EECS 203A - Digital Image Processing

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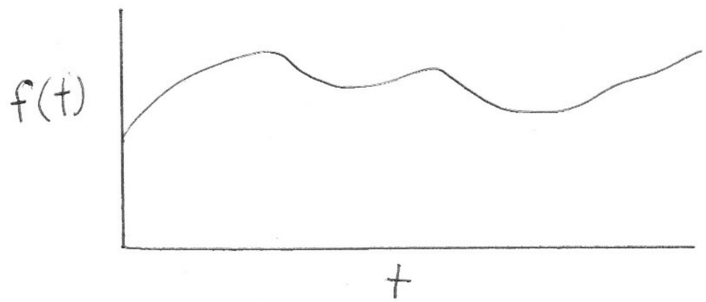
Signal - a function $f(t)$ of the time variable t



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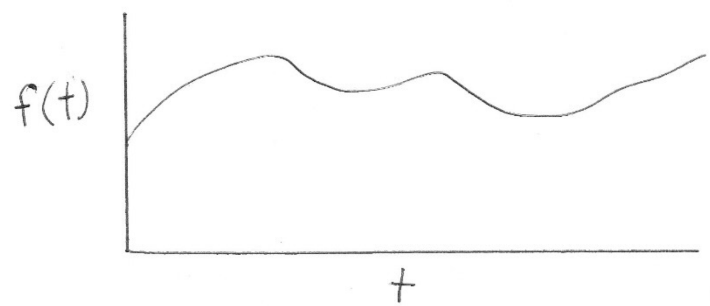
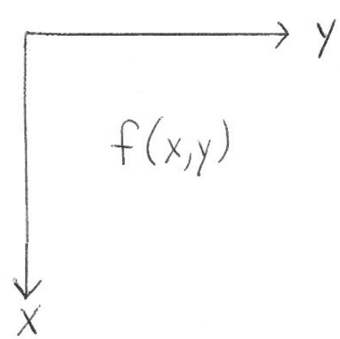


Image - a function $f(x,y)$ of the spatial variables x and y



Digital Image — an image for which x, y, and f can only take on a finite number of discrete values. For example, $x = 0, 1, \ldots, 511$ $y = 0, 1, \ldots, 511$ $f = 0, 1, \ldots, 255$

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Digital Image Processing – the transformation of a digital image to another image or representation

Subareas of Digital Image Processing

Image Acquisition – the process of converting the physical state of the world into an image

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Multispectral Processing – techniques for analyzing images where $f(x,y) = (f_1(x,y), f_2(x,y), \ldots, f_c(x,y))$ is a vector-valued two-dimensional function. For example, a color image is represented by $f(x,y) = (f_1(x,y), f_2(x,y), f_3(x,y))$

Subareas of Digital Image Processing

Multiresolution Processing - methods for representing images at different spatial scales

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Compression – the process of reducing the number of bits used to represent a digital image

Image Acquisition

Images are often formed by focusing energy from a scene onto an image plane

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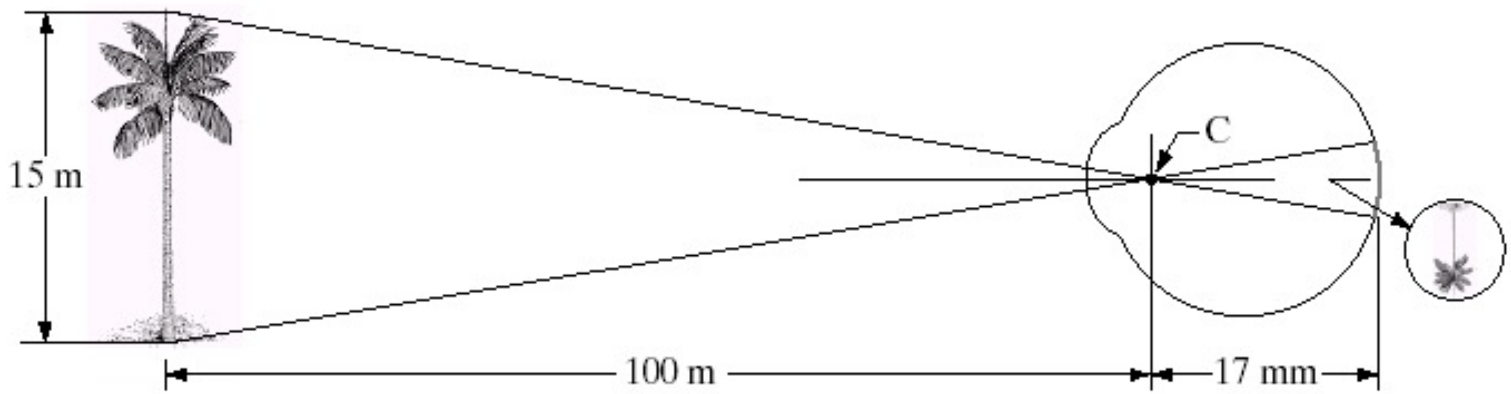

FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point *C* is the optical center of the lens.

Image Acquisition

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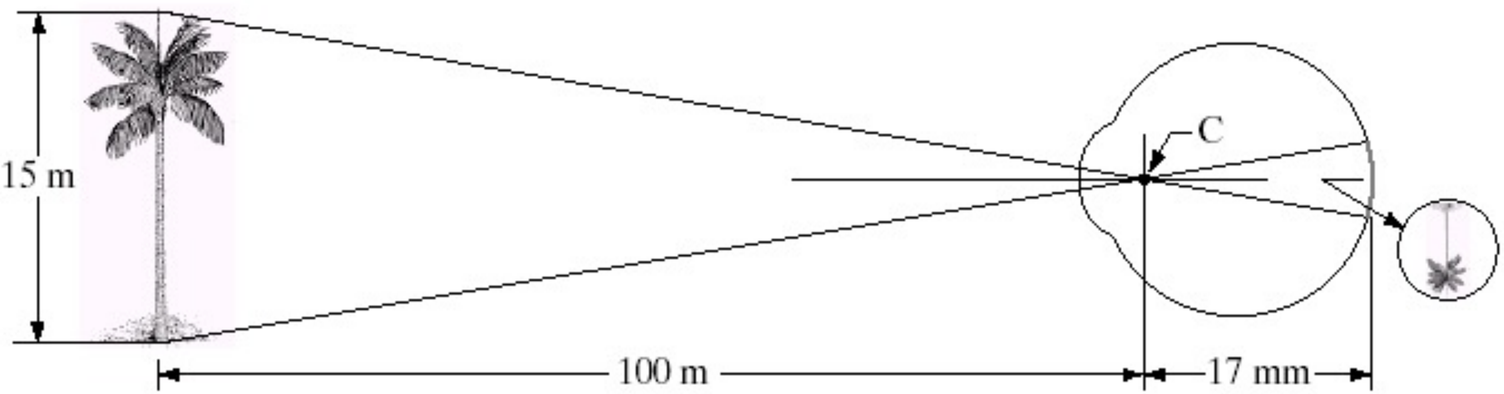
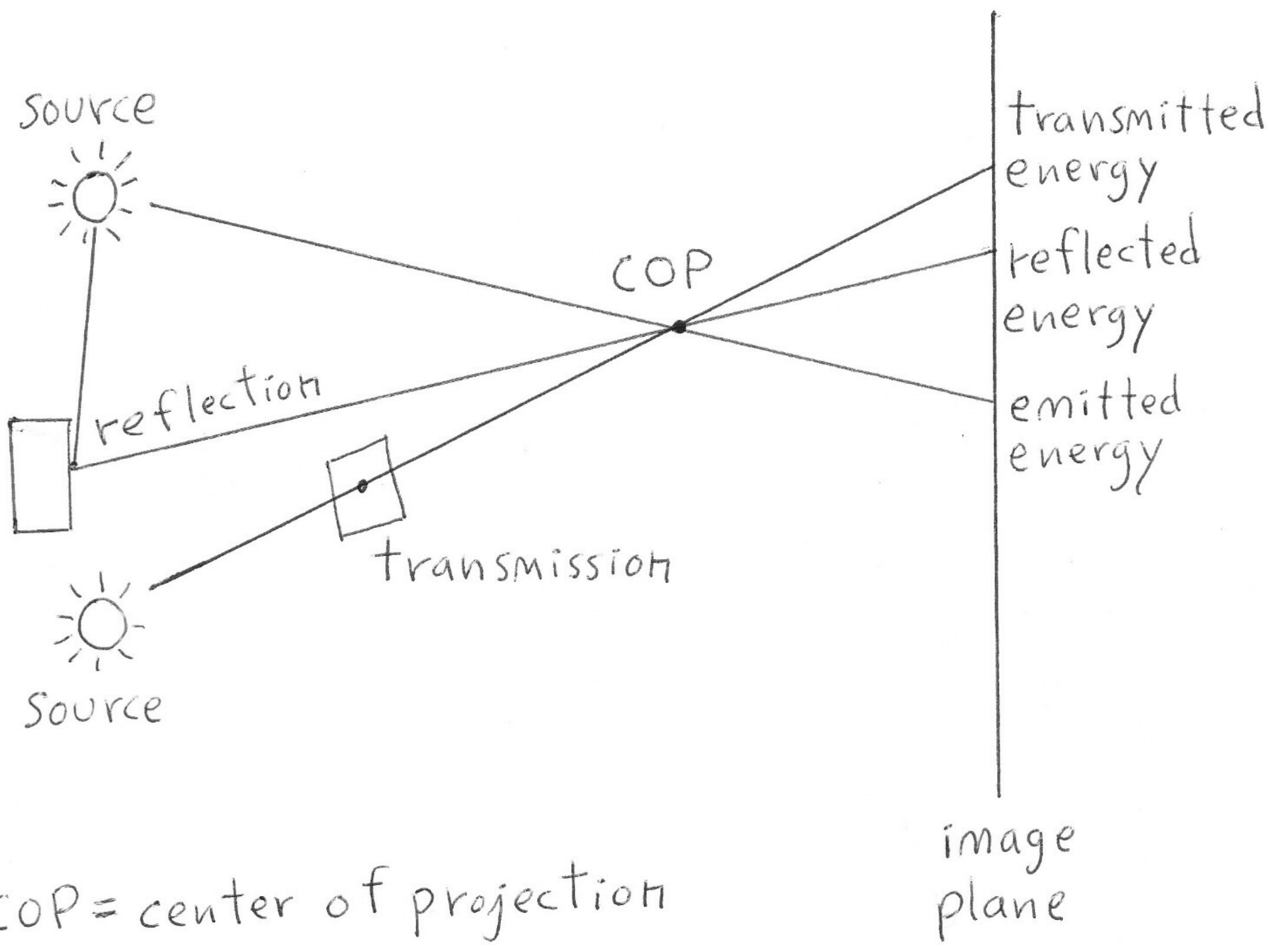


FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

For a focused imaging system, each point on the image plane receives energy from a single direction in the scene. The energy that reaches location (x,y) in the image plane has a relationship with the object(s) in the scene that image to (x,y).

Image Acquisition



COP = center of projection

An image can be sensed using different regions of the electromagnetic spectrum, e.g. x-rays, ultraviolet, visible, infrared, microwave

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Active Imaging – the source is a controlled component of the image acquisition system, e.g. imaging radar

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Passive Imaging - ambient scene energy is used to form an image

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Active Imaging – the source is a controlled component of the image acquisition system, e.g. imaging radar

Passive Imaging – ambient scene energy is used to form an image

For an energy measuring image acquisition system

$$0 \leq f(x,y) < \infty$$

Sampling and Quantization

The energy focused on an image plane provides a continuous image with continuous values of x, y and $f(x,y)$

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Sampling - take $f(x,y)$ at discrete values of x and y to give an image with M rows and N columns

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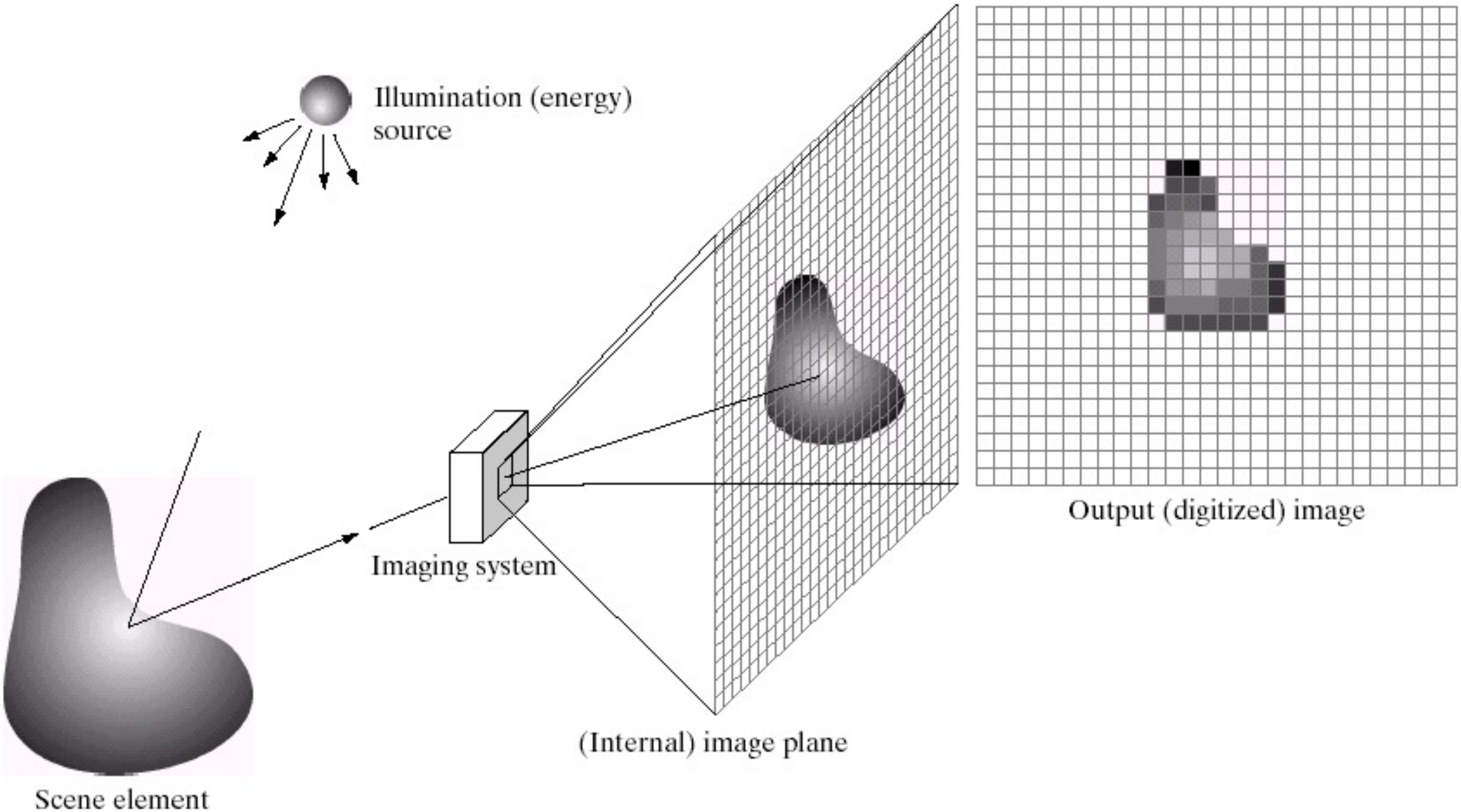
The number of bits required to store an image is MNK

Sampling and Quantization



a
b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Sampling and Quantization

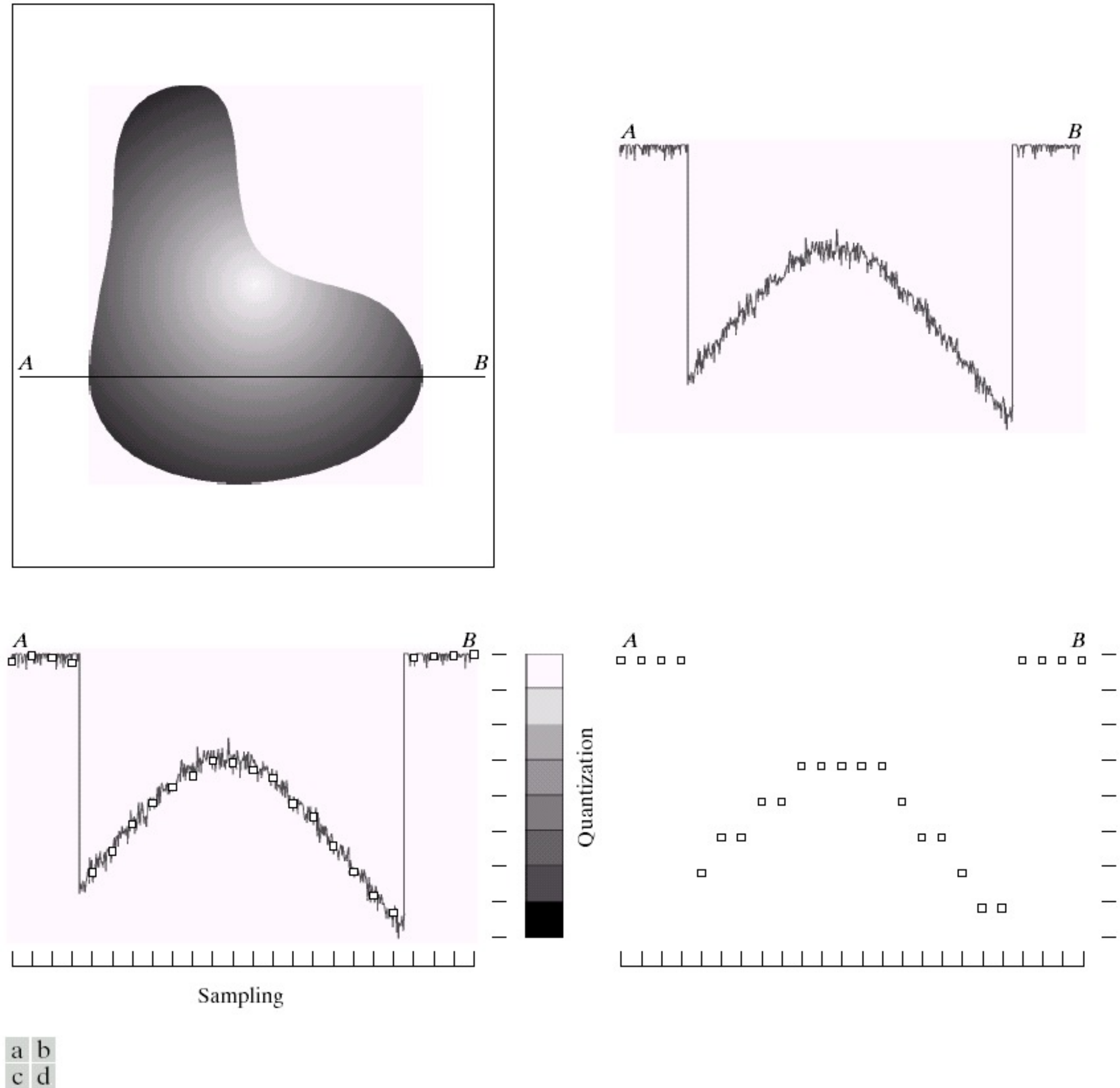
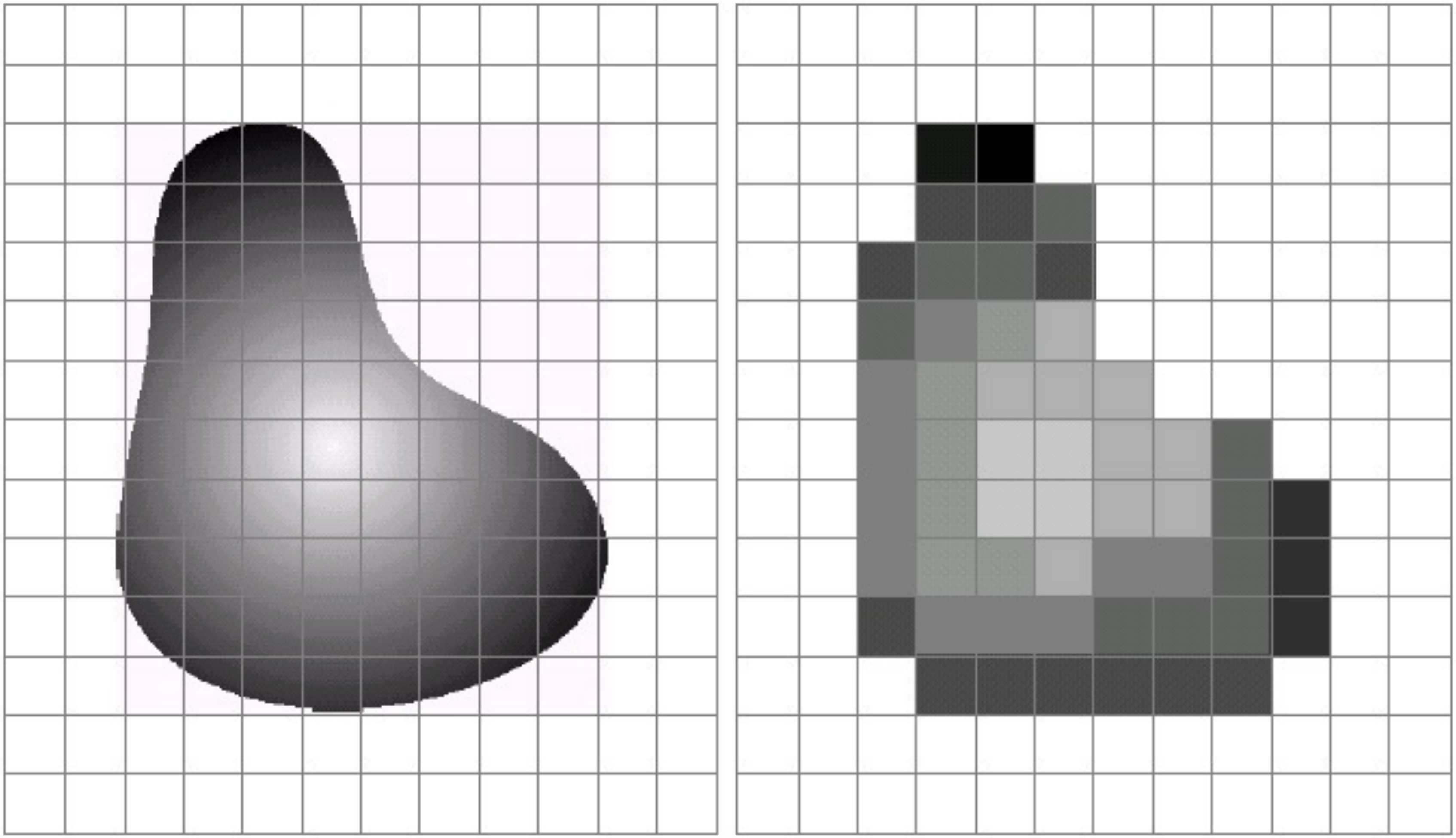


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampling and Quantization



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

After sampling, use integer values to index x and y and let $(x,y)=(0,0)$ be the upper left location in the image.

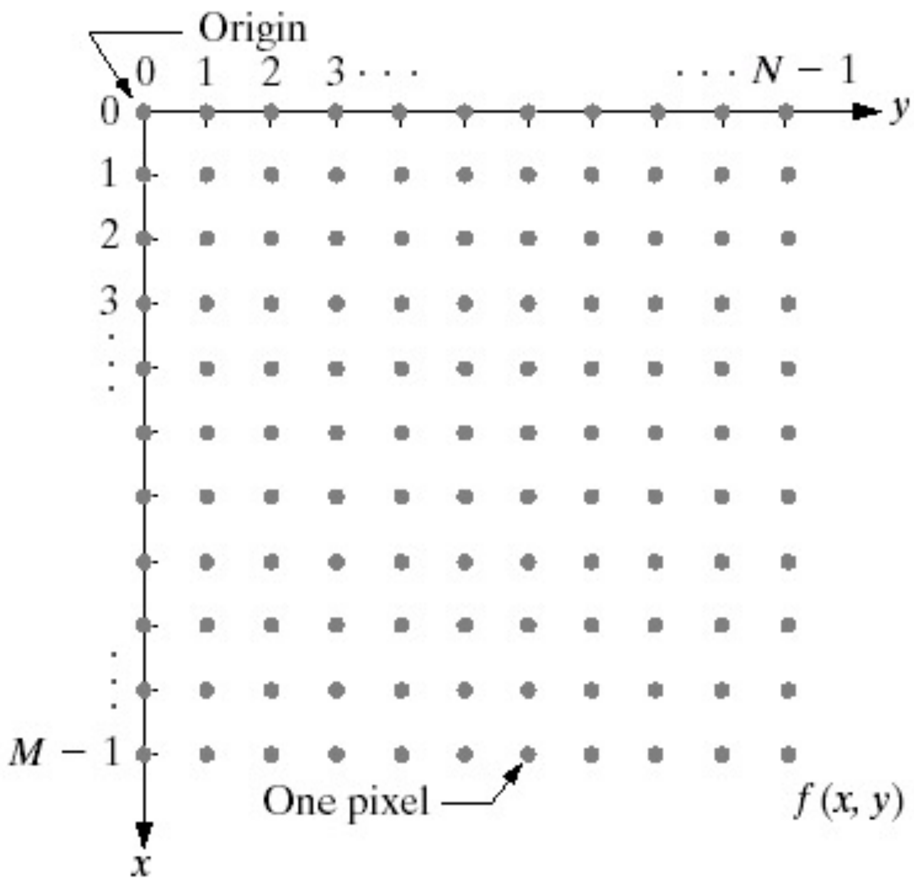


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Image as a Matrix

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

Image as a Matrix

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Each element of this matrix is a pixel and the values taken by pixels are typically represented as integers.

Image as a Matrix

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Each element of this matrix is a pixel and the values taken by pixels are typically represented as integers.

Spatial resolution increases as M and N increase. Gray-level (intensity) resolution increases as L increases.

Changing the Spatial Resolution of a Digital Image

Zoom – increase M and N for a digital image

Shrink – decrease M and N for a digital image

Changing the Spatial Resolution of a Digital Image

Zoom - increase M and N for a digital image

Shrink - decrease M and N for a digital image

Methods for Zoom

Pixel replication - increase M to rM and N to rN for an integer $r > 1$ by replacing each column with r copies of the column and then replacing each row with r copies of the row

Pixel replication

Ex original 3x3 image

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$$

Pixel replication

Ex original 3x3 image

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$$

column replacement

$r = 2$

$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

Pixel replication

Ex original 3x3 image

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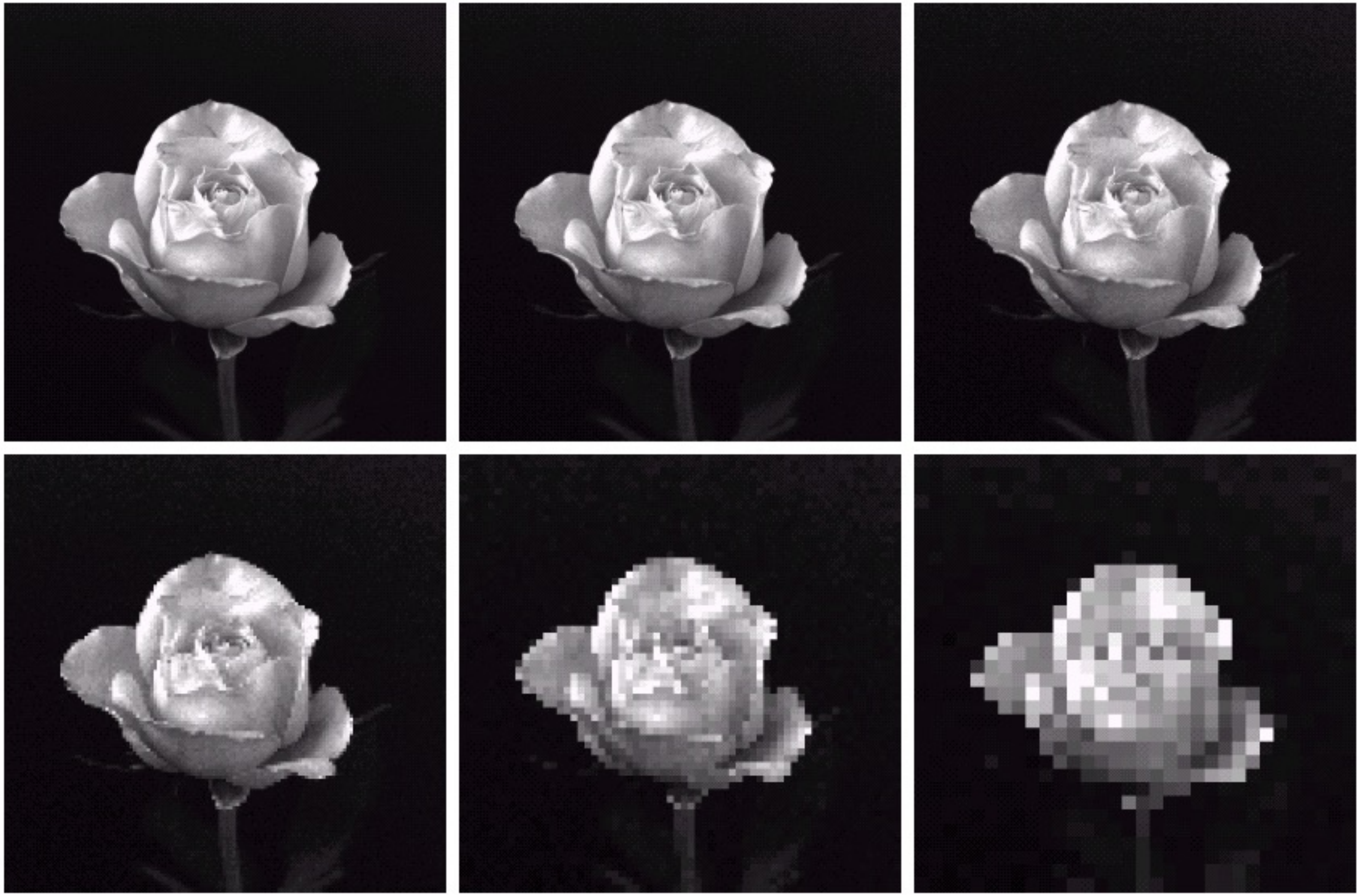
$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

row replacement

$r = 2$

$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

Pixel replication



a b c
d e f

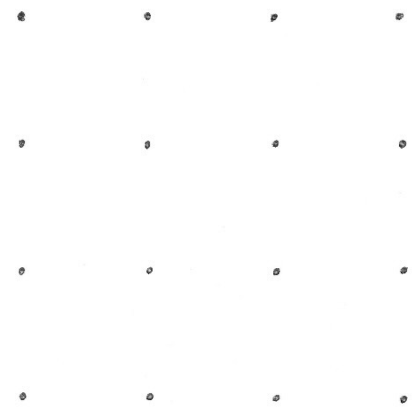
FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

Methods for Zoom

Interpolation - increase M to rM and N to rN for a real number $r > 1$ by generating a $rM \times rN$ sampling grid on the original image and interpolating to get the pixel values on the new sampling grid

Interpolation

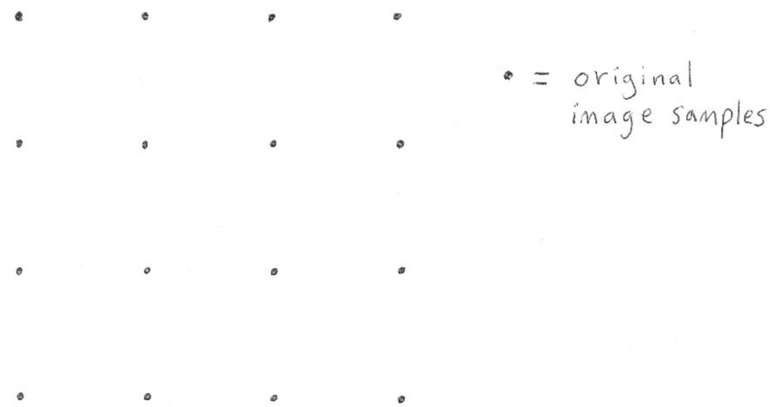
Ex $M = 4,\ N = 4,\ r = 1.25$



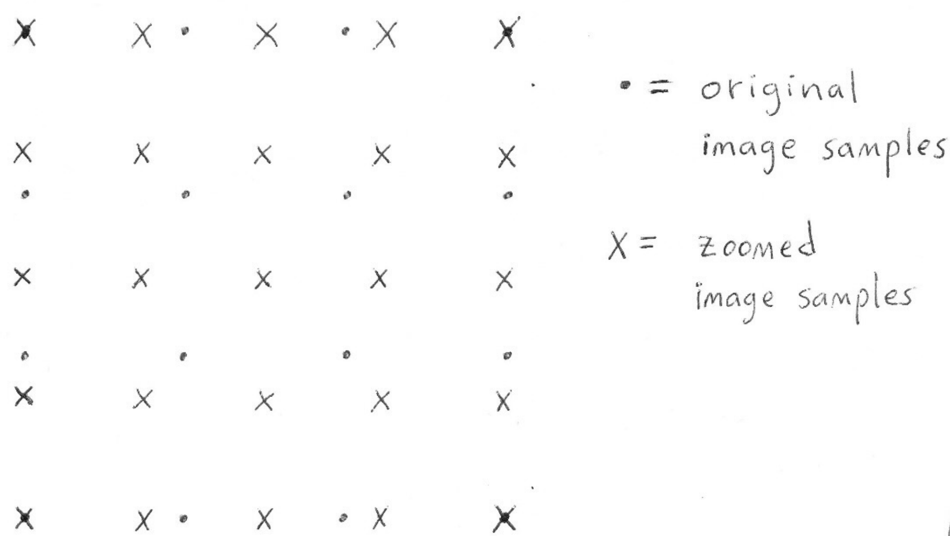
• = original image samples

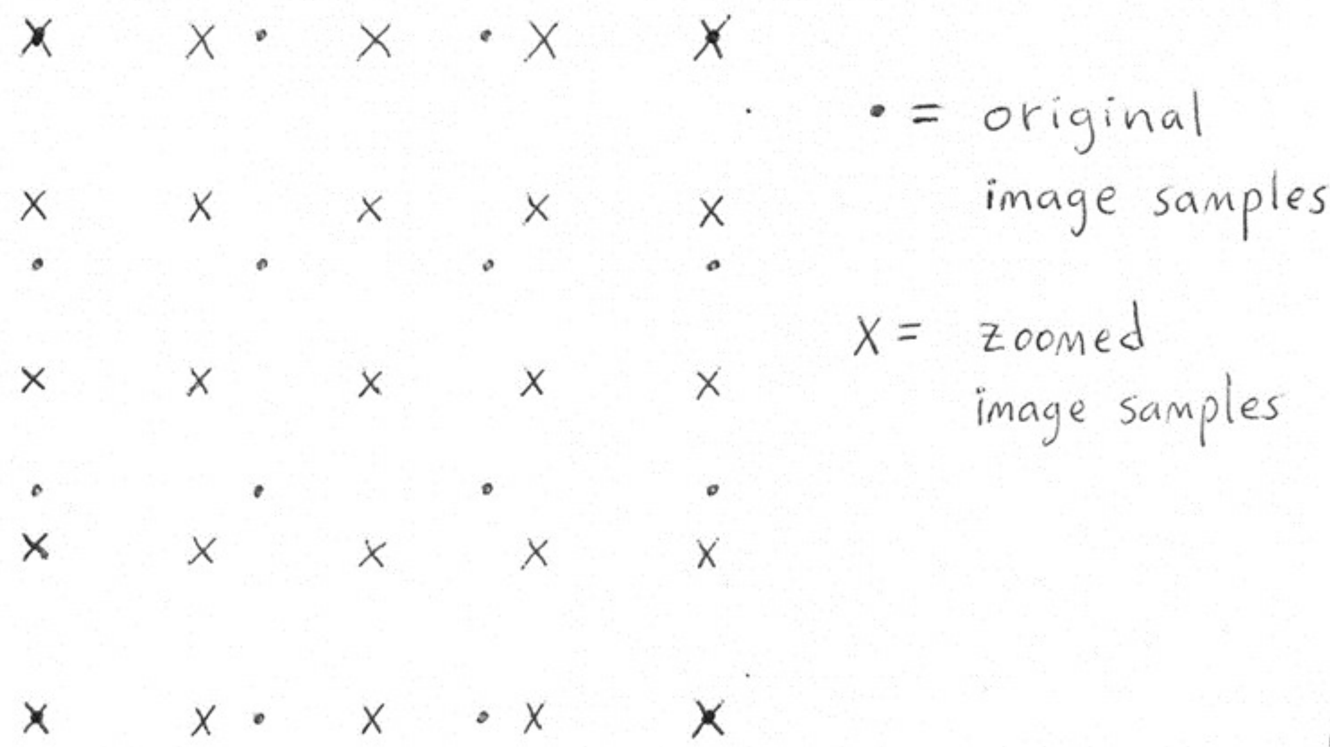
Interpolation

Ex $M = 4,\ N = 4,\ r = 1.25$



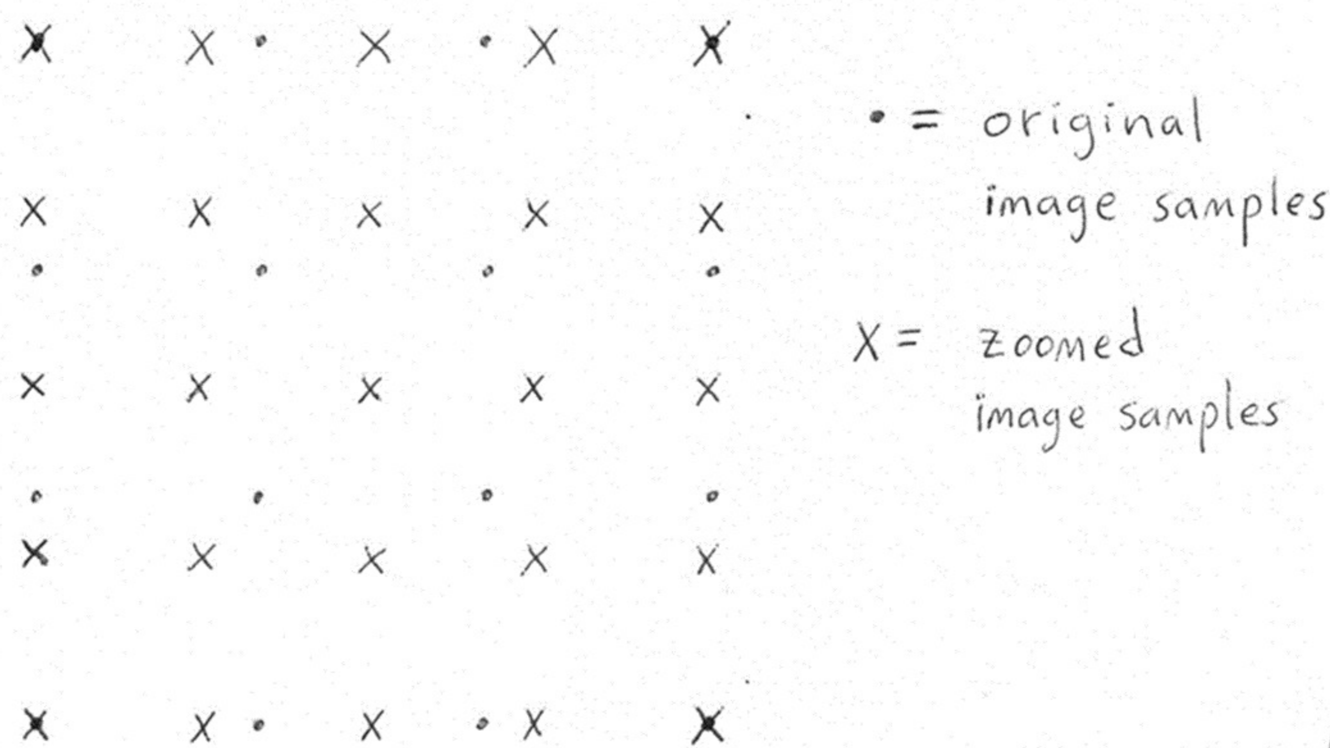
Generate a $rM \times rN$ sampling grid on the original image.



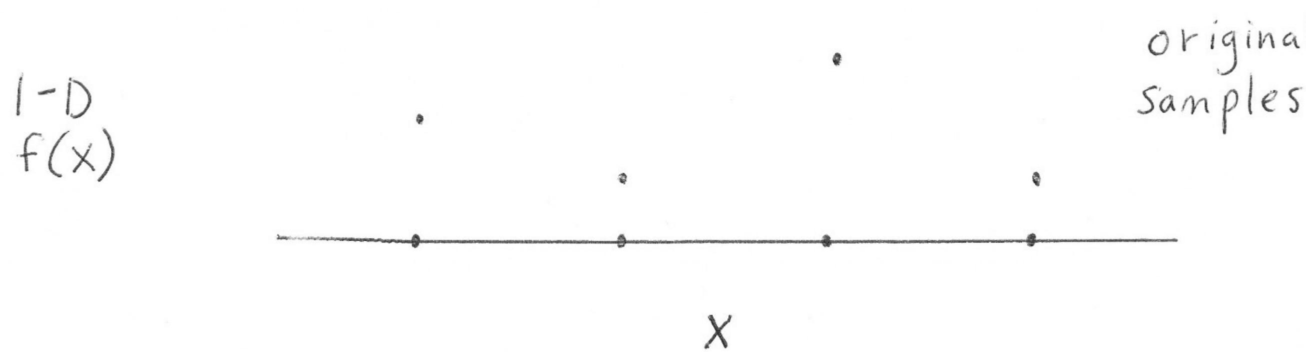


For each x sample (zoomed image) use pixel values at nearby • samples (original image) to interpolate the pixel value at the x sample.

Nearest Neighbor Interpolation – assign the pixel value at each sample in the zoomed image as the pixel value for the closest sample in the original image.



NN Interpolation generates a piecewise constant interpolating function



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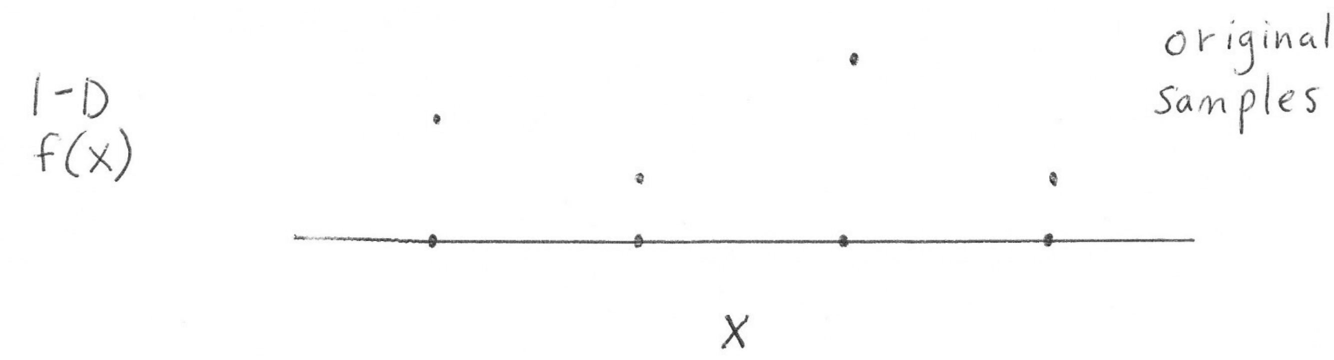
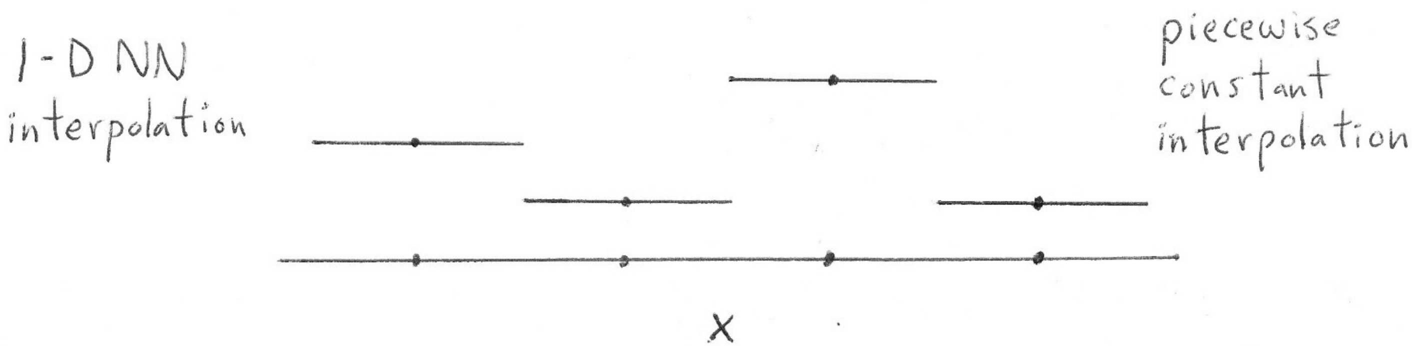
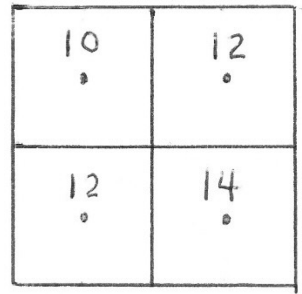




2-D NN
interpolation



piecewise
constant
interpolation

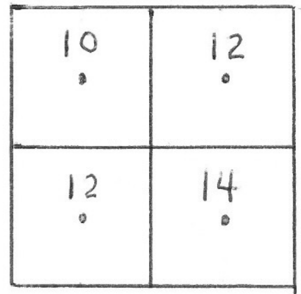
2-D NN interpolation

10	12
12	14

piecewise constant interpolation

If r is large, nearest neighbor interpolation gives a blocky zoomed image.

2-D NN
interpolation

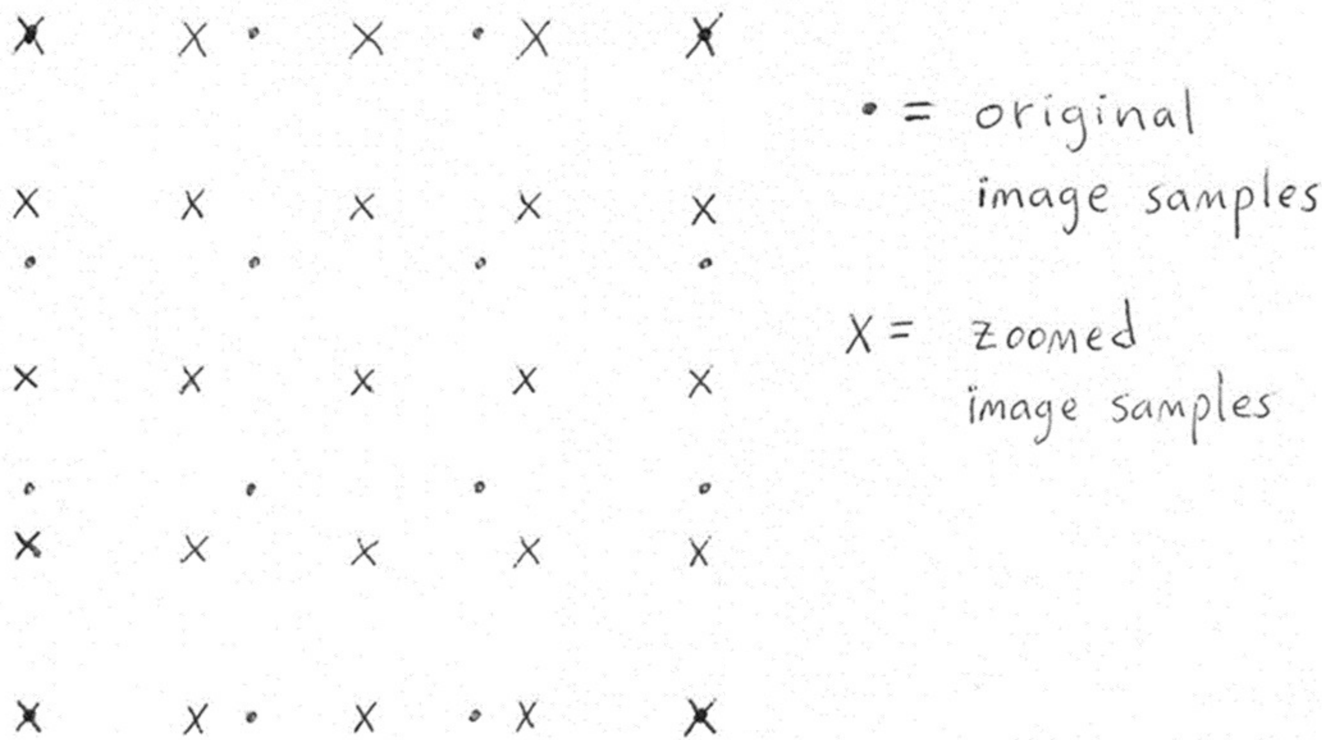


piecewise
constant
interpolation

If r is large, nearest neighbor interpolation gives a blocky zoomed image.

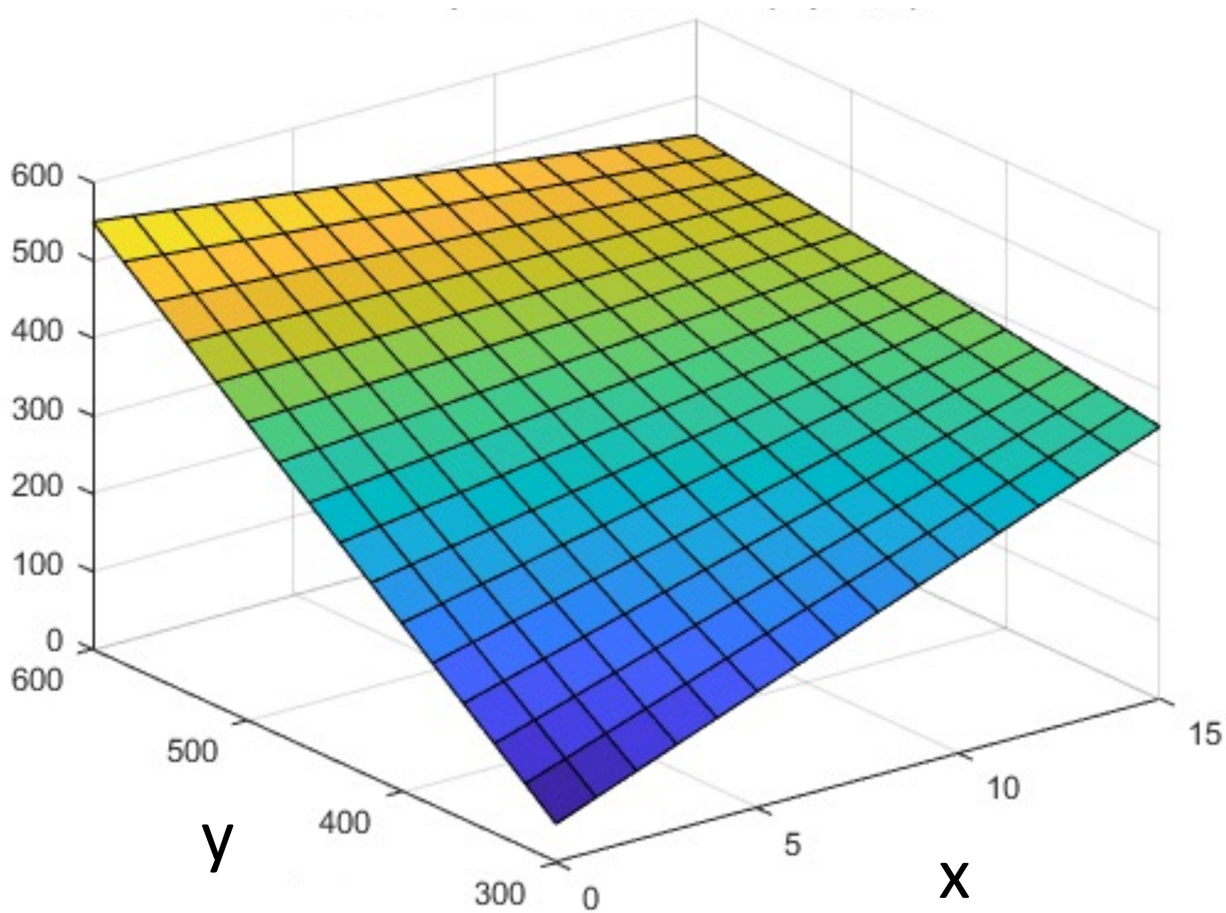
FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Bilinear interpolation – for each sample (x', y') in the zoomed image, find the closest 4 samples $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ in the original image $f_o(x, y)$.



Bilinear interpolation

Assume a continuous model for $f_o(x,y)$ near (x',y') of the form $f_o(x,y) = ax + by + cxy + d$



Bilinear interpolation

Solve for a, b, c, d using the equations

$$f_0(x_1, y_1) = ax_1 + by_1 + cx_1y_1 + d$$
$$f_0(x_2, y_2) = ax_2 + by_2 + cx_2y_2 + d$$
$$f_0(x_3, y_3) = ax_3 + by_3 + cx_3y_3 + d$$
$$f_0(x_4, y_4) = ax_4 + by_4 + cx_4y_4 + d$$

Bilinear interpolation

Solve for a, b, c, d using the equations

$$f_0(x_1, y_1) = ax_1 + by_1 + cx_1y_1 + d$$
$$f_0(x_2, y_2) = ax_2 + by_2 + cx_2y_2 + d$$
$$f_0(x_3, y_3) = ax_3 + by_3 + cx_3y_3 + d$$
$$f_0(x_4, y_4) = ax_4 + by_4 + cx_4y_4 + d$$

Assign the pixel value at x', y' in the zoomed image f_Z as

$$f_Z(x', y') = ax' + by' + cx'y' + d$$

Bilinear interpolation

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Methods for Shrink

Subsampling (Row-column deletion) - reduce M to M/r and N to N/r for an integer $r > 1$ by starting from the first and taking every rth row and every rth column.

Subsampling

Ex original 6x6 image

$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

Subsampling

Ex original 6x6 image

$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

row deletion

$r = 2$

$$\begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 6 & 6 \\ 7 & 7 & 8 & 8 & 9 & 9 \end{matrix}$$

Subsampling

Ex original 6x6 image

1	1	2	2	3	3
1	1	2	2	3	3
4	4	5	5	6	6
4	4	5	5	6	6
7	7	8	8	9	9
7	7	8	8	9	9

row deletion

$r = 2$

1	1	2	2	3	3
4	4	5	5	6	6
7	7	8	8	9	9

column deletion

$r = 2$

1	2	3
4	5	6
7	8	9

Subsampling

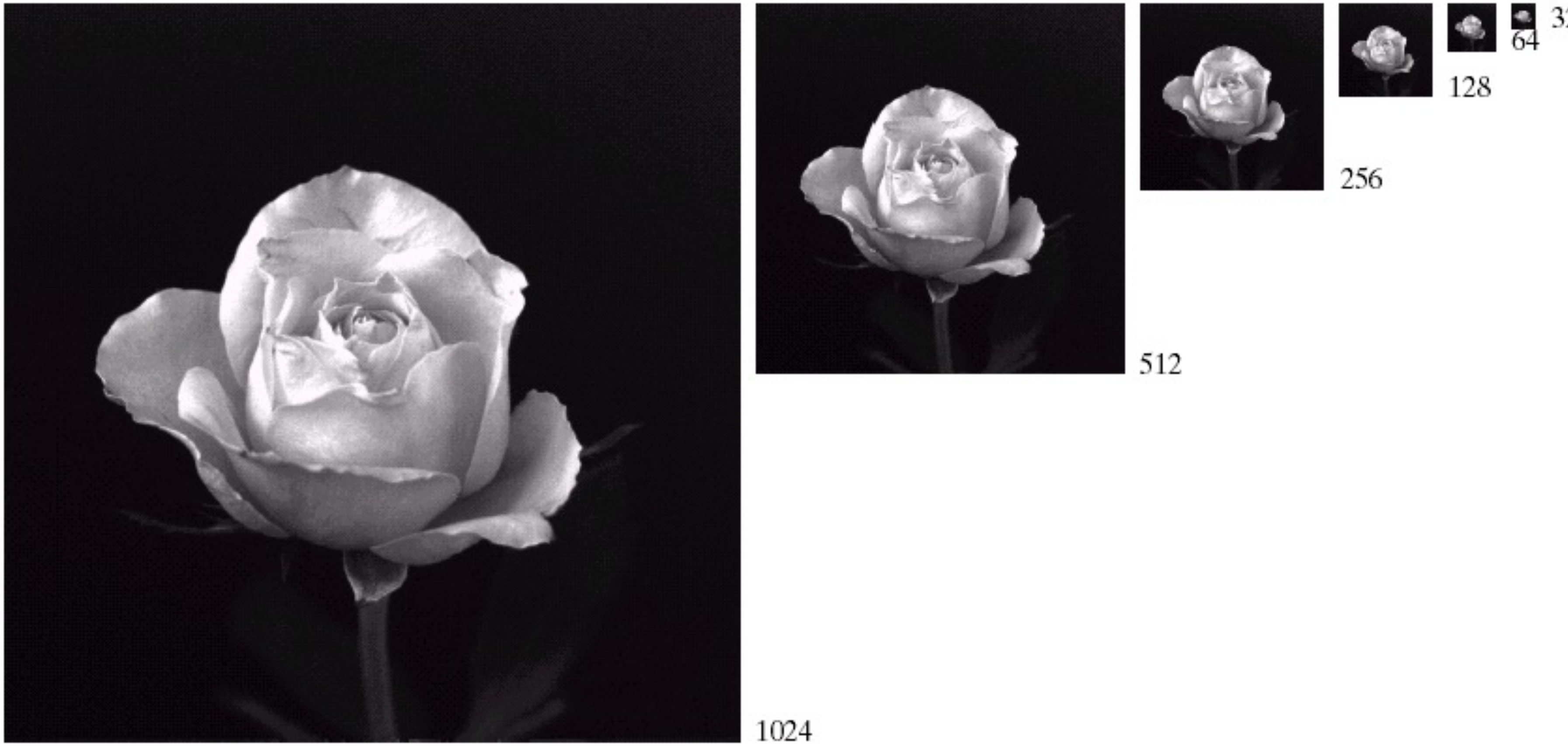
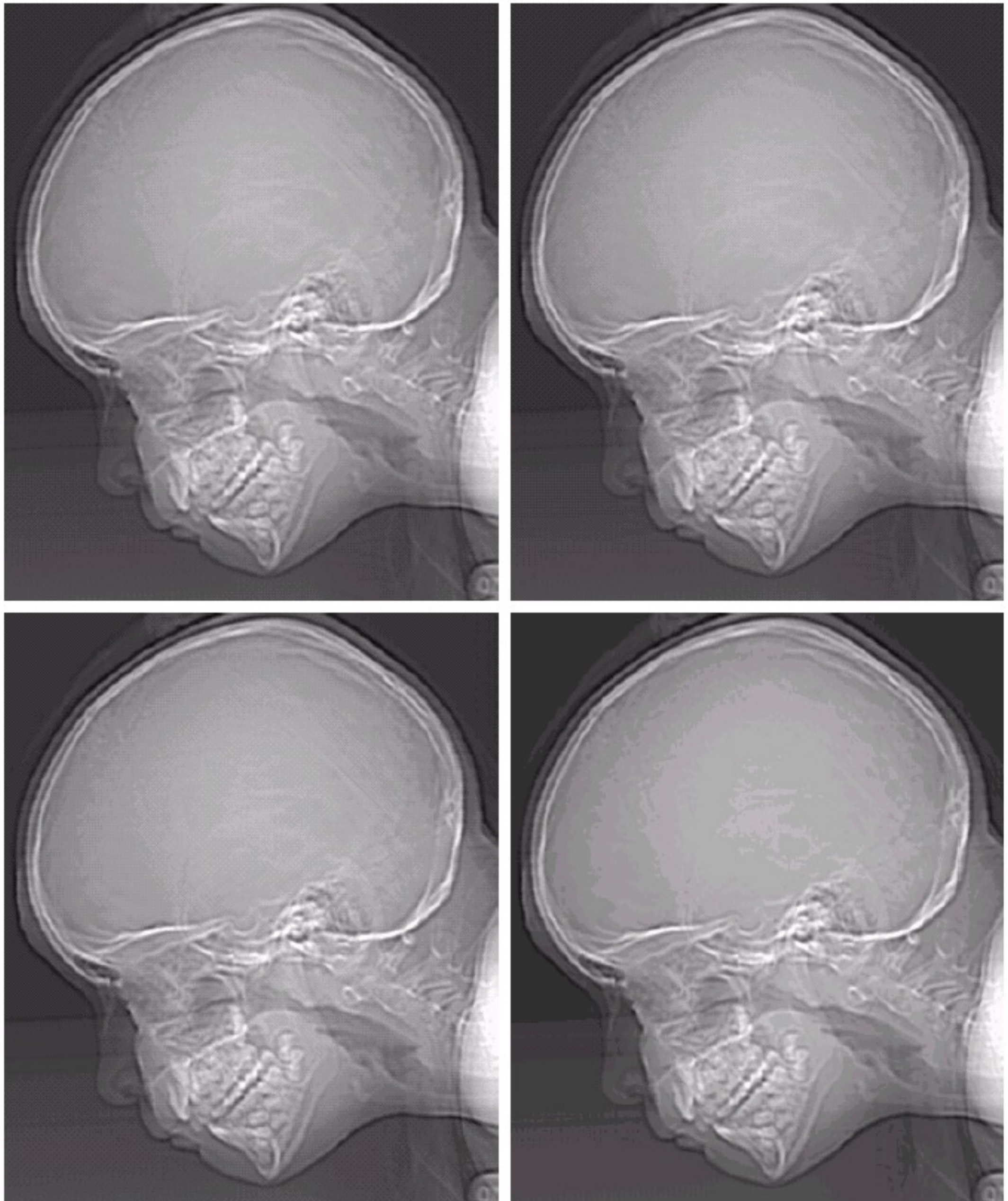


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Methods for Shrink

Interpolation - reduce M to M/r and N to N/r for a real number $r > 1$ by generating a $M/r \times N/r$ sampling grid on the original image and interpolating to get the pixel values on the new sampling grid.

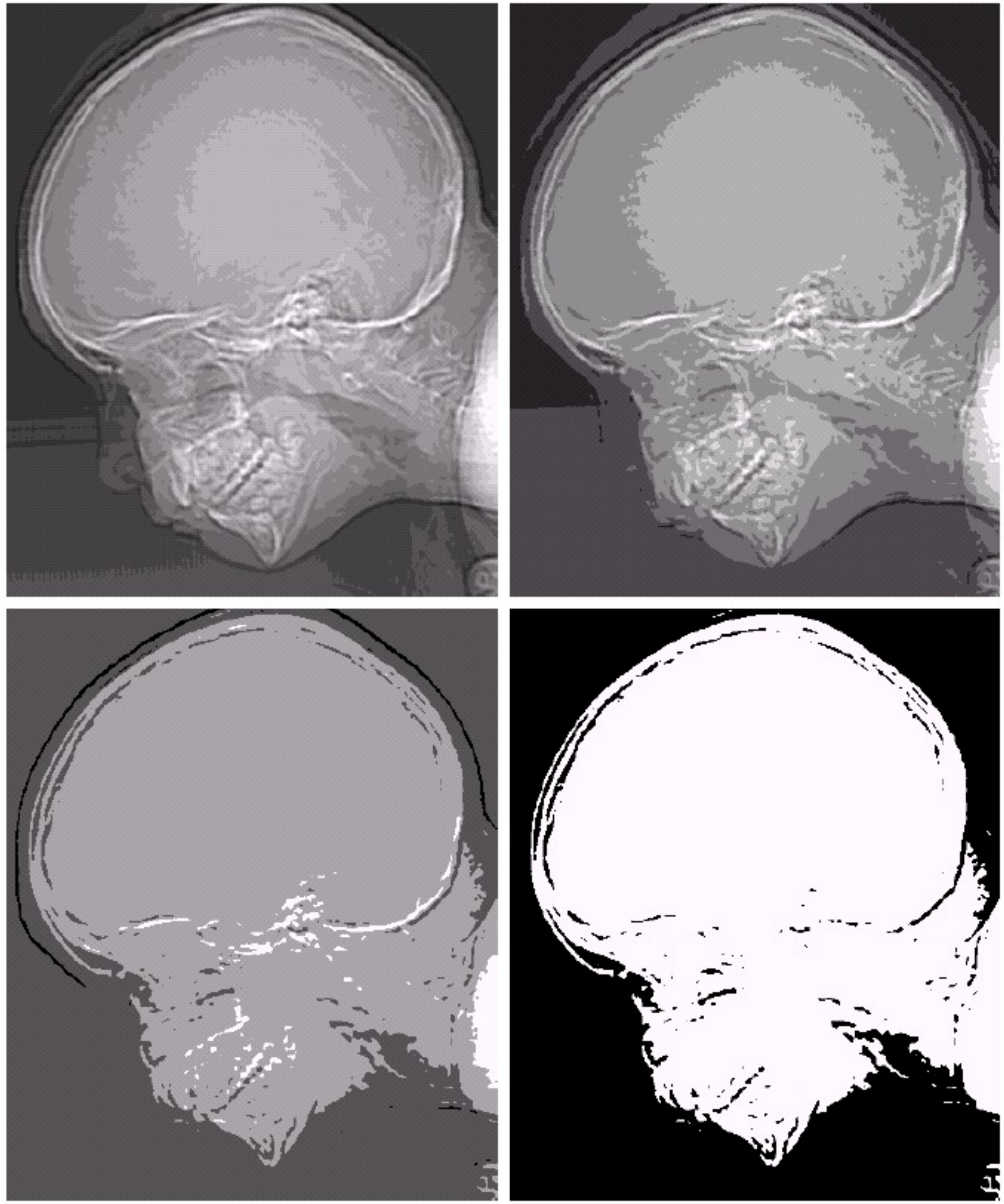
Changing the Gray-Level Resolution of a Digital Image



a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Changing the Gray-Level Resolution of a Digital Image

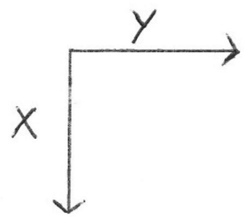


e f
g h

FIGURE 2.21
(Continued)
(e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Neighbor Sets

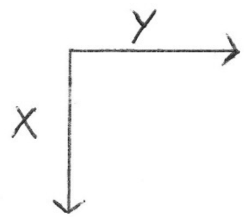
A pixel P at (x,y) has the 4 horizontal and vertical neighbors



	$(x-1,y)$	
$(x,y-1)$	$\bullet P$	$(x,y+1)$
	$(x+1,y)$	

Neighbor Sets

A pixel P at (x,y) has the 4 horizontal and vertical neighbors

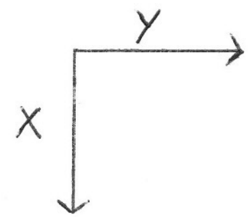


$$\begin{array}{ccc} & (x-1,y) & \\ (x,y-1) & \bullet P & (x,y+1) \\ & (x+1,y) & \end{array}$$

Define $N_4(P)$ to be this set of 4 pixels called 4-neighbors of P

Neighbor Sets

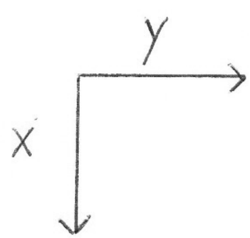
A pixel P at (x,y) has the 4 horizontal and vertical neighbors



$$\begin{array}{ccc} & (x-1,y) & \\ (x,y-1) & \bullet P & (x,y+1) \\ & (x+1,y) & \end{array}$$

Define $N_4(P)$ to be this set of 4 pixels called 4-neighbors of P

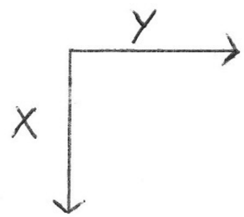
A pixel P at (x,y) has the 4 diagonal neighbors



$$\begin{array}{ccc} (x-1,y-1) & & (x-1,y+1) \\ & \bullet P & \\ (x+1,y-1) & & (x+1,y+1) \end{array}$$

Neighbor Sets

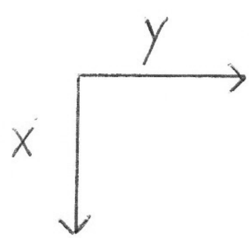
A pixel P at (x,y) has the 4 horizontal and vertical neighbors



$$\begin{array}{ccc} & (x-1,y) & \\ (x,y-1) & \bullet P & (x,y+1) \\ & (x+1,y) & \end{array}$$

Define $N_4(P)$ to be this set of 4 pixels called 4-neighbors of P

A pixel P at (x,y) has the 4 diagonal neighbors



$$\begin{array}{ccc} (x-1,y-1) & & (x-1,y+1) \\ & \bullet P & \\ (x+1,y-1) & & (x+1,y+1) \end{array}$$

Define $N_8(P)$ to be the union of the 4 diagonal neighbors of P and $N_4(P)$

Let V be a set of gray levels that define adjacency

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Pixels P and Q are 4-adjacent if $Q \in N_4(P)$ and P and Q both have values in V.

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Ex Let $V = \{2, 3\}$

$$\begin{bmatrix} f(0,0) & f(0,1) & f(0,2) \\ f(1,0) & f(1,1) & f(1,2) \\ f(2,0) & f(2,1) & f(2,2) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 2 \\ 2 & 7 & 7 \end{bmatrix}$$

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A path from the pixel at (x_0, y_0) to the pixel at (x_n, y_n) is defined by a sequence of distinct coordinates $(x_0, y_0), (x_1, y_1), \cdots, (x_n, y_n)$ where the pixels corresponding to any consecutive pair of coordinates in the sequence are adjacent. Adjacent can be defined as 4-adjacent or 8-adjacent

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An 8-adjacent path from $f(0,0)$ to $f(1,2)$ is defined by $(0,0), (0,1), (1,2)$

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Using 8-adjacency, $f(0,0)$ is connected to $f(1,2)$

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Given a pixel P with a gray level in V, the set consisting of P and all pixels that are connected to P is called a connected component.

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Ex Using 4-adjacency, the image has 3 connected components

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Using 8-adjacency, the image has 2 connected components

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 2 \\ 2 & 7 & 7 \end{bmatrix}$$

Operators

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1) Multiplication of an image by a scalar
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To add, multiply, or divide two images, the images must be the same size.

Operators

Let H be an operator that maps an image to an image. H is a linear operator if

$$H[af(x,y) + bg(x,y)] = aH[f(x,y)] + bH[g(x,y)]$$

for any images f(x,y) and g(x,y) and for any scalars a and b.

An operator that is not linear is called nonlinear.

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where $K(x,y)$ is an image that multiplies the input image.

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$$aH[f(x,y)] + bH[g(x,y)] = af(x,y)f(x,y) + bg(x,y)g(x,y)$$

NONLINEAR