EECS 203A HW1 John Liu 25961868

1. Suppose that a continuous ramp image is defined by

$$c(x,y) = 256x \qquad 0 \le x \le 1 \quad 0 \le y \le 1$$

An $N \times N$ digital image $f(X,Y)$ is formed by sampling $c(x,y)$ at the spatial locations

$$x = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N} \qquad y = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}$$

where N is a power of 2. The value at each pixel is represented using 8 bits where only the b most significant bits are allowed to be nonzero. For example, in 11100000 (= 224 decimal) the three most significant bits are nonzero. If a sampled value of $c(x,y)$ is larger than the largest representable value, then it is represented by the largest representable value. A pixel-to-pixel difference of 6 or more is considered jagged for this ramp image. For what combinations of values of N and b will the digital image $f(X,Y)$ not have any jagged differences? Explain your answer. The values of N and b should be large enough so that $f(X,Y)$ is not a constant image.

$\because 2^2 < 6 < 2^3$

$\therefore \underset{\substack{\uparrow \\ \text{8 bits}}}{8} \ge b \ge (8-2) \Rightarrow b = 6, 7, 8$

$\therefore N \ge 2^6 \Rightarrow N \ge 64$

2. a) Let H be an operator that maps an input image $I(x,y)$ to the output image $2I(x,y)+4$. Is H a linear operator? Prove your answer.

b) Is an operator that replaces every pixel in an image with the median of all of the pixels in the image a linear operator? Prove your answer.

a) $H(I(x,y)) = 2I(x,y) + 4$

$H(af(x,y) + bg(x,y)) = 2(af(x,y) + bg(x,y)) + 4$

$= 2af(x,y) + 2bg(x,y) + 4$ ①

$aH(f(x,y)) + bH(g(x,y)) = 2af(x,y) + 4 + 2bg(x,y) + 4$

$= 2af(x,y) + 2bg(x,y) + 8$ ②

① ≠ ② ∴ Nonlinear

b) Assume $f(x,y) = \{1, 2, 3, 4, 5\}$,

$g(x,y) = \{6, 7, 8, 9, 10\}$,

$a = 1, \quad b = 1$

$H(af(x,y) + bg(x,y)) = 5.5$ ①

$aH(f(x,y)) + bH(g(x,y)) = 3 + 8 = 11$ ②

① ≠ ② ∴ Nonlinear

3. Consider an image $f(x, y)$ with the pixel values

$$f(1,1) = 16 \quad f(1,2) = 11 \quad f(2,1) = 12 \quad f(2,2) = 8$$

a) Find the continuous bilinear function $b(x, y)$ such that $b(x, y) = f(x, y)$ at these four points.
b) Find $b(1.3, 1.7)$.

a) $\left.\begin{array}{l} 16 = a + b + c + d \\ 11 = a + 2b + 2c + d \\ 12 = 2a + b + 2c + d \\ 8 = 2a + 2b + 4c + d \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = -5 \\ b = -6 \\ c = 1 \\ d = 26 \end{array}\right.$

$b(x, y) = -5x - 6y + xy + 26$

b) $b(1.3, 1.7) = -5 \times 1.3 - 6 \times 1.7 + 1.3 \times 1.7 + 26$
$= 11.51$

4. Consider a television standard with 1125 horizontal lines and a width-to-height aspect ratio of 16:9 with full images displayed every 1/30 of a second. Suppose that we create a digital image by sampling each horizontal line so that the horizontal and vertical sample spacing are the same (i.e. the digital image also has a 16:9 aspect ratio). Each pixel is represented using 24 bits. How many bits would it take to store all of the digital images without compression for a 2-hour movie in this format?

$24 \times 1125 \times 1125 \times 16/9 \times 2 \times 60 \times 60 \times 30 = 1.1664 \times 10^{13}$ bits