EECS203A HW6 John Liu 25961868

1. Suppose that $g(x, y)$ is a degraded version of an ideal image $f(x, y)$ with

$$g(x, y) = h(x, y) * f(x, y)$$

where $h(x, y)$ is an ideal bandreject filter with parameters D_0 and W.

a) Can we recover $f(x, y)$ from $g(x, y)$ using inverse filtering? Explain your answer.

b) Can we recover $f(x, y)$ from $g(x, y)$ using Wiener filtering? Explain your answer.

c) Given an input image $f(x, y)$ that gives a corresponding degraded image $g(x, y)$, describe the set of input images that will give the same filtered image $g(x, y)$.

ideal bandreject filter

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

a) $\because$ $F(u,v) = G(u,v)/H(u,v)$ get from inverse filter

Also. due to IBRF, when $D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2}$. $H(u,v) = 0$

$\therefore$ Cannot recover $f(x,y)$ from $g(x,y)$ using inverse filtering

b) $\because$ $n(x,y) = 0$

$\therefore$ $S_n(u,v) = 0$

$\therefore$ Wiener filter becomes inverse filter

$\therefore$ Same as a). cannot recover $f(x,y)$ from $g(x,y)$ using Wiener filtering

c) If $H(u,v) \cdot F(u,v)$ the same as $G(u,v)$ then get the same $g(x,y)$ which means when $D(u,v) < D_0 - \frac{W}{2}$ or $D(u,v) > D_0 + \frac{W}{2}$. i.e. $H(u,v) = 1$, the input image will give the same filtered image $g(x,y)$

2. Suppose that $g(x, y)$ is a noisy version of an ideal image $f(x, y)$

$$g(x, y) = f(x, y) + n(x, y)$$

where the DFT magnitudes have the properties $|N(u, v)| = 1$ and $|F(u, v)|$ decreases as $u^2 + v^2$ increases. Consider the filters H_1 and H_2 defined in the frequency domain by

$$H_1(u, v) = \frac{1}{1 + 0.01(u^2 + v^2)} \quad \text{and} \quad H_2(u, v) = \sqrt{H_1(u, v)}$$

The filters specified by $H_1(u, v)$ and $H_2(u, v)$ are applied to the image $g(x, y)$.

a) Does $H_1(u, v)$ or $H_2(u, v)$ reduce more noise in $g(x, y)$? Explain your answer.

b) Does $H_1(u, v)$ or $H_2(u, v)$ blur the image $g(x, y)$ more? Explain your answer.

a) $u^2 + v^2 \geq 0$

$|H_1(u,v)| < |H_2(u,v)|$

$\therefore$ $H_1(u,v)$ reduces more noise

b) Because $H_2(u,v)$ reduces more high frequencies than $H_1(u,v)$ does.

$H_2(u,v)$ blurs the image $g(x,y)$ more.

3. Suppose that a rectangular area in image $f(x, y)$ with vertices $(x, y) = \{(7, 3), (7, 11), (12, 11), (12, 3)\}$ appears distorted in image $f'(x', y')$ with corresponding vertices $(x', y') = \{(6, 5), (6, 13), (12, 11), (12, 3)\}$ Determine the functions $x'(x, y)$ and $y'(x, y)$ using a bilinear model for the distortion.

$f'(6,5) = f(7,3)$ $f'(6,13) = f(7,11)$. $f'(12,11) = f(12,11)$ $f'(12,3) = f(12,3)$

$f'(x'(x,y), y'(x,y)) = f(x,y)$

$x'(7,3) = 6$, $y'(7,3) = 5$

$x'(7,11) = 6$. $y'(7,11) = 13$

$x'(12,11) = 12$, $y'(12,11) = 11$

$x'(12,3) = 12$, $y'(12,3) = 3$

$x'(x,y) = ax + by + cxy + d$

$$\begin{cases} 6 = 7a + 3b + 21c + d \\ 6 = 7a + 11b + 77c + d \\ 12 = 12a + 11b + 132c + d \\ 12 = 12a + 3b + 36c + d \end{cases} \Rightarrow \begin{cases} a = 1.2 \\ b = 0 \\ c = 0 \\ d = -2.4 \end{cases} \Rightarrow x'(x,y) = 1.2x - 2.4$$

$y'(x,y) = ex + fy + gxy + h$

$$\begin{cases} 5 = 7e + 3f + 21g + h \\ 13 = 7e + 11f + 77g + h \\ 11 = 12e + 11f + 132g + h \\ 3 = 12e + 3f + 36g + h \end{cases} \Rightarrow \begin{cases} e = -0.4 \\ f = 0 \\ g = 1 \\ h = 4.8 \end{cases} \Rightarrow y'(x,y) = -0.4x + xy + 4.8$$

Computer Problem: Define the continuous-space Gaussian function by $G(x, y) = Ae^{-(x^2+y^2)/(2\sigma^2)}$. Generate a 31×31 digital filter $g(i, j)$ over $i = -15, \ldots, 0, \ldots, 15$ and $j = -15, \ldots, 0, \ldots, 15$ by sampling $G(x, y)$ so that $g(0, 0) = A$ and $g(7, 0) = Ae^{-0.5}$. Normalize $g(i, j)$ by finding A so that the sum of the $g(i, j)$ mask values equals one. Degrade the triangle image by convolution with $g(i, j)$. Use the inverse filtering method to restore the image. Submit your code, the $g(i, j)$ mask coefficients, the degraded image, and the restored image. You may use Matlab or other available software to compute DFTs.

$g(0.0) = A$
$g(7.0) = Ae^{-0.5}$ $\Rightarrow \sigma = 7$

code: "EECS203A_HW6.m"
$g(i.j)$: "mask.csv"
degraded image: "triangle_degraded.jpg"
restored image: "triangle_restored.jpg"