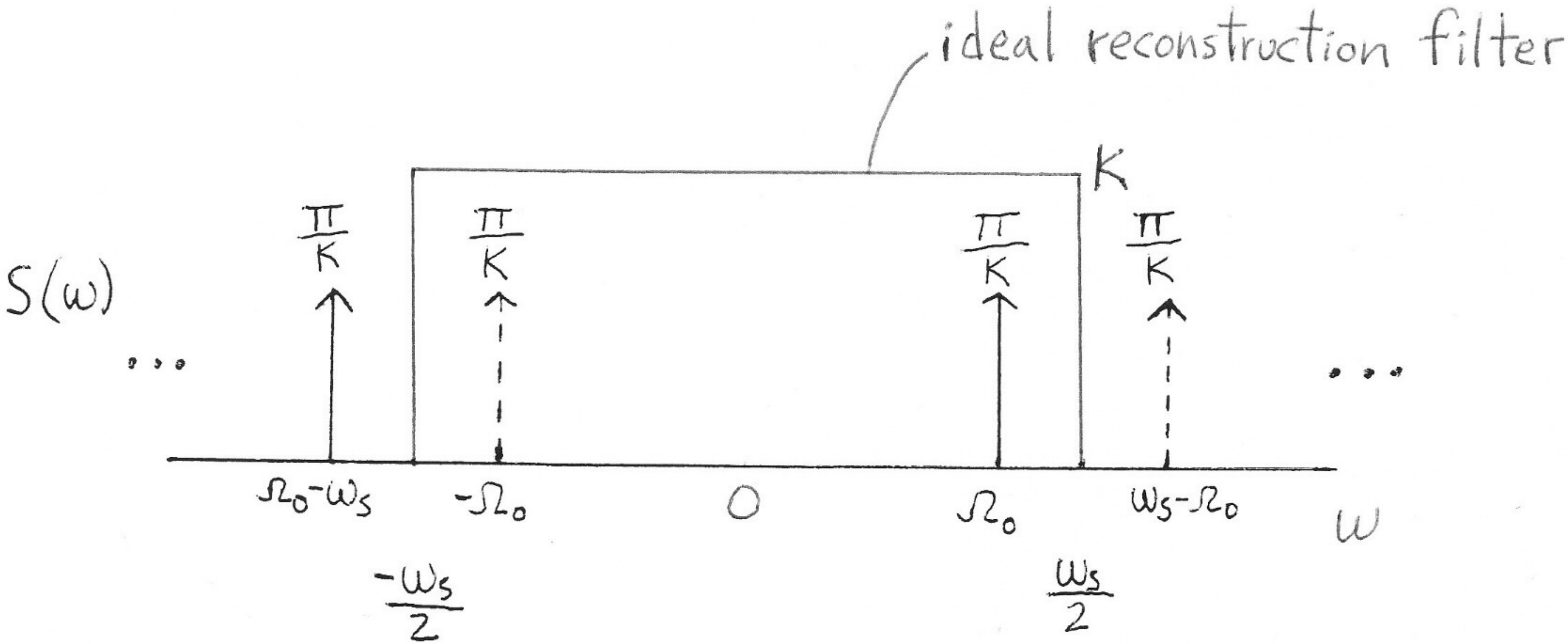

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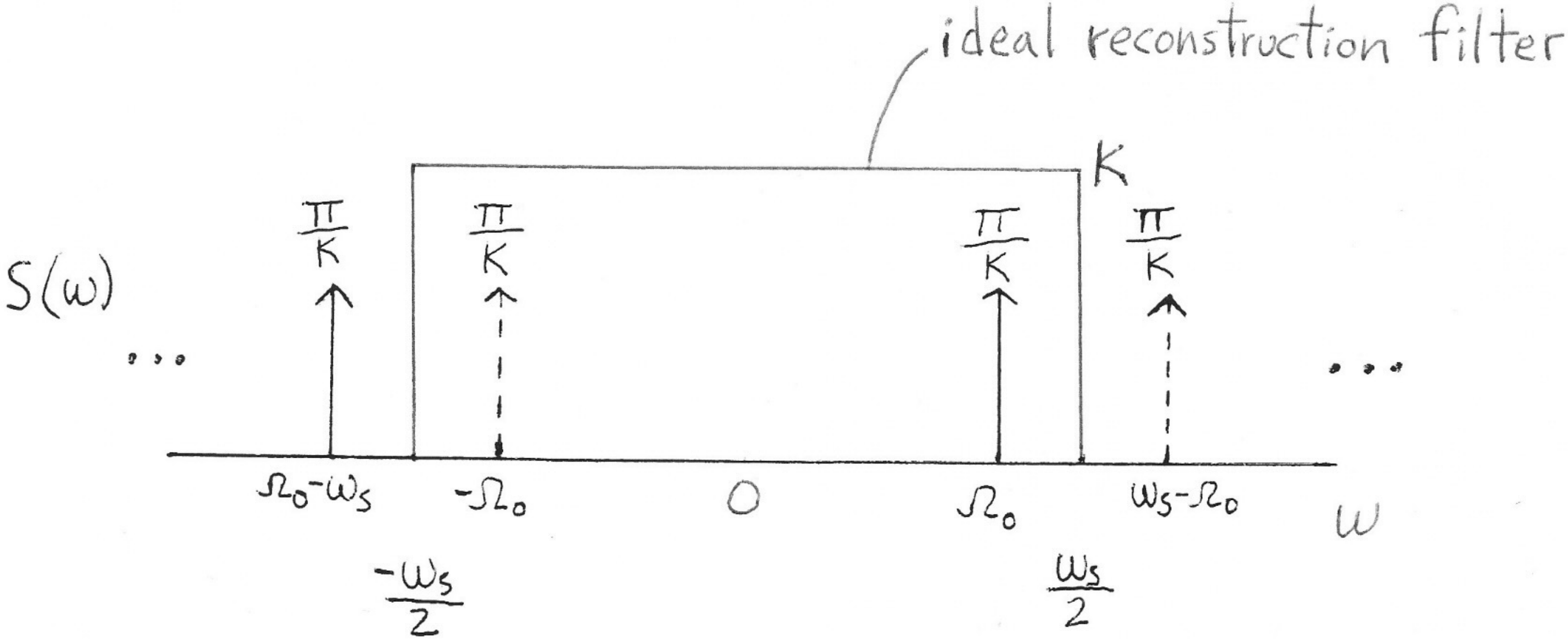

Correct $I(x) = \cos(\Omega_0 x)$ is recovered

Aliasing $(\omega_s \leq 2\Omega_0)$

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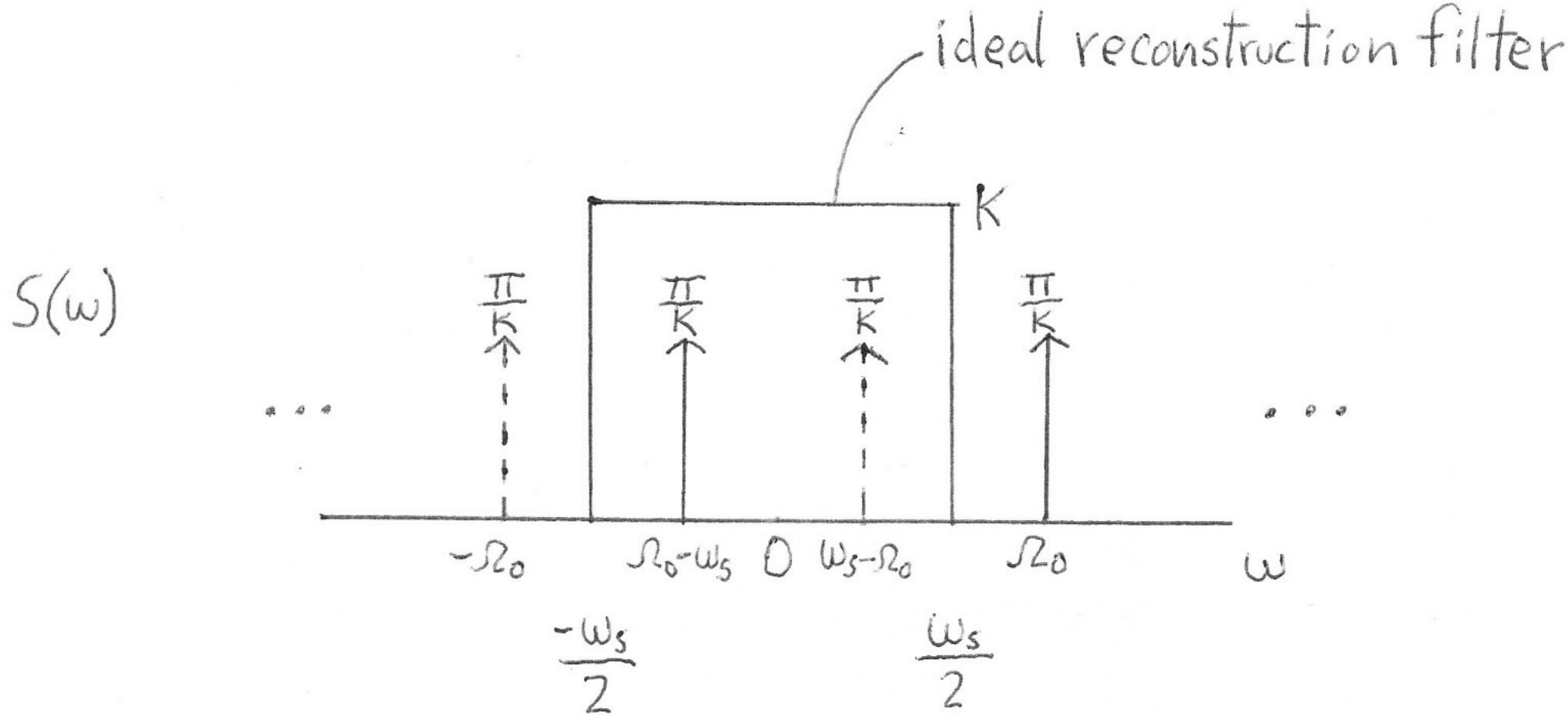

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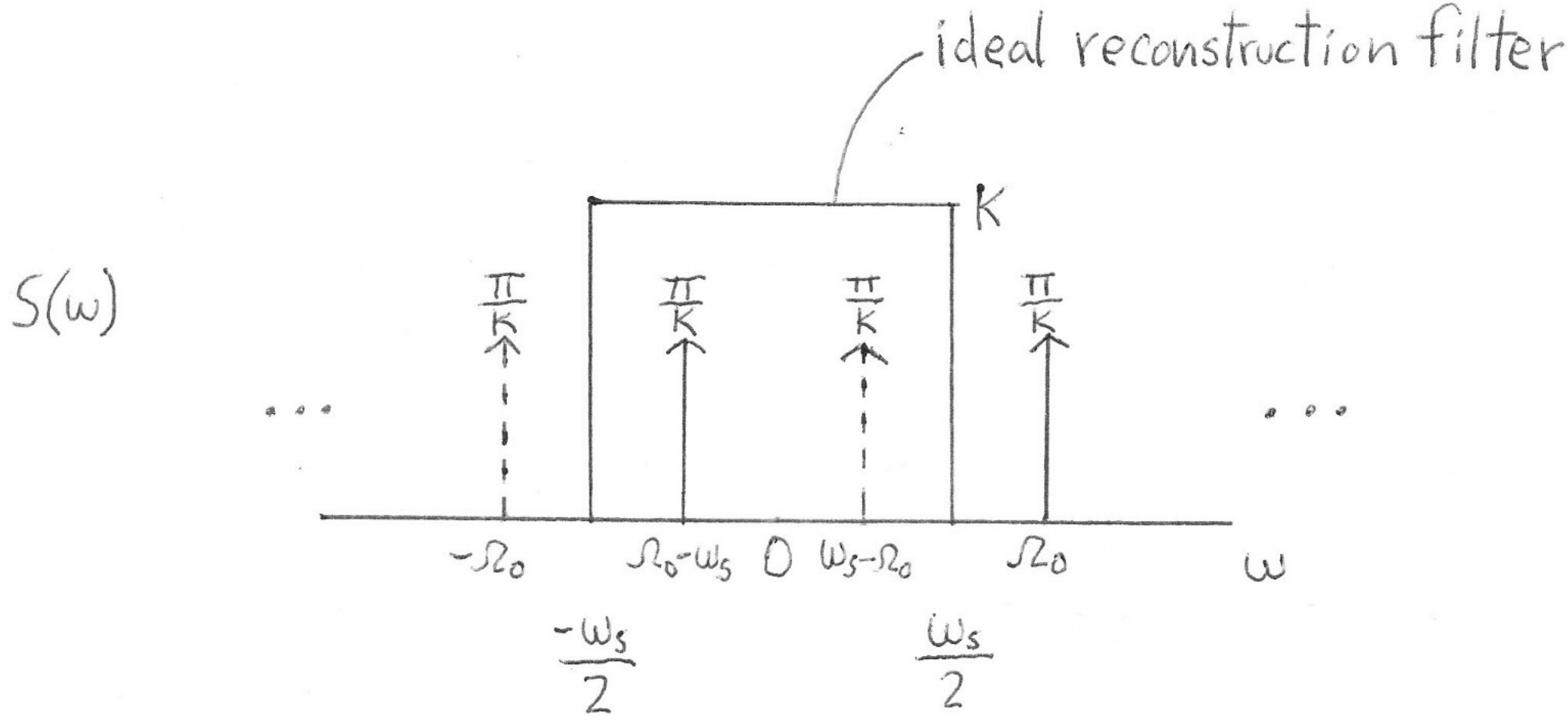

Incorrect lower frequency sinusoid is recovered

Poisson disk sampling – sample $I(x)$ with a nonuniform sampling function so that samples are randomly placed but no two samples are closer than a given distance

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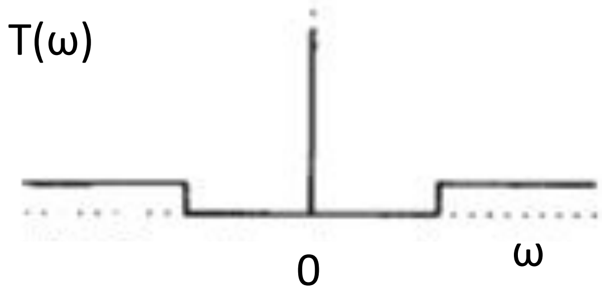

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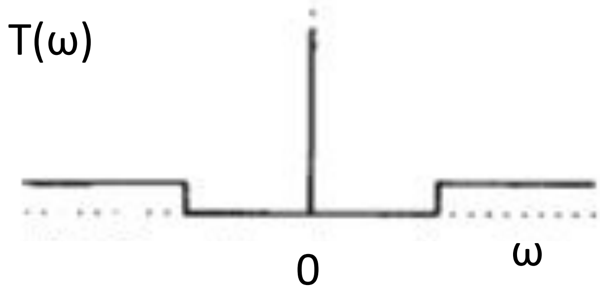

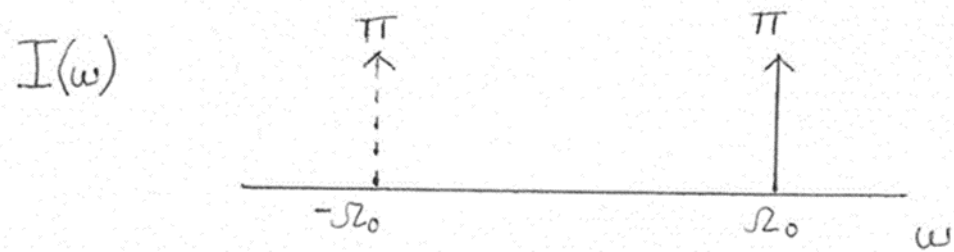

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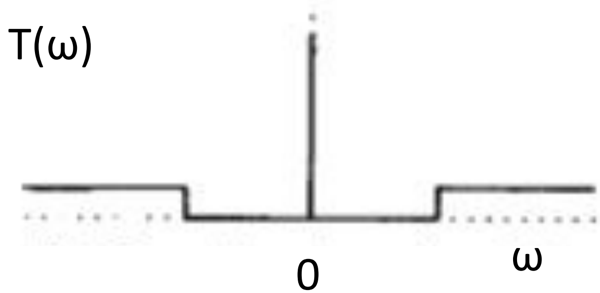
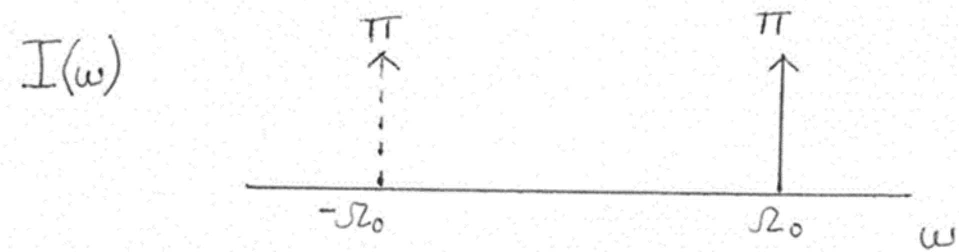
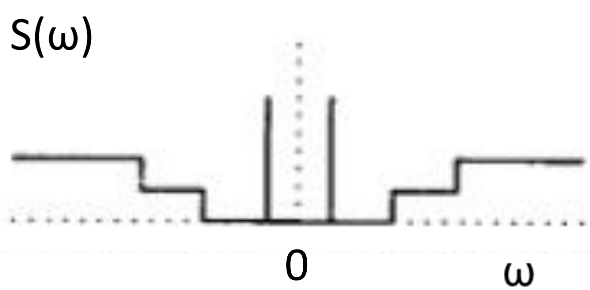




No Aliasing Case



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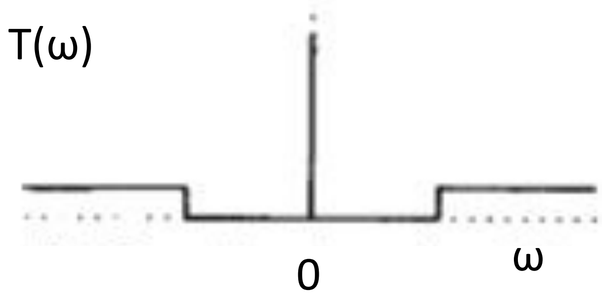

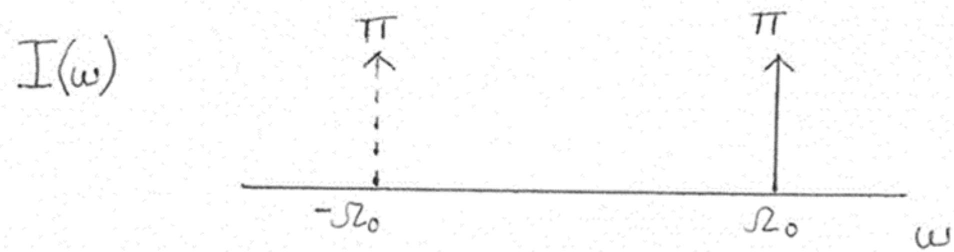

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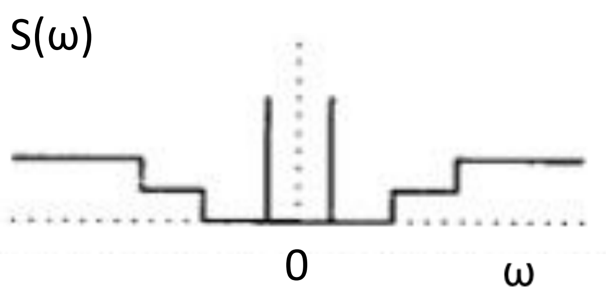

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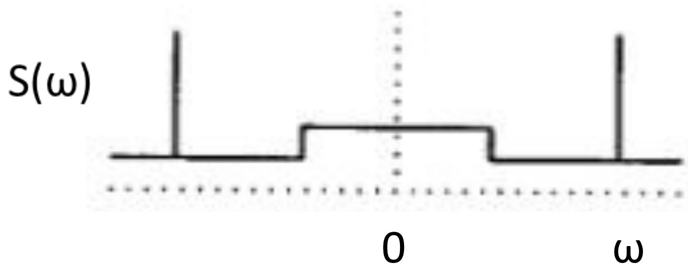

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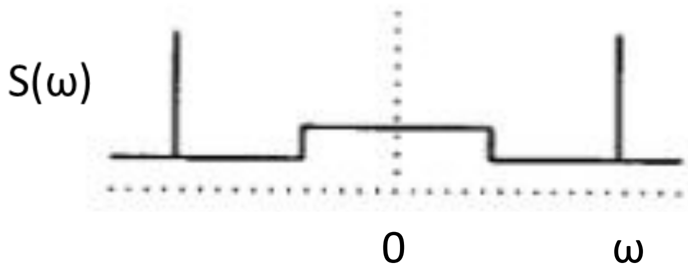

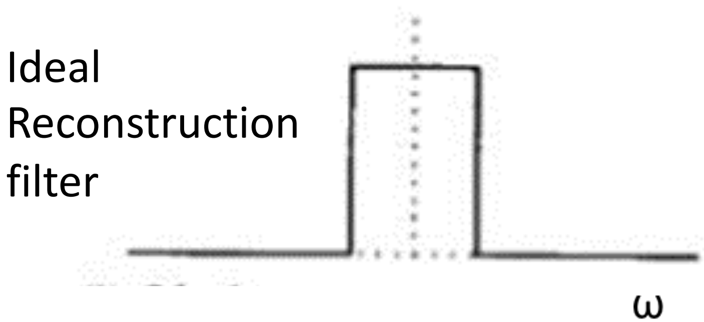

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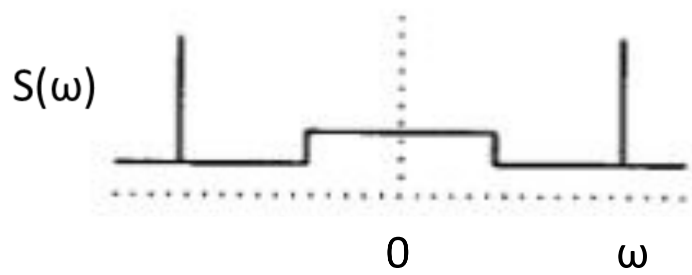
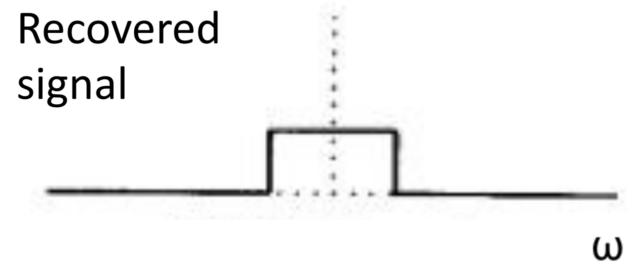
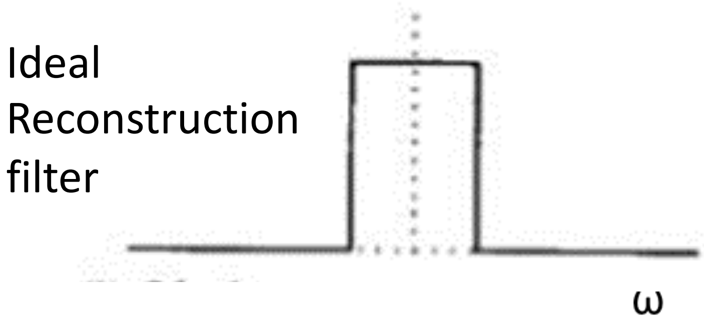
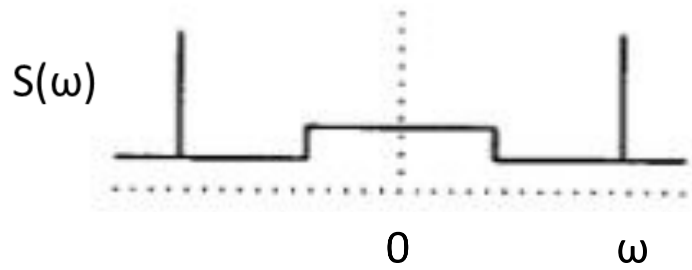
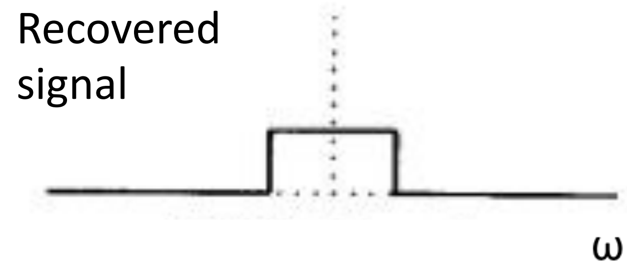


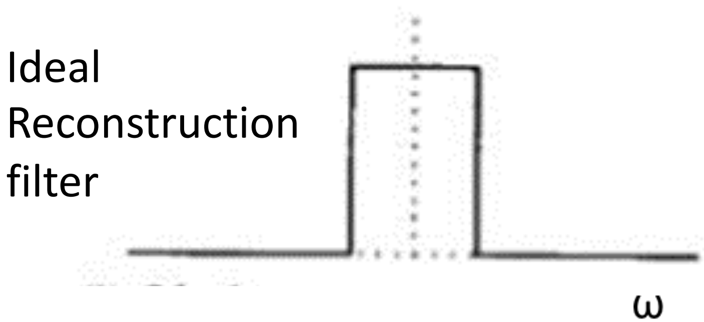




Aliasing Case



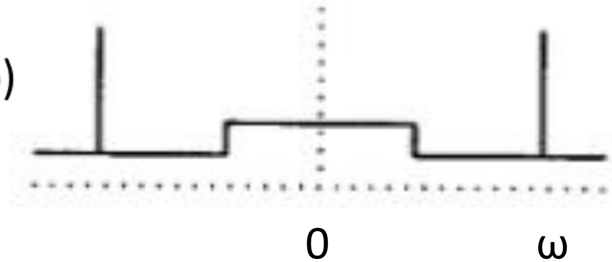




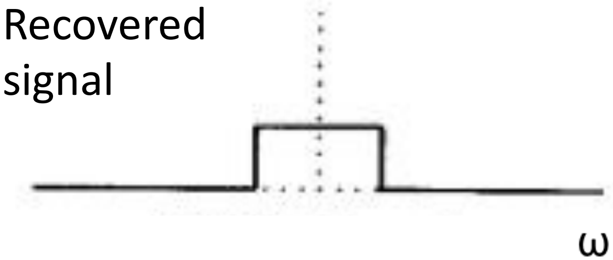
Noise is visually more appealing than incorrect sinusoids from aliasing

Aliasing Case

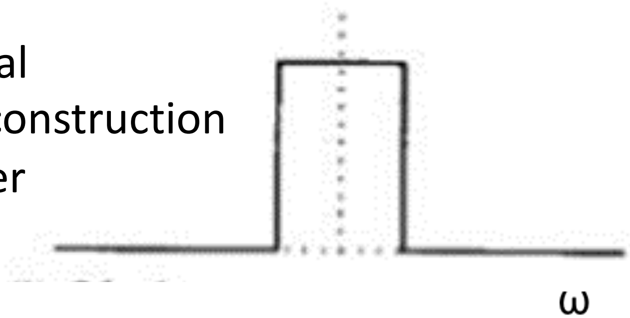
S(ω)



Recovered signal



Ideal Reconstruction filter



Noise is visually more appealing than incorrect sinusoids from aliasing

Use stochastic sampling by jittering a regular grid and interpolating to get the values for a standard display.