EECS203A HW3 John Liu 25961868

1. Let $f(x, y)$ be the 4×4 digital image

$$
\begin{matrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) \end{matrix} = \begin{matrix} 14 & 12 & 10 & 11 \\ 10 & 8 & 6 & 4 \\ 6 & 4 & 4 & 2 \\ 7 & 5 & 3 & 1 \end{matrix}
$$

a) Let $g(x, y)$ be the digital image that results after filtering $f(x, y)$ with the Laplacian mask with -8 in the center. Find $g(1,1), g(1,2), g(2,1), g(2,2)$.

b) Let $h(x, y)$ be the digital image that results after filtering $f(x, y)$ with a 3×3 median filter. Find $h(1,1), h(1,2), h(2,1), h(2,2)$.

a) Laplacian mask with -8 in the center:

$$
\begin{matrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{matrix}
$$

$g(1.1) = 8(-8)+14+12+10+10+6+6+4+4 = 2$

$g(1.2) = 6(-8)+12+10+11+8+4+4+4+2 = 7$

$g(2,1) = 4(-8)+10+8+6+6+4+7+5+3 = 17$

$g(2,2) = 4(-8)+8+6+4+4+2+5+3+1 = 1$

b) $h(1.1) = \text{median}\{4, 4, 6, 6, 8, 10, 10, 12, 14\} = 8$

$h(1.2) = \text{median}\{2, 4, 4, 4, 6, 8, 10, 11, 12\} = 6$

$h(2,1) = \text{median}\{3, 4, 4, 5, 6, 6, 7, 8, 10\} = 6$

$h(2.2) = \text{median}\{1, 2, 3, 4, 4, 4, 5, 6, 8\} = 4$

2. Consider the 100×100 digital image $f(x, y)$ defined by

$$f(x,y) = \begin{cases} 100 & \text{if } 0 \le x \le 50 \text{ and } 0 \le y \le 50 \\ 200 & \text{otherwise} \end{cases}$$

a) Draw the image.

b) Let $g(x, y)$ be the output image that results after processing $f(x, y)$ with a 3×3 median filter? Ignore boundary cases. Is $g(x, y)$ the same as $f(x, y)$? Explain.

a)

b) $g(x,y) \neq f(x,y)$

$f(50,50) = 100$

$$\begin{aligned} g(50,50) &= \text{median}\{f(49,49),\ f(49,50),\ f(49,51), \\ &\qquad f(50,49),\ f(50,50),\ f(50,51), \\ &\qquad f(51,49),\ f(51,50),\ f(51,51)\} \\ &= \text{median}\{100,\ 100,\ 200, \\ &\qquad 100,\ 100,\ 200, \\ &\qquad 200,\ 200,\ 200\} \\ &= 200 \neq f(50,50) \end{aligned}$$

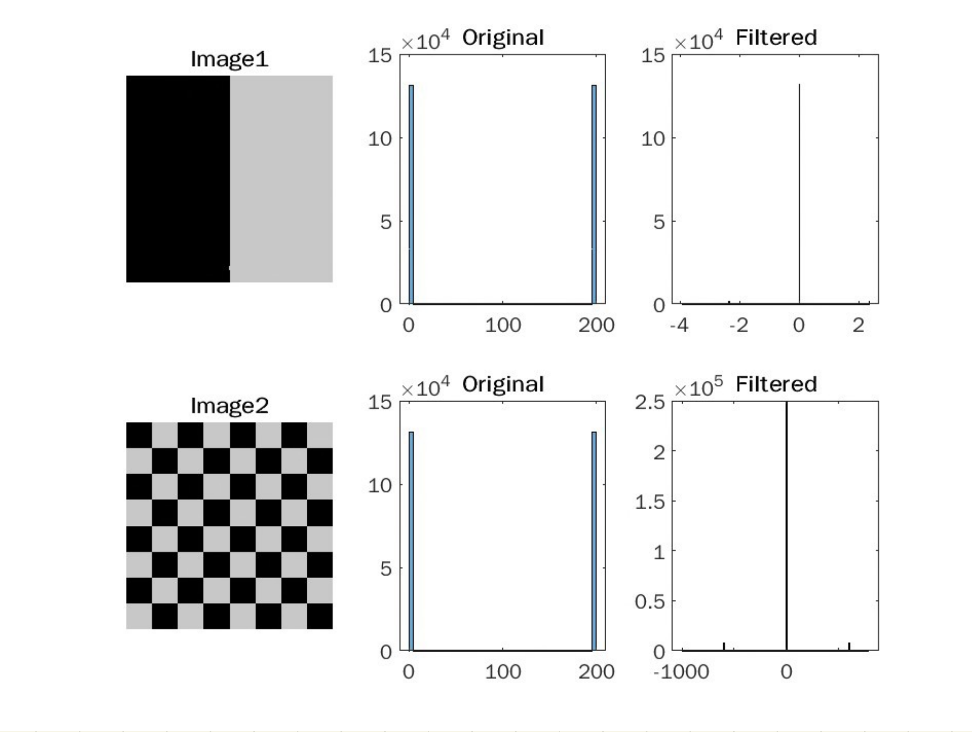
3. Consider two images. Image1 is 512×512 pixels where the first 256 columns have brightness 0 and the last 256 columns have brightness 200. Image2 is 512×512 pixels with the pattern of a chess board with an 8×8 pattern of 64×64 pixel squares that are alternatively brightness 0 and brightness 200. The histograms of Image1 and Image2 are the same. Suppose that each image is filtered by a 3×3 Laplacian filter with -8 in the center of the mask. Ignore boundary cases.

a) Are the histograms of the filtered images the same? Explain.

b) If your answer is no, plot the two histograms.

a) Not the same. Because the number of pixels on the edge of black and white part in image 1 is larger than the chess board image 2. So after Laplacian filter. the result of image 1 is supposed to be brighter than image 2 result.

b)



4. Let $f_1(x,y)$ be the 3×3 smoothing filter with nine elements each having a value of $1/9$. Let $f_2(x,y)$ be the 3×3 Laplacian filter with -8 in the center of the mask.

a) Suppose that we filter an input image using $f_2(x,y)$ and then filter the result with $f_1(x,y)$. Is this double filtering process a linear operation on the input image? Explain.

b) If you answered yes to part a, derive the filter mask that corresponds to the double filtering process. If you answered no to part a, explain why not.

c) Does the result of this double filtering operation depend on the order in which we apply the two filters to an input image? Explain your answer.

a) Yes. Because convolution of a image is linear, same is the convolution of convolution

b)

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \text{ and } \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \Rightarrow \frac{1}{81}\begin{bmatrix}1&2&3&2&1\\2&4&6&4&2\\3&6&9&6&3\\2&4&6&4&2\\1&2&3&2&1\end{bmatrix}$$

c) No. because they are both linear operation, the order doesn't matter