EECS203A Exam #2 June 8, 2021

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This is a 120 minute, open book, open notes exam. Calculators are allowed. Interaction with other people is not allowed. Internet use outside of Zoom is not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Question 11:

Question 12:

Question 13:

Question 14:

TOTAL:

Note: The Euclidean distance $E(v, w)$ between a vector $v = (v_x \; v_y \; v_z)$ and a vector $w = (w_x \; w_y \; w_z)$ is $E(v, w) = \sqrt{(v_x - w_x)^2 + (v_y - w_y)^2 + (v_z - w_z)^2}$.

Question 1 (4 points) Let $I_1(\lambda)$ and $I_2(\lambda)$ be two functions over the visible wavelengths.

2

a) What does it mean to say $I_1(\lambda)$ and $I_2(\lambda)$ are isomers?

$I_1(\lambda) = I_2(\lambda)$

2

b) What does it mean to say $I_1(\lambda)$ and $I_2(\lambda)$ are metamers?

$I_1(\lambda) \neq I_2(\lambda)$ but $I_1(\lambda)$ and $I_2(\lambda)$ appear the same to a human. Can also say $\int I_1(\lambda)x(\lambda)d\lambda = \int I_2(\lambda)x(\lambda)d\lambda$ $\int I_1(\lambda)y(\lambda)d\lambda = \int I_2(\lambda)y(\lambda)d\lambda$ $\int I_1(\lambda)z(\lambda)d\lambda = \int I_2(\lambda)z(\lambda)d\lambda$

Question 2 (4 points) A display device has b bits per pixel to represent gray level. A halftoned image uses an $n \times n$ grid of display device pixels to represent one half-toned pixel. How many different gray levels are possible for each halftoned pixel?

4

$n^2(2^b - 1) + 1$

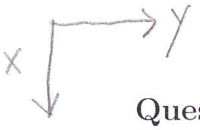
4

Question 3 (4 points) A filter transforms an input image $f(x, y)$ to an output image $g(x, y)$ according to $g(x, y) = \frac{1}{g'(x,y)}$ where

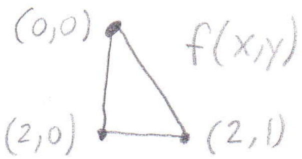
$$g'(x, y) = \sum_{i=x-1}^{x+1} \sum_{j=y-1}^{y+1} (f(i, j))^{-1}$$

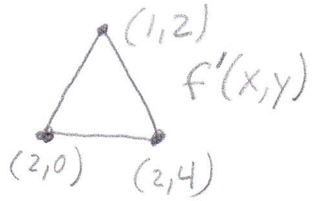
Describe in words the effect of this filter on an input image.

This is a 3x3 harmonic mean filter (HMF) scaled by $\frac{1}{9}$. If we multiply $g(x,y)$ by 9, we get a smoothed version of $f(x,y)$ which is darker than the AMF filtered result in regions with gray level variability. This filter is unstable for small gray levels.



Question 4 (6 points) A triangular region in image $f(x, y)$ with vertices $(x, y) = \{(0, 0), (2, 0), (2, 1)\}$ appears distorted in image $f'(x', y')$ with corresponding vertices $(x', y') = \{(1, 2), (2, 0), (2, 4)\}$. Determine the functions $x'(x, y)$ and $y'(x, y)$ using a bilinear model without the cross xy term for the distortion.





$f'(1,2) = f(0,0)$
$f'(2,0) = f(2,0)$
$f'(2,4) = f(2,1)$

2 $x'(0,0) = 1 \quad x'(2,0) = 2 \quad x'(2,1) = 2$
$y'(0,0) = 2 \quad y'(2,0) = 0 \quad y'(2,1) = 4$

2 $x'(x,y) = c_1 x + c_2 y + c_3 \qquad y'(x,y) = c_4 x + c_5 y + c_6$

at $(0,0)$ $\quad 1 = 0 + 0 + c_3 \qquad 2 = 0 + 0 + c_6$
at $(2,0)$ $\quad 2 = 2c_1 + 0 + c_3 \qquad 0 = 2c_4 + 0 + c_6$
at $(2,1)$ $\quad 2 = 2c_1 + c_2 + c_3 \qquad 4 = 2c_4 + c_5 + c_6$

$c_3 = 1 \quad c_1 = 0.5 \quad c_2 = 0 \qquad c_6 = 2 \quad c_4 = -1 \quad c_5 = 4$

2 $x'(x,y) = 0.5x + 1 \qquad y'(x,y) = -x + 4y + 2$

6

Question 5 (6 points) Let $f(x, y)$ be the 4×4 digital image

```
f(0,0) f(0,1) f(0,2) f(0,3)      6   6   6   6
f(1,0) f(1,1) f(1,2) f(1,3)  =   4   4   4   4
f(2,0) f(2,1) f(2,2) f(2,3)      6   6   6   6
f(3,0) f(3,1) f(3,2) f(3,3)      4   4   4   4
```

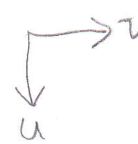
Find the DFT $F(u, v)$ of $f(x, y)$ for $u = 0, 1, 2, 3$ and $v = 0, 1, 2, 3$. Simplify your answer.

$f(x,y) = 5 + \cos(\pi x) \qquad u_0 = \frac{1}{2} \quad v_0 = 0$

$F(u,v) = 5\delta(u,v) + \frac{1}{2}\left[\delta(u+2,v) + \delta(u-2,v)\right]$

$= 5\delta(u,v) + \delta(u-2,v)$ (since periodic) $u = 0,1,2,3$
$v = 0,1,2,3$

$$F(u,v) = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



full points for either

Question 6 (6 points) The digital image $g(x,y)$ is a degraded version of an ideal digital image $f(x,y)$ with

$$g(x,y) = h(x,y) * f(x,y) + n(x,y)$$

2 a) What must be known to apply an inverse filter to $g(x,y)$?

We must Know $H(u,v)$ (or $h(x,y)$)

2 b) What image $f'(x,y)$ is the result of applying an inverse filter to $g(x,y)$?

$$F'(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$f'(x,y) = IDFT[F'(u,v)]$$

2 c) What are the most general conditions on $h(x,y)$ and $n(x,y)$ for which $f'(x,y) = f(x,y)$?

We require $n(x,y) = 0$ and $H(u,v)$ is nonzero for all (u,v)

$x(\lambda), y(\lambda), z(\lambda)$ are CIE 1931 color matching functions.

Question 7 (8 points) We are given three display guns with spectral distributions $D_R(\lambda)$, $D_G(\lambda)$, and $D_B(\lambda)$.

2 a) Find the chromaticity coordinates (x, y) for each of the three display guns.

$X_R = \int x(\lambda) D_R(\lambda) d\lambda \quad Y_R = \int y(\lambda) D_R(\lambda) d\lambda \quad Z_R = \int z(\lambda) D_R(\lambda) d\lambda$

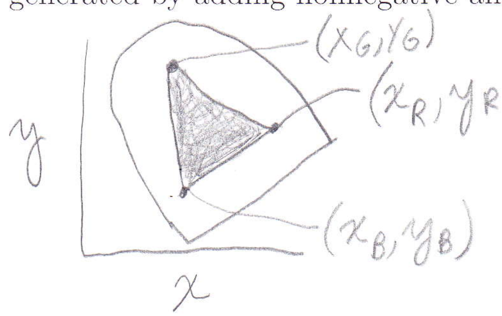
$X_G = \int x(\lambda) D_G(\lambda) d\lambda \quad Y_G = \int y(\lambda) D_G(\lambda) d\lambda \quad Z_G = \int z(\lambda) D_G(\lambda) d\lambda$

$X_B = \int x(\lambda) D_B(\lambda) d\lambda \quad Y_B = \int y(\lambda) D_B(\lambda) d\lambda \quad Z_B = \int z(\lambda) D_B(\lambda) d\lambda$

Chromaticity coordinates

$D_R(\lambda)$	$D_G(\lambda)$	$D_B(\lambda)$
$x_R = X_R/(X_R+Y_R+Z_R)$	$x_G = X_G/(X_G+Y_G+Z_G)$	$x_B = X_B/(X_B+Y_B+Z_B)$
$y_R = Y_R/(X_R+Y_R+Z_R)$	$y_G = Y_G/(X_G+Y_G+Z_G)$	$y_B = Y_B/(X_B+Y_B+Z_B)$

2 b) Draw a labeled chromaticity diagram (x, y) that shows the gamut of chromaticities that can be generated by adding nonnegative amounts of the three display guns.



gamut is chromaticities inside the triangle

2 c) Let $I(\lambda)$ be an arbitray nonnegative function of wavelength. In general, can we find constants r, g, b so that $I(\lambda) = rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$? Explain your answer.

No. The functions can only be equal if $I(\lambda)$ happens to lie in the space of functions spanned by $D_R(\lambda)$, $D_G(\lambda)$, $D_B(\lambda)$

2 d) Let $I(\lambda)$ be an arbitray nonnegative function of wavelength. Find constants r, g, b so that $rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$ looks the same as $I(\lambda)$ to a human observer.

$$[r\ g\ b] = [X_I\ Y_I\ Z_I]\, T^{-1}$$

$$T = \begin{bmatrix} X_R & Y_R & Z_R \\ X_G & Y_G & Z_G \\ X_B & Y_B & Z_B \end{bmatrix}$$

$$X_I = \int x(\lambda) I(\lambda) d\lambda$$

$$Y_I = \int y(\lambda) I(\lambda) d\lambda$$

$$Z_I = \int z(\lambda) I(\lambda) d\lambda$$

Question 8 (8 points) A camera is viewing a flat surface that fills the entire image. The generated digital image $D(x, y)$ is given by

$$D(x, y) = (R(x, y)e + N(x, y))A$$

where e is the constant illumination incident on the viewed surface and $R(x, y)$ is the reflectance of the surface at pixel (x, y). Let $R(x, y) = \overline{r} + r(x, y)$ where $\overline{r}$ is the constant mean reflectance of the surface and $r(x, y)$ describes the variation in the reflectance of the surface. $r(x, y)$ has zero-mean and a variance of σ_r^2 over the pixels (x, y) in the image. $N(x, y)$ is zero-mean noise with variance σ_N^2. A is the constant gain of the system. Assume that the noise and $r(x, y)$ are independent.

2

a) What is the mean of $D(x, y)$ over the image.

$$E[D(x,y)] = Ae E[R(x,y)] + A E[N(x,y)]$$
$$= Ae\overline{r}$$

3

b) What is the total variance of $D(x, y)$ over the image?

$$VAR[D(x,y)] = A^2e^2 VAR[R(x,y)] + A^2 VAR[N(x,y)]$$
$$= A^2e^2\sigma_r^2 + A^2\sigma_N^2$$

3

c) What fraction of the total variance of $D(x, y)$ is due to the variation in the reflectance of the viewed surface?

$$\frac{A^2e^2\sigma_r^2}{A^2e^2\sigma_r^2 + A^2\sigma_N^2} = \frac{e^2\sigma_r^2}{e^2\sigma_r^2 + \sigma_N^2}$$

Question 9 (8 points) A two-dimensional function $I(x, y)$ is defined for continuous variables x and y over the space $0 \le x \le 99,\ 0 \le y \le 99$ by

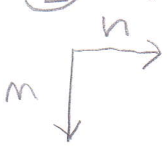
$$\begin{aligned} I(x,y) &= 10 \quad \text{for } x \le E \\ &= 0 \quad \text{for } x > E \end{aligned}$$

where E is a real number. A 100×100 digital image $D(m, n)$ is generated from $I(x, y)$ according to $D(m, n) = I(m, n)$ for all pairs of integers (m, n) in $m = 0, 1, \ldots, 99$ and $n = 0, 1, \ldots, 99$.

a) What is this process for generating $D(m, n)$ from $I(x, y)$ called?

2 Sampling

2 b) If the mean value of the pixels in $D(m, n)$ is 6.0, what are the possible values of E?



$$m \quad \begin{array}{c|cccc} & \multicolumn{4}{c}{n} \\ & 0 & 1 & & 99 \\ \hline 0 & 10 & 10 & \cdots & 10 \\ 1 & 10 & 10 & \cdots & 10 \\ \vdots & \vdots & \vdots & & \vdots \\ 59 & 10 & 10 & \cdots & 10 \\ 60 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 99 & 0 & 0 & \cdots & 0 \end{array}$$

$59.0 \le E < 60.0$

2 c) Can we uniquely recover $I(x, y)$ from $D(m, n)$? Explain.

No. Different $I(x,y)$ images derived using different E values can give the same $D(m,n)$ image as shown in part b. We cannot uniquely recover $I(x,y)$ from $D(m,n)$.

1 point for No

2 d) Will the use of an antialiasing filter improve the appearance of jagged edges in $D(m, n)$? If you answer No, then explain how the use of an antialiasing filter will affect the appearance of the digital image.

No. An edge in $D(m,n)$ will be horizontal and not jagged. The antialiasing filter will smooth the edge.

1 point for No

Question 10 (8 points) We have 5 pixels in a region of a color image with values

$$\Sigma = \begin{pmatrix} 504 & 360 & 180 \\ 360 & 360 & 0 \\ 180 & 0 & 720 \end{pmatrix}$$

pixel	R	G	B
1	90	60	90
2	90	90	30
3	60	60	60
4	60	60	90
5	30	30	30

$\mu_R = 66$

$\mu_G = 60$

$\mu_B = 60$

6

a) Use the method given in class to estimate the color covariance matrix Σ using these 5 pixels.

$$\Sigma = \begin{pmatrix} E[(R-\mu_R)^2] & E[(R-\mu_R)(G-\mu_G)] & E[(R-\mu_R)(B-\mu_B)] \\ E[(R-\mu_R)(G-\mu_G)] & E[(G-\mu_G)^2] & E[(G-\mu_G)(B-\mu_B)] \\ E[(R-\mu_R)(B-\mu_B)] & E[(G-\mu_G)(B-\mu_B)] & E[(B-\mu_B)^2] \end{pmatrix}$$

$$\sigma_R^2 = \frac{1}{5}\left[(90-66)^2+(90-66)^2+(60-66)^2+(60-66)^2+(30-66)^2\right] = 504$$

$$\sigma_G^2 = \frac{1}{5}\left[(60-60)^2+(90-60)^2+(60-60)^2+(60-60)^2+(30-60)^2\right] = 360$$

$$\sigma_B^2 = \frac{1}{5}\left[(90-60)^2+(30-60)^2+(60-60)^2+(90-60)^2+(30-60)^2\right] = 720$$

$$\sigma_{RG} = \frac{1}{5}\left[(90-66)(60-60)+(90-66)(90-60)+(60-66)(60-60)+(60-66)(60-60)+(30-66)(30-60)\right] = 360$$

$$\sigma_{RB} = \frac{1}{5}\left[(90-66)(90-60)+(90-66)(30-60)+(60-66)(60-60)+(60-66)(90-60)+(30-66)(30-60)\right] = 180$$

$$\sigma_{GB} = \frac{1}{5}\left[(60-60)(90-60)+(90-60)(30-60)+(60-60)(60-60)+(60-60)(90-60)+(30-60)(30-60)\right] = 0$$

2

b) Would you expect to observe these R,G,B values for 5 pixels in the region of a single material in a color image? Explain.

No. The (R,G,B) values have more scatter than we would expect for a single material.

1 point for No

Question 11 (8 points) The pixels $z = (z_r\ z_g\ z_b)^T$ in a color image have a mean vector $\mu = (\mu_R\ \mu_G\ \mu_B)^T = (50\ 100\ 200)^T$ and a covariance matrix $\Sigma = E[(z-\mu)(z-\mu)^T] = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix}$.

2 a) If the distribution of color pixel values in this image is multivariate Gaussian, then find the probability density function $p(z)$. Simplify.

$$p(z) = \frac{1}{(2\pi)^{3/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(z-\mu)^T\Sigma^{-1}(z-\mu)}$$

$$|\Sigma|^{1/2} = \sqrt{6000}$$

$$\Sigma^{-1} = \begin{pmatrix} \frac{1}{10} & 0 & 0 \\ 0 & 1/20 & 0 \\ 0 & 0 & 1/30 \end{pmatrix}$$

$$= \frac{1}{\sqrt{6000}(2\pi)^{3/2}} e^{-\frac{1}{20}\left((z_r-50)^2 + \frac{1}{2}(z_g-100)^2 + \frac{1}{3}(z_b-200)^2\right)}$$

2 b) Find the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of Σ where $\lambda_1 > \lambda_2 > \lambda_3$.

$\lambda_1 = 30 \quad \lambda_2 = 20 \quad \lambda_3 = 10$

2 c) Find the unit length eigenvectors e_1, e_2, e_3 that correspond to $\lambda_1, \lambda_2, \lambda_3$ respectively.

$e_1 = (0\ 0\ 1) \quad e_2 = (0\ 1\ 0) \quad e_3 = (1\ 0\ 0)$

2 d) Find the matrix A so that the operation Az implements the principal components transform.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Question 12 (10 points) The pixels $z = (z_r\ z_g\ z_b)^T$ in a color image region have a mean $\mu = (\mu_R\ \mu_G\ \mu_B)^T = (50\ 100\ 200)^T$ and a covariance matrix $\Sigma = E[(z-\mu)(z-\mu)^T] = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix}$.

2

a) Find the set of color pixel vectors $(R\ G\ B)$ that have a Euclidean distance of 10 from the point $(50\ 100\ 200)^T$ in color space.

$$\sqrt{(R-50)^2 + (G-100)^2 + (B-200)^2} = 10$$

$$(R-50)^2 + (G-100)^2 + (B-200)^2 = 100$$

2

b) Use μ and Σ to find the set of color pixel vectors (R, G, B) that have a Mahalanobis distance of 10 from the color pixel distribution for the region.

$$\sqrt{(R-50\ \ G-100\ \ B-200)\begin{bmatrix} 1/10 & 0 & 0 \\ 0 & 1/20 & 0 \\ 0 & 0 & 1/30 \end{bmatrix}\begin{pmatrix} R-50 \\ G-100 \\ B-200 \end{pmatrix}} = 10$$

$$\frac{1}{10}(R-50)^2 + \frac{1}{20}(G-100)^2 + \frac{1}{30}(B-200)^2 = 100$$

3

c) Consider the set of color pixel vectors (R, G, B) that have a Euclidean distance of 10 from the point $\mu = (50\ 100\ 200)^T$ in color space. Find the color pixel vector(s) (R, G, B) in this set with the smallest Mahalanobis distance from the color pixel distribution for the region.

Blue is the direction of maximum variance so we want points at a Euclidean distance of 10 in the blue direction.

Points are (50 100 190) and (50 100 210).

3

d) Consider the set of color pixel vectors (R, G, B) that have a Euclidean distance of 10 from the point $\mu = (50\ 100\ 200)^T$ in color space. Find the color pixel vector(s) (R, G, B) in this set with the largest Mahalanobis distance from the color pixel distribution for the region.

Red is the direction of minimum variance so we want points at a Euclidean distance of 10 in the red direction.

Points are (40 100 200) and (60 100 200)

Question 13 (10 points) Consider a spatial domain filter that transforms an input image $f(x, y)$ with N rows and N columns where N is divisible by 4 into an output image $g(x, y)$ according to

$$g(x, y) = \frac{1}{6}\left[f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)\ \right] + \frac{1}{3}f(x, y)$$

Let $G(u, v)$ be the DFT of $g(x, y)$ and let $F(u, v)$ be the DFT of $f(x, y)$.

3 a) For this filter, find $H(u, v)$ so that $G(u, v) = H(u, v)F(u, v)$. Simplify your answer.

$$G(u,v) = \frac{1}{6}\left[F(u,v)e^{j2\pi u/N} + F(u,v)e^{-j2\pi u/N} + F(u,v)e^{j2\pi v/N} + F(u,v)e^{-j2\pi v/N}\right] + \frac{F(u,v)}{3}$$

$$= \frac{F(u,v)}{6}\left[2\cos\left(\frac{2\pi u}{N}\right) + 2\cos\left(\frac{2\pi v}{N}\right) + 2\right]$$

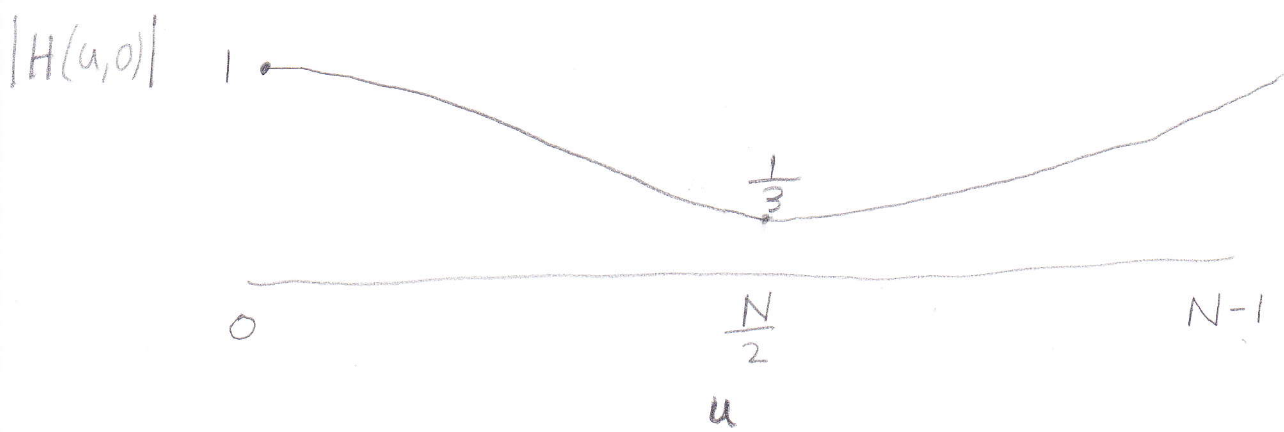
$$H(u,v) = \frac{G(u,v)}{F(u,v)} = \frac{1}{3}\left[\cos\left(\frac{2\pi u}{N}\right) + \cos\left(\frac{2\pi v}{N}\right) + 1\right]$$

1 b) Find $|H(u, v)|$

$$|H(u,v)| = \frac{1}{3}\left|\cos\left(\frac{2\pi u}{N}\right) + \cos\left(\frac{2\pi v}{N}\right) + 1\right|$$

$$|H(u,0)| = \frac{1}{3}\left|2 + \cos\left(\frac{2\pi u}{N}\right)\right|$$

2 c) Plot $|H(u, 0)|$ for the range $u = 0, 1, \ldots, N-1$.



2 d) What is the maximum value of $|H(u,v)|$? For what value(s) of (u,v) over $u = 0, 1, \ldots, N-1$ and $v = 0, 1, \ldots, N-1$ does $|H(u,v)|$ take on this maximum value?

The maximum value is 1 at $(u,v) = (0,0)$.

one point for 1

2 e) What is the minimum value of $|H(u,v)|$? For what value(s) of (u,v) over $u = 0, 1, \ldots, N-1$ and $v = 0, 1, \ldots, N-1$ does $|H(u,v)|$ take on this minimum value?

minimum value is zero when

$$1 + \cos\left(\frac{2\pi u}{N}\right) + \cos\left(\frac{2\pi v}{N}\right) = 0$$

This occurs for

$$(u,v) = \left(\frac{N}{2}, \frac{N}{4}\right), \left(\frac{N}{2}, \frac{3N}{4}\right), \left(\frac{N}{4}, \frac{N}{2}\right), \left(\frac{3N}{4}, \frac{N}{2}\right)$$

one point for zero

Question 14 (10 points) An input digital image $f(x, y)$ consists of only a bright rectangle of constant brightness on a dark background of constant brightness. We process $f(x, y)$ with a linear space-invariant filter defined by a 3×3 mask where the nine mask coefficients add up to one. The central part of the output image is given below. Find the $f(x, y)$ pixel values corresponding to the 11×11 region shown below and find the 3×3 mask.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	4	4	4	4	4	4	0	0
0	0	4	12	16	16	16	16	12	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	16	20	20	20	20	16	4	0
0	0	4	12	16	16	16	16	12	4	0
0	0	0	4	4	4	4	4	4	0	0
0	0	0	0	0	0	0	0	0	0	0

Since the mask coefficients add to 1, the rectangle has brightness 20 and the background has brightness 0.

From the symmetry of the output image the mask has the form

$$\boxed{\begin{matrix} a & b & a \\ b & c & b \\ a & b & a \end{matrix}}$$

input

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 20 \\ 0 & 0 & 20 & 20 \\ 0 & 0 & 20 & 20 \end{matrix}$$

output

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 20a & 20(a+b) & 20(2a+b) \\ 0 & 20(a+b) & 20(a+2b+c) & 20(2a+3b+c) \\ 0 & 20(2a+b) & 20(2a+3b+c) & 20 \end{matrix} = \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 4 & 12 & 16 \\ 0 & 4 & 16 & 20 \end{matrix}$$

$a = 0 \quad b = 0.2 \quad c = 0.2$

mask

$$\begin{matrix} 0 & 0.2 & 0 \\ 0.2 & 0.2 & 0.2 \\ 0 & 0.2 & 0 \end{matrix}$$

6 points

$f(x,y)$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	20	20	20	20	20	20	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

4 points