EECS203A HW4 John Liu 25961868

1. Let $h(x, y)$ be the 64×64 filter defined by

$$h(x, y) = 2 + \cos(0.25\pi y) \quad x = 0, 1, 2, \ldots, 63 \quad y = 0, 1, 2, \ldots, 63$$

a) Compute the DFT $H(u, v)$ for $u = 0, 1, 2, \ldots, 63 \quad v = 0, 1, 2, \ldots, 63$.

b) If your answer to part a) is not defined for (u, v) in the range $u = 0, 1, 2, \ldots, 63$ and $v = 0, 1, 2, \ldots, 63$ then write your answer so that $H(u, v)$ is defined for $u = 0, 1, 2, \ldots, 63$ and $v = 0, 1, 2, \ldots, 63$.

a) $h(x,y) = 2 + \cos(0.25\pi y) = 2 + \cos(\frac{1}{8} \cdot 2\pi y)$

$M = N = 64, \quad u_0 = 0, \quad v_0 = \frac{1}{8}$

$H(u,v) = 2\delta(u,v) + \frac{1}{2}[\delta(u, v+8) + \delta(u, v-8)]$

b) Since periodic

$H(u,v) = 2\delta(u,v) + \frac{1}{2}[\delta(u, v-56) + \delta(u, v-8)]$

2. Consider a transform from an $M \times N$ input digital image $f(x, y)$ to an $M \times N$ output digital image $g(x, y)$ defined by

$$g(x, y) = f(x, y) - f(x + 1, y) + f(x, y) - f(x, y + 1)$$

a) Prove that this transform is linear.

b) Write down the convolution mask associated with this linear transform.

c) Find the frequency domain transfer function $H(u, v) = G(u, v)/F(u, v)$ for the transform.

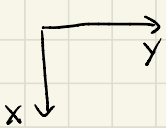
d) Find the magnitude response $|H(u, v)|$ for the transform.

e) Generate a surface plot of $|H(u, v)|$.

f) Using $|H(u, v)|$, is this transform best described as lowpass or highpass or neither? Explain.

a) Since this transform is implemented by a convolutional mask, it is linear

b)



$$\begin{matrix} 0 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 0 \end{matrix}$$

c) $g(x,y) = 2f(x,y) - f(x+1,y) - f(x,y+1)$

$G(u,v) = 2F(u,v) - F(u,v)e^{j2\pi \frac{u}{M}} - F(u,v)e^{j2\pi \frac{v}{N}}$

$H(u,v) = G(u,v)/F(u,v) = 2 - e^{j2\pi \frac{u}{M}} - e^{j2\pi \frac{v}{N}}$

$= 2 - \cos 2\pi \frac{u}{M} - j \sin 2\pi \frac{u}{M} - \cos 2\pi \frac{v}{N} - j \sin 2\pi \frac{v}{N}$

d) $|H(u,v)| = \sqrt{(2 - \cos 2\pi \frac{u}{M} - \cos 2\pi \frac{v}{N})^2 + (\sin 2\pi \frac{u}{M} + \sin 2\pi \frac{v}{N})^2}$

e) "hw4p2.jpg"

f) Highpass

$|H(u,v)|_{lowest} = |H(u,v)|_{u=0,v=0} = \sqrt{0+0} = 0$

$|H(u,v)|_{highest} = |H(u,v)|_{u=\frac{M}{2},v=\frac{N}{2}} = \sqrt{16+0} = 4$

3. Consider the Laplacian filter with -4 at the center of the mask.

a) Find the frequency response $H(u,v)$ of the filter.

b) Find the magnitude response $|H(u,v)|$ of the filter.

c) Generate a surface plot of $|H(u,v)|$.

d) Using $|H(u,v)|$, is this filter best described as lowpass or highpass or neither? Explain.

a) $g(x,y) = -4f(x,y) + f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1)$

$g(x,y) = h(x,y) * f(x,y)$

$G(u,v) = -4F(u,v) + F(u,v)e^{-j2\pi\frac{u}{M}} + F(u,v)e^{j2\pi\frac{u}{M}} + F(u,v)e^{-j2\pi\frac{v}{N}} + F(u,v)e^{j2\pi\frac{v}{N}}$

$G(u,v) = H(u,v) \cdot F(u,v)$

$H(u,v) = G(u,v)/F(u,v) = -4 + e^{-j2\pi\frac{u}{M}} + e^{j2\pi\frac{u}{M}} + e^{-j2\pi\frac{v}{N}} + e^{j2\pi\frac{v}{N}}$

$= -4 + \cos 2\pi\frac{u}{M} - j\sin 2\pi\frac{u}{M} + \cos 2\pi\frac{u}{M} + j\sin 2\pi\frac{u}{M}$

$+ \cos 2\pi\frac{v}{N} - j\sin 2\pi\frac{v}{N} + \cos 2\pi\frac{v}{N} + j\sin 2\pi\frac{v}{N}$

$= -4 + 2\cos 2\pi\frac{u}{M} + 2\cos 2\pi\frac{v}{N}$

b) $|H(u,v)| = \sqrt{(-4+2\cos 2\pi\frac{u}{M} + 2\cos 2\pi\frac{v}{N})^2}$

c) "hw4p3.jpg"

d) Highpass

$|H(u,v)|_{lowest} = |H(u,v)|_{u=0,v=0} = 0$

$|H(u,v)|_{highest} = |H(u,v)|_{u=\frac{M}{2},v=\frac{N}{2}} = \sqrt{64} = 8$