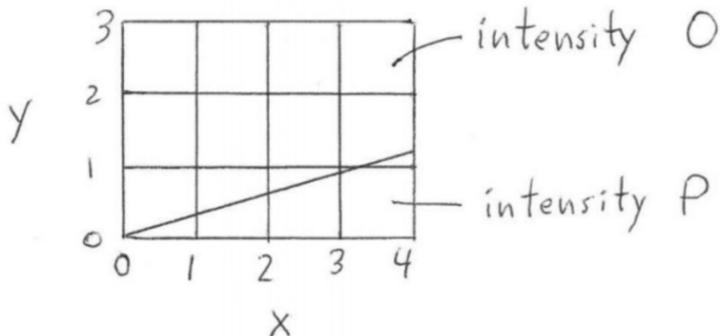


1. Consider a scene with a polygon of intensity  $P$  on a background of intensity 0. The edge of the polygon in continuous space is described by the equation  $y = 0.3x$ . We will generate a  $10 \times 10$  pixel image of this scene where the figure below depicts 12 pixels in the bottom left of the image.



- a) Compute the  $10 \times 10$  image pixel values that are generated if we sample the continuous data at pixel centers (the center of the drawn boxes). Explain why this polygon might appear visually undesirable.
- b) Compute the pixel values using the antialiasing algorithm that averages intensity over each pixel area (box filter).
- c) Will this antialiasing scheme used for part b) prevent aliasing in the Nyquist sense? Explain.

a)

	<u>X</u>	<u>Y</u>
1	0.3	
2	0.6	
3	0.9	
4	1.2	
5	1.5	
6	1.8	
7	2.1	
8	2.4	
9	2.7	
10	3	

10	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	P	P
1	0	0	0	0	P	P	P	P	P
0	0	P	P	P	P	P	P	P	P

Because the edge of polygon is jagged so its visually undesirable

b)

10	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0.01P	0.25P	0.55P	0.85P		
1	0	0	0	0.01P	0.35P	0.65P	0.93P	P	P	P	
0	0.15P	0.45P	0.75P	0.98P	P	P	P	P	P	P	
	0	1	2	3	4	5	6	7	8	9	10

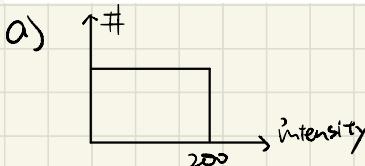
c) No. Nyquist requests a continuous signal can be recovered from its samples iff it is sampled at a rate greater than twice the highest frequency in the signal. which doesn't match here.

2. Suppose that we have a color image  $C_1$  where every pixel has a color given by  $(R, G, B) = (50K, 50K, 200K)$  where  $K$  is a random variable that has a uniform distribution between 0 and 1. Thus each pixel in  $C_1$  has a blue hue and an intensity that varies with  $K$ . Assume that the intensity levels in the images vary continuously (i.e. are not constrained to be integers).

a) Plot the gray level histogram for the blue band of the image.

b) Suppose that we apply histogram equalization to each of the three bands of  $C_1$  individually to generate the transformed bands  $R'(x, y)$ ,  $G'(x, y)$ , and  $B'(x, y)$ . Assume that the maximum gray level for each band is 255. The three transformed bands can be combined to form a transformed color image  $C_2$  where each pixel in  $C_2$  is described by a color vector  $(R, G, B)$ . What color vectors  $(R, G, B)$  will occur in  $C_2$ ? Explain your answer.

c) Suppose that we have a color image  $C_3$  for which the red, green, and blue band histograms are identical to those for  $C_1$ . Will images  $C_1$  and  $C_3$  contain the same set of color vectors  $(R, G, B)$ ? Explain your answer.



b) Since the maximum gray level for each band is 255.

$$\therefore C_2 = (R', G', B') = (255K, 255K, 255K)$$

c) No. Same histograms does not mean the combination of RGB are the same, so that  $C_1, C_3$  might contain different sets of color vectors

3. Suppose that at a pixel  $(x, y)$  on a color monitor we would like to generate a spectral power distribution  $I(\lambda)$ . We are given the spectral power distributions  $D_R(\lambda), D_G(\lambda), D_B(\lambda)$  for the three display guns.

a) Is it possible, in general, to find constants  $r, g, b$  such that  $I(\lambda) = rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$ . Explain your answer.

b) Given  $I(\lambda)$ , find constants  $r, g, b$  such that  $rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$  looks the same as  $I(\lambda)$  to a standard human observer. Explain all symbols in your equations.

c) Suppose that the display guns are related by  $D_G(\lambda) = 3D_R(\lambda) + 2D_B(\lambda)$ . How will this relationship affect the computation of the  $r, g, b$  values in part b? Be specific.

a) No, because even though it may look the same for human, it can be isomeric but cannot be isomers.

$$I(\lambda) \stackrel{M}{=} rD_R(\lambda) + gD_G(\lambda) + bD_B(\lambda)$$

$$D_R(\lambda) \stackrel{M}{=} X_R R(\lambda_1) + Y_R G(\lambda_2) + Z_R B(\lambda_3)$$

$$D_G(\lambda) \stackrel{M}{=} X_G R(\lambda_1) + Y_G G(\lambda_2) + Z_G B(\lambda_3)$$

$$D_B(\lambda) \stackrel{M}{=} X_B R(\lambda_1) + Y_B G(\lambda_2) + Z_B B(\lambda_3)$$

$$X_R = \int x(\lambda) D_R(\lambda) d\lambda \quad Y_R = \int y(\lambda) D_R(\lambda) d\lambda \quad Z_R = \int z(\lambda) D_R(\lambda) d\lambda$$

$$X_G = \int x(\lambda) D_G(\lambda) d\lambda \quad Y_G = \int y(\lambda) D_G(\lambda) d\lambda \quad Z_G = \int z(\lambda) D_G(\lambda) d\lambda$$

$$X_B = \int x(\lambda) D_B(\lambda) d\lambda \quad Y_B = \int y(\lambda) D_B(\lambda) d\lambda \quad Z_B = \int z(\lambda) D_B(\lambda) d\lambda$$

$$\begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix} \stackrel{M}{=} \underbrace{\begin{bmatrix} X_R & Y_R \\ X_G & Y_G \\ X_B & Y_B \end{bmatrix}}_T \begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix}$$

depends on monitor and properties of the human vision system

$$\begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix} \stackrel{M^{-1}}{=} T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

$$I(\lambda) \stackrel{M}{=} [X_I \ Y_I \ Z_I] \begin{bmatrix} R(\lambda_1) \\ G(\lambda_2) \\ B(\lambda_3) \end{bmatrix} \stackrel{M^{-1}}{=} [X_I \ Y_I \ Z_I] T^{-1} \begin{bmatrix} D_R(\lambda) \\ D_G(\lambda) \\ D_B(\lambda) \end{bmatrix}$$

$$[r \ g \ b] = [X_I \ Y_I \ Z_I] T^{-1}$$

c) If  $D_G$  is related to  $D_R, D_B$ , then we cannot represent  $D_G$  using  $X, Y, Z$  system  
then  $r, g, b$  are different