Color Corrections

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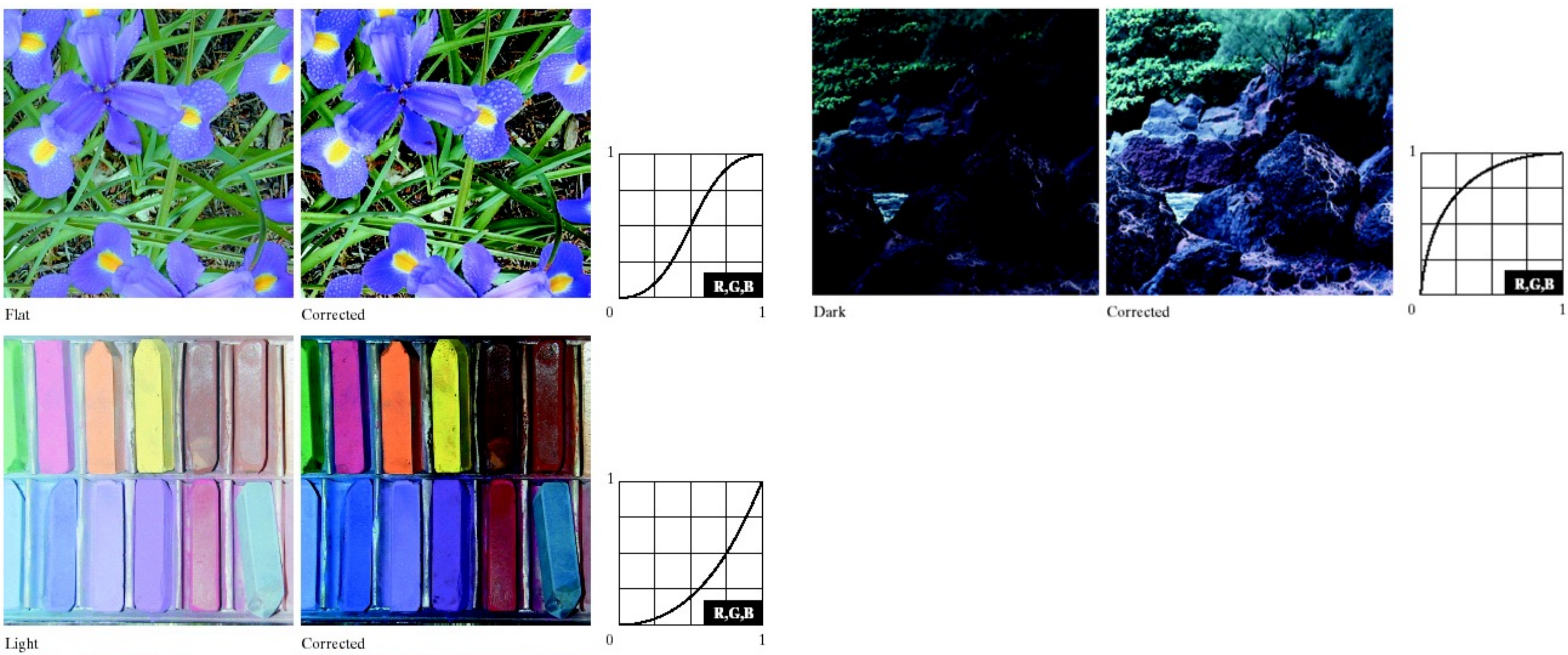

FIGURE 6.35 Tonal corrections for flat, light (high key), and dark (low key) color images. Adjusting the red, green, and blue components equally does not alter the image hues.

Color balancing – can apply different gray level transforms to different primaries to adjust the color properties of an image.

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Original/Corrected

FIGURE 6.36 Color balancing corrections for CMYK color images.

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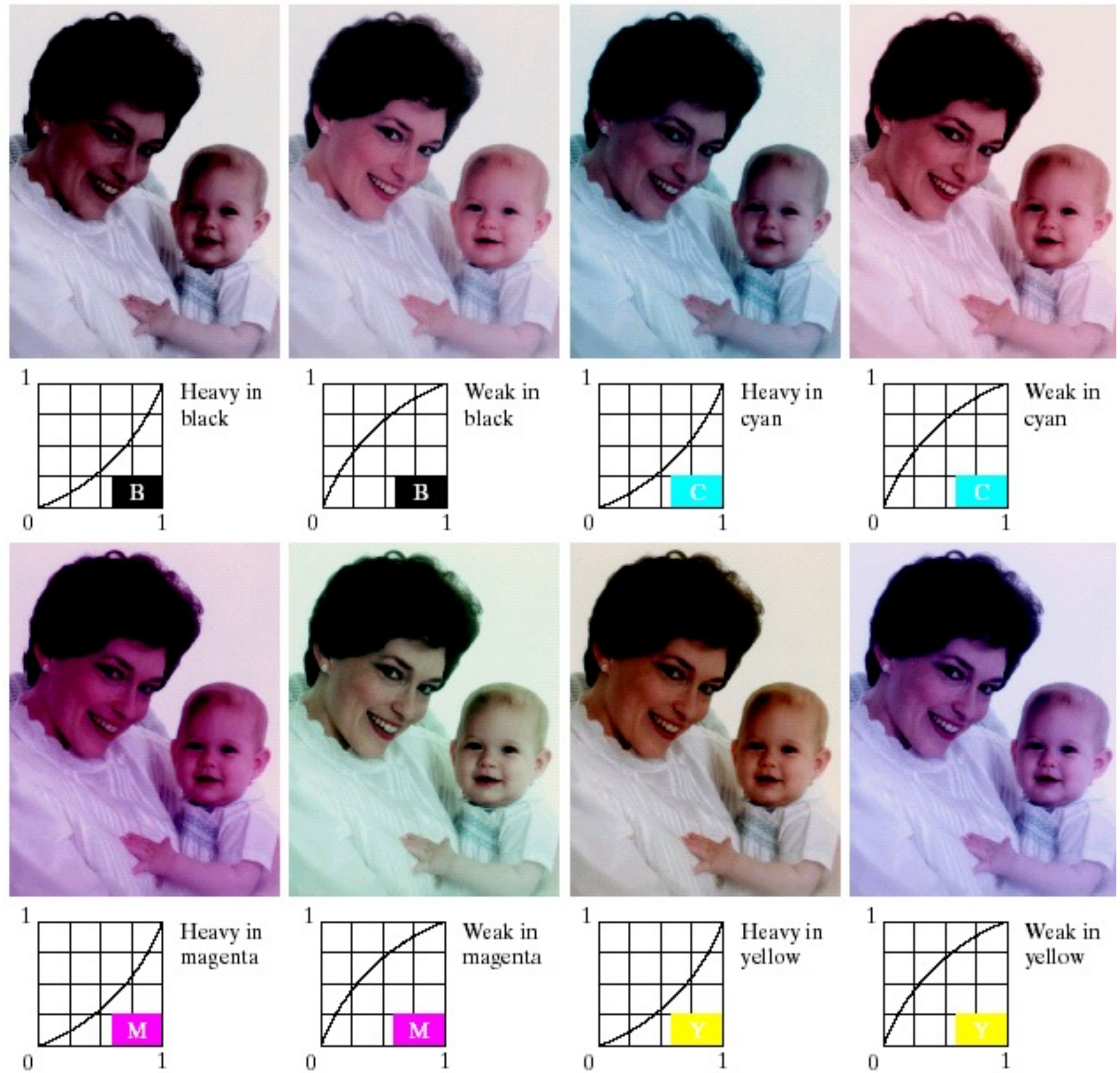
Original/Corrected

FIGURE 6.36 Color balancing corrections for CMYK color images.



Spatial Operations

We can apply a spatial filter to each band of a color image in the same way that a spatial filter is applied to a gray level image

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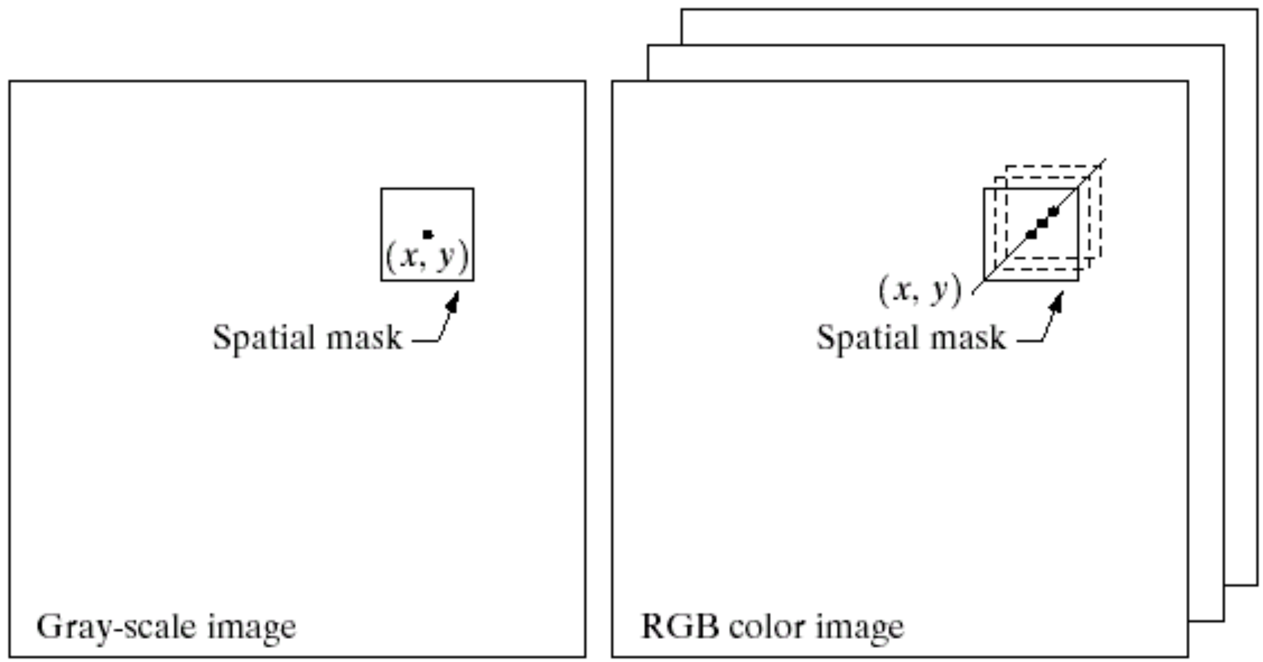


a b

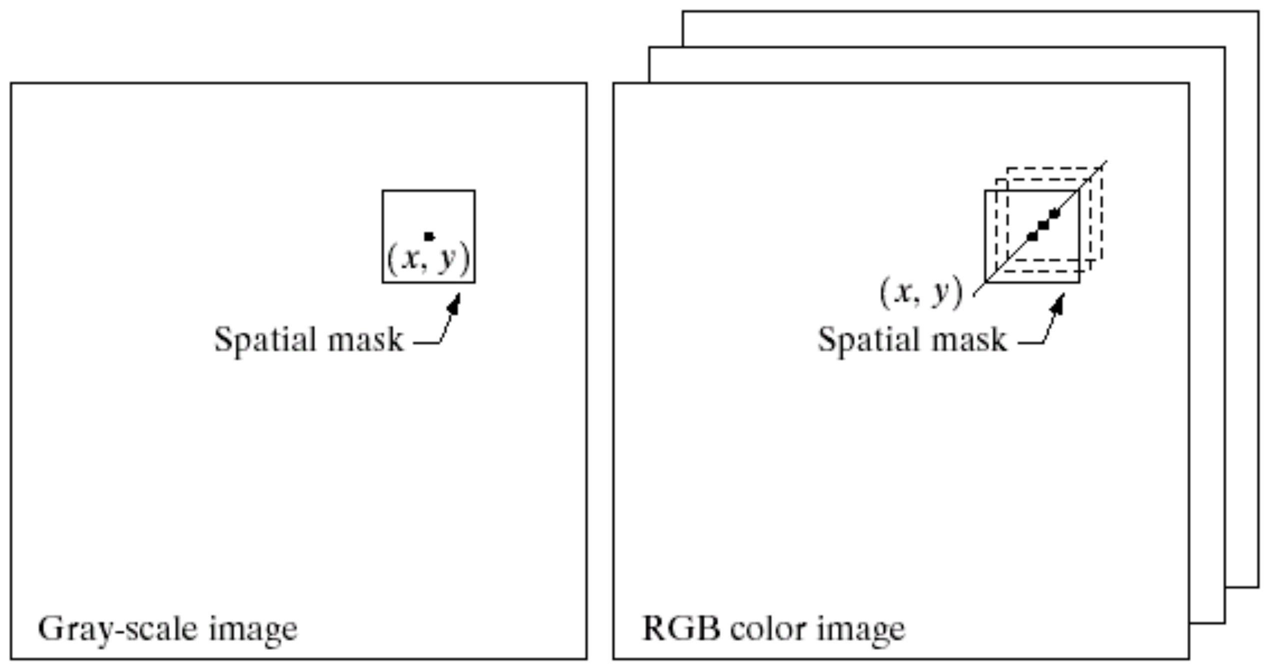
FIGURE 6.29 Spatial masks for gray-scale and RGB color images.

Spatial Operations

We can apply a spatial filter to each band of a color image in the same way that a spatial filter is applied to a gray level image

a b

FIGURE 6.29 Spatial masks for gray-scale and RGB color images.



We can combine the 3 filtered bands to form the filtered color image.

Ex Color image smoothing

Input image $[R(x,y)\ G(x,y)\ B(x,y)]$

Output image

$$C'(x,y) = \begin{bmatrix} R'(x,y) \\ G'(x,y) \\ B'(x,y) \end{bmatrix} = \begin{bmatrix} \frac{1}{K} \sum\limits_{\substack{(s,t) \\ \in S_{xy}}} R(s,t) \\ \frac{1}{K} \sum\limits_{\substack{(s,t) \\ \in S_{xy}}} G(s,t) \\ \frac{1}{K} \sum\limits_{\substack{(s,t) \\ \in S_{xy}}} B(s,t) \end{bmatrix}$$

S_{xy} is a window of K pixels centered at (x,y).

a	b
c	d

FIGURE 6.38
(a) RGB image.
(b) Red component image.
(c) Green component.
(d) Blue component.

FIGURE 6.40 Image smoothing with a 5 × 5 averaging mask.

Alternate Method using HSI

1. Convert the input image $[R(x,y)\ G(x,y)\ B(x,y)]$ to the HSI image $[H(x,y)\ S(x,y)\ I(x,y)]$
2. Apply the filter to $I(x,y)$ only. to get

$$[H(x,y)\ S(x,y)\ I'(x,y)]$$

3. Convert $[H(x,y)\ S(x,y)\ I'(x,y)]$ back to RGB to get filtered image $\hat{C}(x,y) = [\hat{R}(x,y)\ \hat{G}(x,y)\ \hat{B}(x,y)]$

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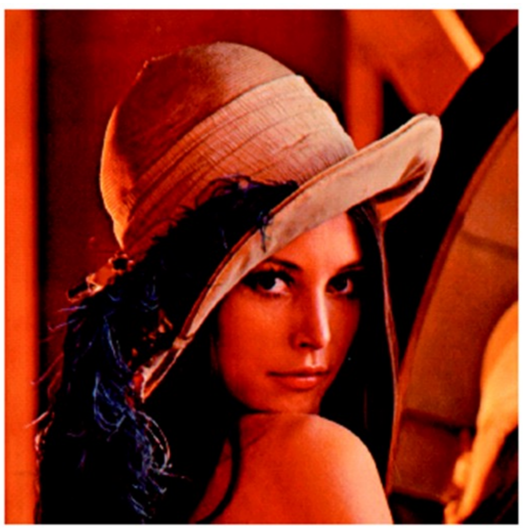
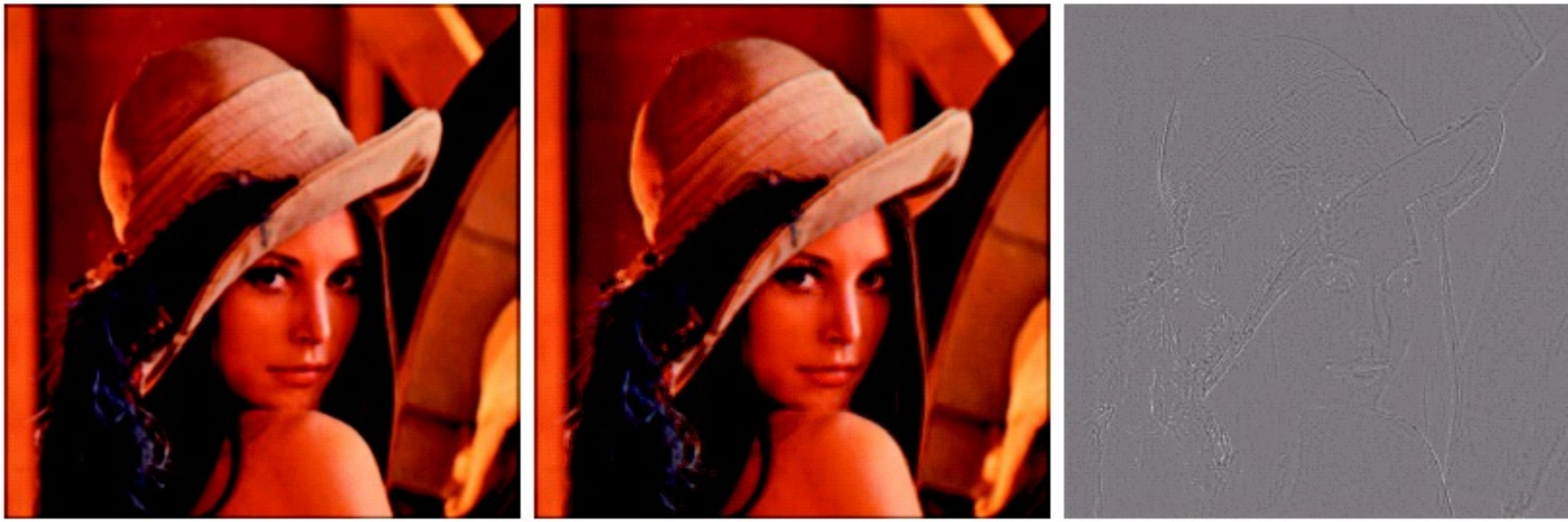


a b c

FIGURE 6.39 HSI components of the RGB color image in Fig. 6.38(a). (a) Hue. (b) Saturation. (c) Intensity.



a b c

FIGURE 6.40 Image smoothing with a 5×5 averaging mask. (a) Result of processing each RGB component image. (b) Result of processing the intensity component of the HSI image and converting to RGB. (c) Difference between the two results.

Ex Color image sharpening

Input image $[R(x,y) \; G(x,y) \; B(x,y)]$

Output image

$$C'(x,y) = \begin{bmatrix} R'(x,y) \\ G'(x,y) \\ B'(x,y) \end{bmatrix} = \begin{bmatrix} R(x,y) - \nabla^2 R(x,y) \\ G(x,y) - \nabla^2 G(x,y) \\ B(x,y) - \nabla^2 B(x,y) \end{bmatrix}$$

FIGURE 6.41 Image sharpening with the Laplacian.

a b c

FIGURE 6.41 Image sharpening with the Laplacian. (a) Result of processing each RGB channel. (b) Result of processing the intensity component and converting to RGB. (c) Difference between the two results.

Image Segmentation

Segmentation is the process of partitioning an image into regions that correspond to materials of interest in a scene.

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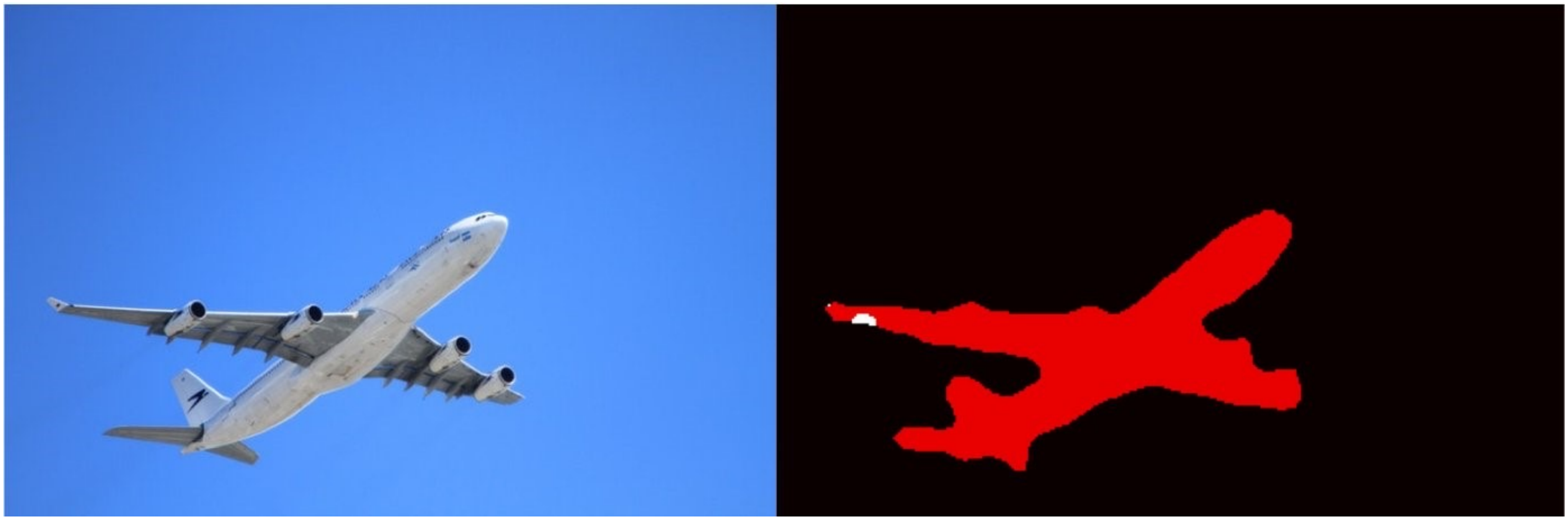

In the image of a surface, color information has a strong relationship with surface material while intensity has a strong relationship with surface geometry.

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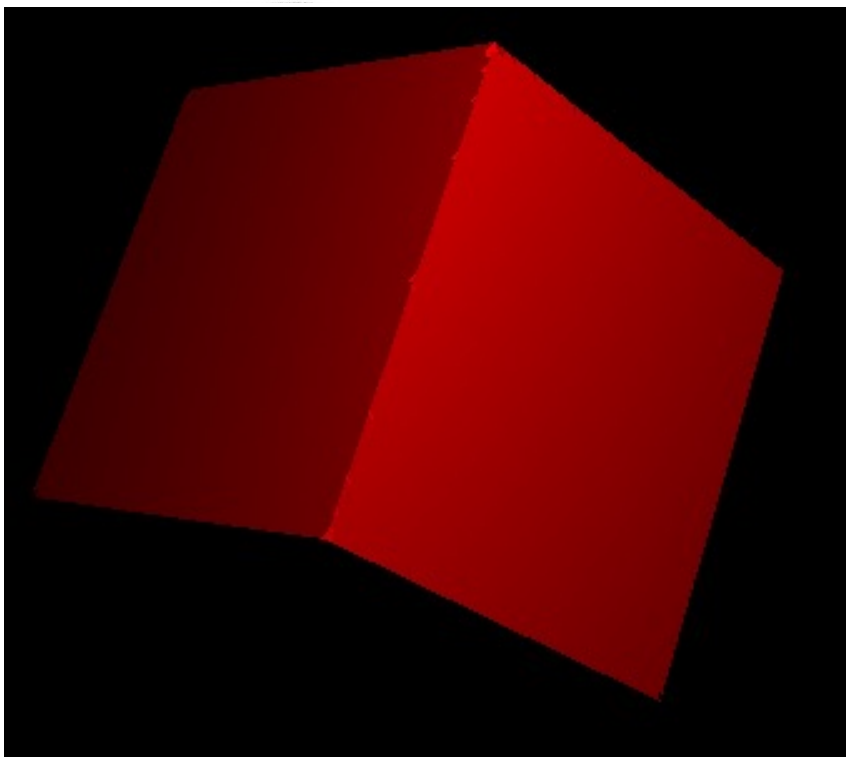
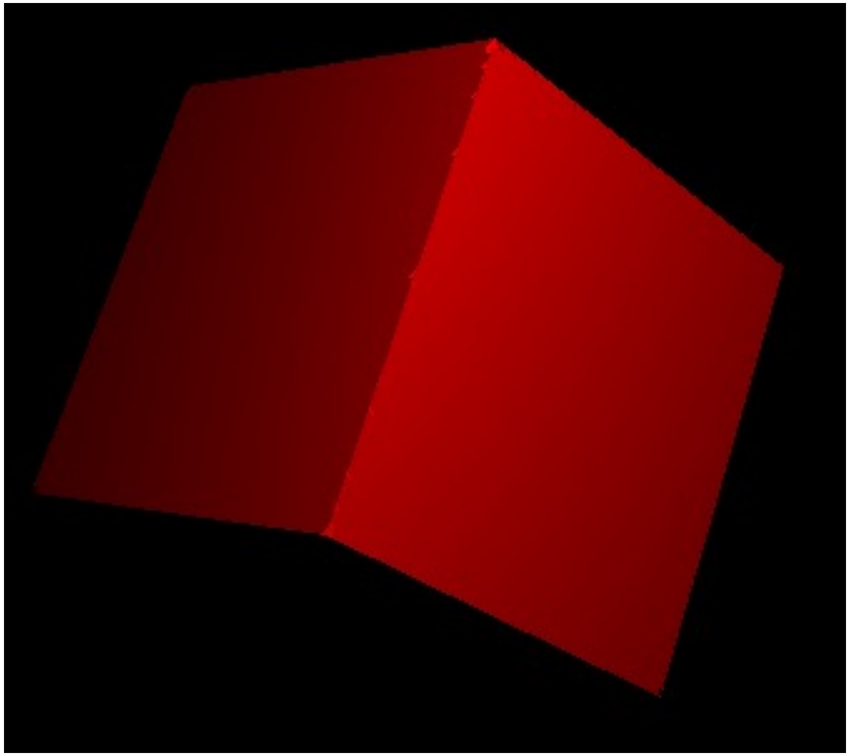
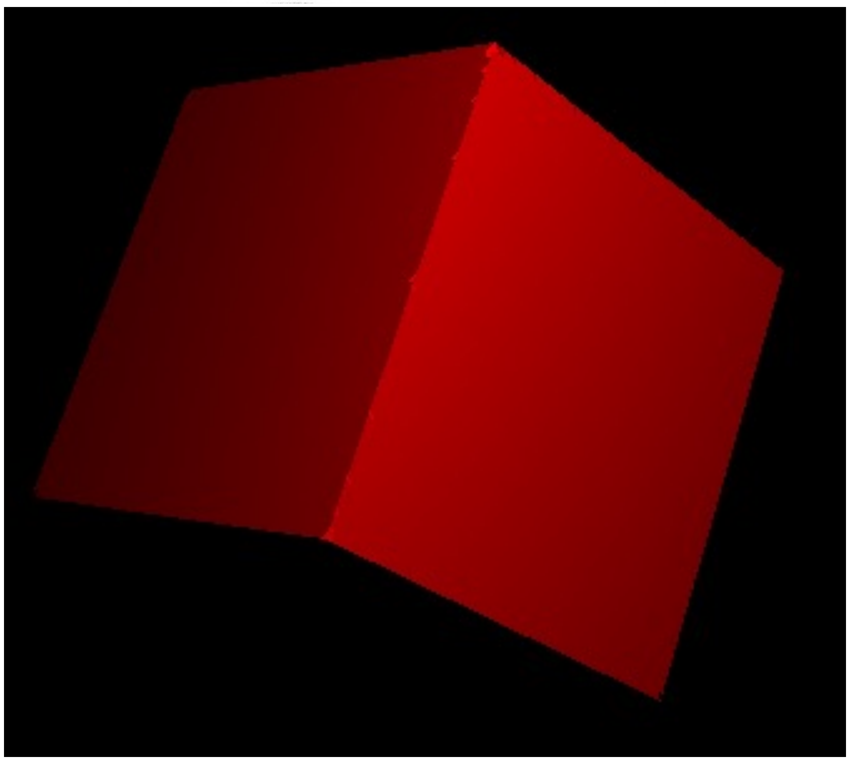


In the image of a surface, color information has a strong relationship with surface material while intensity has a strong relationship with surface geometry.



Color information is often more useful for segmentation than intensity.

In the image of a surface, color information has a strong relationship with surface material while intensity has a strong relationship with surface geometry.



Color information is often more useful for segmentation than intensity.

Color is represented by hue and saturation.

Example : Segment the red region

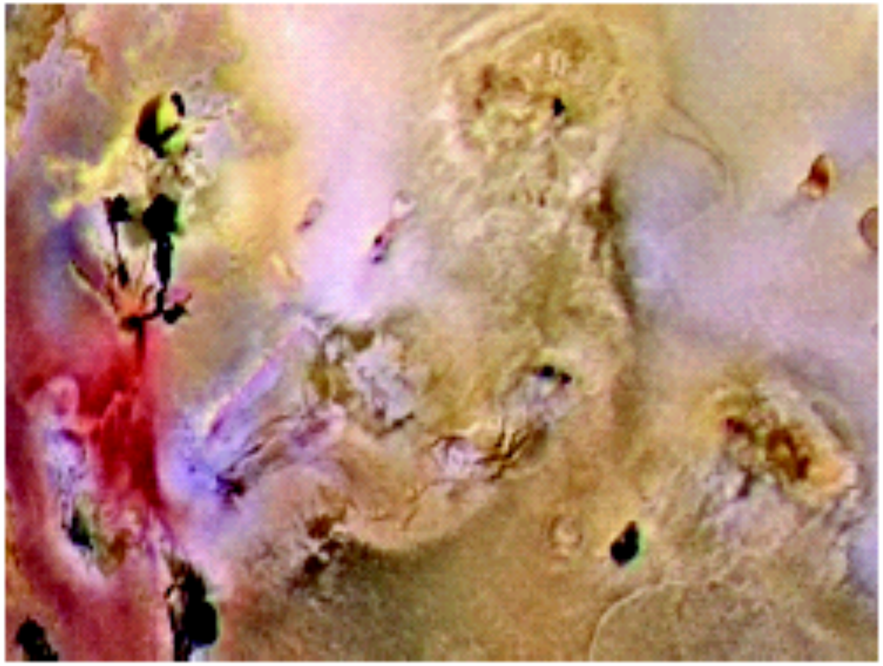


FIGURE 6.42 Image segmentation in HSI space.

Example : Segment the red region

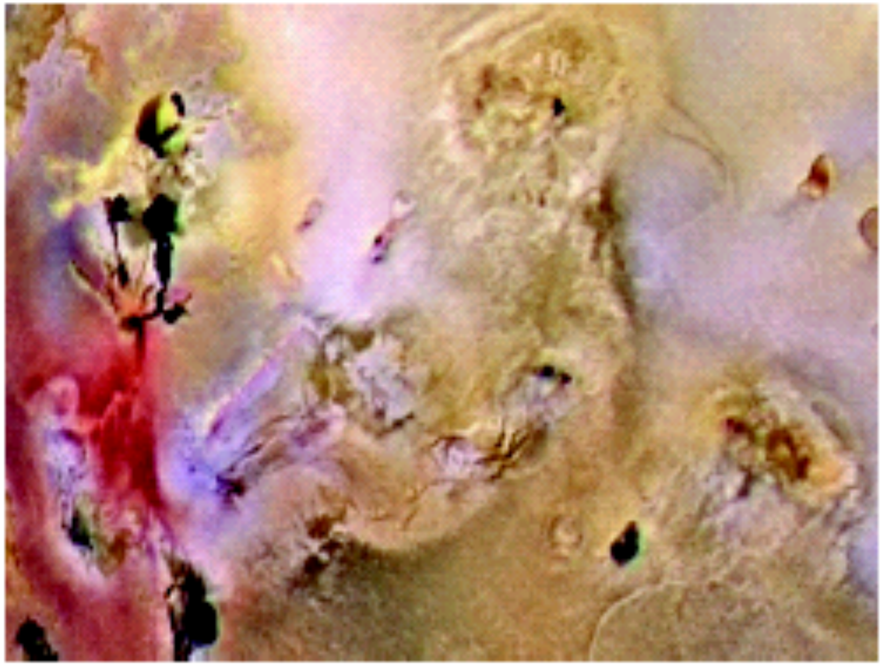


FIGURE 6.42 Image segmentation in HSI space.

Hue H(x,y)

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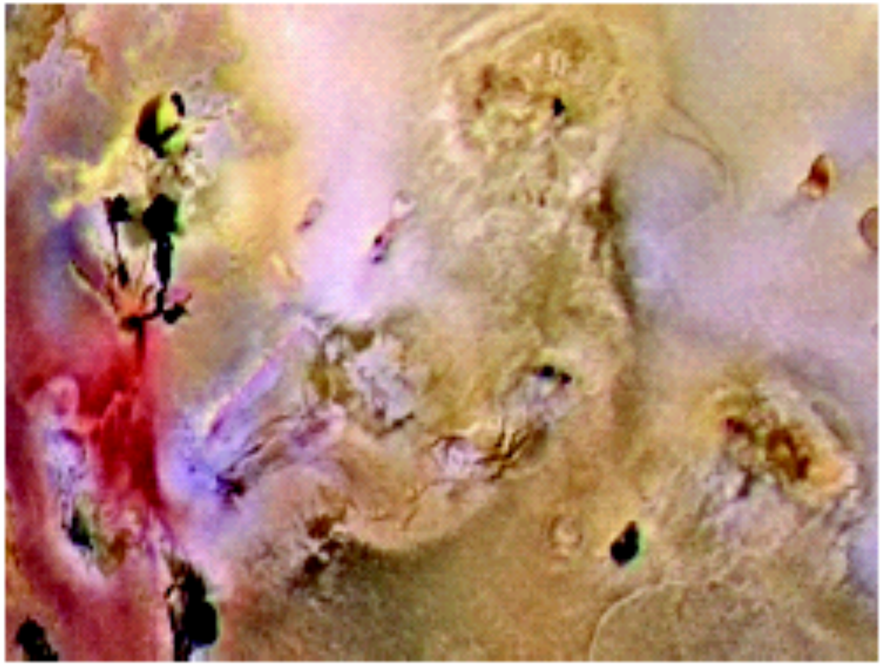

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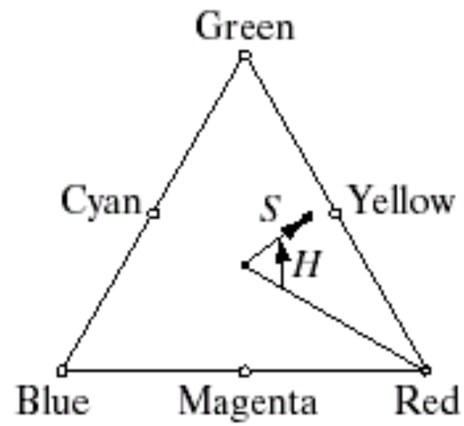
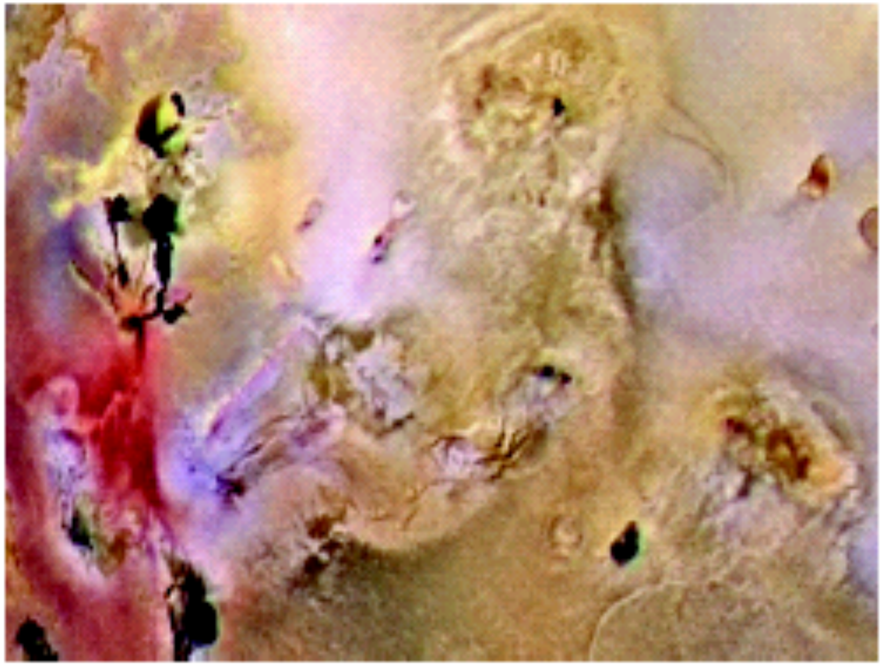


FIGURE 6.13 Hue and saturation in the HSI color model.

Hue H(x,y)

Example : Segment the red region



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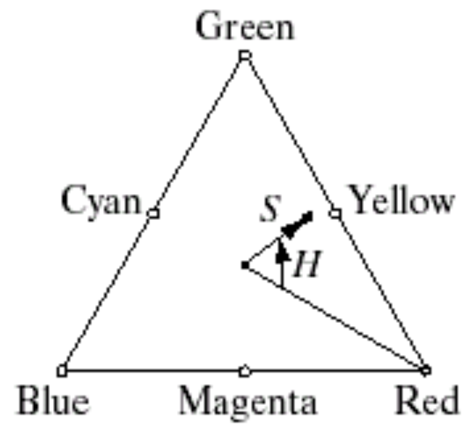

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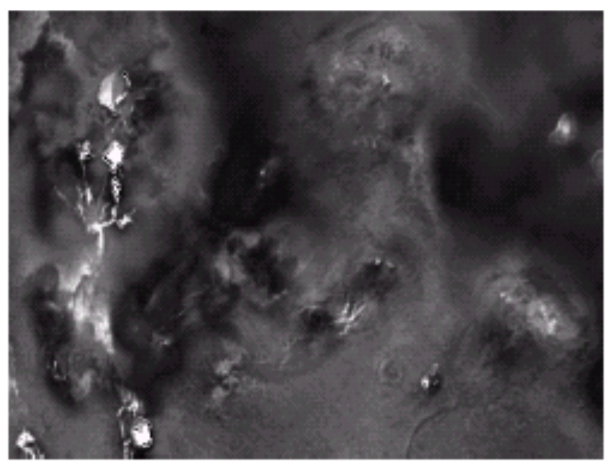
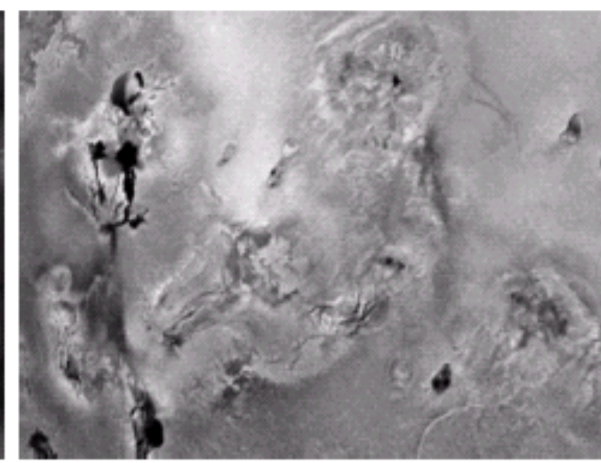
Hue H(x,y)



Saturation S(x,y)



Intensity I(x,y)

Red region has a large hue and a large saturation.

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Generate a binary saturation mask $b(x,y)$

MAX = maximum saturation in image

If $S(x,y) \leq 0.1 * MAX$ then $b(x,y) = 0$

If $S(x,y) > 0.1 * MAX$ then $b(x,y) = 1$

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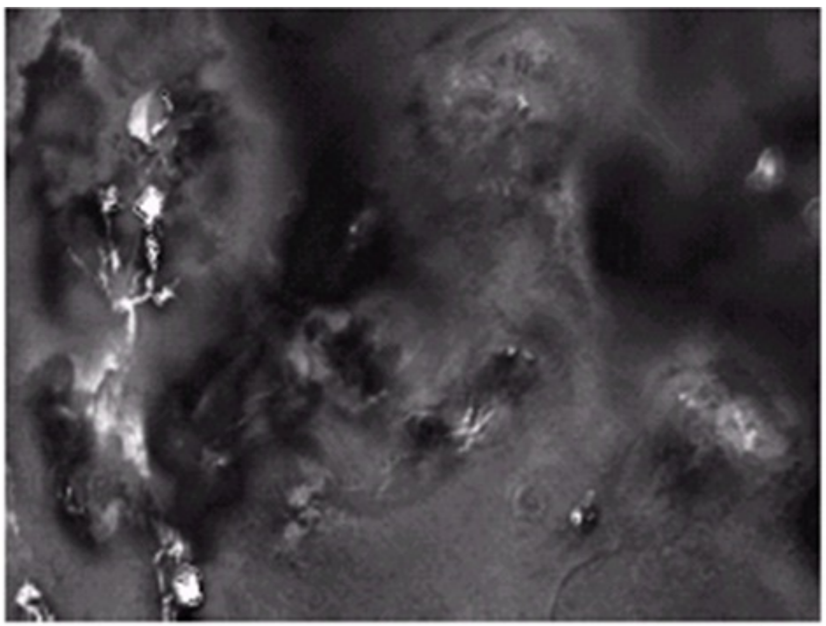
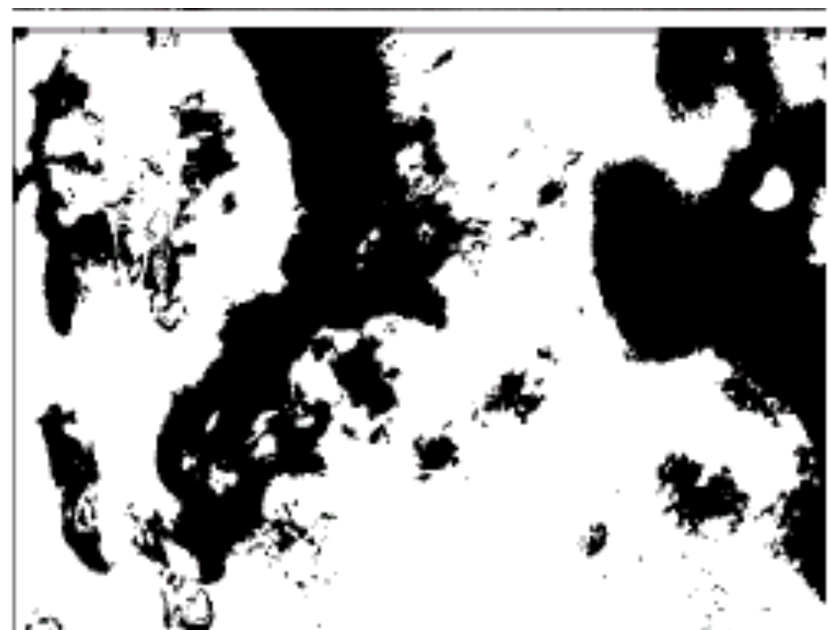
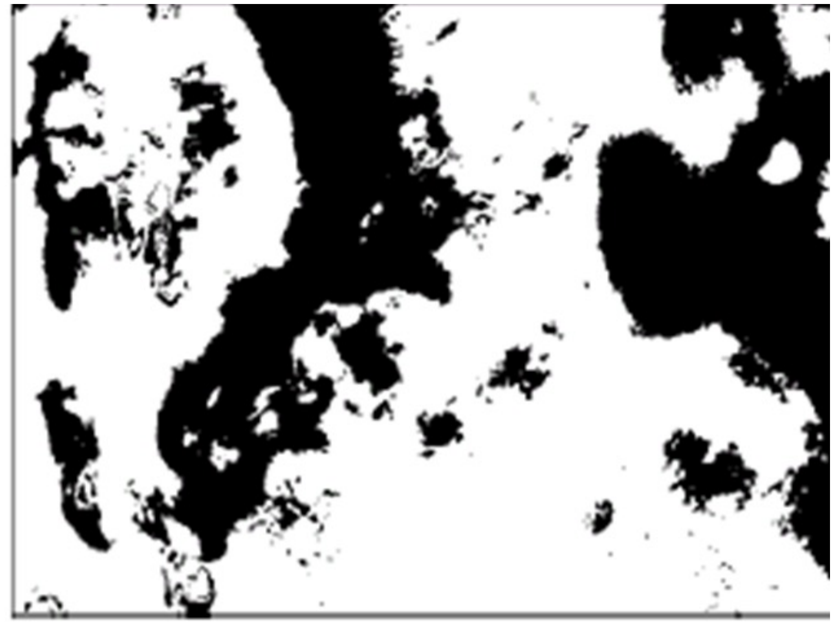


Saturation S(x,y)



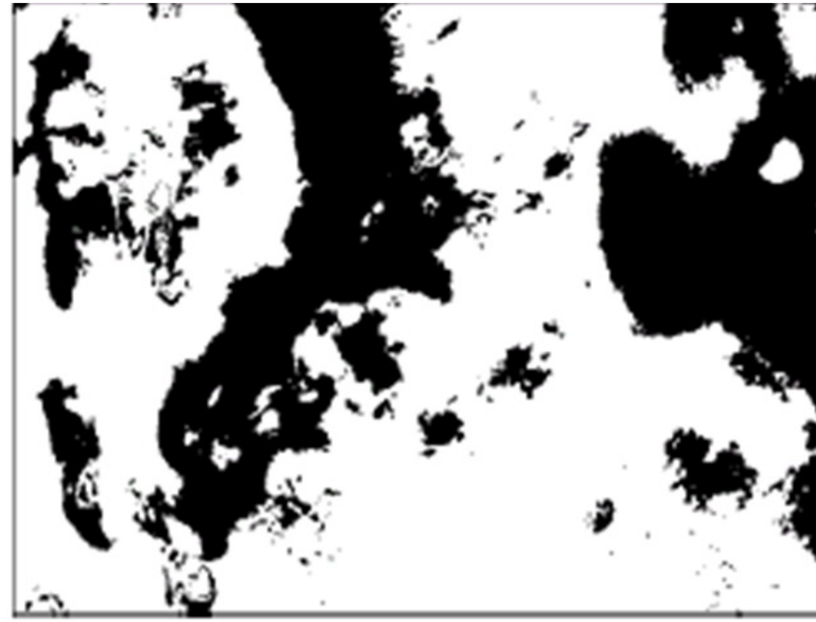
Binary Saturation Mask b(x,y)

Hue H(x,y)

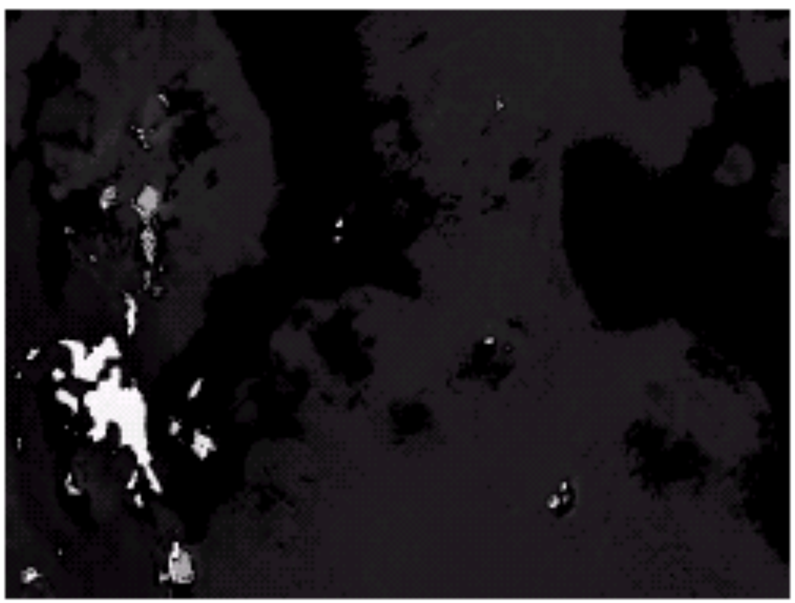


Binary Saturation Mask b(x,y)

Hue H(x,y)

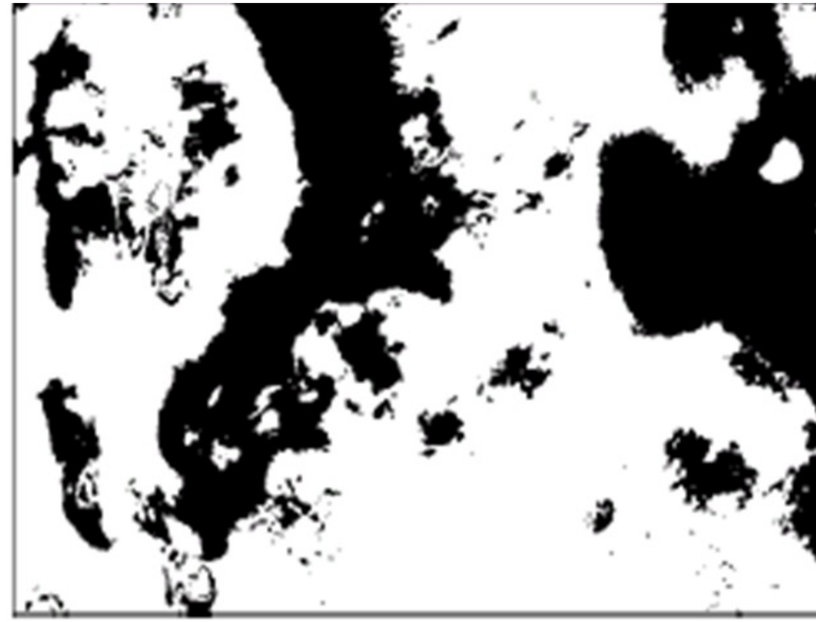


Binary Saturation Mask b(x,y)

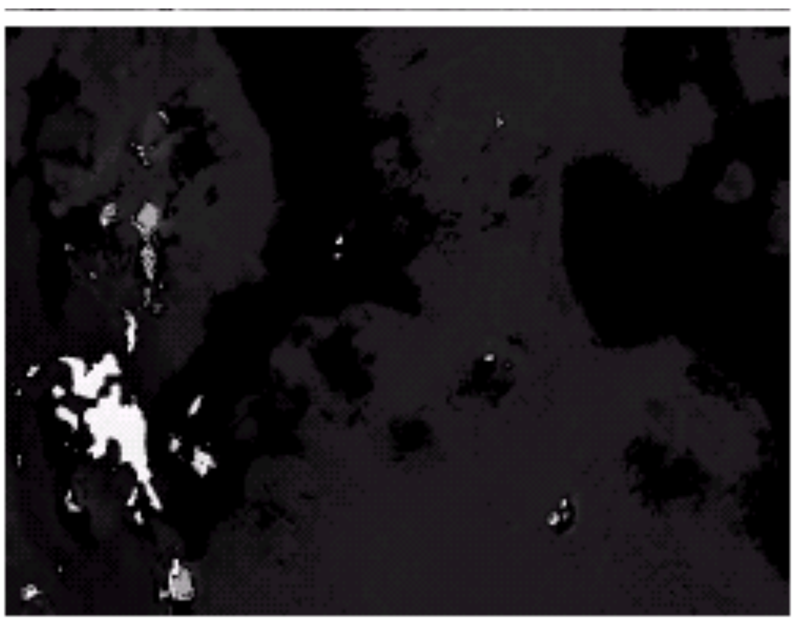


H(x,y)b(x,y)

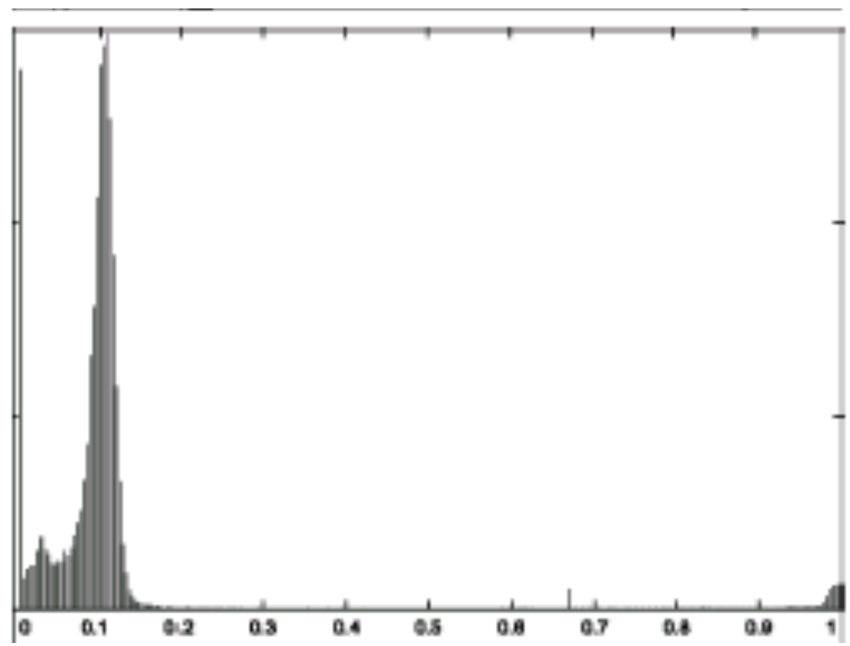
Hue H(x,y)



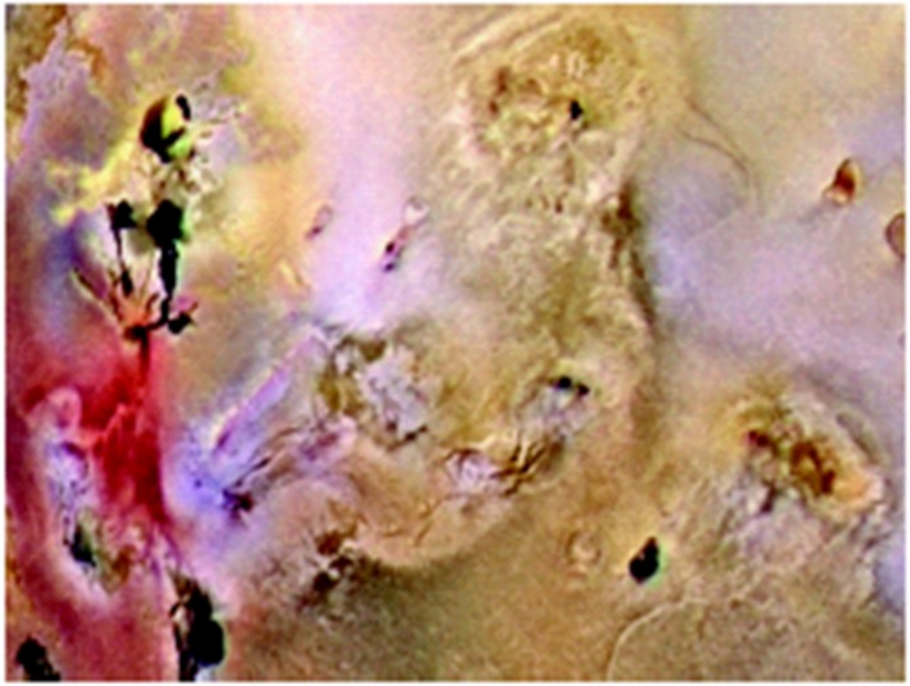
Binary Saturation Mask b(x,y)



H(x,y)b(x,y)



Histogram of H(x,y)b(x,y)



Input Color Image

Threshold H(x,y)b(x,y) at 0.9

FIGURE 6.42 Image segmentation in HSI space.

Segmentation in RGB Space

Find color pixels $a = (a_r, a_g, a_b)^T$ in an image with colors that are similar to a specified color $z = (z_r, z_g, z_b)^T$.

Segmentation in RGB Space

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Cube

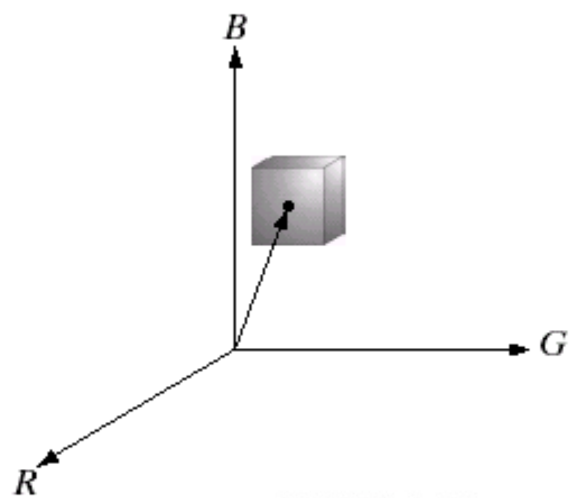


FIGURE 6.43

Segmentation in RGB Space

Find color pixels $a=(a_r, a_g, a_b)^T$ in an image with colors that are similar to a specified color $z=(z_r, z_g, z_b)^T$.

We can define different regions in color space as similar to z.

Cube

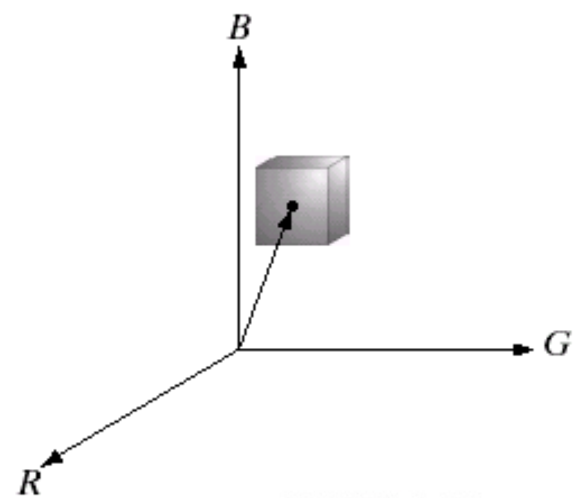


FIGURE 6.43

a is similar to z if

$z_r - d \le a_r \le z_r + d$ AND $z_g - d \le a_g \le z_g + d$
AND $z_b - d \le a_b \le z_b + d$

Sphere

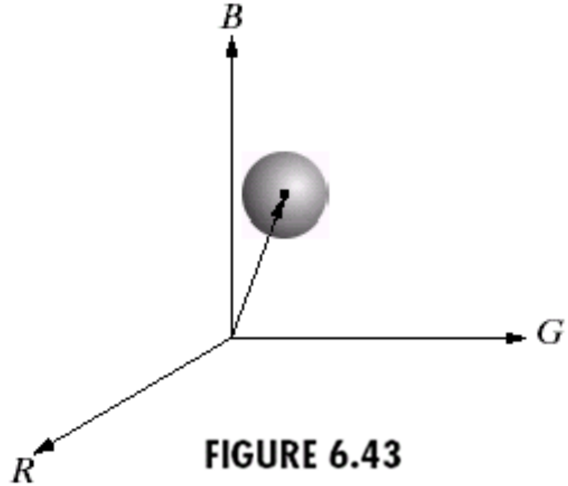


FIGURE 6.43

Sphere

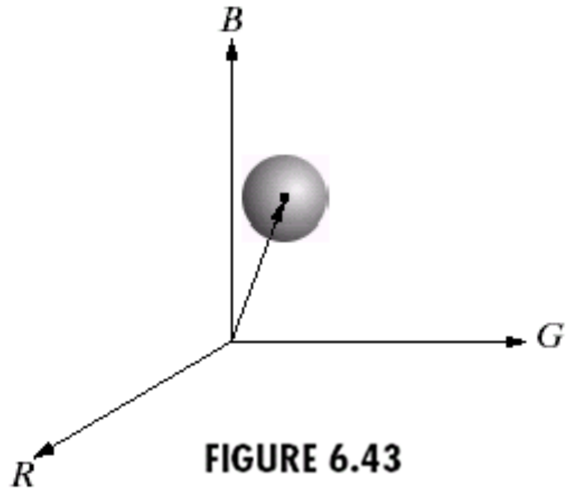


FIGURE 6.43

a is similar to z if

$$(z_r - a_r)^2 + (z_g - a_g)^2 + (z_b - a_b)^2 \le d^2$$

Sphere

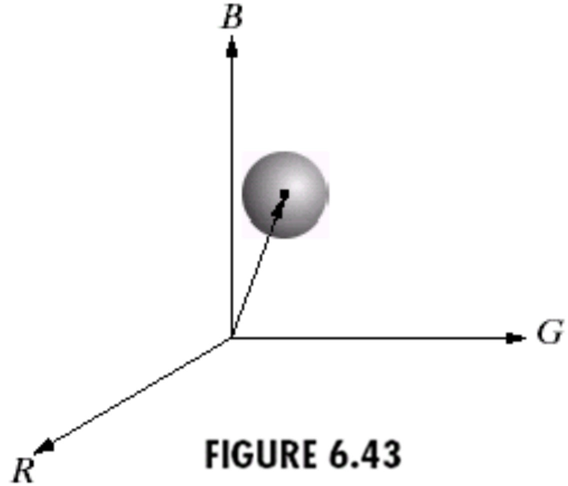
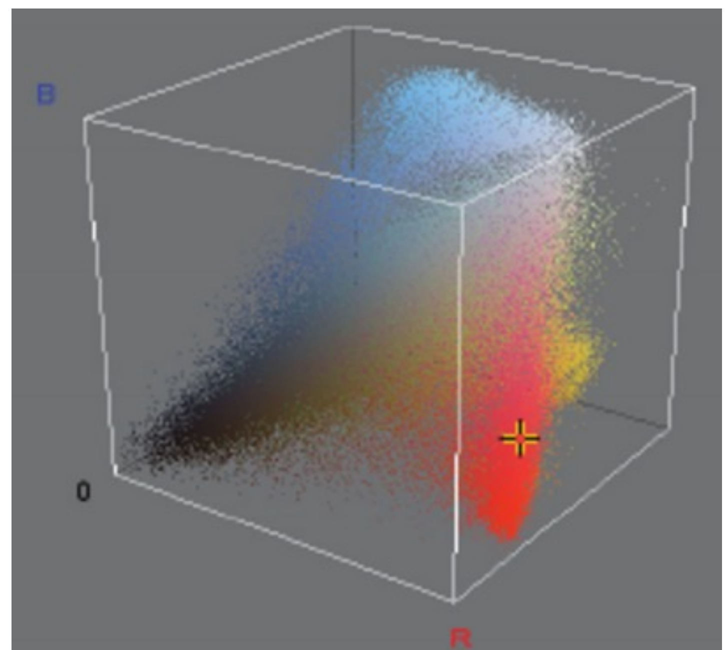


FIGURE 6.43

a is similar to z if

$$(z_r - a_r)^2 + (z_g - a_g)^2 + (z_b - a_b)^2 \leq d^2$$



Ellipsoid

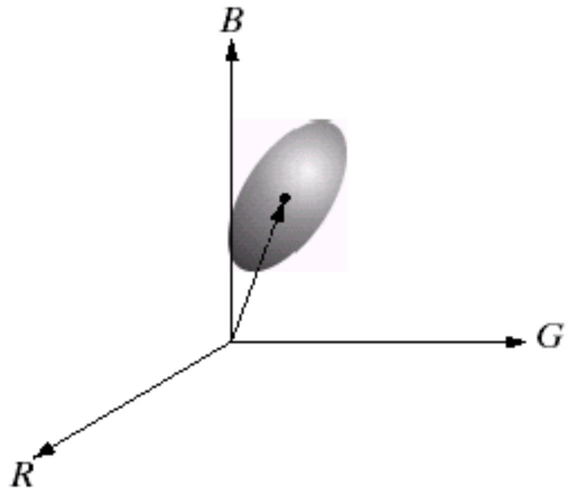


FIGURE 6.43

A Gaussian pdf of color vectors $z = (z_r, z_g, z_b)^T$ is

$$p(z) = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(z-\mu)^T \Sigma^{-1} (z-\mu)}$$

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$$\mu = E[z] = \begin{bmatrix} E[z_r] \\ E[z_g] \\ E[z_b] \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix}$$ mean vector

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$$\Sigma = E\left[(z-\mu)(z-\mu)^T\right]$$ covariance matrix

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$$\Sigma = E\left[(z-\mu)(z-\mu)^T\right] \quad \text{covariance matrix}$$

$$= \begin{bmatrix} E\left[(z_r-\mu_r)^2\right] & E\left[(z_r-\mu_r)(z_g-\mu_g)\right] & E\left[(z_r-\mu_r)(z_b-\mu_b)\right] \\ E\left[(z_r-\mu_r)(z_g-\mu_g)\right] & E\left[(z_g-\mu_g)^2\right] & E\left[(z_g-\mu_g)(z_b-\mu_b)\right] \\ E\left[(z_r-\mu_r)(z_b-\mu_b)\right] & E\left[(z_g-\mu_g)(z_b-\mu_b)\right] & E\left[(z_b-\mu_b)^2\right] \end{bmatrix}$$

$$\text{covariance}[x,y] = \sigma_x \sigma_y \cdot \text{correlation}[x,y]$$

(covariance and correlation have the same sign)

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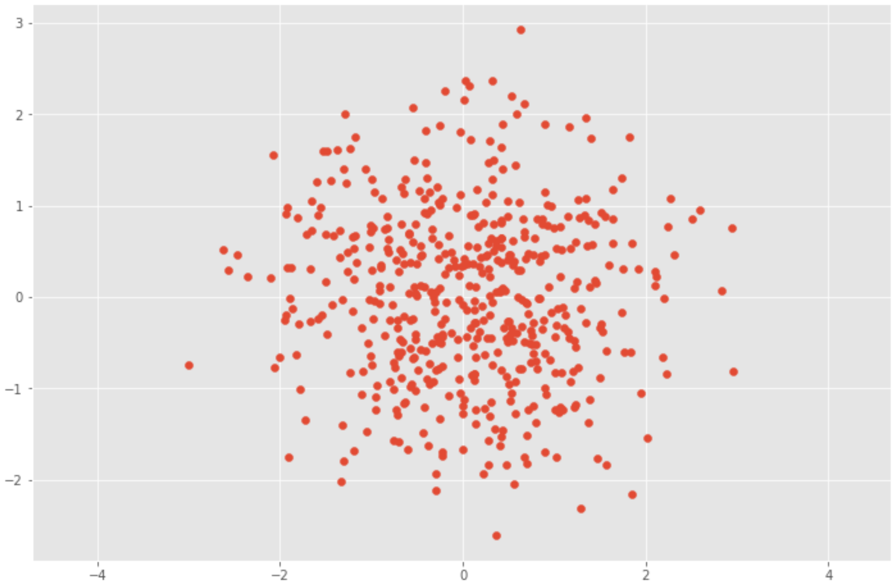
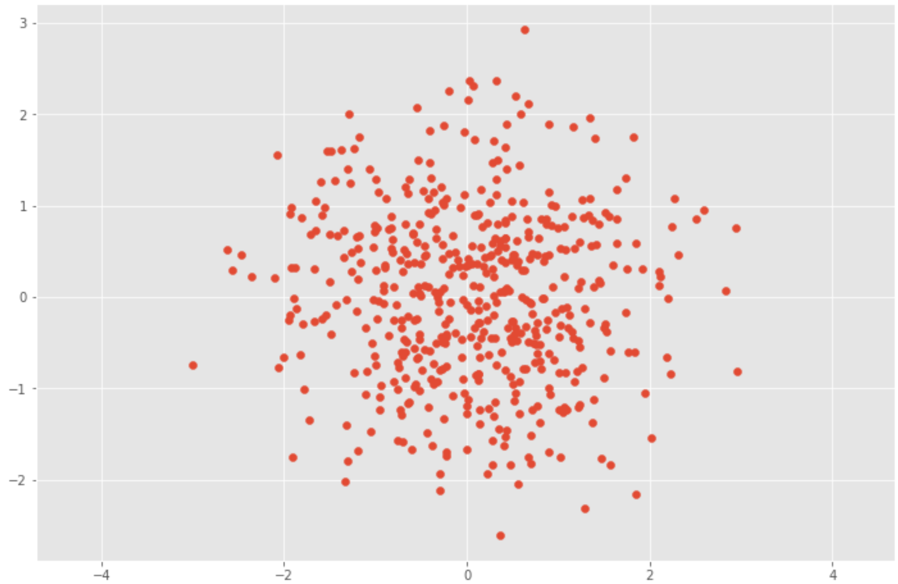


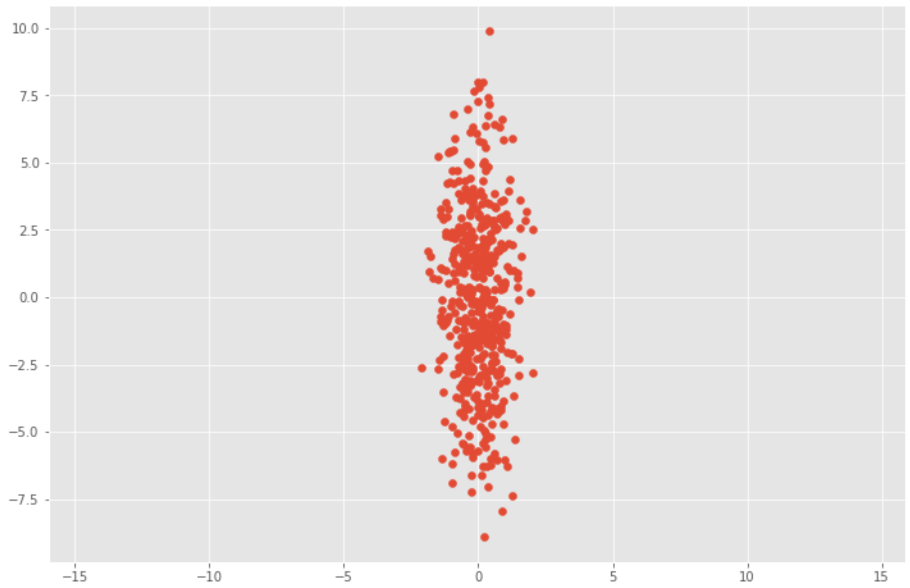
$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$covariance[x, y] = \sigma_x \sigma_y \cdot correlation[x, y]$$

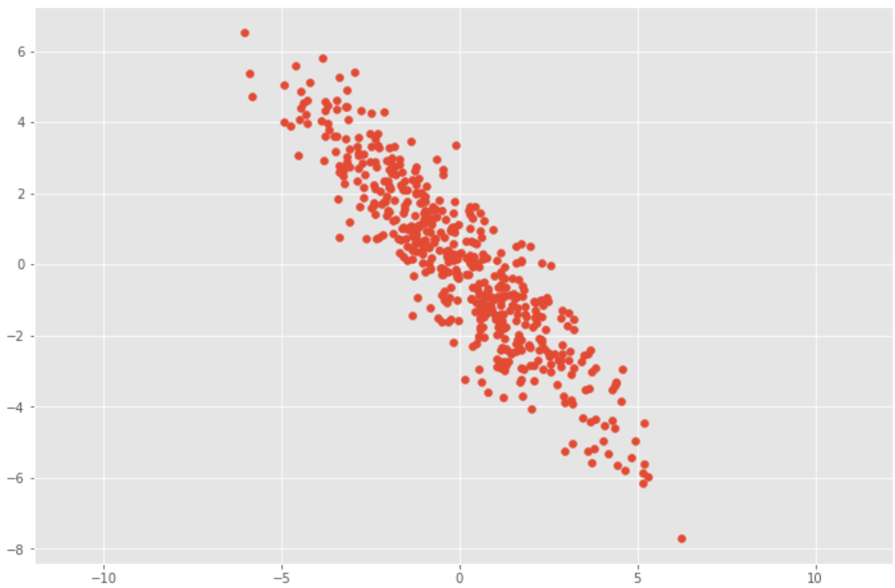
(covariance and correlation have the same sign)



$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} 0.5 & 0 \\ 0 & 10 \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} 5 & -5 \\ -5 & 6 \end{bmatrix}$$

Ex Learning the mean vector μ and covariance matrix Σ

sample	z_r	z_g	z_b
1	9	51	102
2	10	48	98
3	11	51	100

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1	9	51	102
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$$\text{variance}[z_r] = E\left[(z_r - \mu_r)^2\right] = \frac{1}{3}\left((-1)^2 + 0^2 + 1^2\right) = \frac{2}{3}$$

$$\text{variance}[z_g] = E\left[(z_g - \mu_g)^2\right] = \frac{1}{3}\left(1^2 + (-2)^2 + 1^2\right) = 2$$

$$\text{variance}[z_b] = E\left[(z_b - \mu_b)^2\right] = \frac{1}{3}\left(2^2 + (-2)^2 + 0^2\right) = \frac{8}{3}$$

Ex Learning the mean vector μ and covariance matrix Σ

sample	z_r	z_g	z_b
1	9	51	102
2	10	48	98
3	11	51	100

mean vector $\mu = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix} = \begin{bmatrix} 10 \\ 50 \\ 100 \end{bmatrix}$

$$\text{variance}[z_r] = E[(z_r-\mu_r)^2] = \frac{1}{3}\left((-1)^2 + 0^2 + 1^2\right) = \frac{2}{3}$$

$$\text{variance}[z_g] = E[(z_g-\mu_g)^2] = \frac{1}{3}\left(1^2 + (-2)^2 + 1^2\right) = 2$$

$$\text{variance}[z_b] = E[(z_b-\mu_b)^2] = \frac{1}{3}\left(2^2 + (-2)^2 + 0^2\right) = \frac{8}{3}$$

$$\text{covariance}[z_r, z_g] = E[(z_r-\mu_r)(z_g-\mu_g)] = \frac{1}{3}\left((-1)(1) + 0(-2) + (1)(1)\right) = 0$$

$$\text{covariance}[z_r, z_b] = E[(z_r-\mu_r)(z_b-\mu_b)] = \frac{1}{3}\left((-1)(2) + 0(-2) + (1)(0)\right) = \frac{-2}{3}$$

$$\text{covariance}[z_g, z_b] = E[(z_g-\mu_g)(z_b-\mu_b)] = \frac{1}{3}\left(1(2) + (-2)(-2) + (1)(0)\right) = 2$$

Ex Learning the mean vector μ and covariance matrix Σ

sample	z_r	z_g	z_b
1	9	51	102
2	10	48	98
3	11	51	100

mean vector $\mu = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix} = \begin{bmatrix} 10 \\ 50 \\ 100 \end{bmatrix}$

$$\text{variance}[z_r] = E[(z_r-\mu_r)^2] = \frac{1}{3}\left((-1)^2 + 0^2 + 1^2\right) = \frac{2}{3}$$

$$\text{variance}[z_g] = E[(z_g-\mu_g)^2] = \frac{1}{3}\left(1^2 + (-2)^2 + 1^2\right) = 2$$

$$\text{variance}[z_b] = E[(z_b-\mu_b)^2] = \frac{1}{3}\left(2^2 + (-2)^2 + 0^2\right) = \frac{8}{3}$$

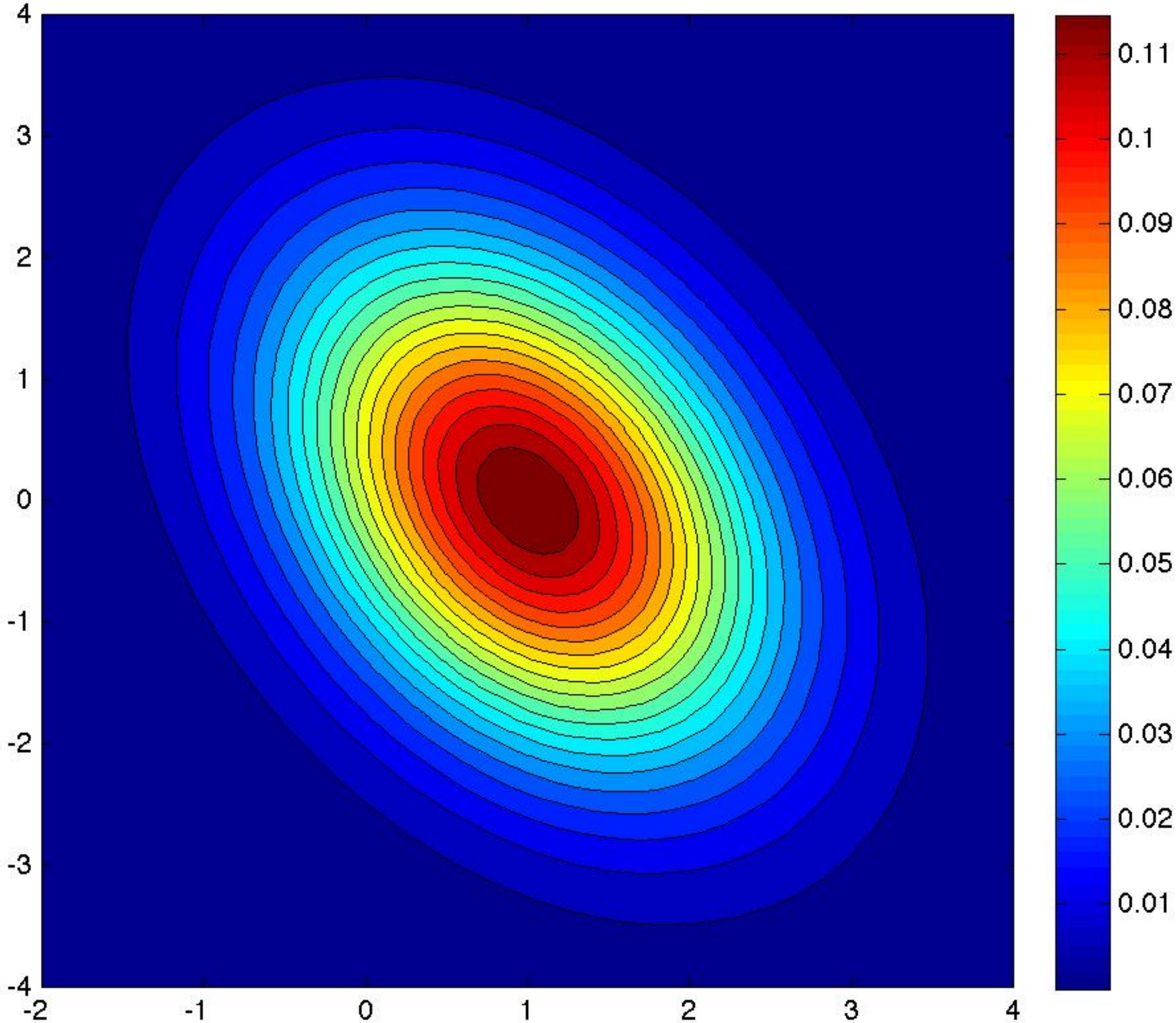
$$\Sigma = \frac{1}{3}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 6 & 6 \\ -2 & 6 & 8 \end{bmatrix}$$

$$\text{covariance}[z_r, z_g] = E[(z_r-\mu_r)(z_g-\mu_g)] = \frac{1}{3}\left((-1)(1) + 0(-2) + (1)(1)\right) = 0$$

$$\text{covariance}[z_r, z_b] = E[(z_r-\mu_r)(z_b-\mu_b)] = \frac{1}{3}\left((-1)(2) + 0(-2) + (1)(0)\right) = \frac{-2}{3}$$

$$\text{covariance}[z_g, z_b] = E[(z_g-\mu_g)(z_b-\mu_b)] = \frac{1}{3}\left(1(2) + (-2)(-2) + (1)(0)\right) = 2$$

(Ex) Gaussian pdf for $\mu = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 3 \end{bmatrix}$



a is similar to z if

$$p(z)\Big|_{z=a} = \frac{1}{(2\pi)^{3/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(a-\mu)^T \Sigma^{-1}(a-\mu)} \geq T$$

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or equivalently $M = (a-\mu)^T\Sigma^{-1}(a-\mu) \leq d^2$

a is inside an ellipsoid centered at μ

$\sqrt{M}$ is the Mahalanobis distance between $a = (a_r, a_g, a_b)^T$ and $z = (z_r, z_g, z_b)^T$ where z has mean vector μ and covariance matrix Σ.

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu) \leq d^2$$

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If $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix, then

$$M = \begin{bmatrix} a_r - \mu_r & a_g - \mu_g & a_b - \mu_b \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_r - \mu_r \\ a_g - \mu_g \\ a_b - \mu_B \end{bmatrix}$$

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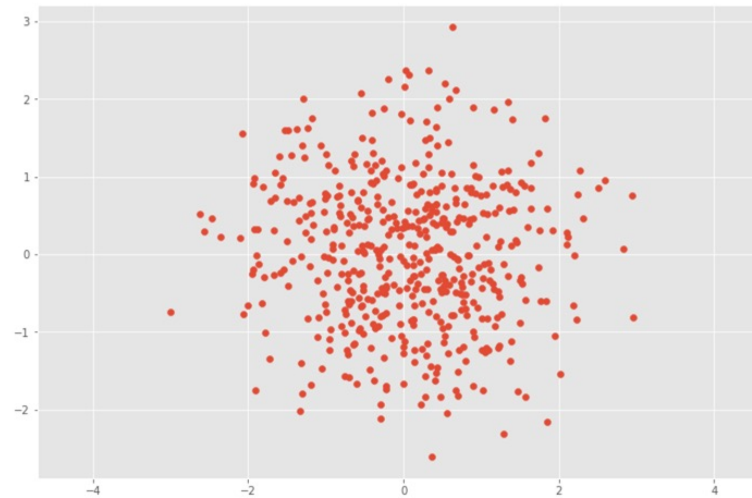

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If $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix, then

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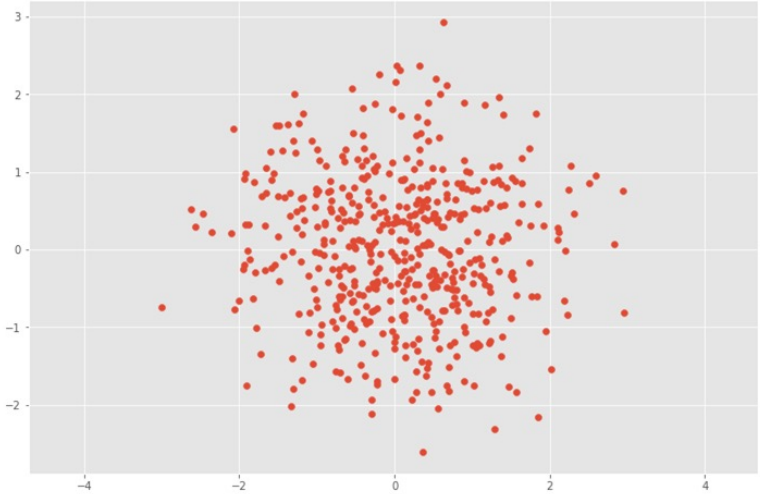
$$= \begin{bmatrix} a_r - \mu_r & a_g - \mu_g & a_b - \mu_b \end{bmatrix} \begin{bmatrix} a_r - \mu_r \\ a_g - \mu_g \\ a_b - \mu_b \end{bmatrix}$$



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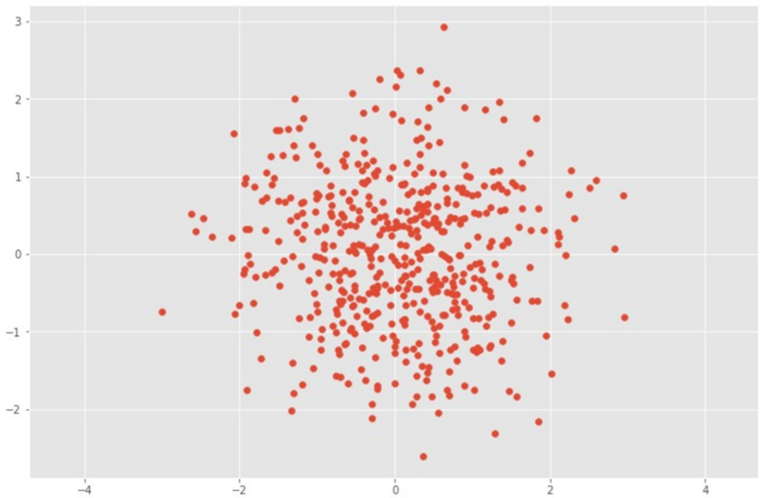
If $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix, then

$$M = \begin{bmatrix} a_r - \mu_r & a_g - \mu_g & a_b - \mu_b \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_r - \mu_r \\ a_g - \mu_g \\ a_b - \mu_B \end{bmatrix}$$

$$= \begin{bmatrix} a_r - \mu_r & a_g - \mu_g & a_b - \mu_b \end{bmatrix} \begin{bmatrix} a_r - \mu_r \\ a_g - \mu_g \\ a_b - \mu_b \end{bmatrix}$$



$$= (a_r - \mu_r)^2 + (a_g - \mu_g)^2 + (a_b - \mu_b)^2$$

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu) \leq d^2$$

If $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix, then

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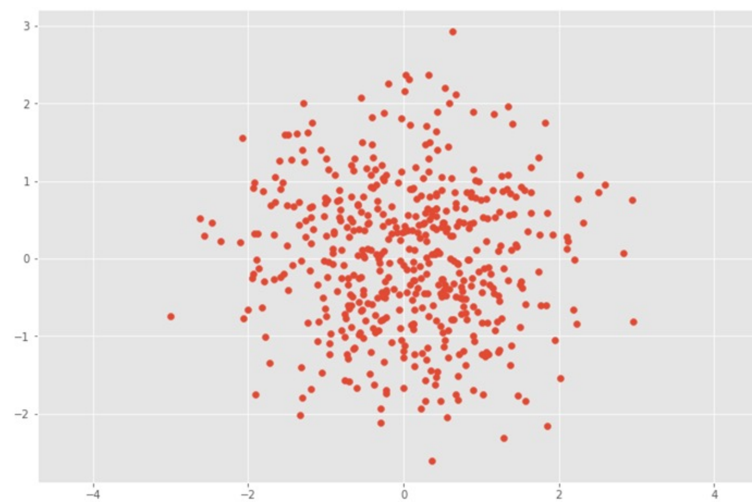
$$= \begin{bmatrix} a_r - \mu_r & a_g - \mu_g & a_b - \mu_b \end{bmatrix} \begin{bmatrix} a_r - \mu_r \\ a_g - \mu_g \\ a_b - \mu_b \end{bmatrix}$$

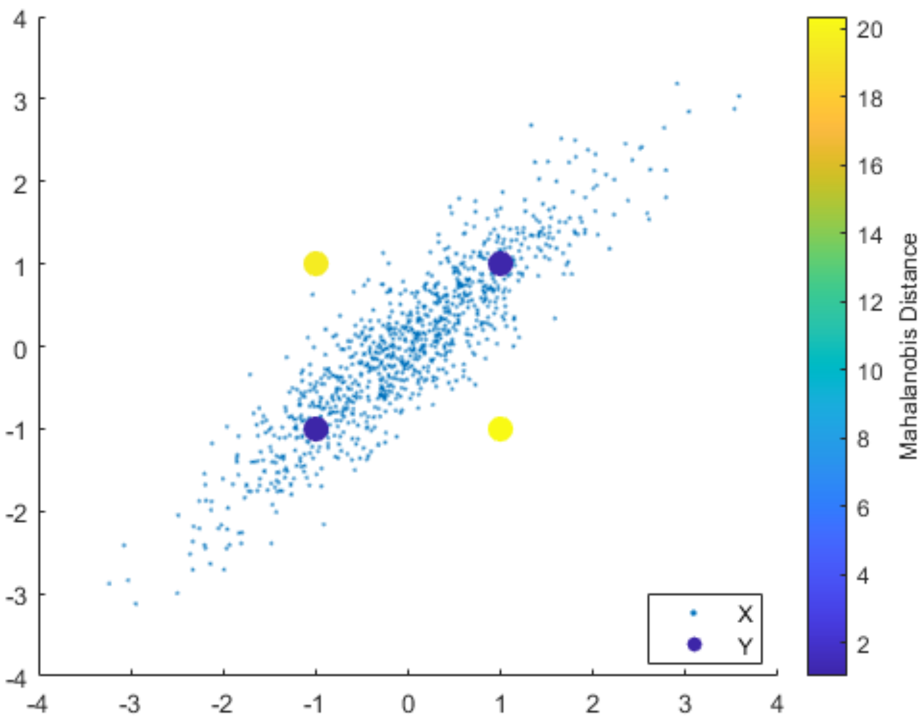


$$= (a_r - \mu_r)^2 + (a_g - \mu_g)^2 + (a_b - \mu_b)^2$$

If Σ is the identity matrix, then the Mahalanobis distance becomes the Euclidean distance.

(Ex) Z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$

Ex Z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$



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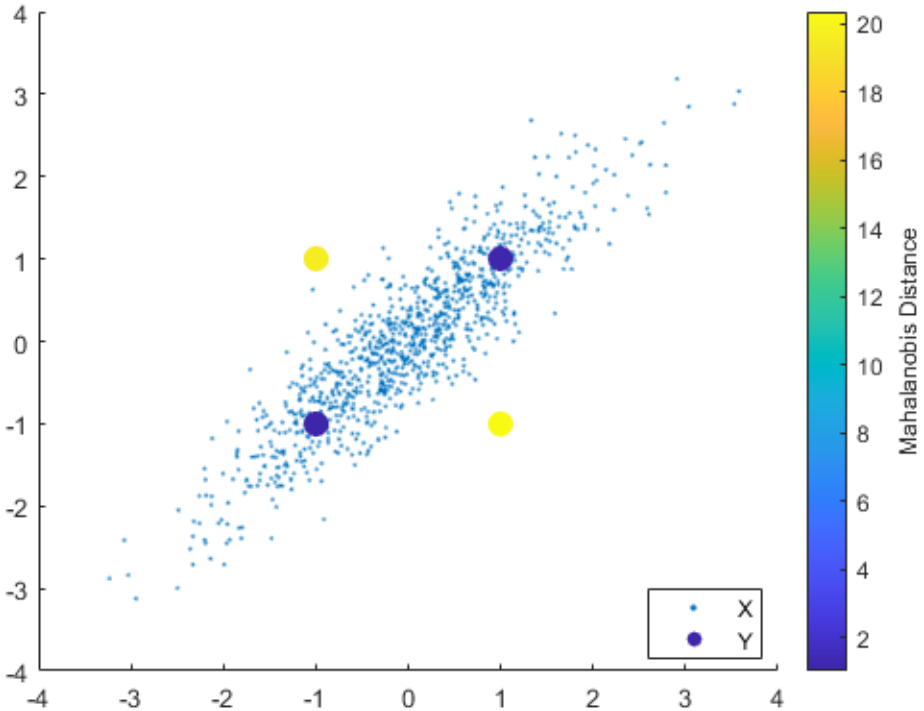
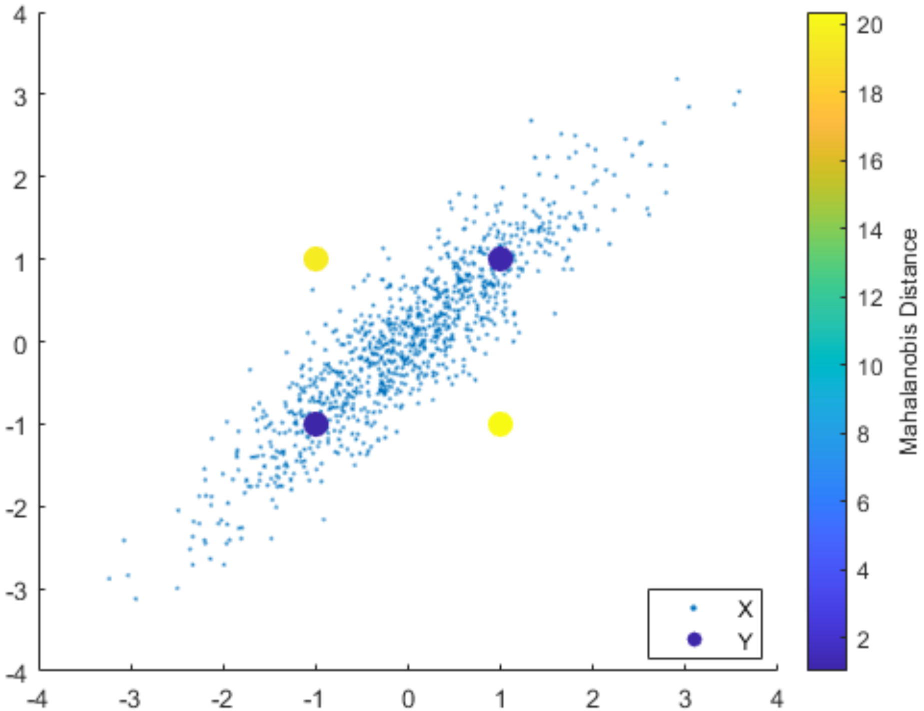


Point 1 a=(-1,-1)

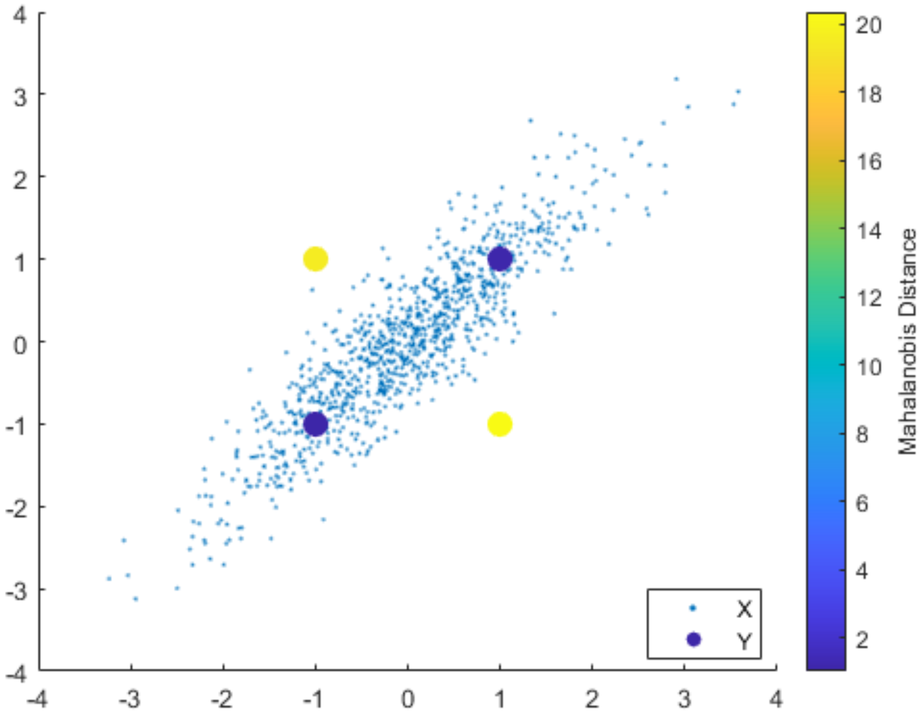
Ex z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$

Point 1 a=(-1,-1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$



Ex Z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$

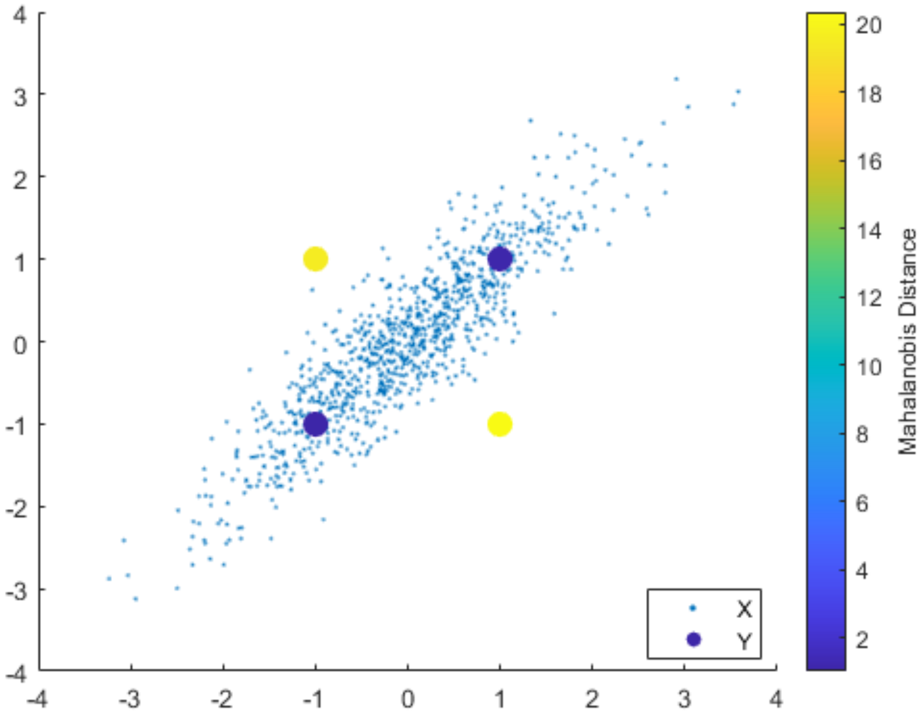


Point 1 a=(-1,-1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

Ex z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$



Point 1 a=(-1,-1)

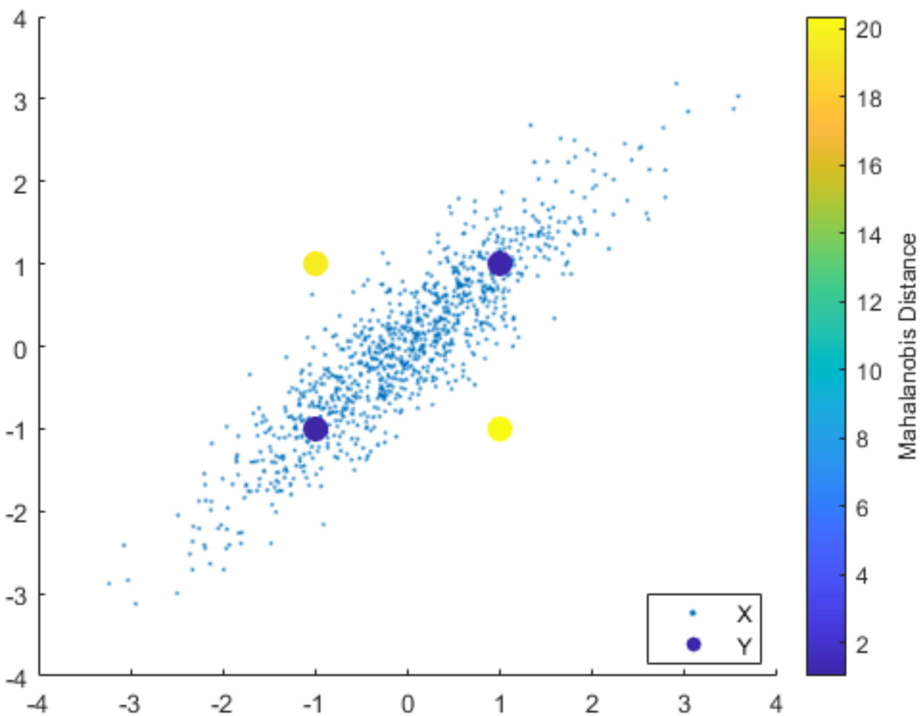
$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

$$\sqrt{M} = 1.026$$

Ex z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$

$$\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$$



Point 1 a=(-1,-1)

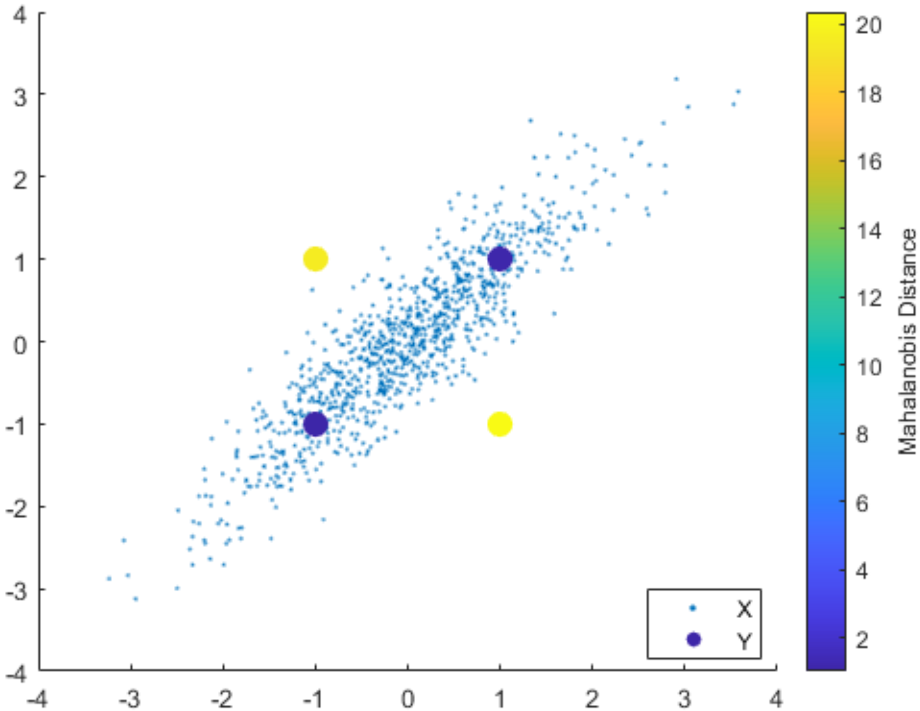
$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

$$\sqrt{M} = 1.026$$

Point 2 a=(-1,1)

Ex Z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$



Point 1 a=(-1,-1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

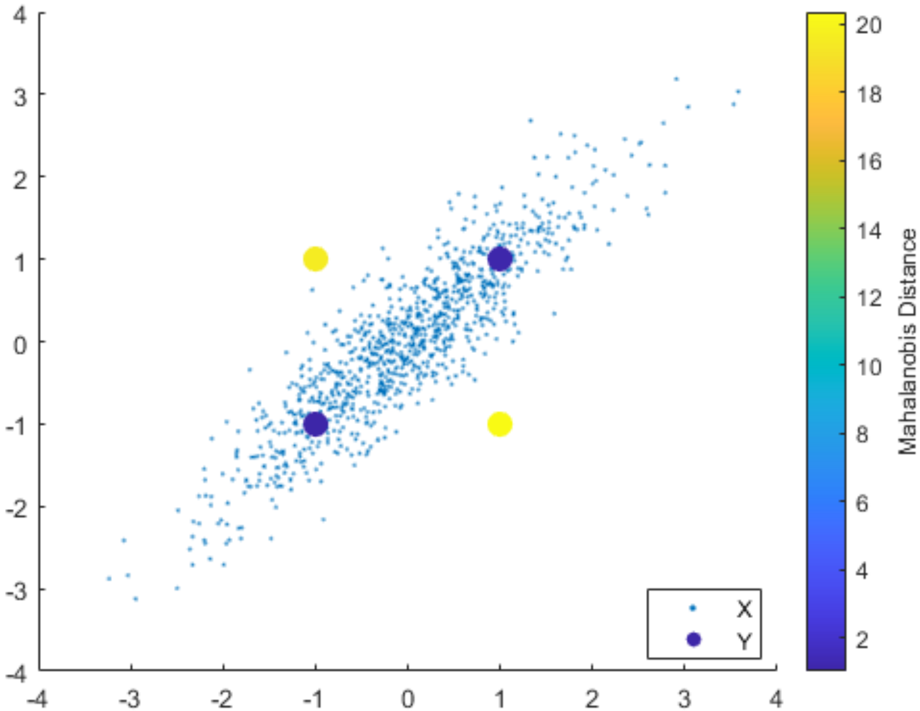
$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

$$\sqrt{M} = 1.026$$

Point 2 a=(-1,1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

Ex z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\quad \Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$



Point 1 a=(-1,-1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

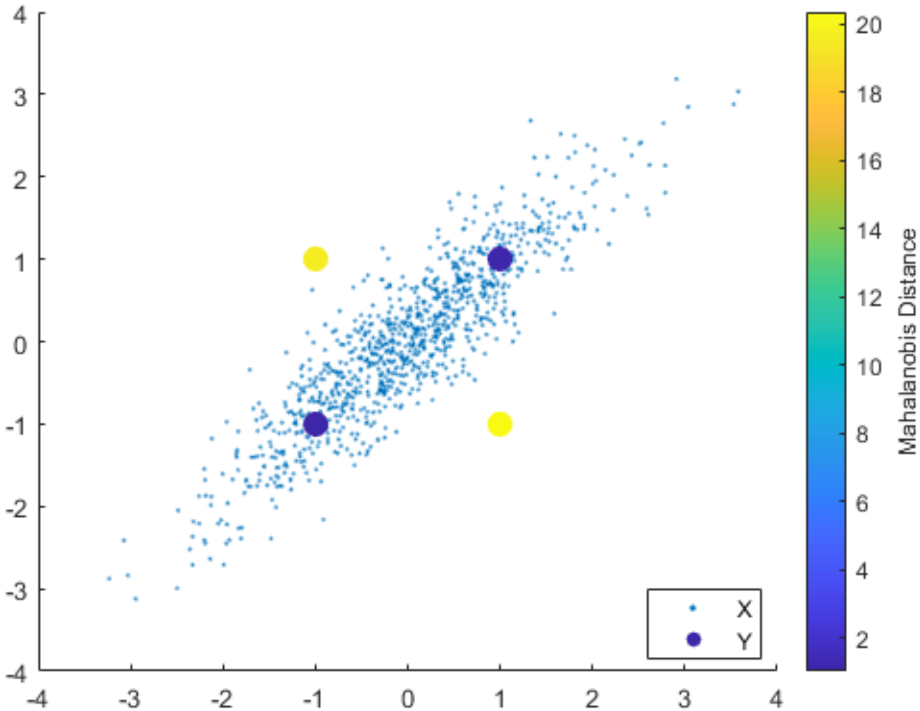
$$\sqrt{M} = 1.026$$

Point 2 a=(-1,1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 20$$

Ex z has $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ $\Sigma^{-1} = \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix}$



Point 1 a=(-1,-1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.052$$

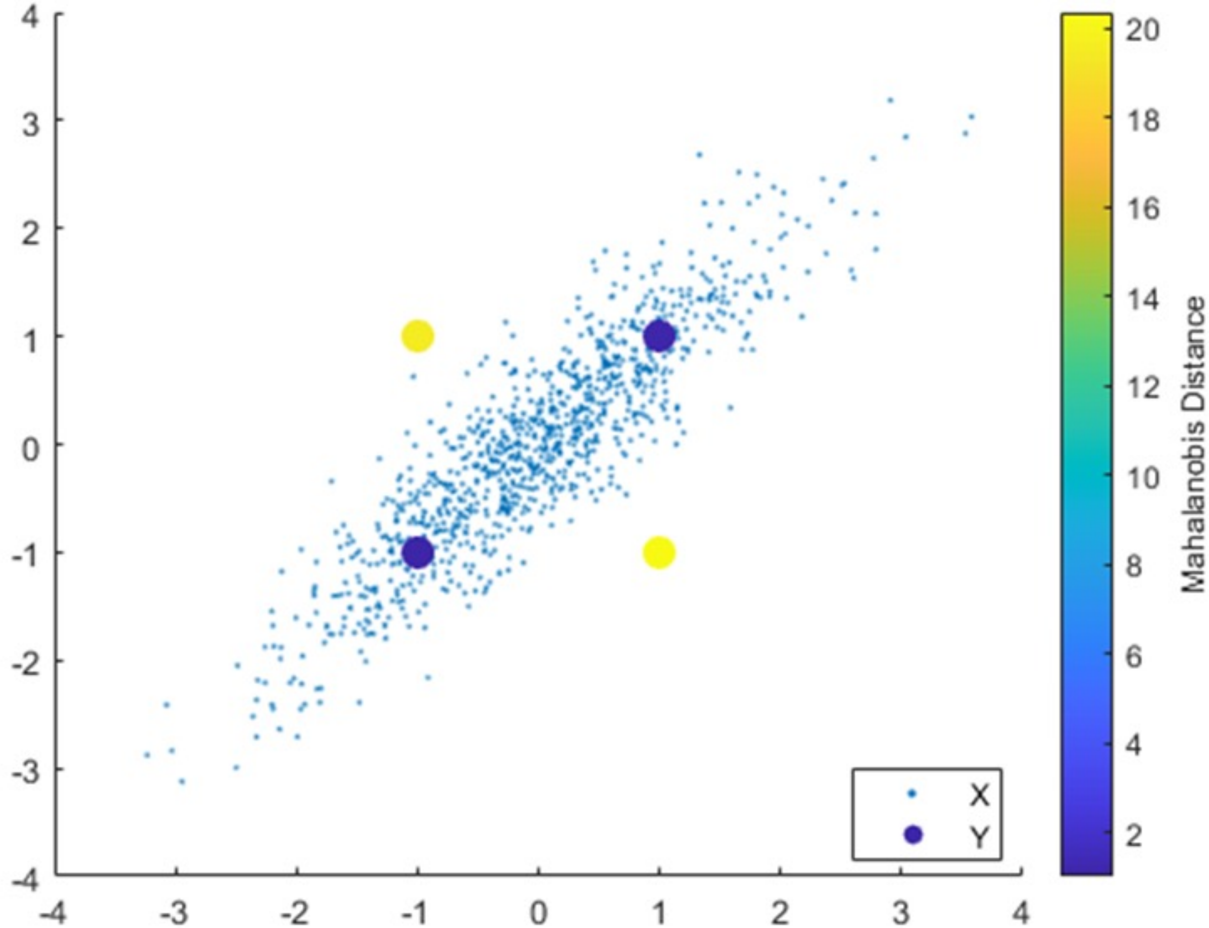
$$\sqrt{M} = 1.026$$

Point 2 a=(-1,1)

$$M = (a-\mu)^T \Sigma^{-1} (a-\mu)$$

$$= \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 5.263 & -4.737 \\ -4.737 & 5.263 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 20$$

$$\sqrt{M} = 4.472$$



Point	a	Euclidean distance to $u = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	Mahalanobis distance to $u = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
1	(-1,-1)	$\sqrt{2}$	1.026
2	(-1,1)	$\sqrt{2}$	4.472
3	(1,-1)	$\sqrt{2}$	4.472
4	(1,1)	$\sqrt{2}$	1.026