EECS203A Exam #1 April 29, 2021

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This is an 80 minute, open book, open notes exam. Calculators are allowed. Interaction with other people is not allowed. Internet use outside of Zoom is not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

TOTAL:

Question 1 (5 points) Consider a 2-D spatial filter $h(x, y)$ defined by the 3×3 mask

0.01	0.10	0.01
0.10	0.56	0.10
0.01	0.10	0.01

a) Is $h(x, y)$ best described as a lowpass or highpass filter? Explain.

Lowpass filter. This computes a weighted average over 9 pixels.

2 for Lowpass 1 for explanation

b) What is the output image if $h(x, y)$ is applied to the constant input image $I(x, y) = 100$?

2 Output is a constant image of brightness 100.

Question 2 (6 points) Suppose that we generate an output image $g(x, y)$ from an input image $f(x, y)$ according to

$$g(x, y) = 5f(x, y) - f(x + 1, y) - f(x - 1, y) - f(x, y + 1) - f(x, y - 1)$$

3 a) Find a mask that implements this operation.

$$\begin{matrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{matrix} \qquad \begin{matrix} & (x-1,y) & \\ (x,y-1) & (x,y) & (x,y+1) \\ & (x+1,y) & \end{matrix}$$

3 b) Describe the appearance of the filtered image $g(x, y)$ compared to the input image $f(x, y)$.

This filter implements the input image minus its Laplacian. The filtered image will appear similar to the input image but the high frequencies will be enhanced.

Question 3 (8 points) Consider a 2-D spatial filter $h(x, y)$ that has the frequency response

$$
\begin{aligned}
H(u,v) &= 0 \text{ if } D(u,v) \leq D_0 \\
&= 1 \text{ if } D(u,v) > D_0
\end{aligned}
$$

where $D(u,v) = \sqrt{u^2 + v^2}$.

2

a) What is this filter called?

Ideal Highpass Filter

b) Will ringing effects in an image filtered with $h(x, y)$ become more prominent as D_0 increases? Explain.

No. As D_0 increases, the lobes of the sinc function in the space domain become closer together and ringing effects are less prominent.

2 for No   1 for explanation

c) Will a filtered image $f(x, y) * h(x, y)$ have a larger fraction of the total power in the input image $f(x, y)$ for small D_0 or for large D_0? Explain.

Small D_0. As D_0 becomes smaller, the filter will pass more low frequencies and the filtered image will have more total power.

2 for Small D_0 1 for explanation

8

Question 4 (8 points) Let H be an operator that maps an input image $I(x, y)$ to an output image $O(x, y)$ according to

$$O(x, y) = c[I(x, y)]^A$$

where c and A are real constants. For what values of c and A is H a linear operator?

LHS linear definition

$$H[af(x,y) + bg(x,y)] = c(af(x,y) + bg(x,y))^A$$

RHS linear definition

$$aH[f(x,y)] + bH[g(x,y)] = ac(f(x,y))^A + bc(g(x,y))^A$$

LHS = RHS for $c = 0$ or $A = 1$

Question 5 (8 points) Suppose that we capture a sequence of images

$$g(x, y, t_i) = f(x, y) + n(x, y, t_i)$$

where $f(x, y)$ is a noise-free image and $n(x, y, t_i)$ is a zero-mean additive noise source with variance $\sigma_n^2(x, y)$. Assume that the noise at any time is independent of the noise at any other time. Suppose that we form the image

$$h(x, y) = \frac{1}{4}(g(x, y, t_1) + g(x, y, t_2) - g(x, y, t_3) - g(x, y, t_4))$$

(4) a) What is the expected value of $h(x, y)$?

$$E[h(x,y)] = \frac{1}{4}E[f(x,y) + n(x,y,t_1)] + \frac{1}{4}E[f(x,y) + n(x,y,t_2)] - \frac{1}{4}E[f(x,y) + n(x,y,t_3)] - \frac{1}{4}E[f(x,y) + n(x,y,t_4)]$$

$$= 0$$

(4) b) What is the variance of $h(x, y)$?

$$VAR[h(x,y)] = \frac{1}{16} VAR[n(x,y,t_1) + n(x,y,t_2) - n(x,y,t_3) - n(x,y,t_4)]$$

$$= \frac{4\sigma_n^2(x,y)}{16} = \frac{\sigma_n^2(x,y)}{4}$$

Question 6 (9 points) Let $I(x, y)$ be an input digital image and let $O(x, y)$ be the output digital image obtained by processing $I(x, y)$ with a 3×3 median filter. Let N_I be the number of different gray levels that occur in $I(x, y)$ and let N_O be the number of different gray levels that occur in $O(x, y)$. For each part of this question, if you answer YES then give an example that satisifes the condition. If you answer NO then explain why not.

3 a) Can we have $N_I > N_O$?

YES

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$I(x,y)$ $\quad$ $O(x,y)$

$N_I = 2$ $\quad$ $N_o = 1$

3 b) Can we have $N_I = N_O$?

YES

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \longrightarrow \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$I(x,y)$ $\quad$ $O(x,y)$

$N_I = 1$ $\quad$ $N_o = 1$

3 c) Can we have $N_I < N_O$?

No. For the median filter, any gray level in the output image must also occur in the input image.

On each part, 1 for correct YES/NO and 2 for example/explanation

Question 7 (10 points) Consider a 2-D spatial filter $h(x, y)$ defined by the 3×3 mask

-1	-2	0
-2	0	3
0	3	1

Suppose that we are given a digital image $I(x, y)$ with gray levels represented using 6 bits so that pixels have the possible values 0,1,2,...,63. Let $O(x, y)$ be the output image after $h(x, y)$ is applied to $I(x, y)$. Assume pixels in $O(x, y)$ can take any integer values including negatives.

a) What is the minimum possible value MIN of a pixel in $O(x, y)$?

3

$$-5(63) + 7(0) = -315$$

b) What is the maximum possible value MAX of a pixel in $O(x, y)$?

3

$$7(63) + (-5)(0) = 441$$

c) Find a gray-level transform $T(r)$ that maps a gray level r in $O(x, y)$ to a gray level in the 6-bit range 0,1,2,...,63 where $T(\text{MIN}) = 0$, $T(\text{MAX}) = 63$, and gray levels r in $O(x, y)$ with $\text{MIN} < r < \text{MAX}$ are mapped to the 6-bit range so that $T(r)$ is a monotonically increasing linear function of r. Note that with rounding error, $T(r)$ may deviate slightly from linear.

4

$$T(r) = \frac{63(r - \text{MIN})}{\text{MAX} - \text{MIN}}$$

where $T(r)$ is rounded to the nearest integer for each r.

Question 8 (10 points) Let $f(x, y)$ be the 4×4 digital image with DFT $F(u, v)$ given by

```
F(0,0) F(0,1) F(0,2) F(0,3)       6   0   0   0
F(1,0) F(1,1) F(1,2) F(1,3)   =   0   0   0   1
F(2,0) F(2,1) F(2,2) F(2,3)       0   0   0   0
F(3,0) F(3,1) F(3,2) F(3,3)       0   1   0   0
```

$M=4,\ N=4$

Find the 4×4 digital image $f(x, y)$ for $x = 0, 1, 2, 3$ and $y = 0, 1, 2, 3$. Simplify your answer.

$$F(u,v) = 6\delta(u,v) + \delta(u-1, v-3) + \delta(u-3, v-1)$$
$$= 6\delta(u,v) + \delta(u+3, v+1) + \delta(u-3, v-1)$$

since DFT periodic

$$f(x,y) = \cos(2\pi u_0 x + 2\pi v_0 y)$$
$$\rightarrow F(u,v) = \frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$$

$$f(x,y) = 6 + 2\cos\left(2\pi\left(\tfrac{3}{4}\right)x + 2\pi\left(\tfrac{1}{4}\right)y\right)$$
$$= \underbrace{6 + 2\cos(1.5\pi x + 0.5\pi y) \quad x = 0,1,2,3 \quad y = 0,1,2,3}$$

$$\begin{matrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) \end{matrix} = \begin{bmatrix} 8 & 6 & 4 & 6 \\ 6 & 8 & 6 & 4 \\ 4 & 6 & 8 & 6 \\ 6 & 4 & 6 & 8 \end{bmatrix}$$

full points for either

Question 9 (12 points) Consider an $N \times N$ digital image f with 8 gray levels from 0 to 7. Suppose that the gray level histogram for f is given by

$$h(r_k) = 2r_k + 1 \qquad r_k = 0, 1, 2, \ldots, 7$$

3

a) Find N.

cumulative $T(r_k) = \sum_{i=0}^{r_k} (2i+1)$

$T(7) = \sum_{i=0}^{7} (2i+1) = \frac{2(7)(8)}{2} + 8 = 64 \rightarrow N = 8$

b) Use the method described in class to determine the gray level transformation $M(r_k)$ for $r_k = 0, 1, 2, \ldots, 7$ that corresponds to histogram equalization.

r_k	$h(r_k)$	$T(r_k)$
0	1	1
1	3	4
2	5	9
3	7	16
4	9	25
5	11	36
6	13	49
7	15	64

r_k	$d(r_k)$	$G(r_k)$
0	8	8
1	8	16
2	8	24
3	8	32
4	8	40
5	8	48
6	8	56
7	8	64

r_k	$M(r_k)$
0	0
1	0
2	0
3	1
4	2
5	3 or 4
6	5
7	7

6

3

c) Find the histogram $h'(r_k)$ $\quad r_k = 0, 1, 2, \ldots, 7$ for the transformed image that results after applying histogram equalization to f.

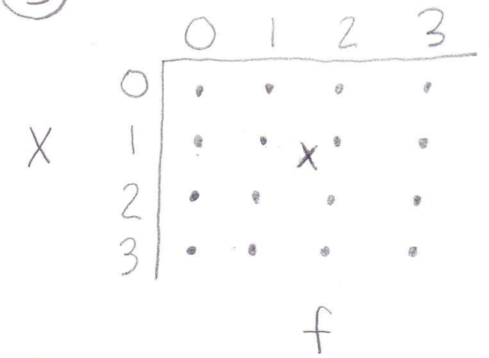
r_k	$h'(r_k)$	
0	9	from 0,1,2
1	7	from 3
2	9	from 4
3	0 or 11	from 5
4	0 or 11	from 5
5	13	from 6
6	0	
7	15	from 7

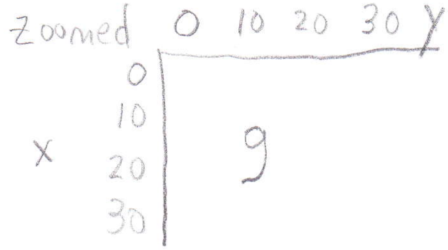
Question 10 (12 points) Consider the 4×4 digital image $f(x, y) = 6x^2y$ defined for $x = 0, 1, 2, 3,\quad y = 0, 1, 2, 3$. Let $g(x, y)$ be a 31×31 zoomed version of $f(x, y)$ defined for $x = 0, 1, \ldots, 30,\quad y = 0, 1, \ldots, 30$ as described in class.

$x = 0,1,2,3 \quad y = 0,1,2,3$

$g(10x, 10y) = f(x,y)$

3 a) Find $g(11, 17)$ using nearest neighbor interpolation.





$g(11,17)$ is closest to $f(1,2)$

$g(11,17) = 6 \cdot 1 \cdot 2$
$= 12$

3 b) Find the coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ for the pixels in $f(x, y)$ that will contribute to the bilinear interpolation value for $g(11, 17)$.

$(1,1), (1,2), (2,1), (2,2)$

6 c) Find $g(11, 17)$ using bilinear interpolation. model is $f(x,y) = ax + by + cxy + d$

$f(1,1) = 6 = a \cdot 1 + b \cdot 1 + c \cdot 1 \cdot 1 + d \quad (1)$

$f(1,2) = 12 = a \cdot 1 + b \cdot 2 + c \cdot 1 \cdot 2 + d \quad (2)$

$f(2,1) = 24 = a \cdot 2 + b \cdot 1 + c \cdot 2 \cdot 1 + d \quad (3)$

$f(2,2) = 48 = a \cdot 2 + b \cdot 2 + c \cdot 2 \cdot 2 + d \quad (4)$

$(1),(2) \rightarrow b + c = 6$
$(3),(4) \rightarrow b + 2c = 24$
$\rightarrow c = 18,\ b = -12$

$(1) \rightarrow a - 12 + 18 + d = 6$
$(3) \rightarrow 2a - 12 + 36 + d = 24$
$\rightarrow a = 0,\ d = 0$

$g(11,17) = f(1.1, 1.7) = -12(1.7) + 18(1.1)(1.7) = 13.26$

Question 11 (12 points) Define the masks for two 3×3 spatial filters $h_1(x, y)$ and $h_2(x, y)$ and define the 5×5 digital image $f(x, y)$ as shown below

```
                                              0  0  0  0  0
            0 1 0            0 0 0            0  0  0  0  0
   h1 =     0 2 0     h2 =   4 5 6     f =    0  0  1  0  0
            0 3 0            0 0 0            0  0  0  0  0
                                              0  0  0  0  0
```

where you may assume for filtering that $f(x, y)$ has all zeros outside the boundaries.

4 a) Find the 5×5 output image $g_1(x, y)$ if we apply $h_1(x, y)$ to $f(x, y)$.

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

4 b) Find the 5×5 output image $g_2(x, y)$ if we apply $h_2(x, y)$ to $g_1(x, y)$. You may assume that $g_1(x, y)$ has all zeros outside the boundaries.

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 18 & 15 & 12 & 0 \\ 0 & 12 & 10 & 8 & 0 \\ 0 & 6 & 5 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

4 c) Find a mask that when applied to an input image is equivalent to the double filtering operation of applying $h_1(x, y)$ to an input image and then applying $h_2(x, y)$ to the result.

Mask should give answer to part b when applied to f.

$$\begin{matrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{matrix}$$