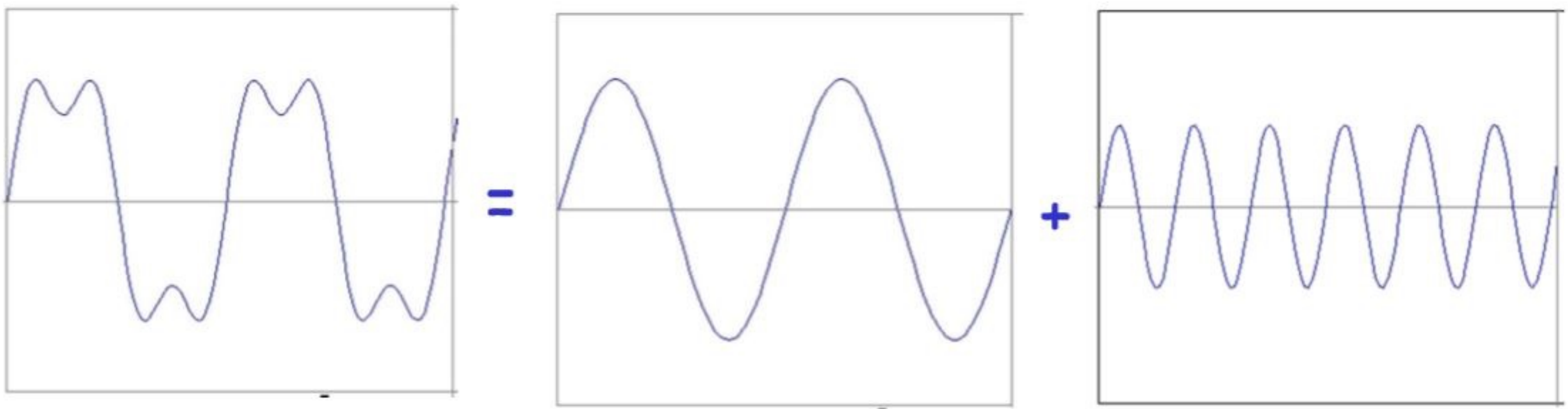
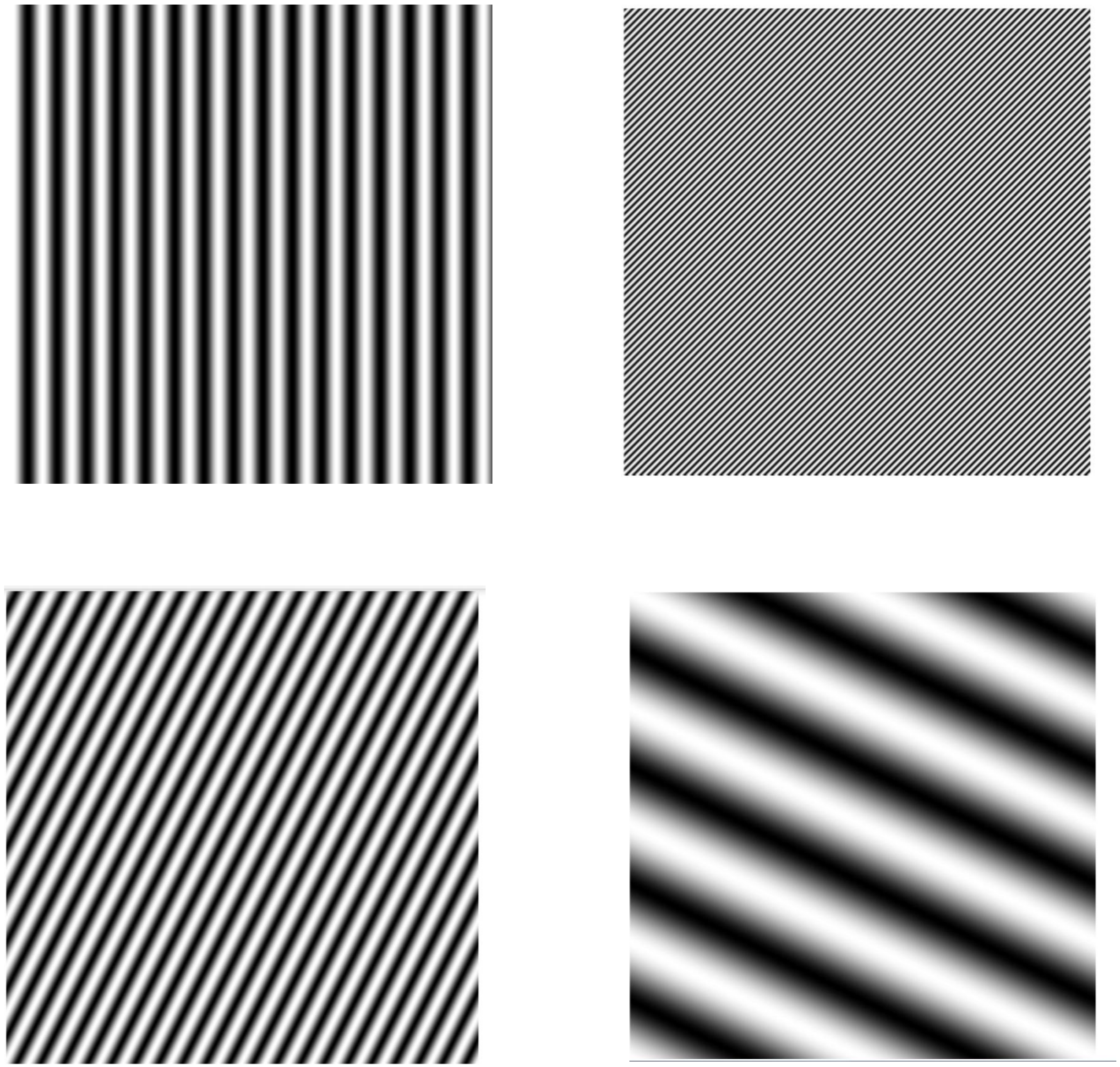
Frequency Domain Image Enhancement

1-D Sinusoids

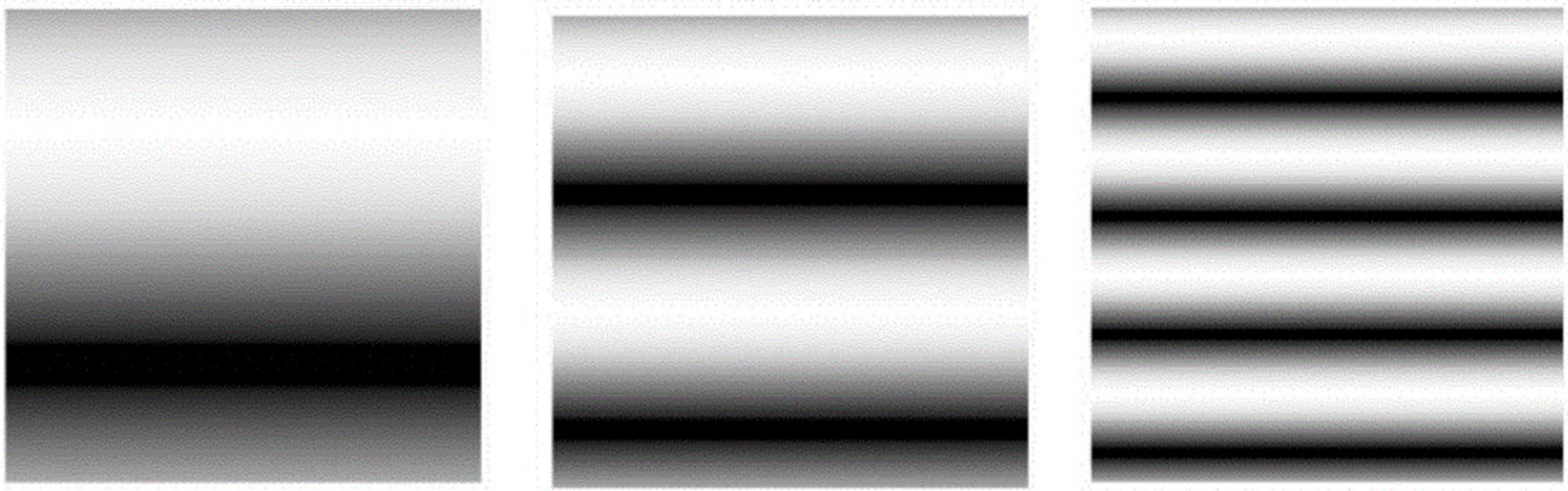


$$f(x) = \sin x + \frac{1}{3}\sin 3x + \ldots$$

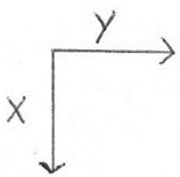
2-D Sinusoids

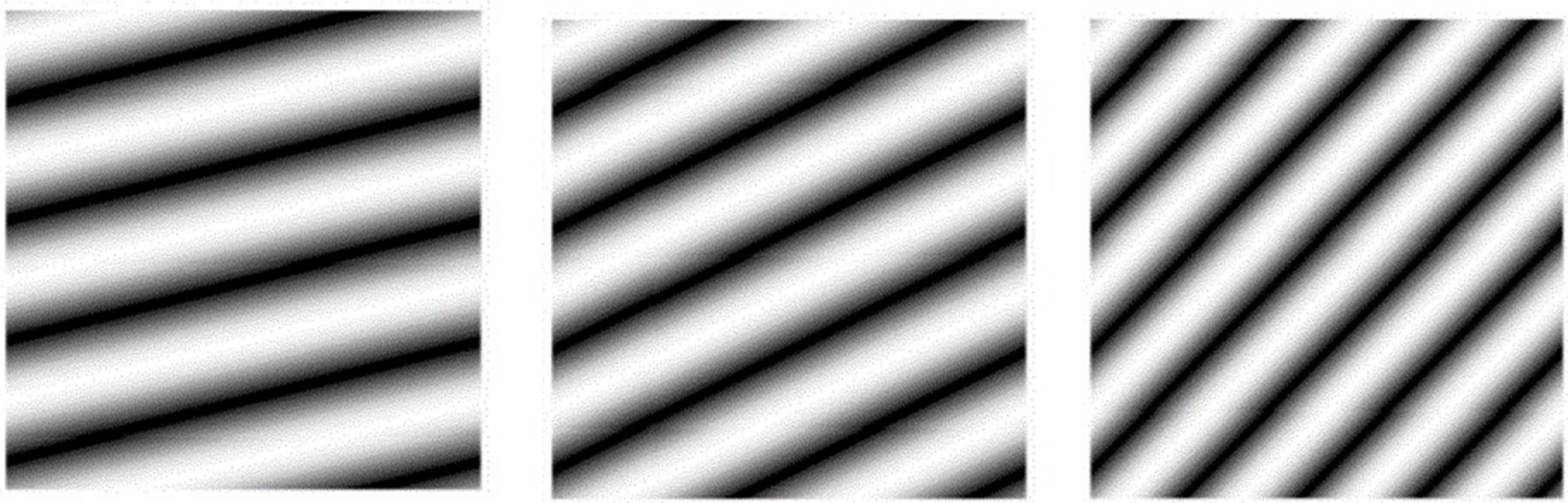


- Intensity images for $s(x,y)=\sin\left[2\pi\left(u_0x+v_0y\right)\right]$



$u_0 = 1, v_0 = 0$ $u_0 = 2, v_0 = 0$ $u_0 = 4, v_0 = 0$





$u_0 = 4, v_0 = 1$ $u_0 = 4, v_0 = 2$ $u_0 = 4, v_0 = 4$

Frequency Domain Image Enhancement

$f(x,y)$ is an $M \times N$ digital image

DFT analysis

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$u = 0, 1, \cdots, M-1 \quad v = 0, 1, \cdots, N-1 \qquad j = \sqrt{-1}$

Frequency Domain Image Enhancement

$f(x,y)$ is an $M \times N$ digital image

DFT analysis $$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

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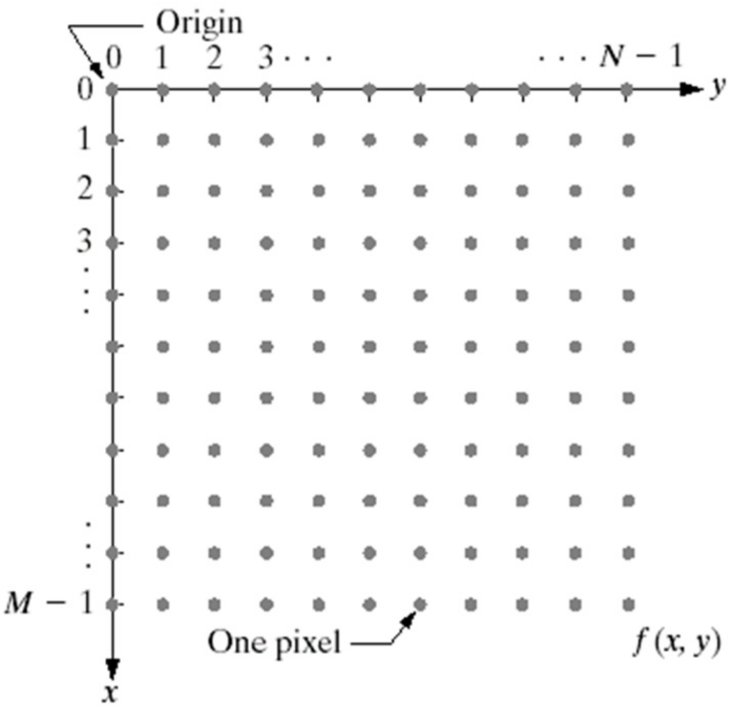
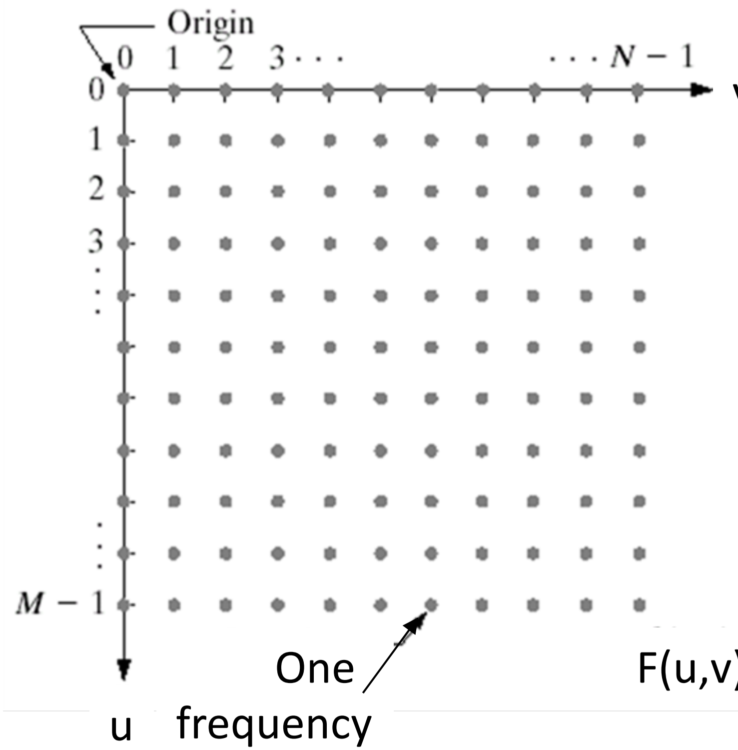
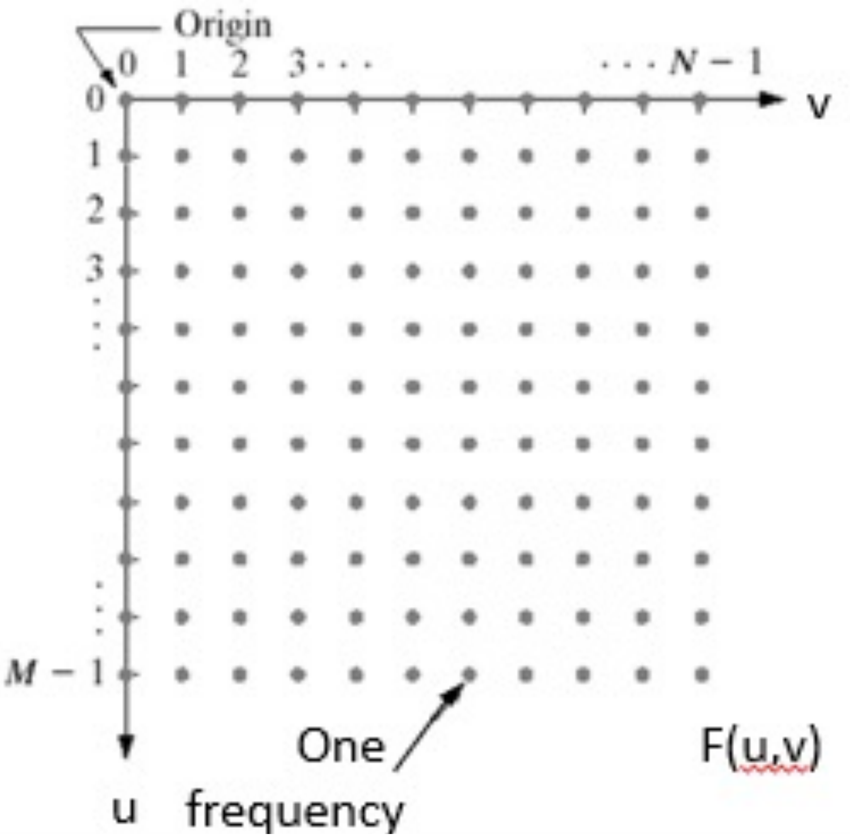


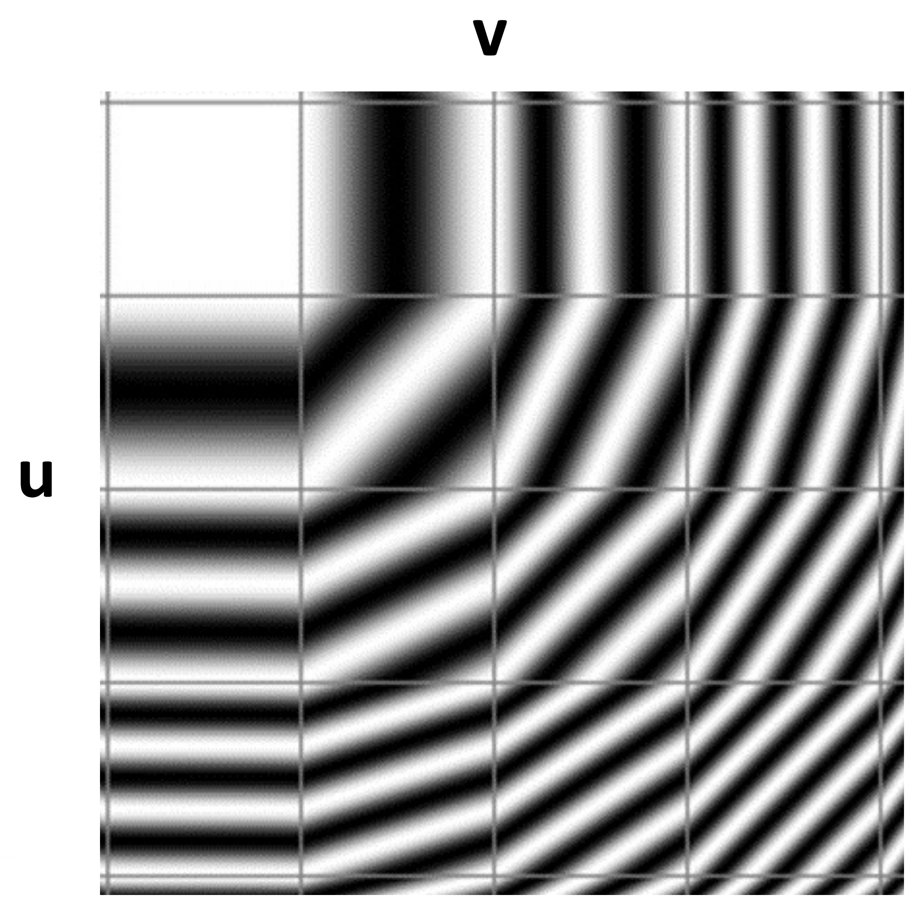


F(u,v) is how much of frequency (u,v) is in f(x,y)

F(u,v)



(u,v) frequencies



Frequency Domain Image Enhancement

$f(x,y)$ is an $M \times N$ digital image

DFT
analysis

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$u = 0, 1, \cdots, M-1 \qquad v = 0, 1, \cdots, N-1 \qquad j = \sqrt{-1}$$

IDFT
synthesis

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$x = 0, 1, \cdots, M-1 \qquad y = 0, 1, \cdots, N-1$$

Frequency Domain Image Enhancement

$f(x,y)$ is an $M \times N$ digital image

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$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$u = 0, 1, \cdots, M-1 \quad v = 0, 1, \cdots, N-1 \qquad j = \sqrt{-1}$

IDFT synthesis

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$x = 0, 1, \cdots, M-1 \quad y = 0, 1, \cdots, N-1$

DC $\quad F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$

Magnitude spectrum $|F(u,v)| = \left(R^2(u,v) + I^2(u,v)\right)^{1/2}$

$R(u,v) = \text{real}\left[F(u,v)\right]$ $\quad I(u,v) = \text{imaginary}\left[F(u,v)\right]$

Magnitude spectrum $|F(u,v)| = \left(R^2(u,v) + I^2(u,v)\right)^{1/2}$

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If $f(x,y)$ is real, then $F(u,v) = F^*(-u,-v)$

$$|F(u,v)| = |F(-u,-v)|$$

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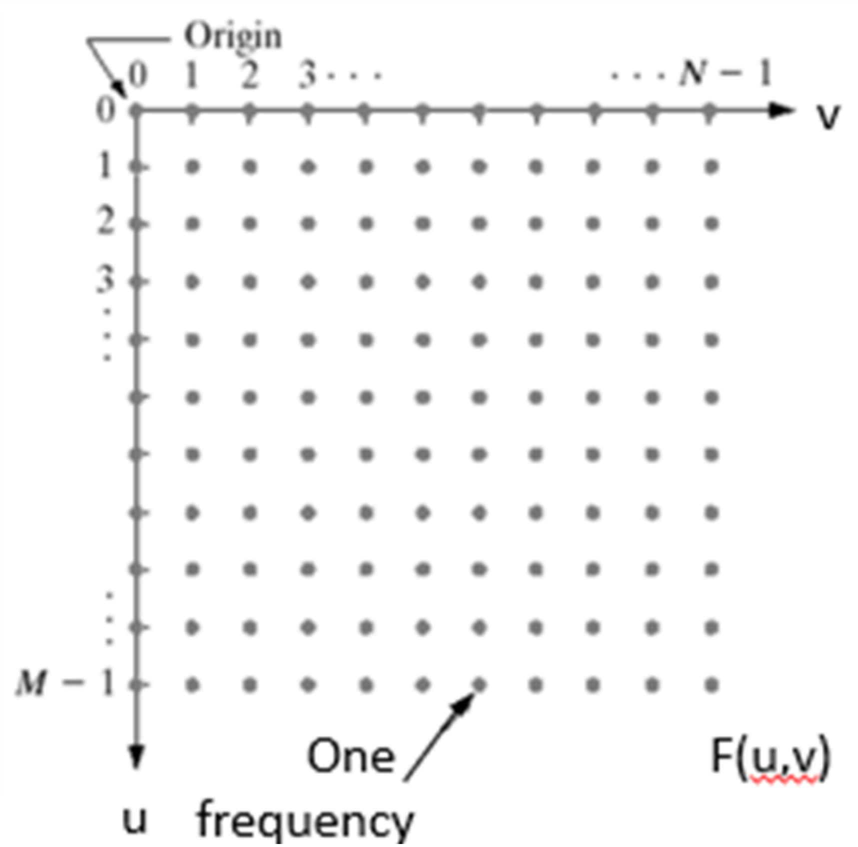
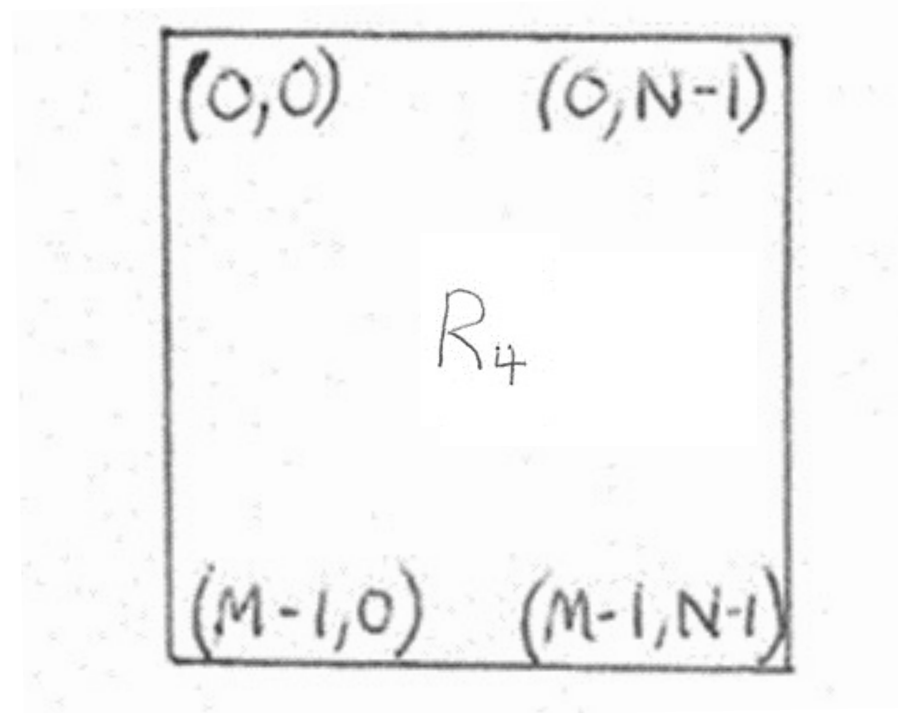
If $f(x,y)$ is real, then $F(u,v) = F^*(-u,-v)$

$|F(u,v)| = |F(-u,-v)|$

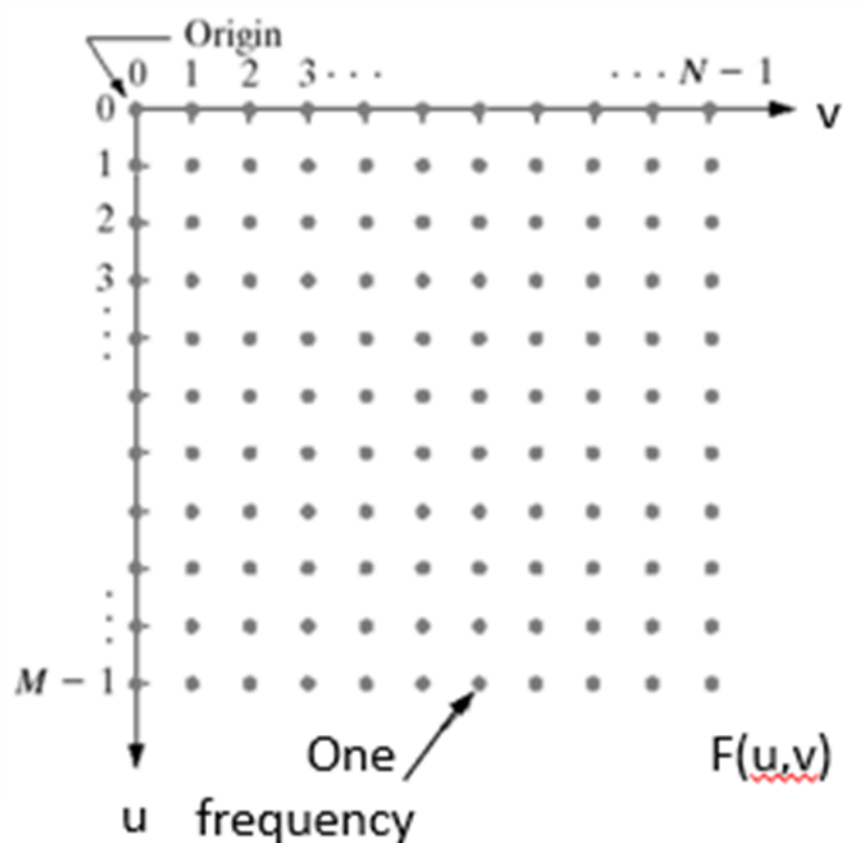
DFT is periodic meaning $F(u + K_1 M, v + K_2 N) = F(u,v)$
for integers K_1 and K_2

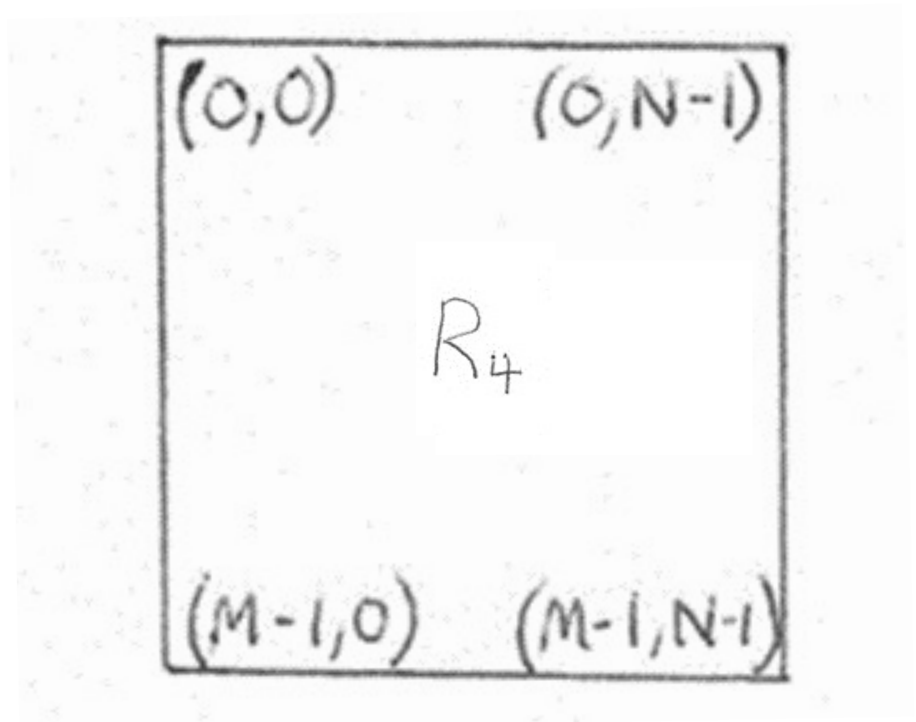
F(u,v)



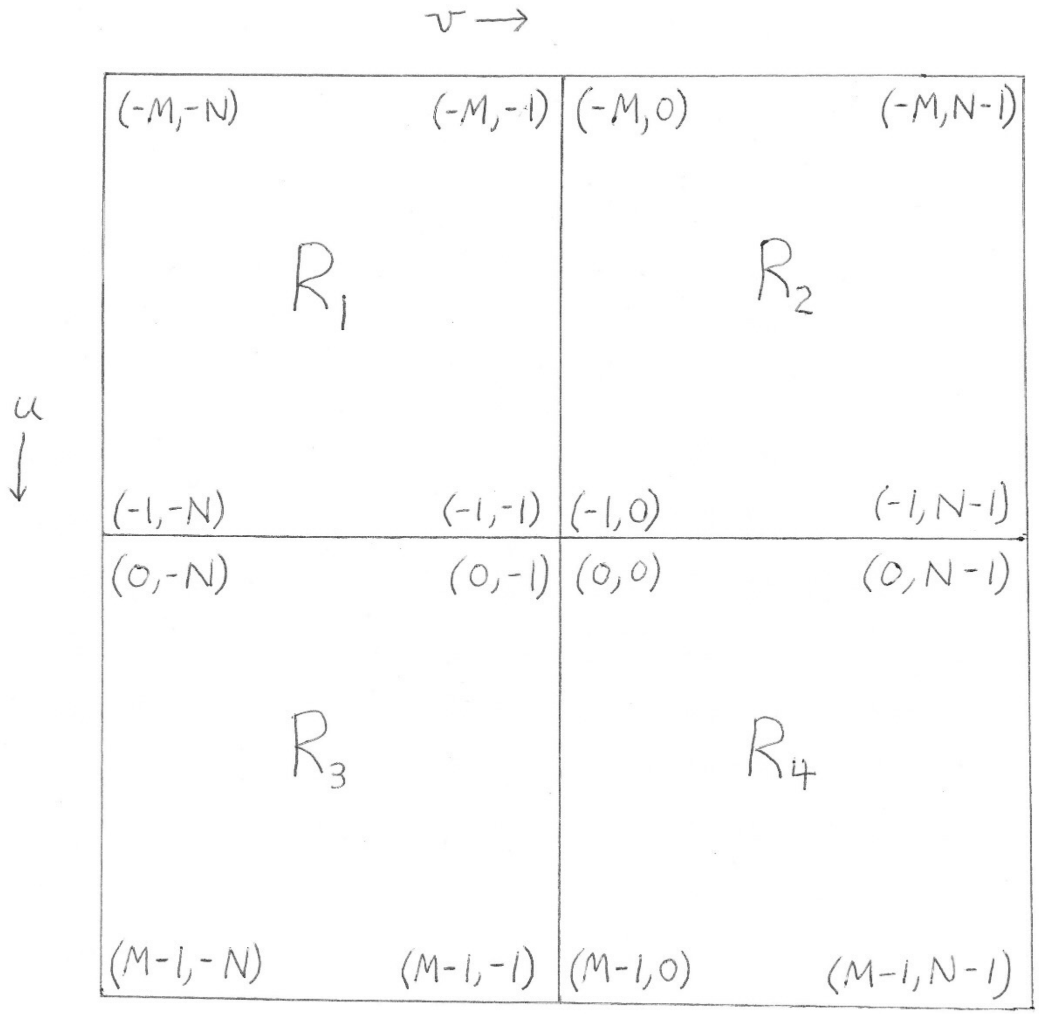


F(u,v)





$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$



We compute the DFT over region R_4. Periodic means the DFT computed over any of the regions R_i will be the same.

Symmetry and Periodicity

$$F(u) = F(u+M) \qquad \text{periodic}$$

$$|F(u)| = |F(-u)| \qquad \text{symmetric}$$

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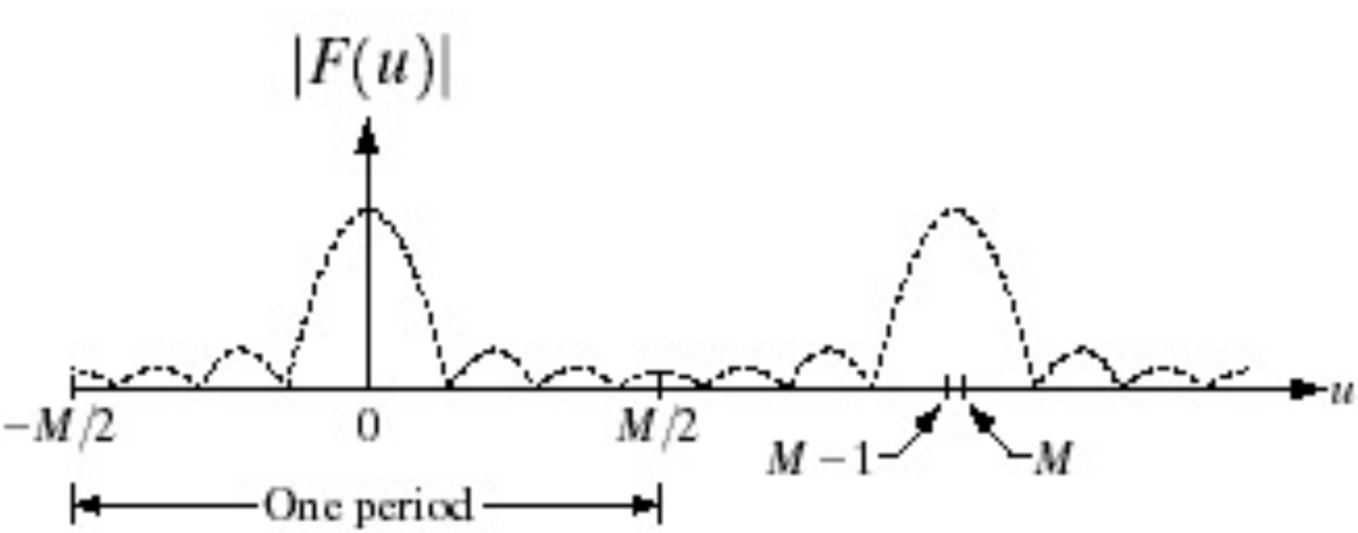

FIGURE 4.34

Symmetry and Periodicity

$F(u) = F(u+M)$ periodic

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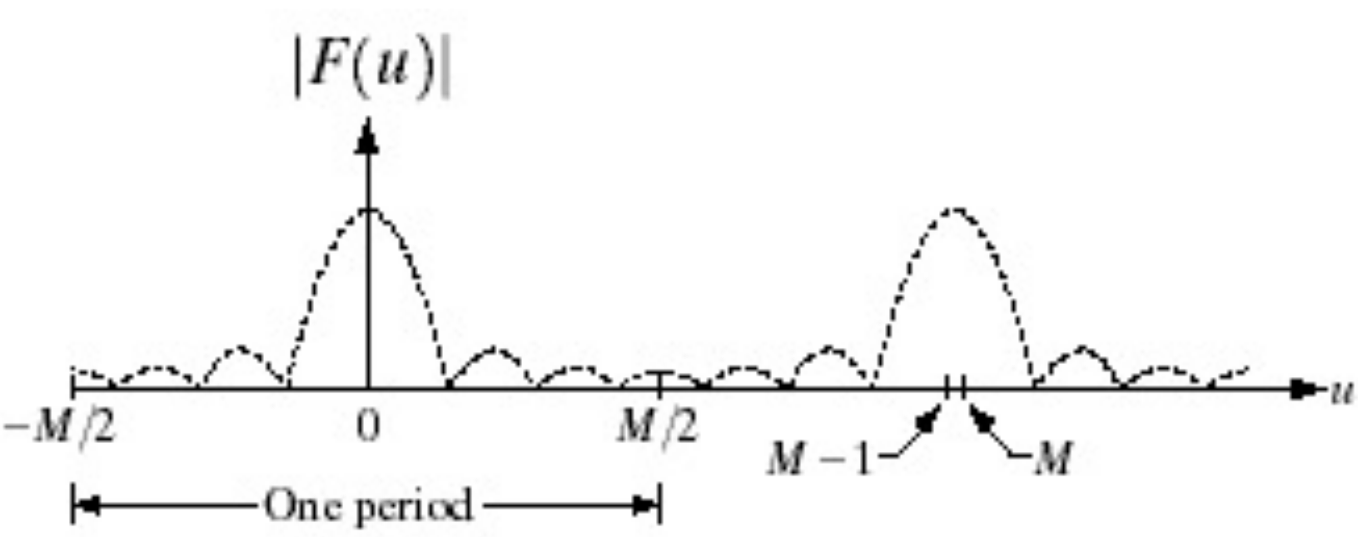
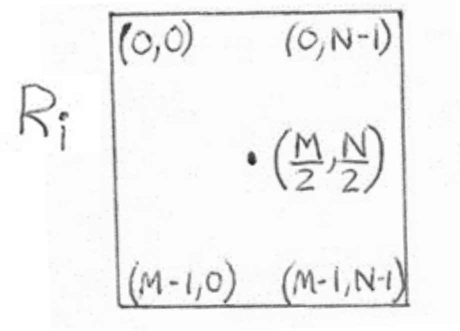


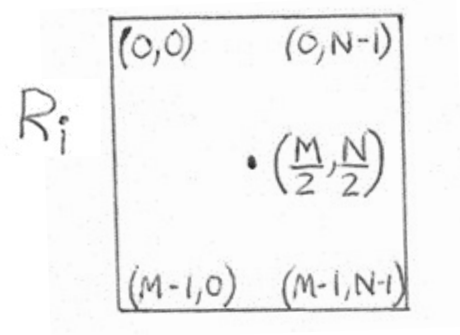
FIGURE 4.34

The highest frequency is at $u = \frac{M}{2}$ because frequencies between $\frac{M}{2}$ and M give copies of $|F(u)|$ for frequencies between $-\frac{M}{2}$ and 0



Center is maximum frequency.

4 corners are DC or adjacent to DC.



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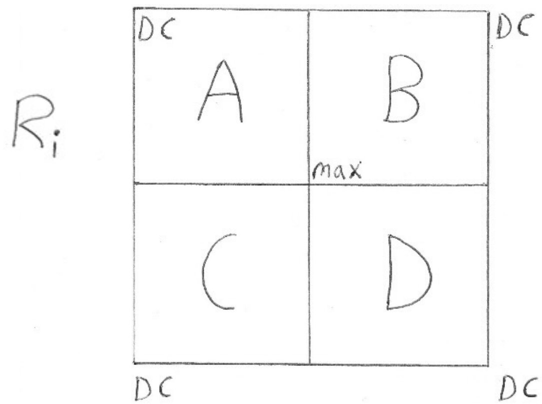
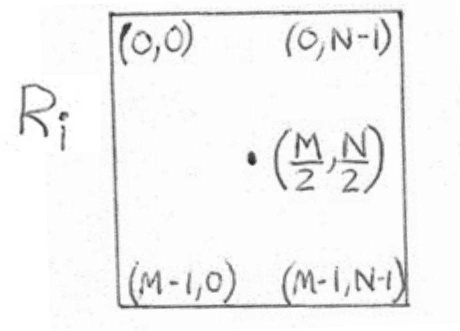
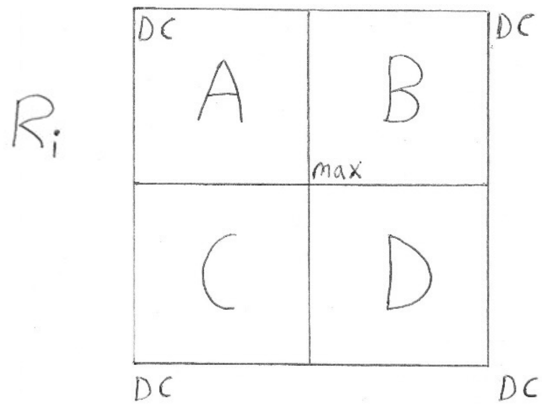
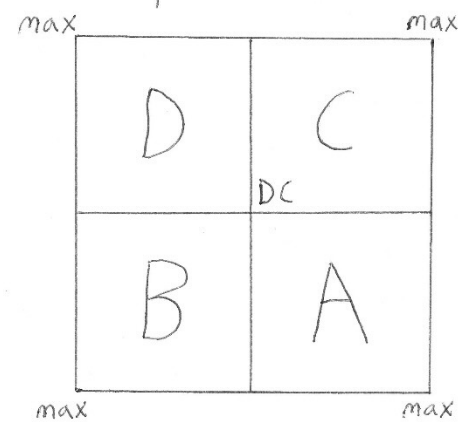




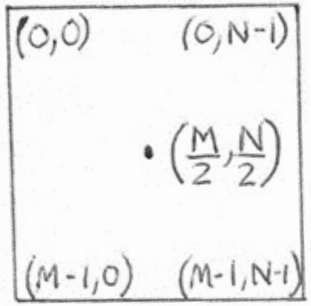
Center is maximum frequency.
4 corners are DC or adjacent to DC.

Move A, B, C, D to put DC in center, max at corners





R_i



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Move A, B, C, D to put DC in center, max at corners

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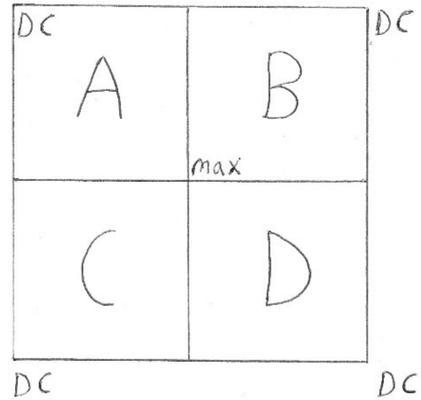
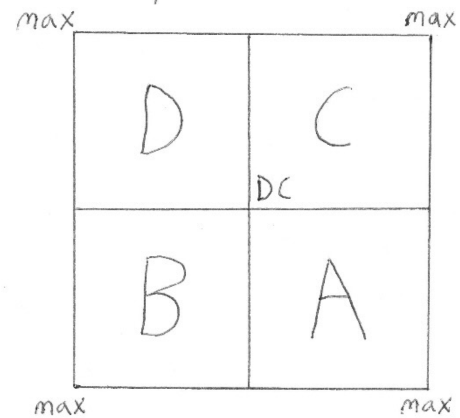
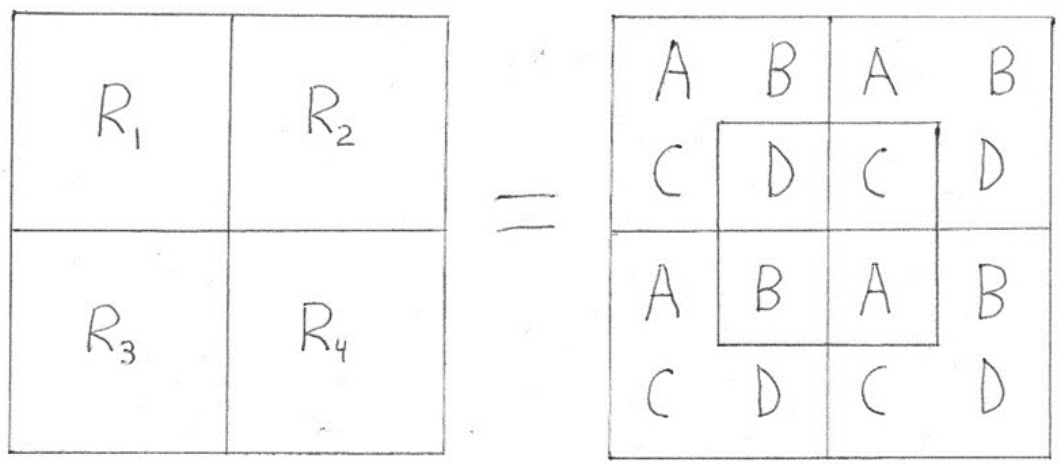
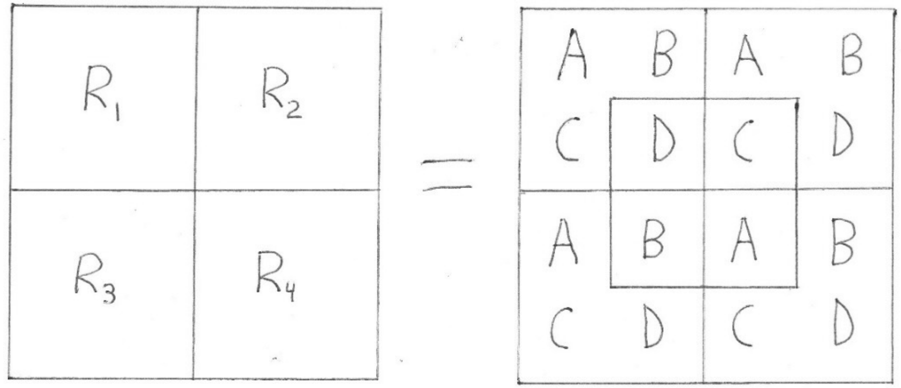








The central region contains the same information as R_4.
We usually display the central region to bring $F(0,0)$
to the center of the displayed image.

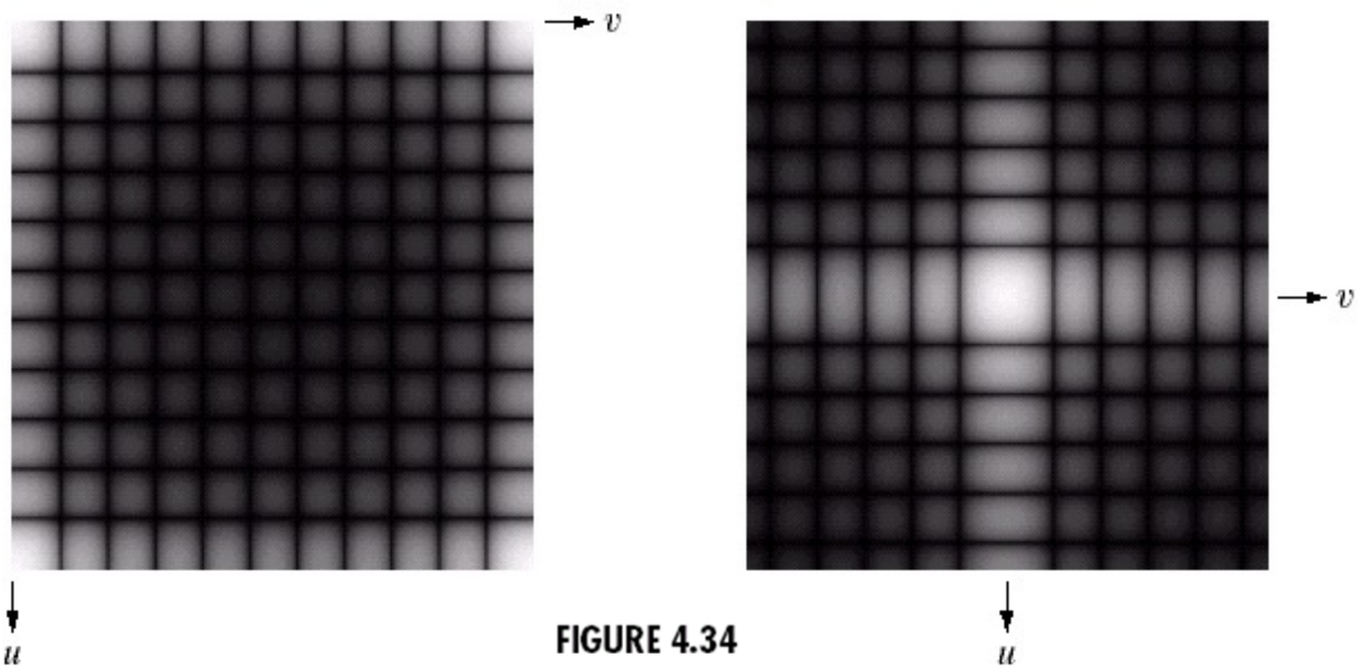


FIGURE 4.34

origin shift

$\mathcal{F}[f(x,y)] = F(u,v)$ DC at $(u,v) = (0,0)$

origin shift

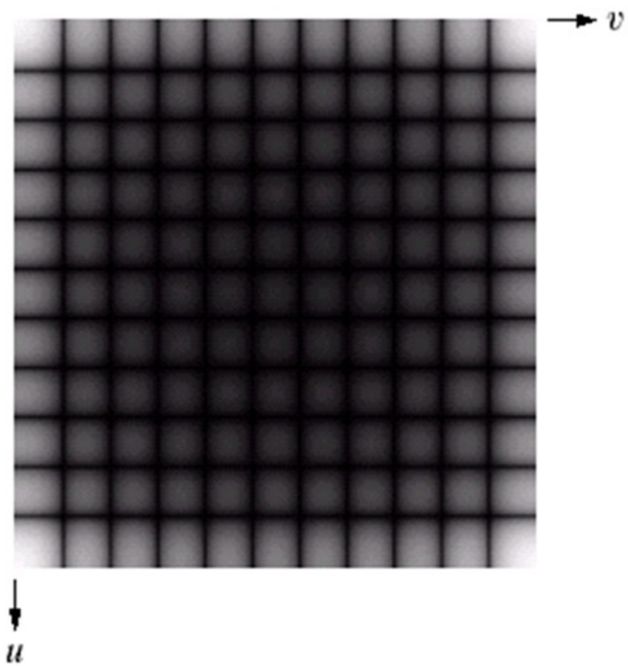
$\mathcal{F}[f(x,y)] = F(u,v)$ DC at $(u,v)=(0,0)$

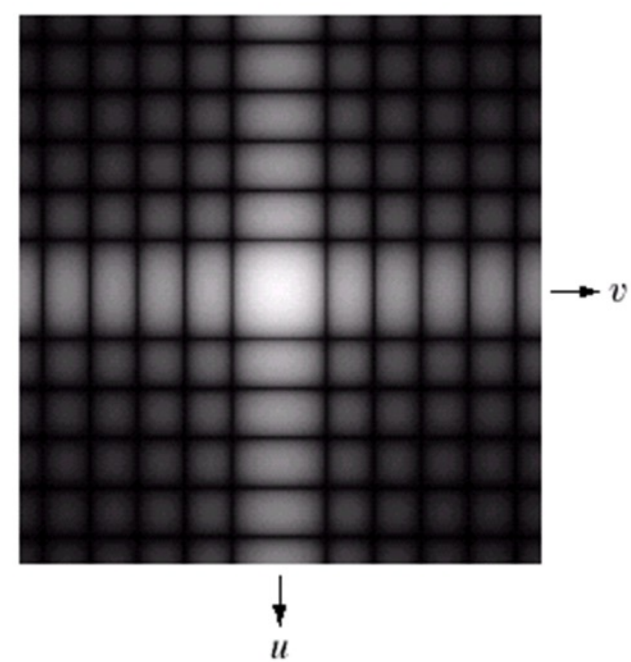
$\mathcal{F}[f(x,y)(-1)^{x+y}] = F(u-\frac{M}{2}, v-\frac{N}{2})$ DC at $(u,v)=(\frac{M}{2}, \frac{N}{2})$

origin shift

$\mathcal{F}[f(x,y)] = F(u,v)$ DC at $(u,v)=(0,0)$

$\mathcal{F}[f(x,y)(-1)^{x+y}] = F(u-\frac{M}{2}, v-\frac{N}{2})$ DC at $(u,v)=(\frac{M}{2}, \frac{N}{2})$





Ex $f(x,y) = \cos(2\pi u_0 x + 2\pi v_0 y)$

Ex $f(x,y) = \cos(2\pi u_0 x + 2\pi v_0 y)$

$$F(u,v) = \frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$$

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$$F(u,v) = \frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$$

$v_0 = 0 \quad f(x,y) = \cos(2\pi u_0 x)$

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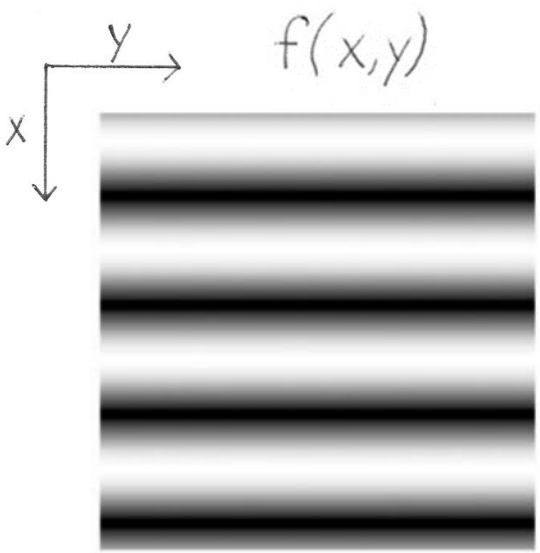

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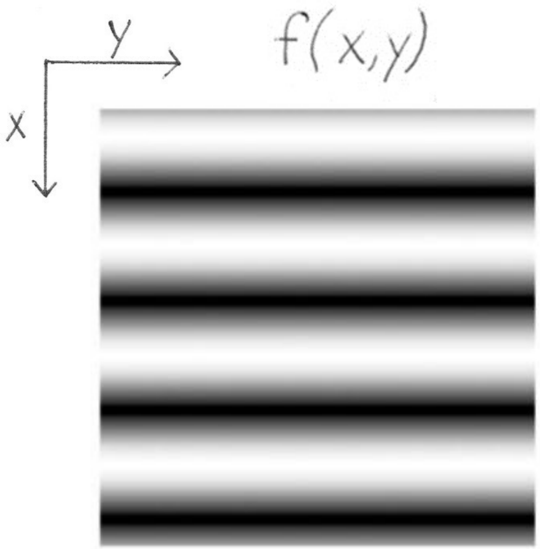
$$v_0 = 0 \qquad f(x,y) = \cos(2\pi u_0 x)$$

$$F(u,v) = \frac{1}{2}\left[\delta(u+Mu_0, v) + \delta(u-Mu_0, v)\right]$$



(Ex) $f(x,y) = \cos(2\pi u_0 x + 2\pi v_0 y)$

$$F(u,v) = \frac{1}{2}\left[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\right]$$

$$v_0 = 0 \qquad f(x,y) = \cos(2\pi u_0 x)$$

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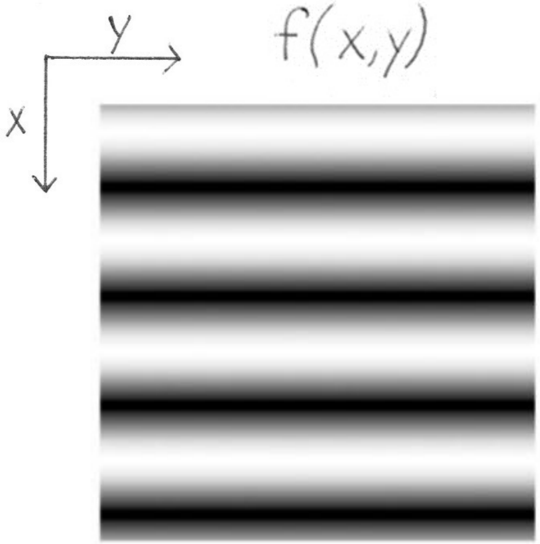

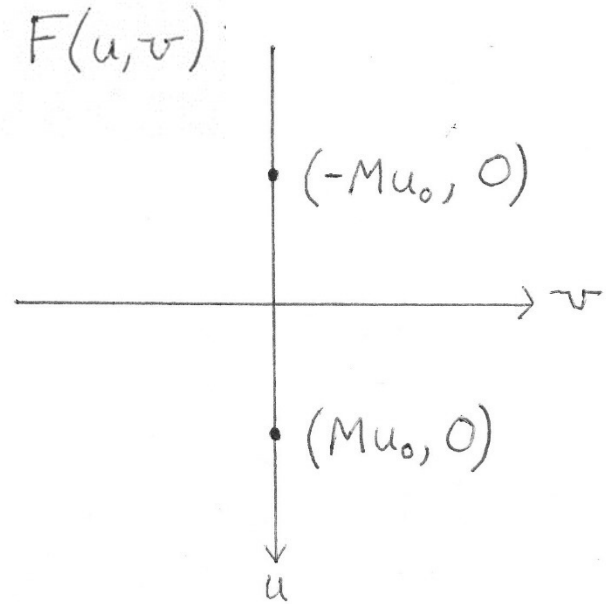

$u_0 = 0 \qquad f(x,y) = \cos(2\pi v_0 y)$

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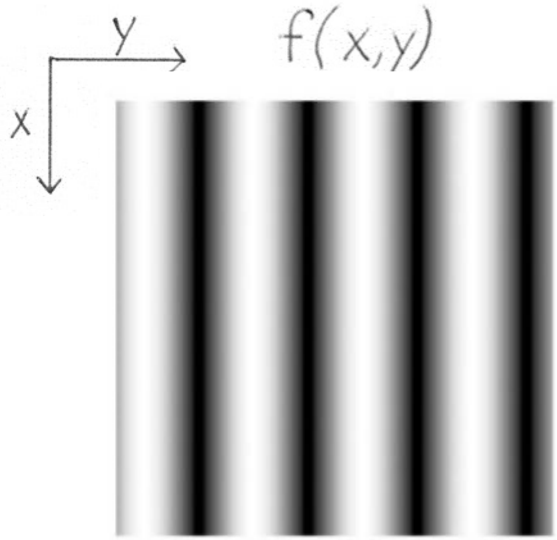
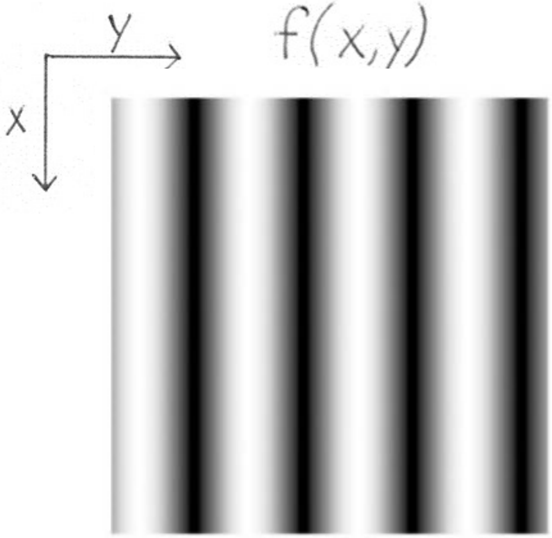


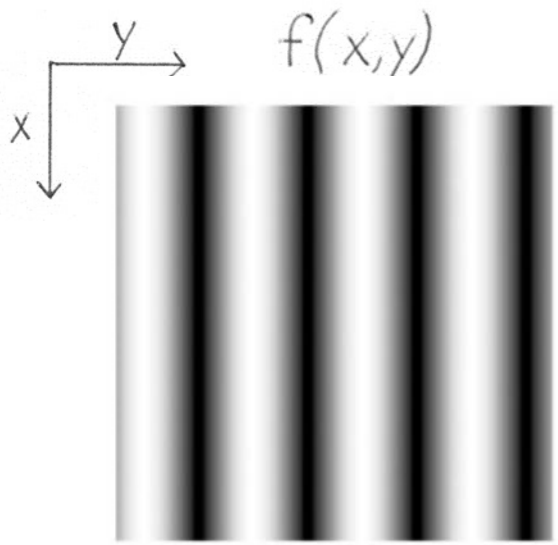
$u_0 = 0 \qquad f(x,y) = \cos(2\pi v_0 y)$

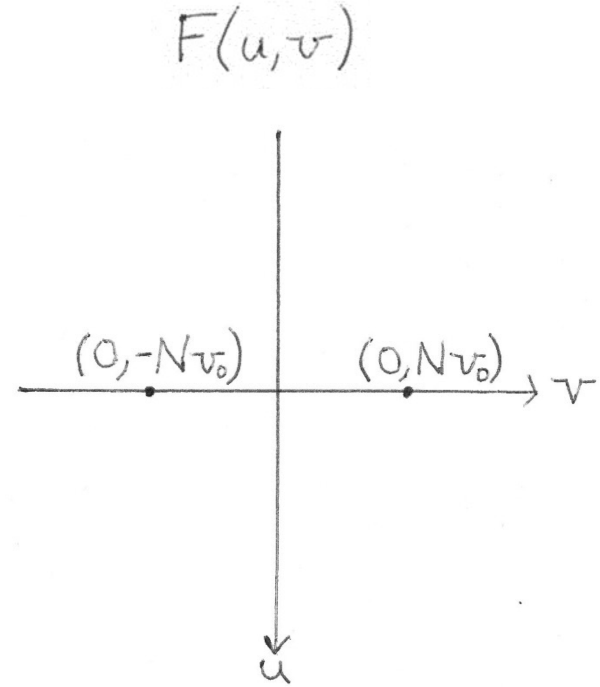
$$F(u,v) = \frac{1}{2}\left[\delta(u, v + N v_0) + \delta(u, v - N v_0)\right]$$



$$u_0 = 0 \qquad f(x,y) = \cos(2\pi v_0 y)$$

$$F(u,v) = \frac{1}{2}\left[\delta(u, v + N v_0) + \delta(u, v - N v_0)\right]$$





Frequency domain components tend to be aligned perpendicular to the orientation of lines in an image

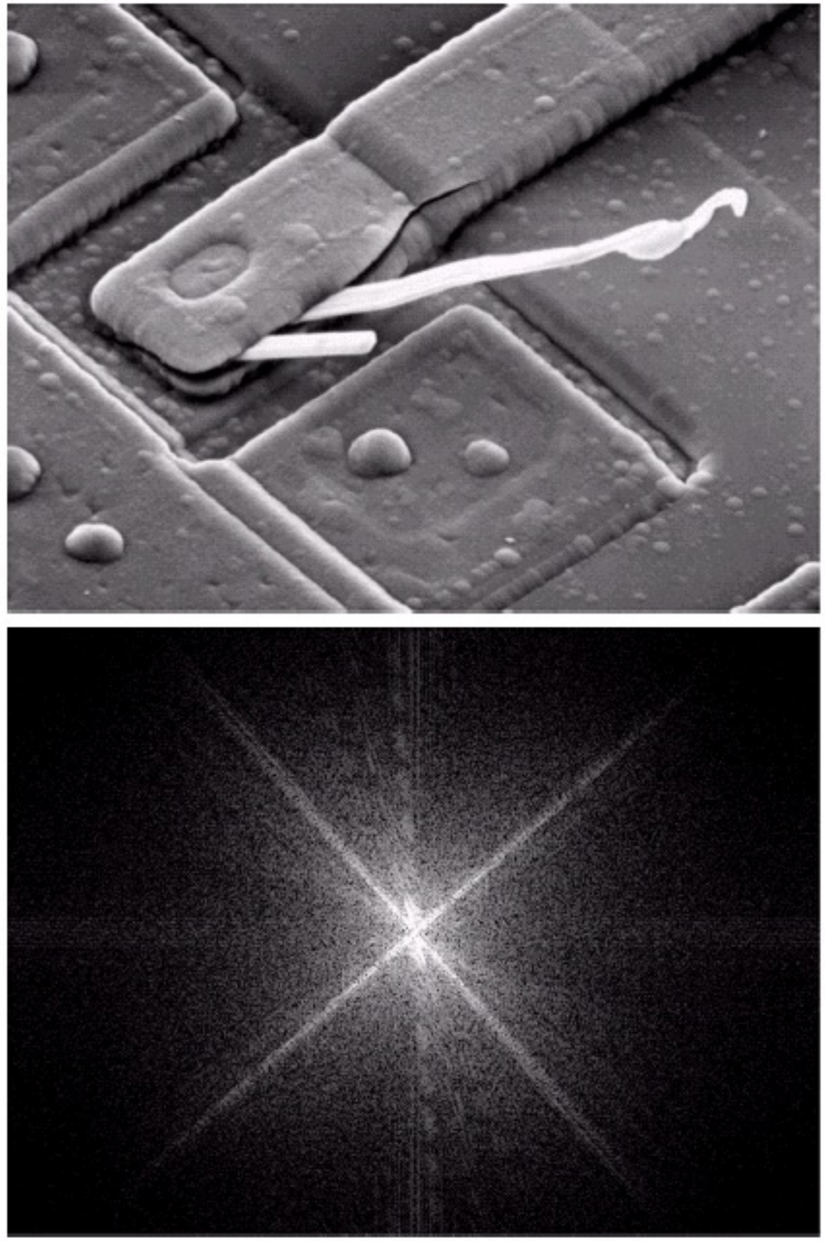
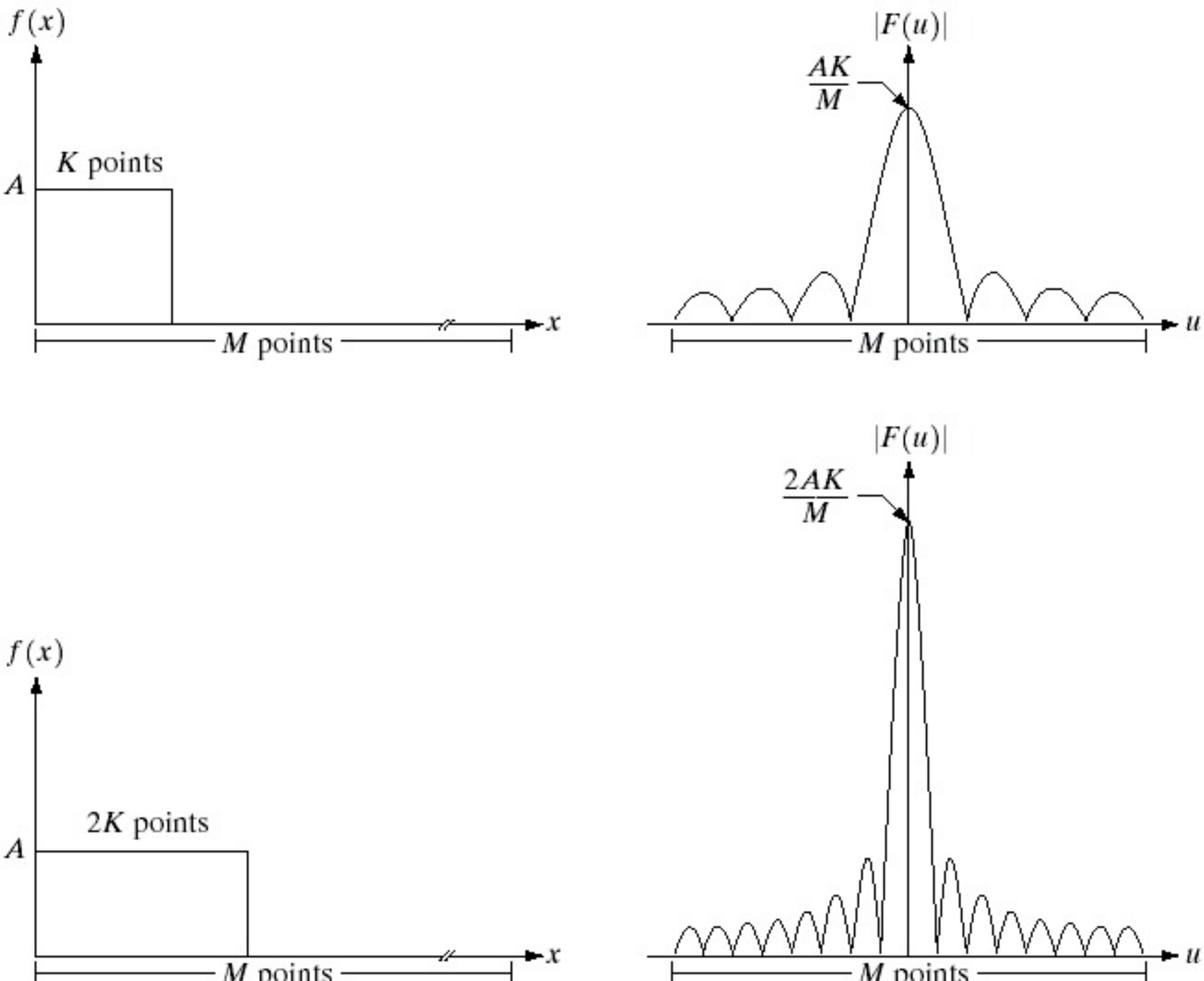


FIGURE 4.4
(a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)



a b
c d

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

a b

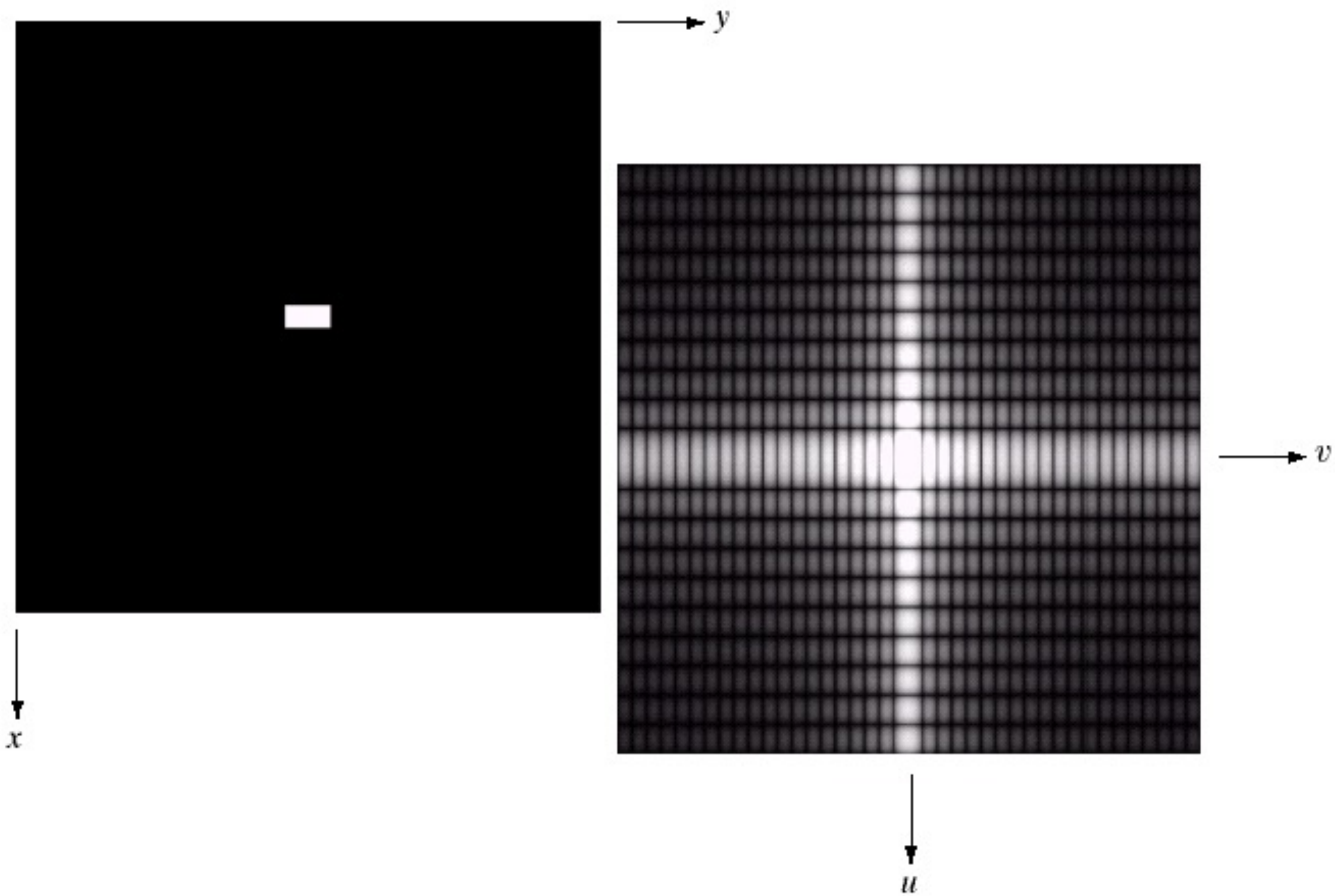


FIGURE 4.3
(a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

Filtering in the Frequency Domain

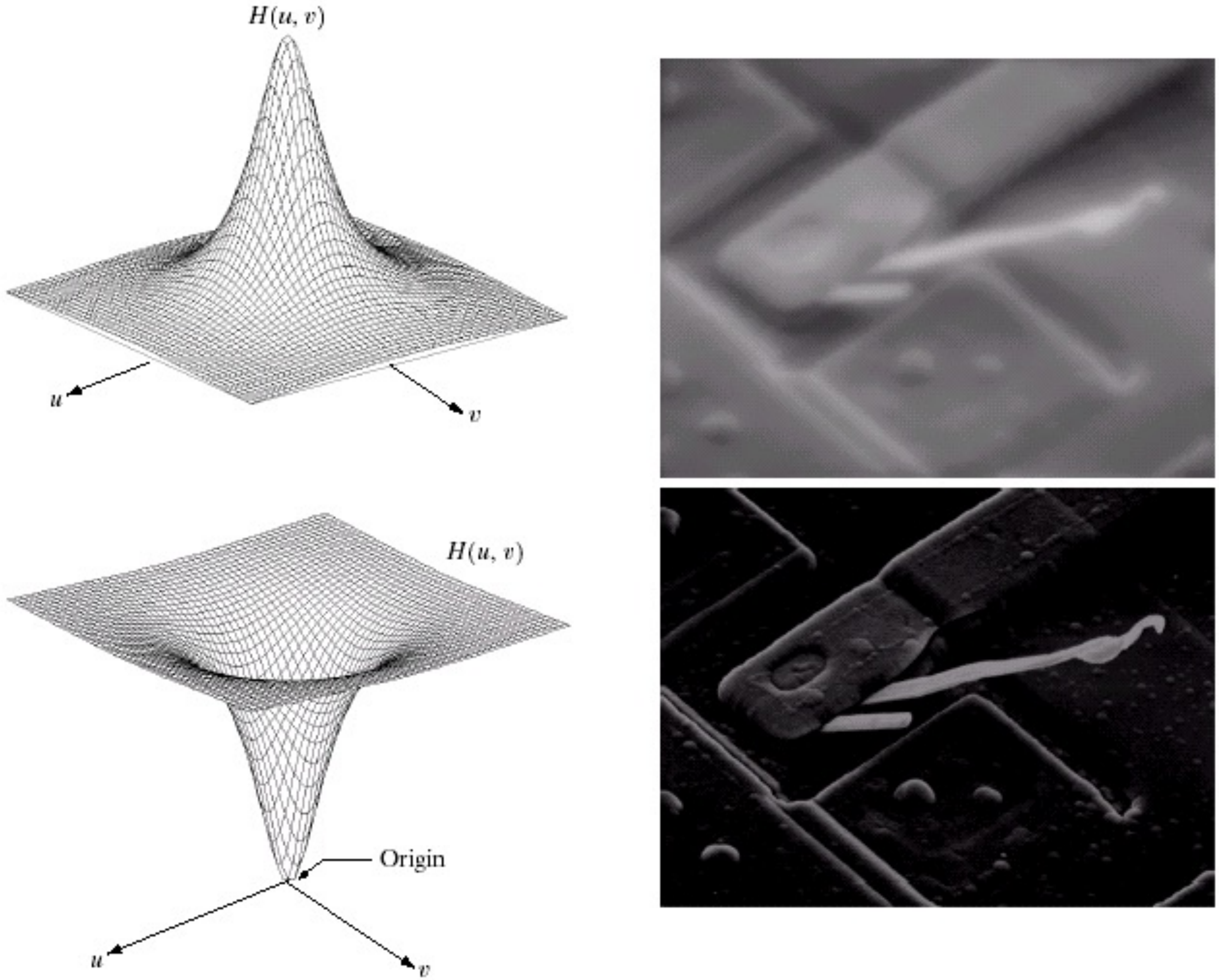
$f(x,y)$ is an $M \times N$ image with DFT $F(u,v)$

$h(x,y)$ is an $M \times N$ filter with DFT $H(u,v)$

$$\mathcal{F}[f(x,y) * h(x,y)] = F(u,v)H(u,v)$$

If we have a filter specified in the frequency domain by $H(u,v)$, we can obtain the filtered image $f(x,y) * h(x,y)$ by $\mathcal{F}^{-1}[F(u,v)H(u,v)]$

We can also compute $h(x,y) = \mathcal{F}^{-1}[H(u,v)]$ and then filter in the space domain using $f(x,y) * h(x,y)$



a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Ex Gaussian Filters

Lowpass $H(u) = A e^{-u^2/2\sigma^2}$ frequency

Ex Gaussian Filters

Lowpass $H(u) = Ae^{-u^2/2\sigma^2}$ frequency

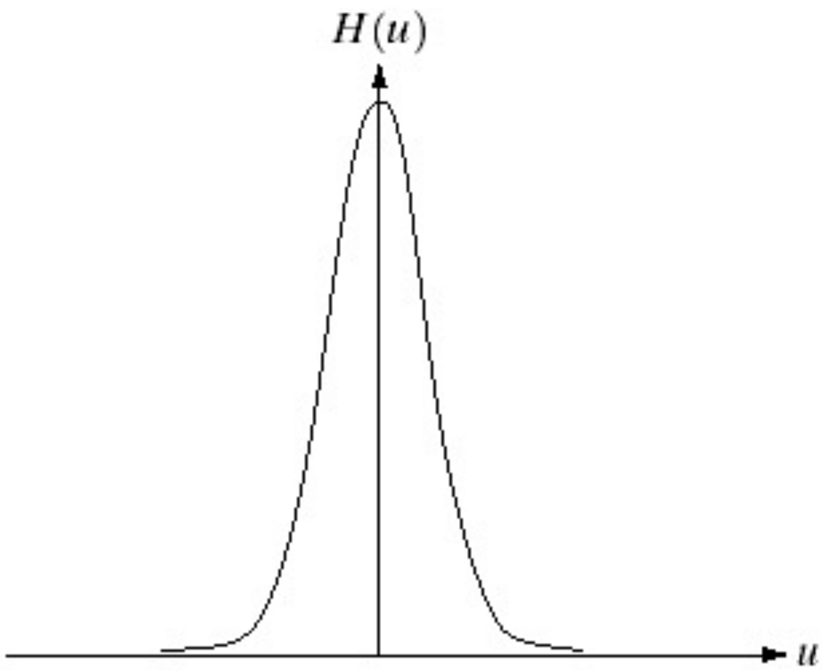


FIGURE 4.9

Ex Gaussian Filters

Lowpass $H(u) = Ae^{-u^2/2\sigma^2}$ frequency

$h(x) = \sqrt{2\pi}\,\sigma A e^{-2\pi^2\sigma^2 x^2}$ space

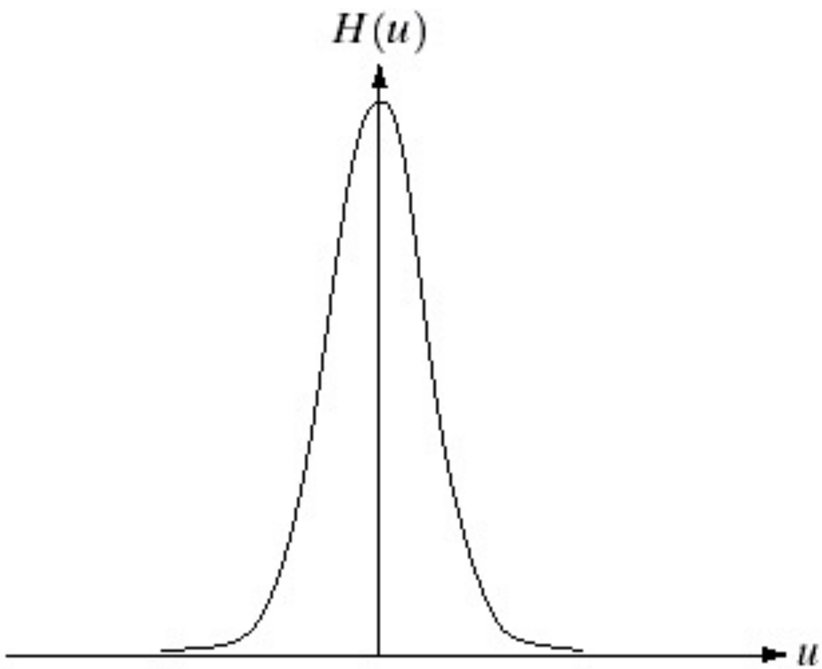


FIGURE 4.9

Ex Gaussian Filters

Lowpass $H(u) = Ae^{-u^2/2\sigma^2}$ frequency

$h(x) = \sqrt{2\pi}\,\sigma A e^{-2\pi^2\sigma^2 x^2}$ space

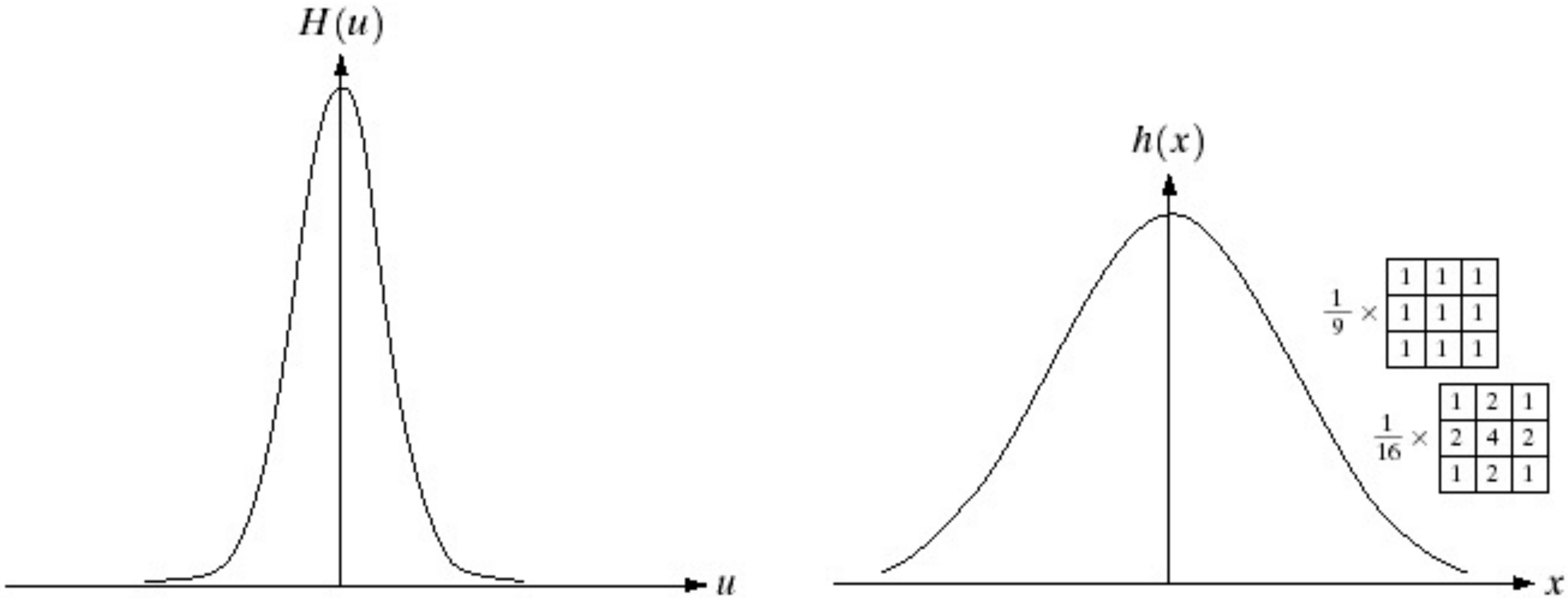


FIGURE 4.9

Highpass difference of Gaussians

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2} \quad \text{frequency}$$

$$A \geq B, \ \sigma_1 > \sigma_2$$

Highpass difference of Gaussians

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$ frequency

$$A \geq B, \; \sigma_1 > \sigma_2$$

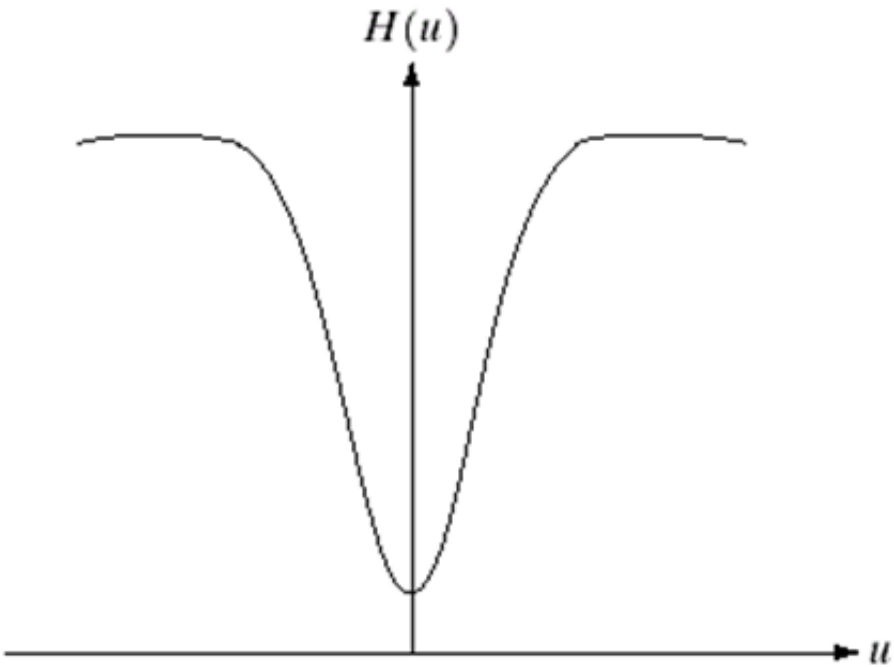


FIGURE 4.9

Highpass difference of Gaussians

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2} \quad \text{frequency}$$

$$A \geq B, \; \sigma_1 > \sigma_2$$

$$h(x) = \sqrt{2\pi}\,\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\,\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2} \quad \text{space}$$

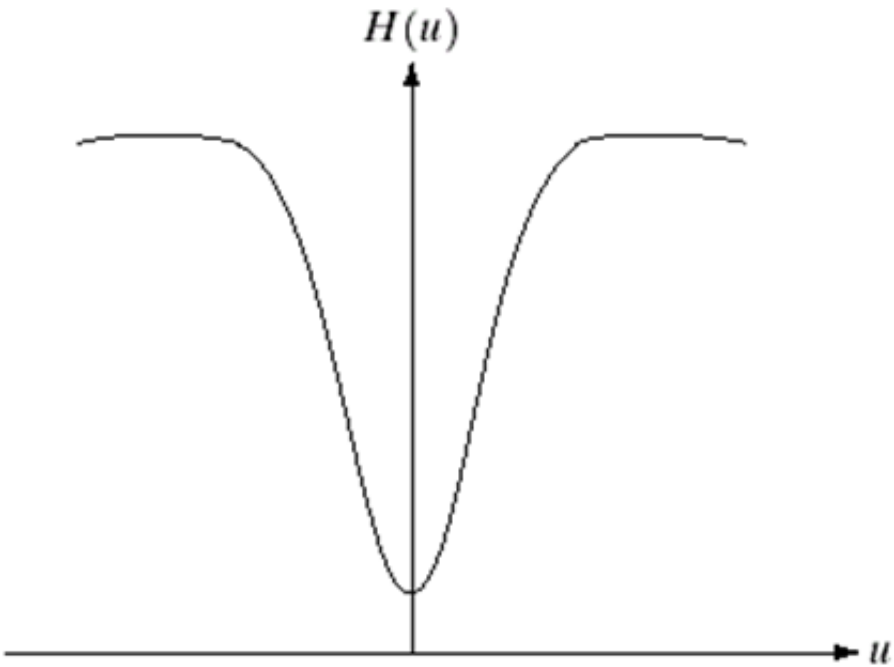


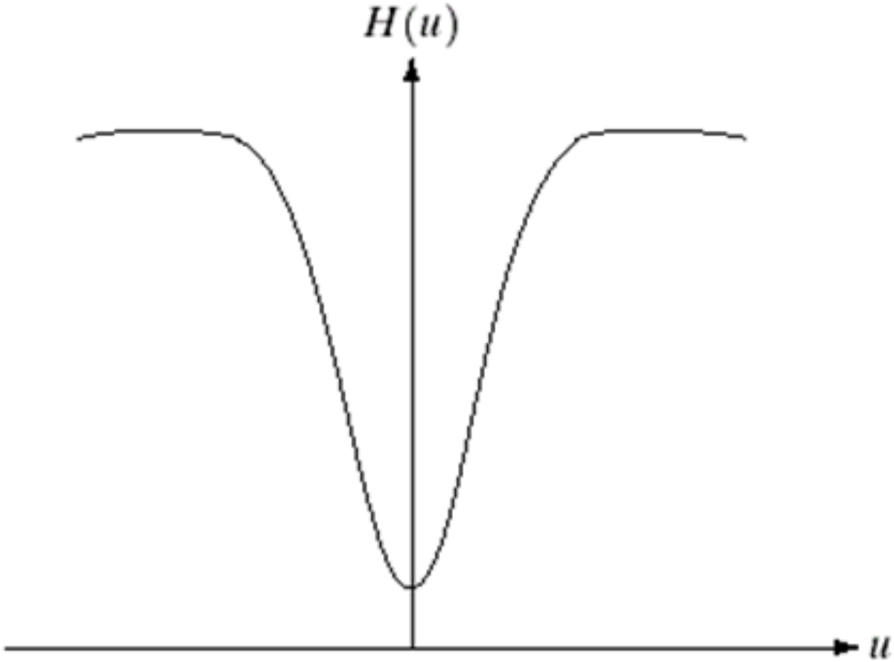
FIGURE 4.9

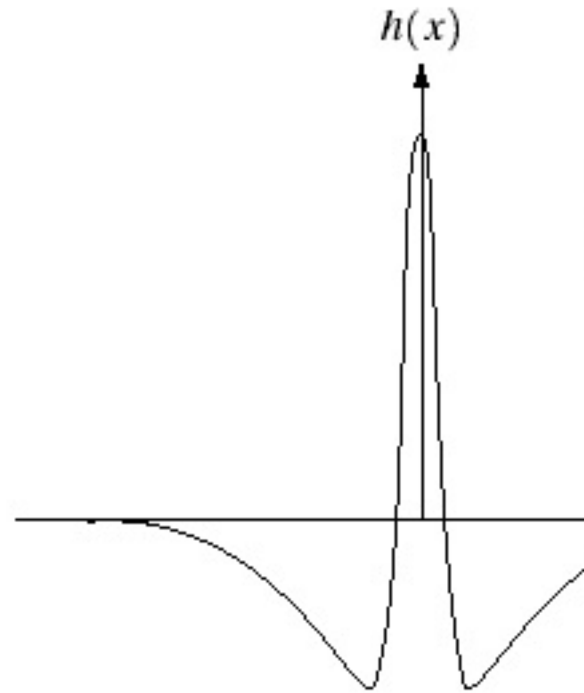
Highpass difference of Gaussians

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2} \quad \text{frequency}$$

$$A \geq B, \; \sigma_1 > \sigma_2$$

$$h(x) = \sqrt{2\pi}\,\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\,\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2} \quad \text{space}$$





−1	−1	−1
−1	8	−1
−1	−1	−1

0	−1	0
−1	4	−1
0	−1	0

FIGURE 4.9

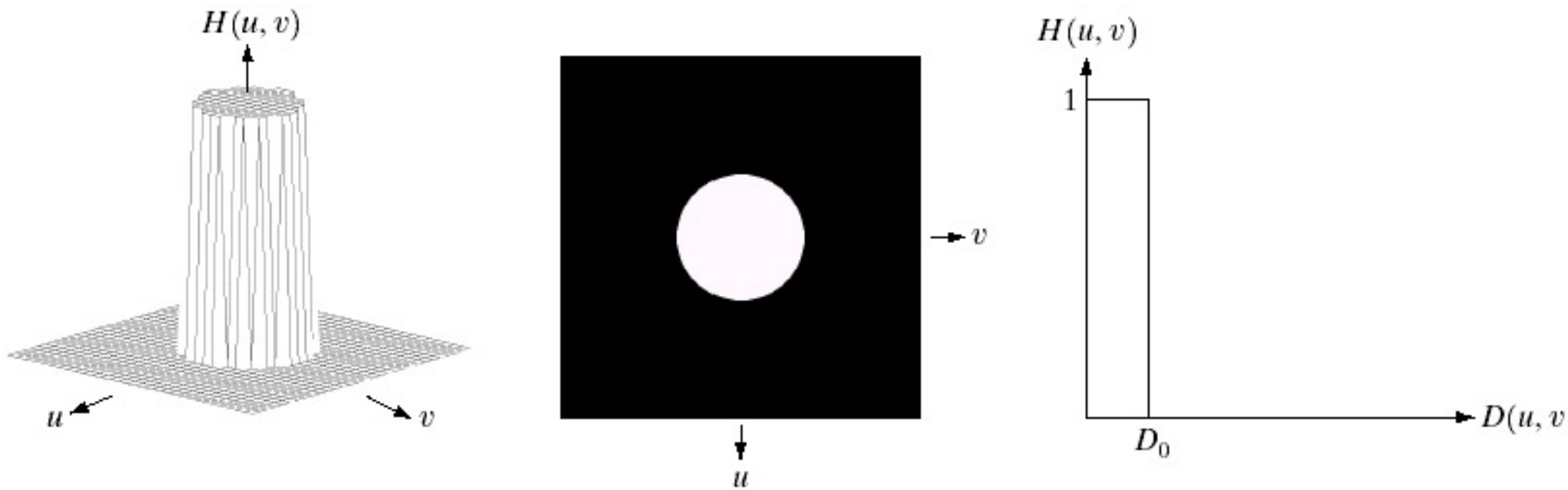
Smoothing Frequency Domain Filters

Smoothing (lowpass) filters pass low frequencies and attenuate high frequencies

Smoothing Frequency Domain Filters

Smoothing (lowpass) filters pass low frequencies and attenuate high frequencies

Ideal Lowpass Filter (ILPF)



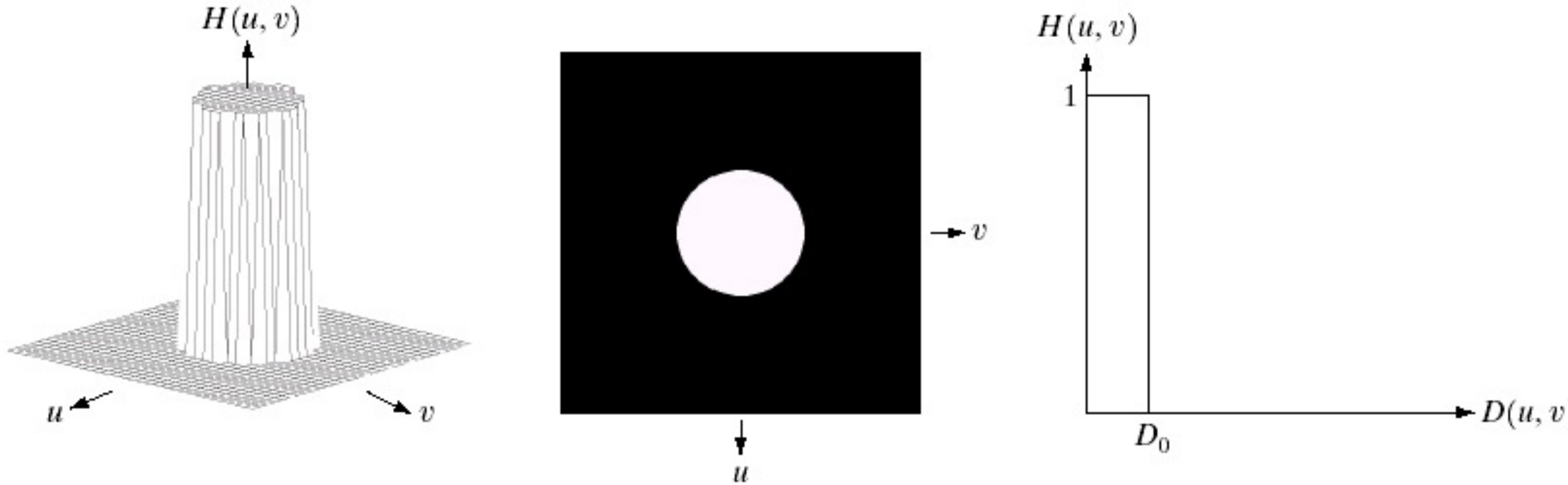
a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Smoothing Frequency Domain Filters

Smoothing (lowpass) filters pass low frequencies and attenuate high frequencies

Ideal Lowpass Filter (ILPF)



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

ILPF is specified by radius D_0 called cutoff frequency

For a given input image $f(x,y)$, we can examine the fraction of power that the ILPF passes.

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Power Spectrum $\quad P(u,v) = |F(u,v)|^2$

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Power Spectrum $\quad P(u,v) = |F(u,v)|^2$

Total Power $\quad P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$

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A circle of radius r in (u,v) space contains a fraction α of the power in an image.

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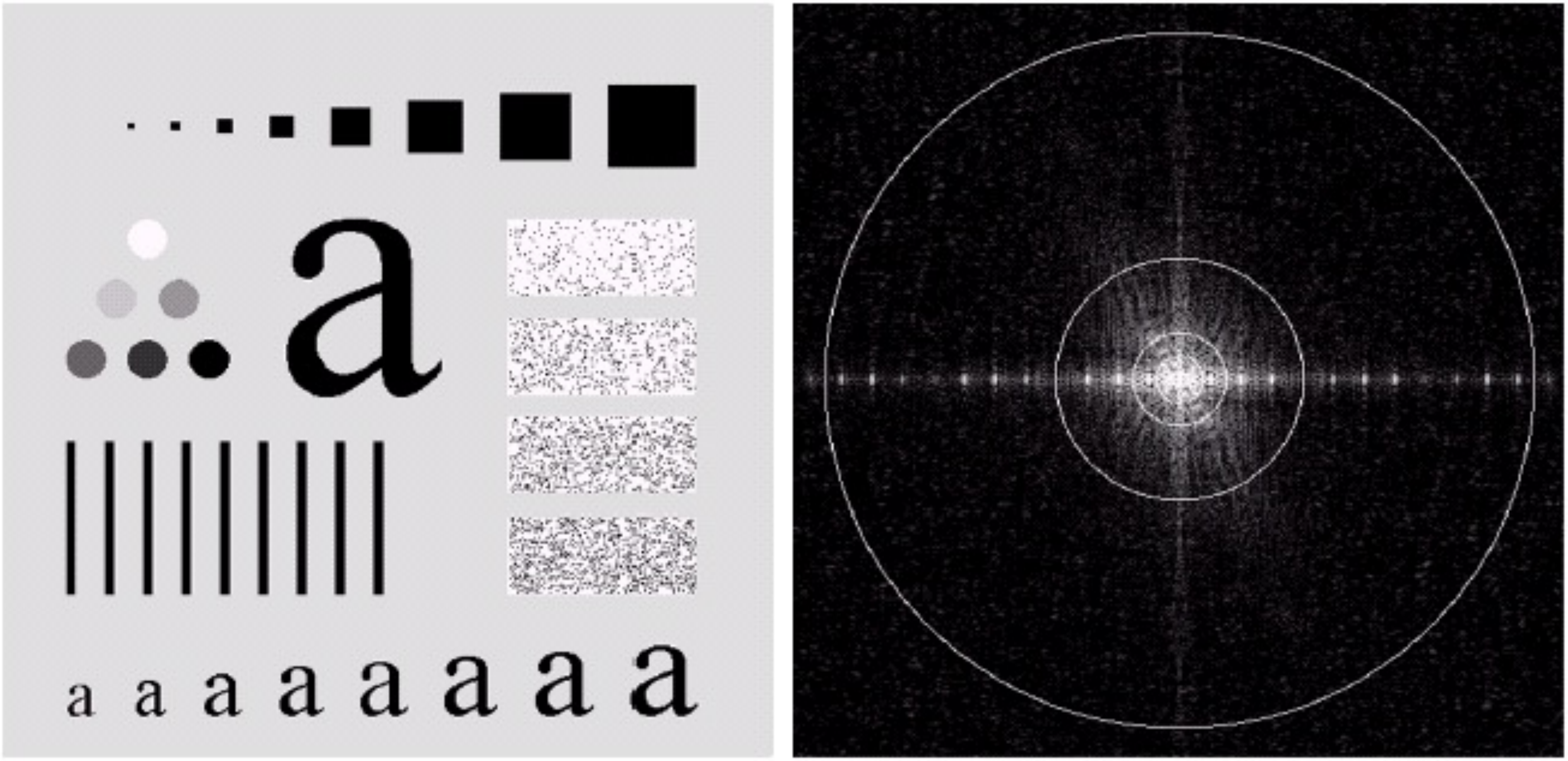
Power Spectrum $\quad P(u,v) = |F(u,v)|^2$

Total Power $\quad P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$

A circle of radius r in (u,v) space contains a fraction α of the power in an image.

$$\alpha = \frac{1}{P_T} \sum_{\substack{u,v \\ \text{such that} \\ \sqrt{u^2+v^2} \leq r}} P(u,v)$$

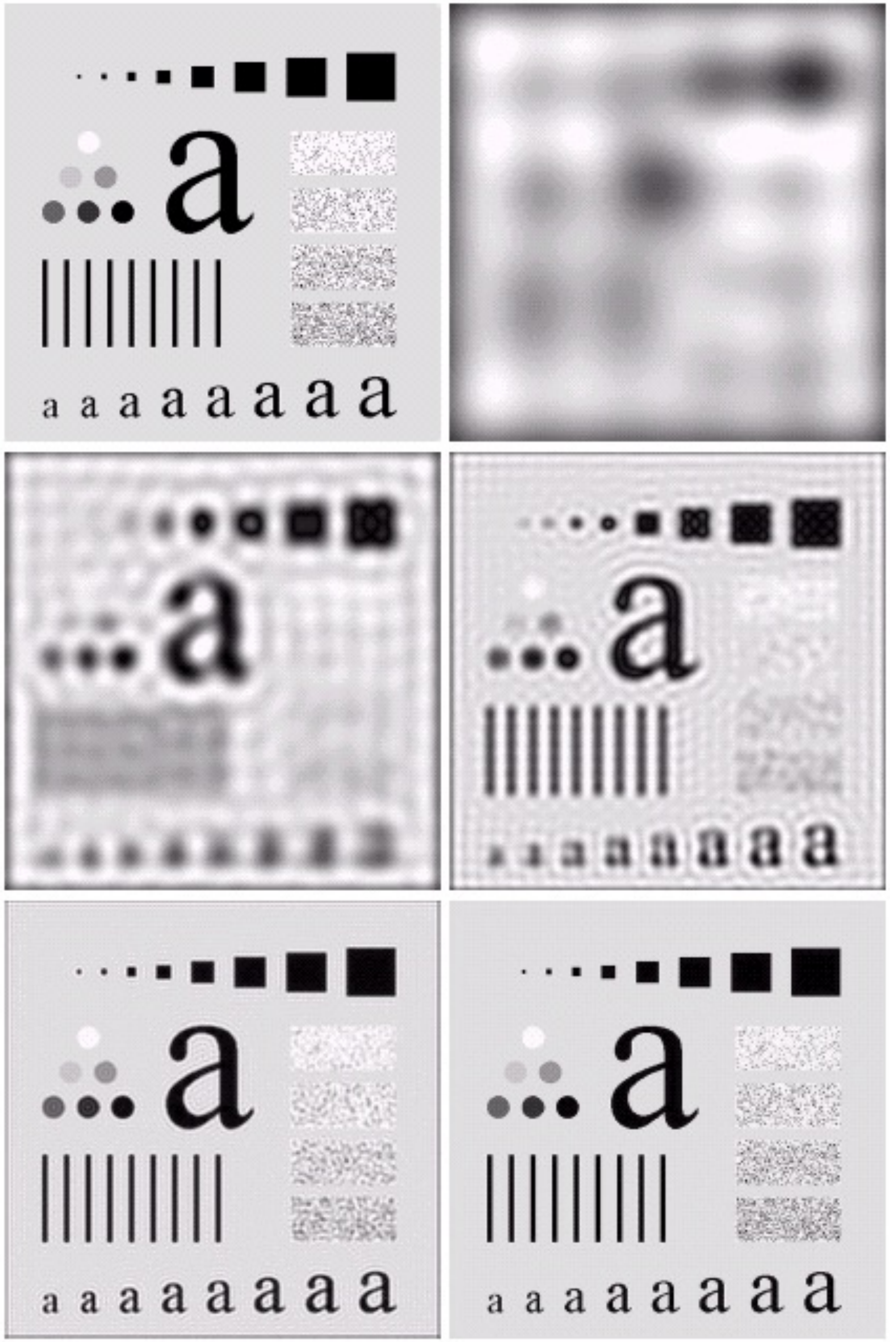


a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

r	5	15	30	80	230
α	0.92	0.946	0.964	0.980	0.995

FIGURE 4.12



r	5	15	30	80	230
α	0.92	0.946	0.964	0.980	0.995

The output image from an ILPF can include ringing and negative values.

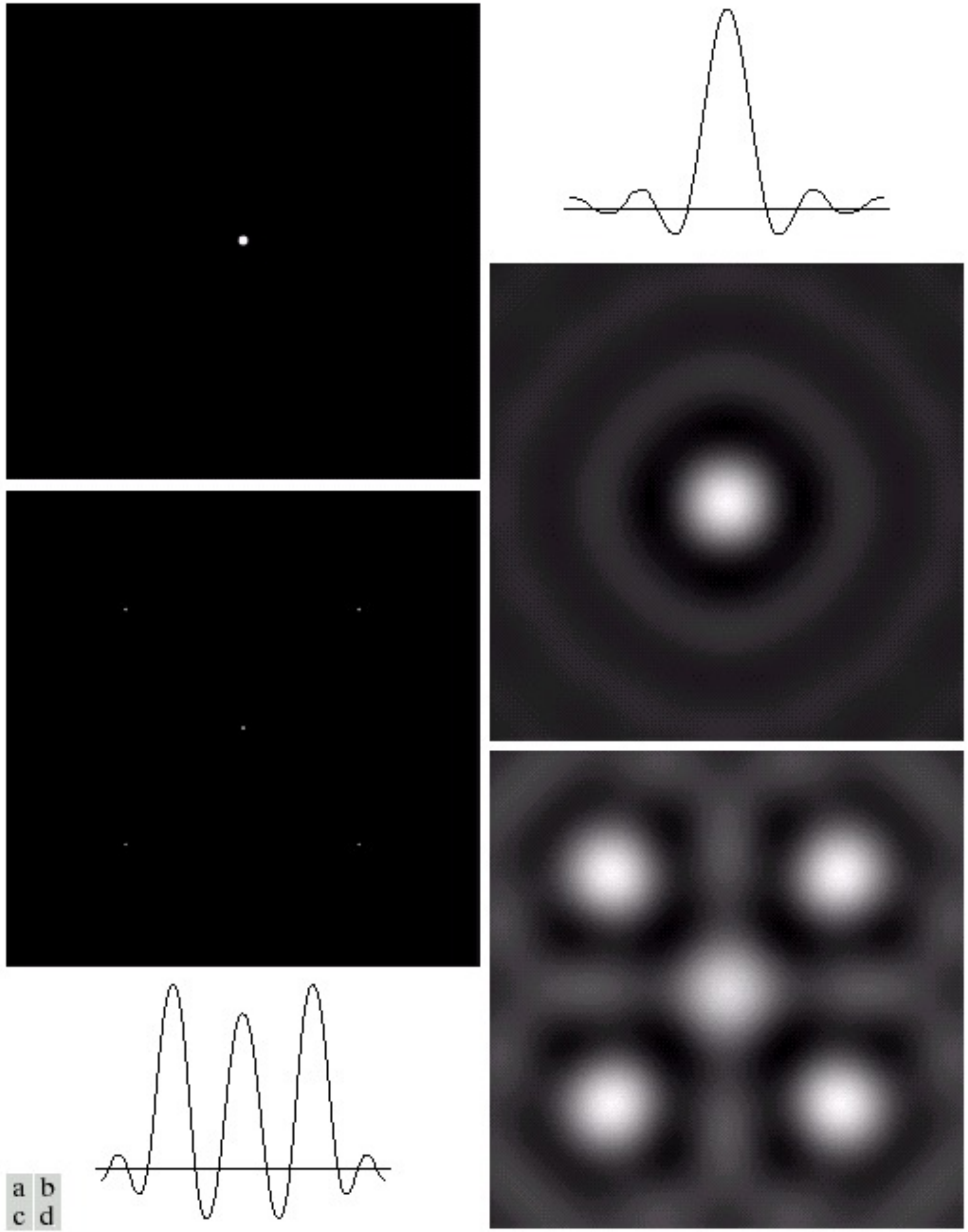


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

Butterworth Lowpass Filter (BLPF)

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

$D(u,v) = \sqrt{u^2 + v^2}$ cutoff frequency D_0, order n

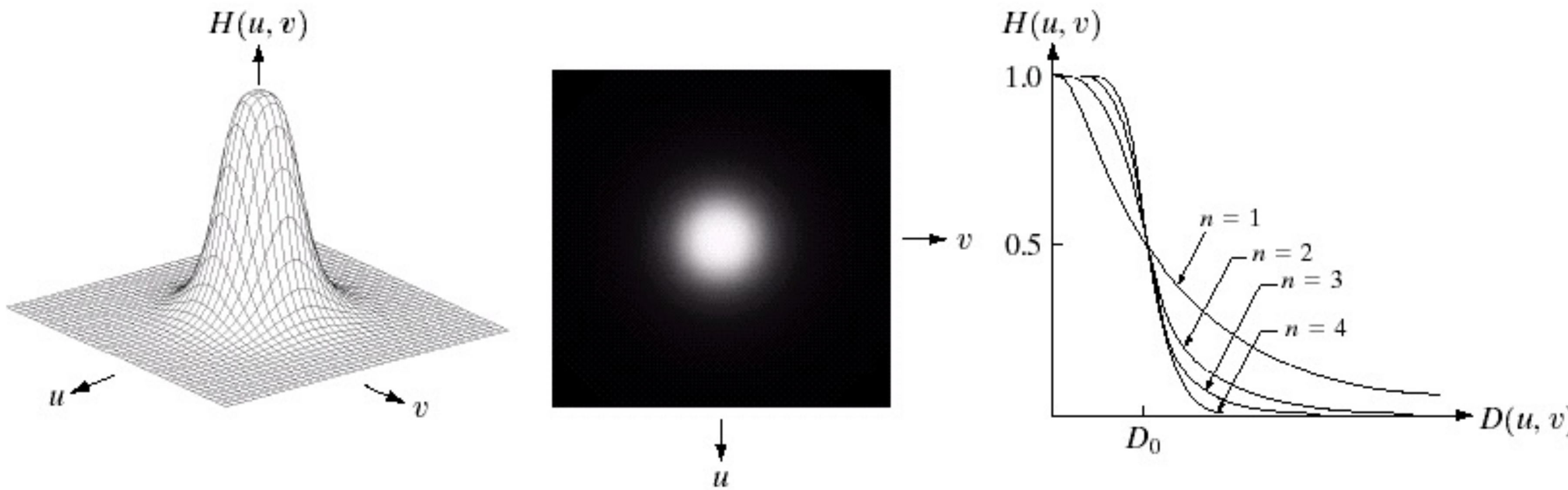
When $D(u,v) = D_0$, $H(u,v) = 0.5$

Butterworth Lowpass Filter (BLPF)

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

$D(u,v) = \sqrt{u^2 + v^2}$ cutoff frequency D_0, order n

When $D(u,v) = D_0$, $H(u,v) = 0.5$



a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

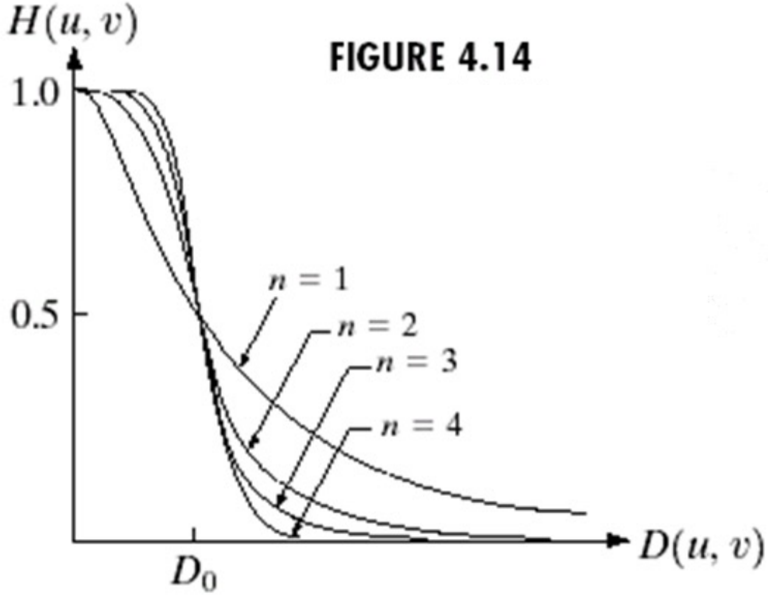


FIGURE 4.14

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

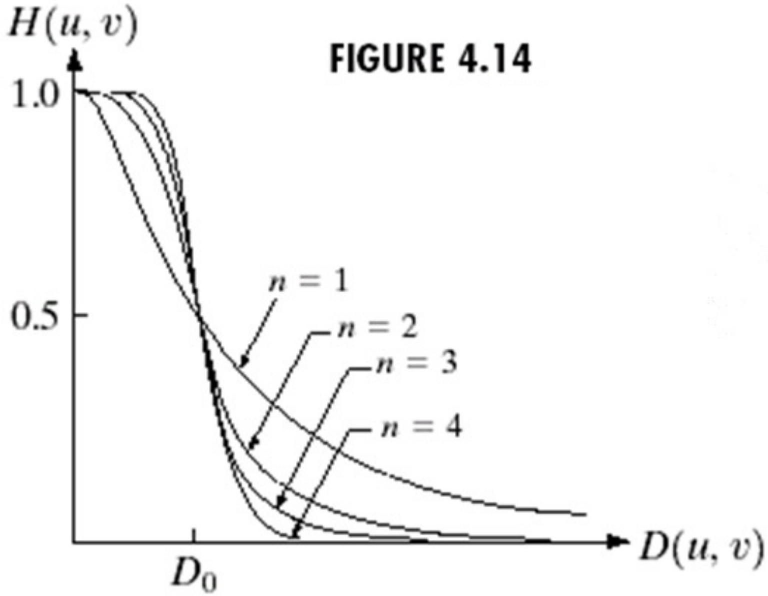


FIGURE 4.14

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

As n increases, BLPF approaches ILPF.

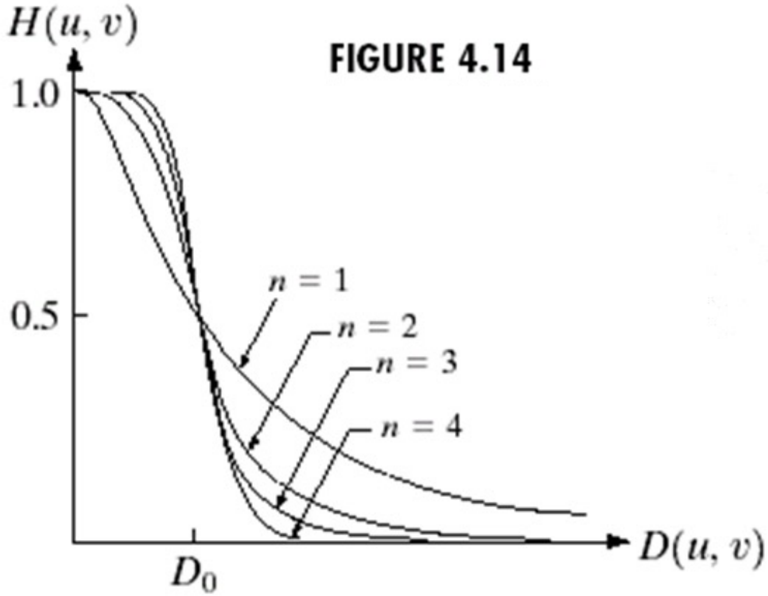


FIGURE 4.14

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

As n increases, BLPF approaches ILPF.

Ringing effects are most significant for filters with hard cutoffs in frequency.

Ringing effects are less significant for filters with a smooth frequency response.

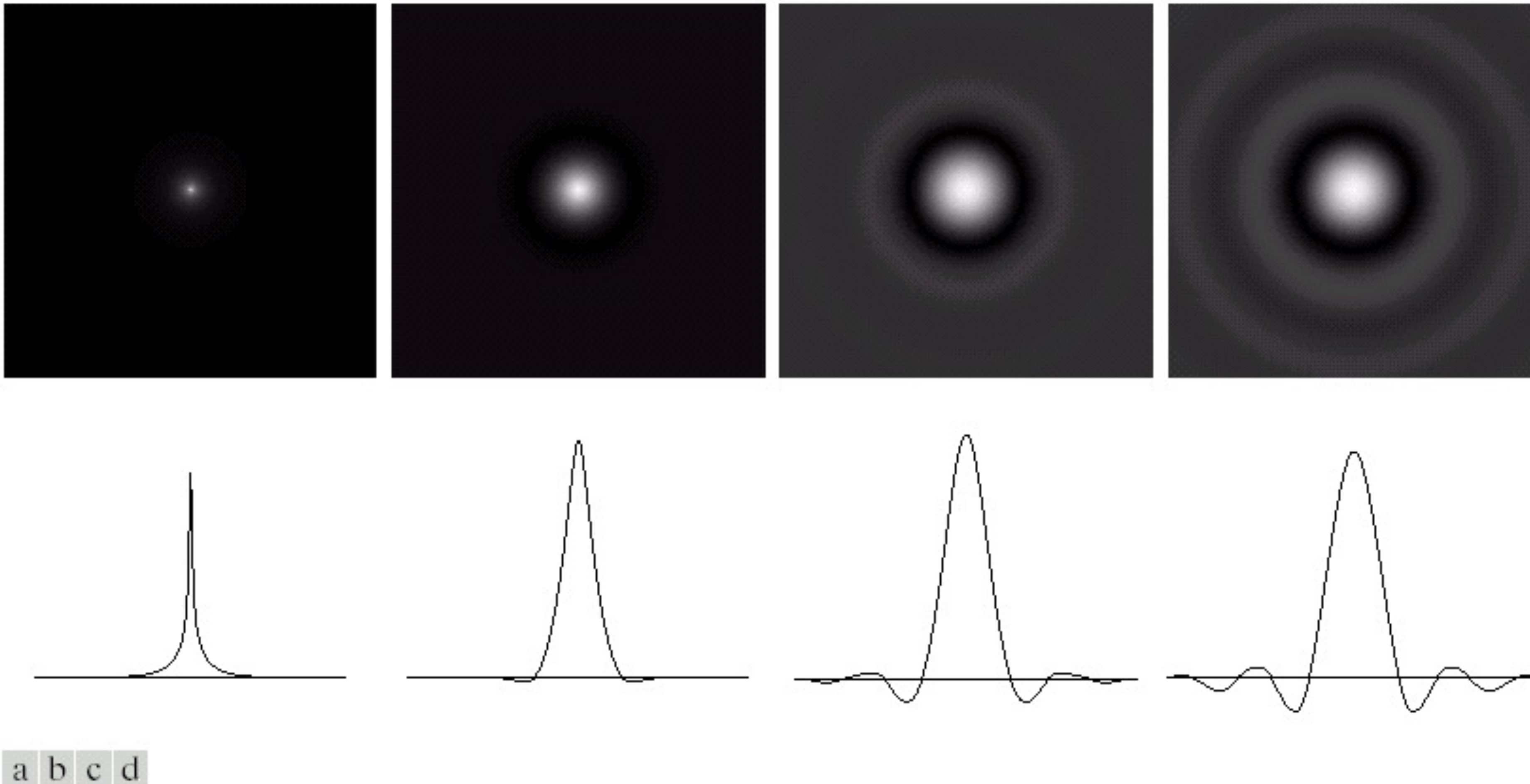
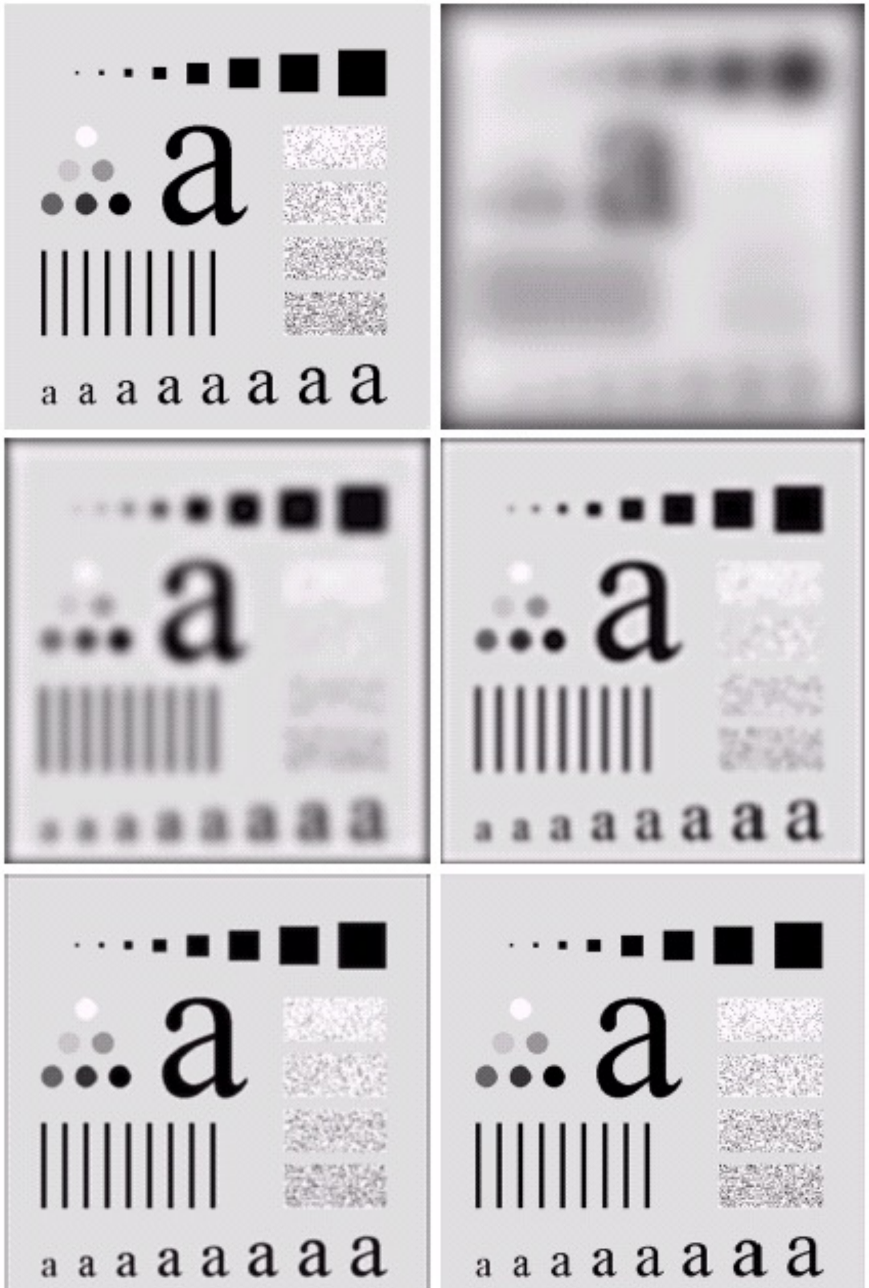


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

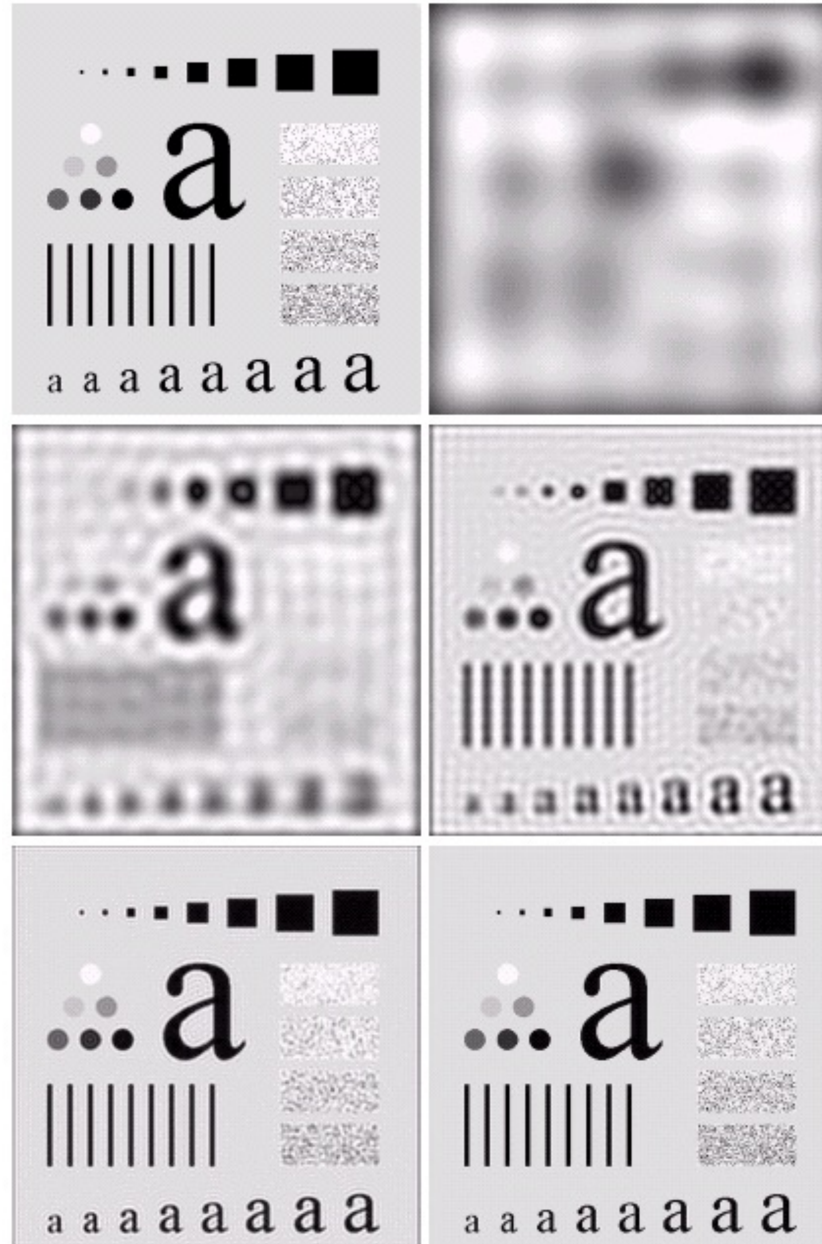


a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

ILPF

FIGURE 4.12



Gaussian Lowpass Filter (GLPF)

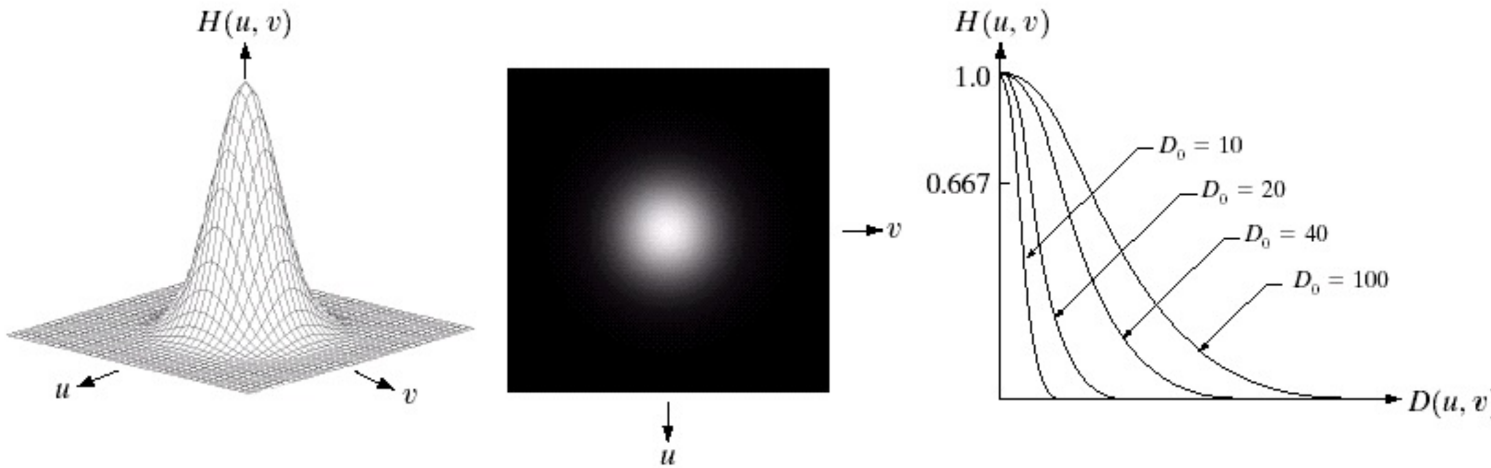
$$H(u,v) = e^{-D^2(u,v)/2D_0^2} \qquad D(u,v) = \sqrt{u^2+v^2}$$

When $D(u,v) = D_0$, $H(u,v) = 0.607$

Gaussian Lowpass Filter (GLPF)

$$H(u,v) = e^{-D^2(u,v)/2D_0^2} \qquad D(u,v) = \sqrt{u^2+v^2}$$

When $D(u,v) = D_0$, $H(u,v) = 0.607$



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian Lowpass Filter (GLPF)

$$H(u,v) = e^{-D^2(u,v)/2D_0^2} \qquad D(u,v) = \sqrt{u^2+v^2}$$

When $D(u,v) = D_0$, $H(u,v) = 0.607$

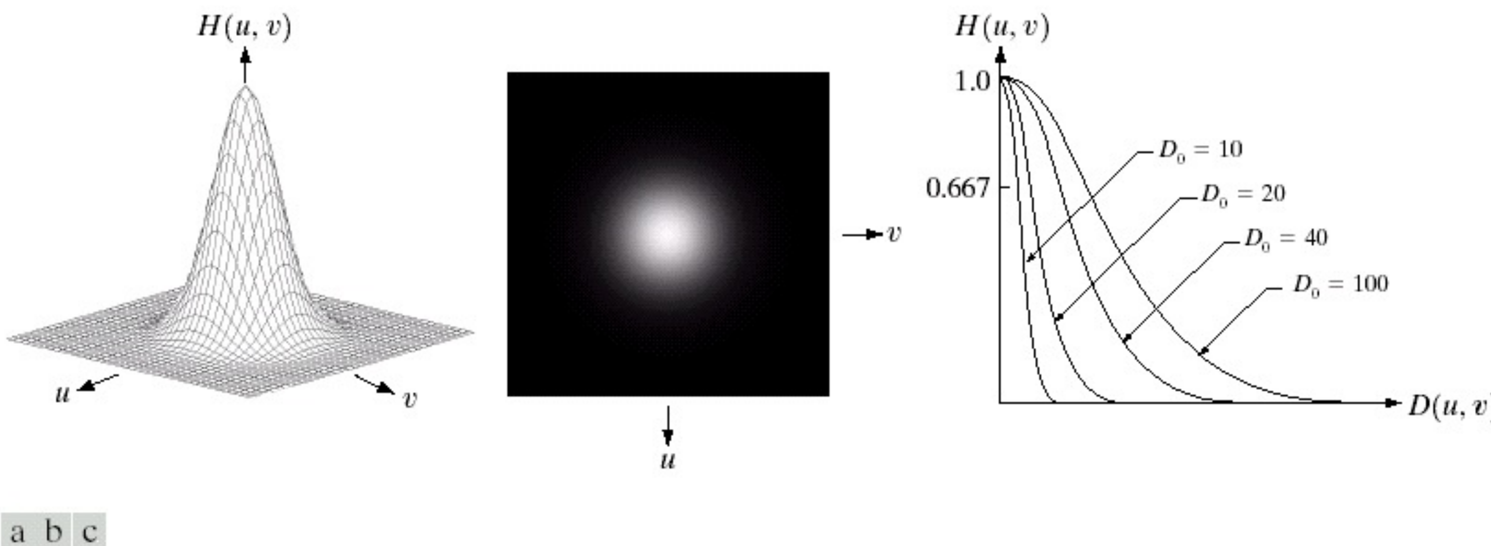


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Space domain filter for GLPF is also a Gaussian.
Thus, GLPF will have no ringing and no possibility of negative values in the filtered image.

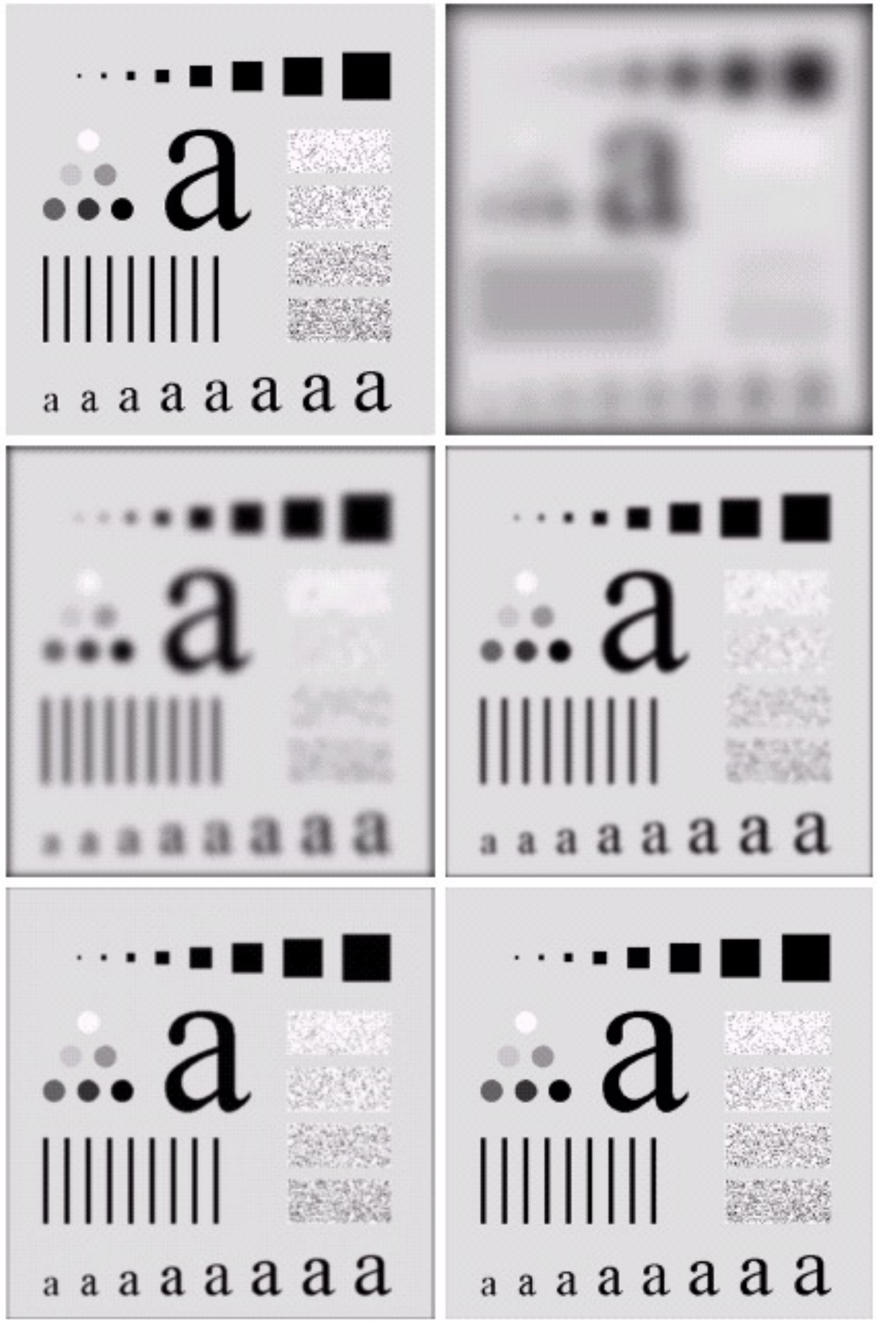
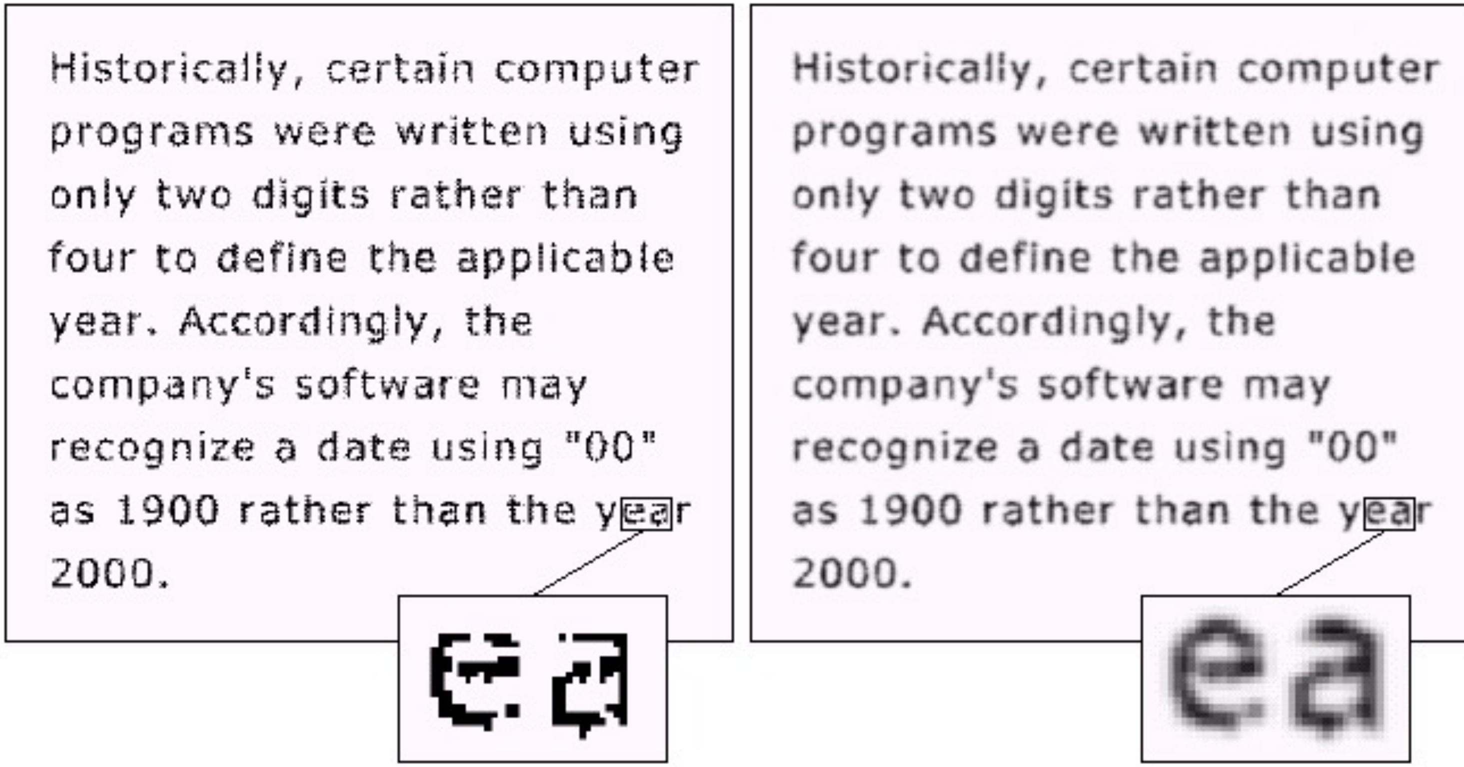


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

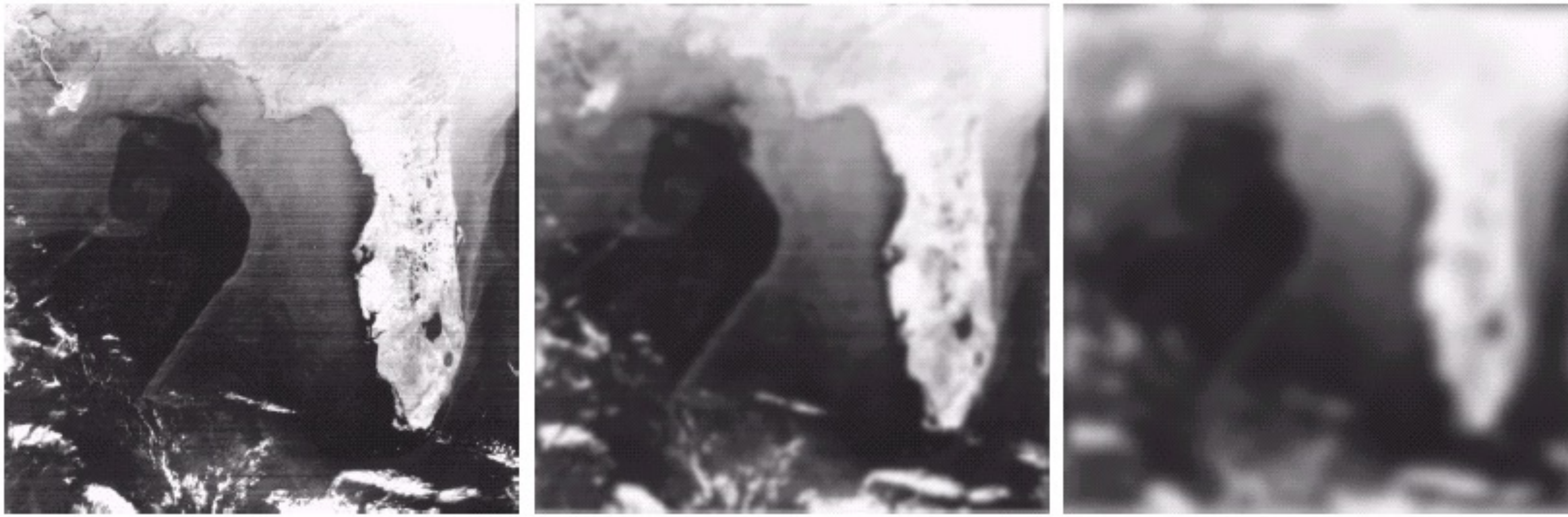
a b

FIGURE 4.19
(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



a b c

FIGURE 4.20 (a) Original image (1028×732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).



a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

ILPF, BLPF, GLPF are radially symmetric and defined by transfer functions $H(u,v)$ that are real.

If $H(u,v)$ is real, then the phase of the input image is unchanged by filtering. Such a filter is called zero-phase.

Sharpening Frequency Domain Filters

Sharpening (highpass) filters pass high frequencies and attenuate low frequencies.

Can obtain a highpass filter $H_{hp}(u,v)$ from a lowpass filter $H_{lp}(u,v)$ by

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

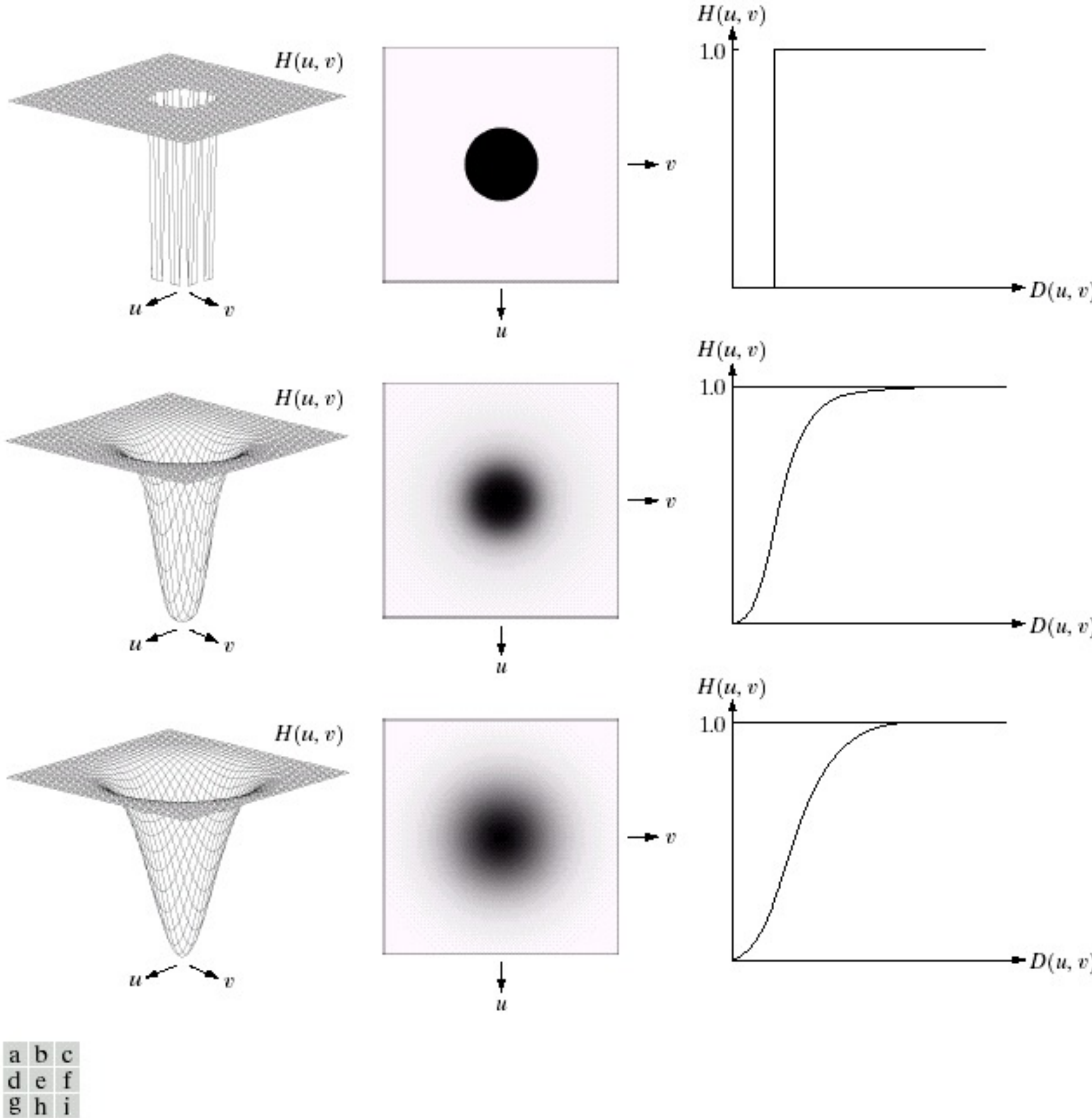
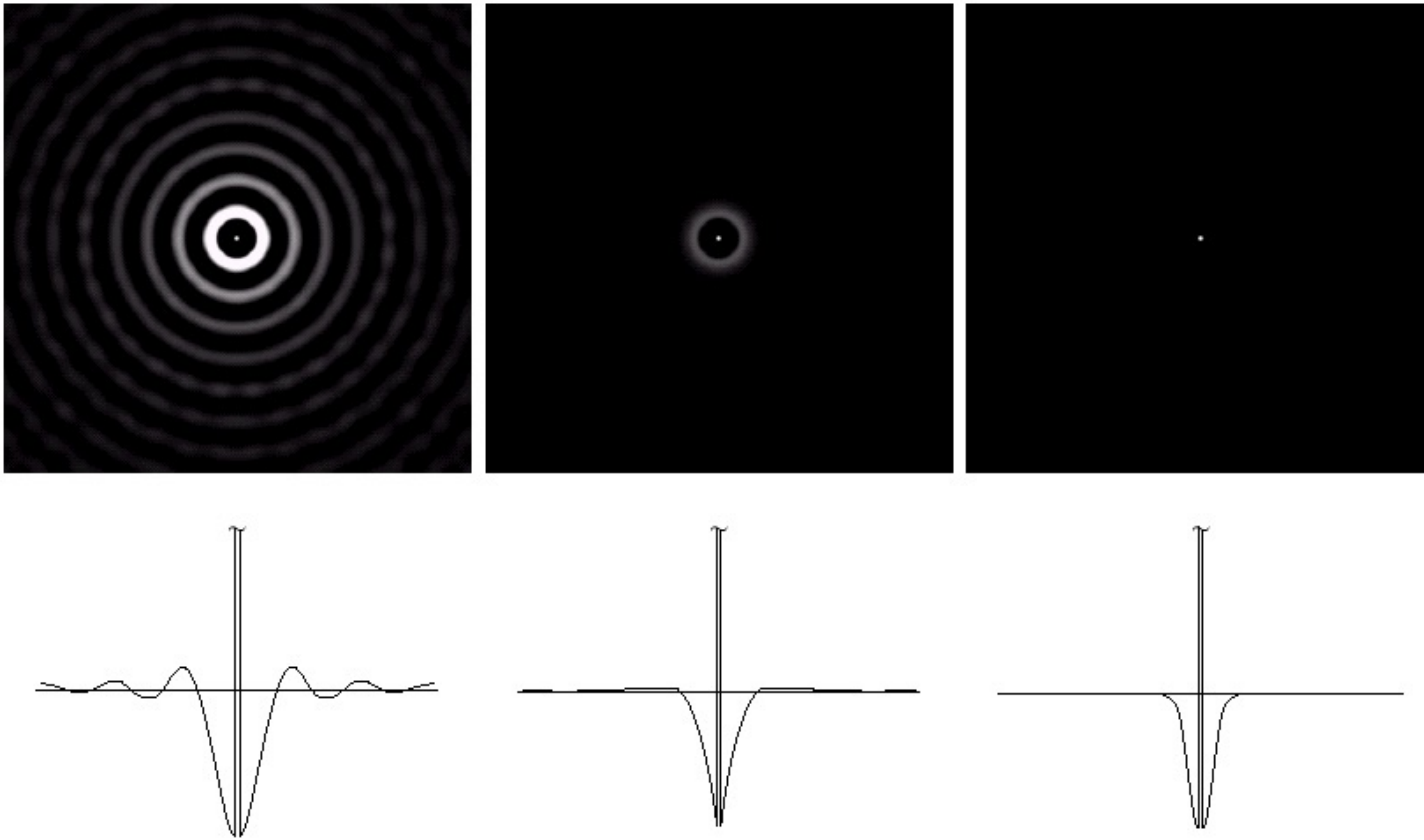


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

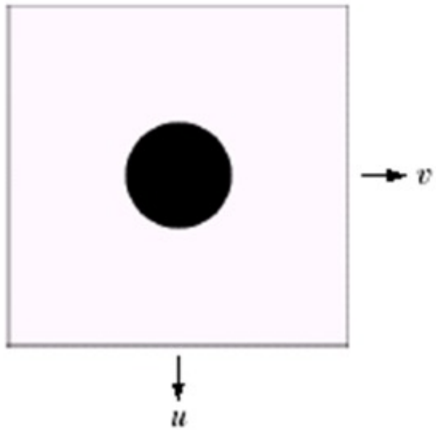


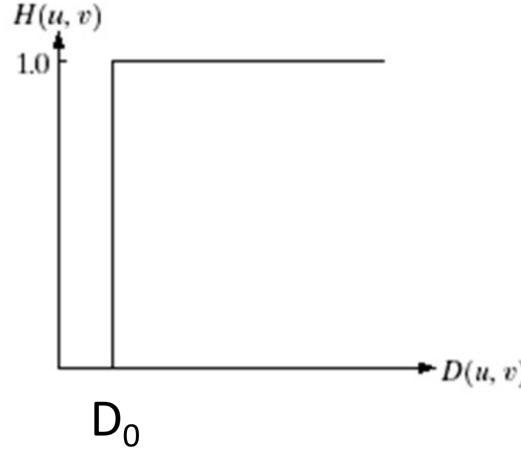
a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

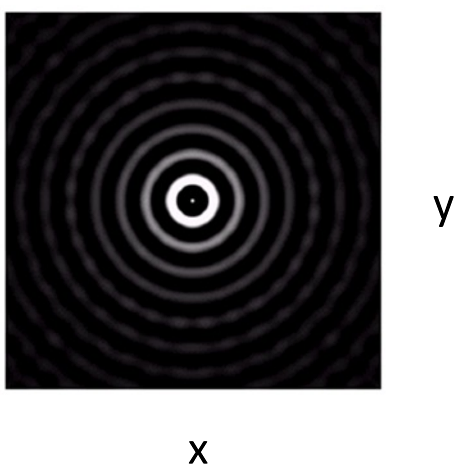
Ideal Highpass Filter (IHPF)

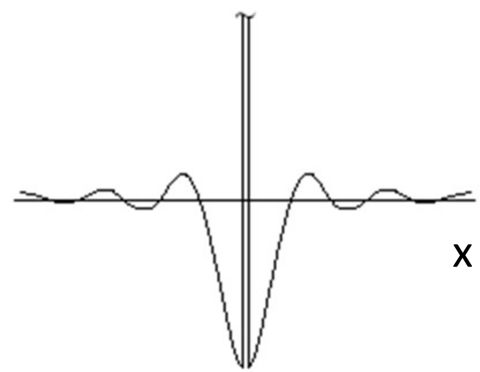
frequency IHPF is specified by radius D_0

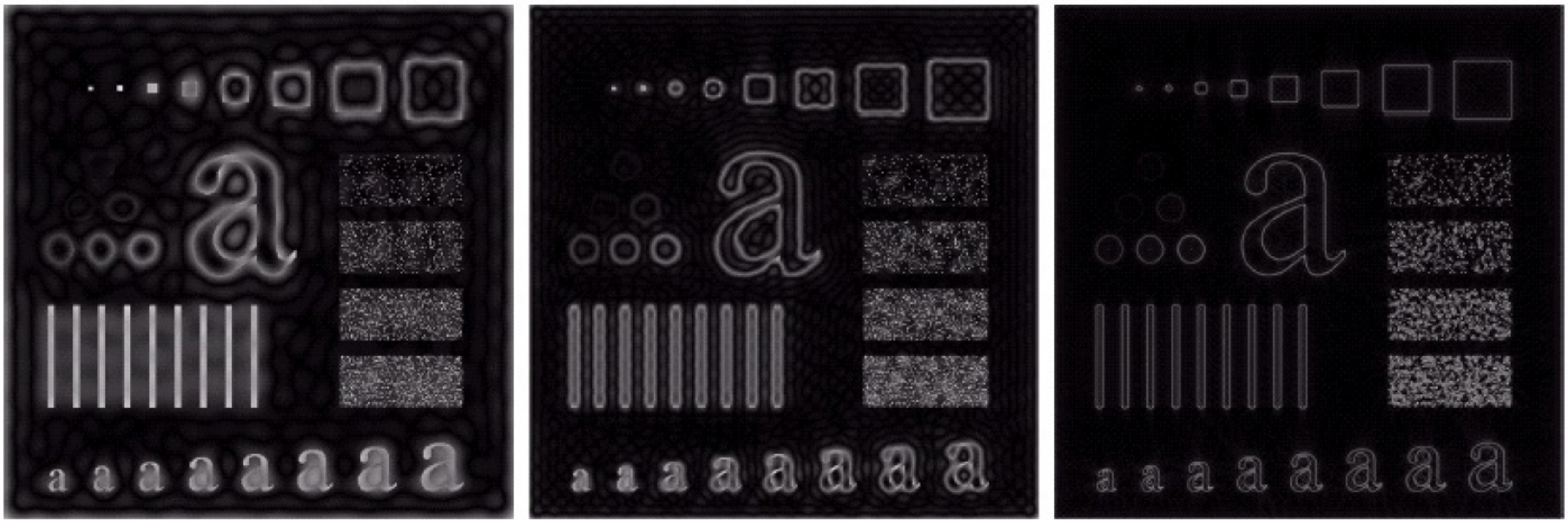




space IHPF can produce ringing effects like ILPF.







a b c

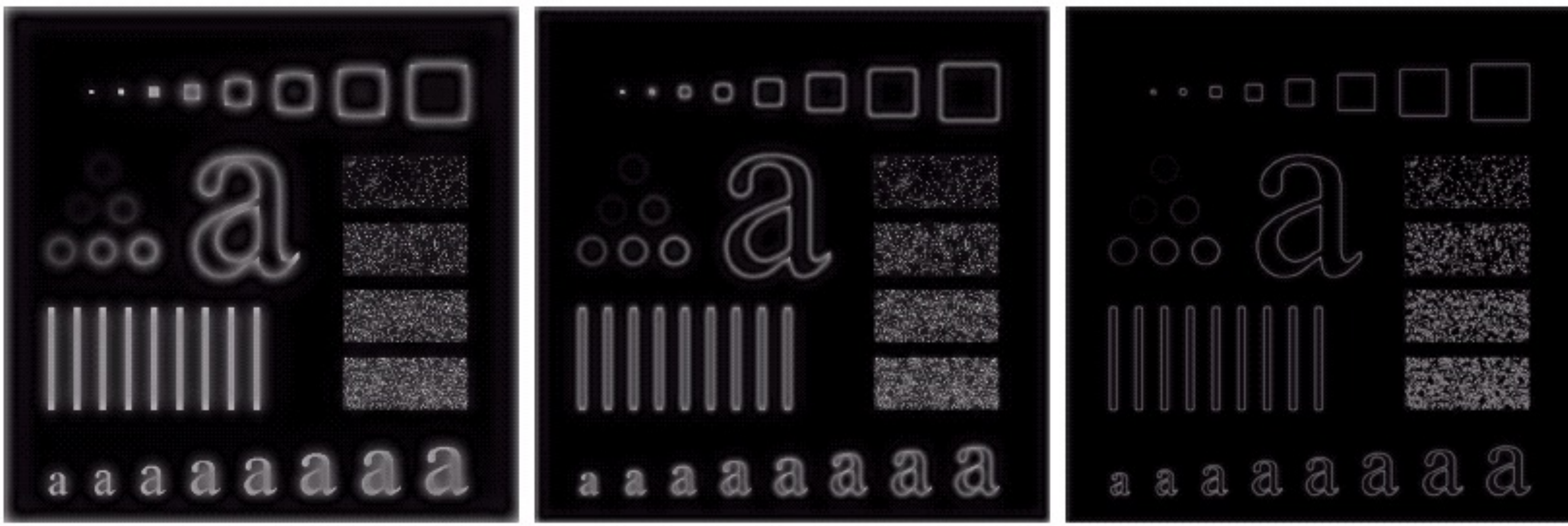
FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

Butterworth Highpass Filter (BHPF)

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

cutoff frequency D_0, order n

Smoother transition in frequency than IHPF. Less ringing than IHPF.



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Gaussian Highpass Filter (GHPF)

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$i(x,y)$ = illumination on surface visible at (x,y)

$r(x,y)$ = fraction of light reflected by surface visible at (x,y)

$r(x,y)$ is called the reflectance of the surface

Homomorphic Filtering

$$f(x,y) = i(x,y)\,r(x,y)$$

$i(x,y) =$ illumination on surface visible at (x,y)

$r(x,y) =$ fraction of light reflected by surface visible at (x,y)

$r(x,y)$ is called the reflectance of the surface

Can enhance an input image by deemphasizing the effects of $i(x,y)$ and emphasizing the structure of $r(x,y)$

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$i(x,y)$ = illumination on surface visible at (x,y)

$r(x,y)$ = fraction of light reflected by surface visible at (x,y)

$r(x,y)$ is called the reflectance of the surface

Can enhance an input image by deemphasizing the effects of $i(x,y)$ and emphasizing the structure of $r(x,y)$

$r(x,y)$ typically contains higher frequencies than $i(x,y)$

Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$i(x,y)$ = illumination on surface visible at (x,y)

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$r(x,y)$ is called the reflectance of the surface

Can enhance an input image by deemphasizing the effects of $i(x,y)$ and emphasizing the structure of $r(x,y)$

$r(x,y)$ typically contains higher frequencies than $i(x,y)$

Homomorphic Filtering Steps

1) Separate $i(x,y)$ and $r(x,y)$
2) Highpass filter to emphasize $r(x,y)$ relative to $i(x,y)$
3) Recombine $i(x,y)r(x,y)$

$$f(x,y) = i(x,y)\,r(x,y)$$

Separation $\quad \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$f(x,y) = i(x,y)\,r(x,y)$$

Separation $\quad \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$L(x,y) = \text{lowpass}[\ln f(x,y)] \approx \ln i(x,y)$$

$$P(x,y) = \text{highpass}[\ln f(x,y)] \approx \ln r(x,y)$$

$$f(x,y) = i(x,y)\,r(x,y)$$

Separation $\quad \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$L(x,y) = \text{lowpass}[\ln f(x,y)] \approx \ln i(x,y)$$

$$P(x,y) = \text{highpass}[\ln f(x,y)] \approx \ln r(x,y)$$

Emphasize $r(x,y)$ relative to $i(x,y)$

$$f(x,y) = i(x,y)\,r(x,y)$$

Separation $\quad \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$L(x,y) = \text{lowpass}[\ln f(x,y)] \approx \ln i(x,y)$$
$$P(x,y) = \text{highpass}[\ln f(x,y)] \approx \ln r(x,y)$$

Emphasize $r(x,y)$ relative to $i(x,y)$

$$s(x,y) = \gamma_L L(x,y) + \gamma_H P(x,y) \qquad \gamma_L < 1,\ \gamma_H > 1$$
$$\approx \gamma_L \ln i(x,y) + \gamma_H \ln r(x,y)$$

$$f(x,y) = i(x,y)\, r(x,y)$$

Separation $\quad \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$L(x,y) = \text{lowpass}[\ln f(x,y)] \approx \ln i(x,y)$$

$$P(x,y) = \text{highpass}[\ln f(x,y)] \approx \ln r(x,y)$$

Emphasize $r(x,y)$ relative to $i(x,y)$

$$s(x,y) = \gamma_L L(x,y) + \gamma_H P(x,y) \qquad \gamma_L < 1,\ \gamma_H > 1$$

$$\approx \gamma_L \ln i(x,y) + \gamma_H \ln r(x,y)$$

Recombine $\quad g(x,y) = e^{s(x,y)} \approx e^{\gamma_L \ln i(x,y)}\, e^{\gamma_H \ln r(x,y)}$

$$\doteq [i(x,y)]^{\gamma_L} [r(x,y)]^{\gamma_H}$$

Can generate $s(x,y)$ using a single filter $H(u,v)$

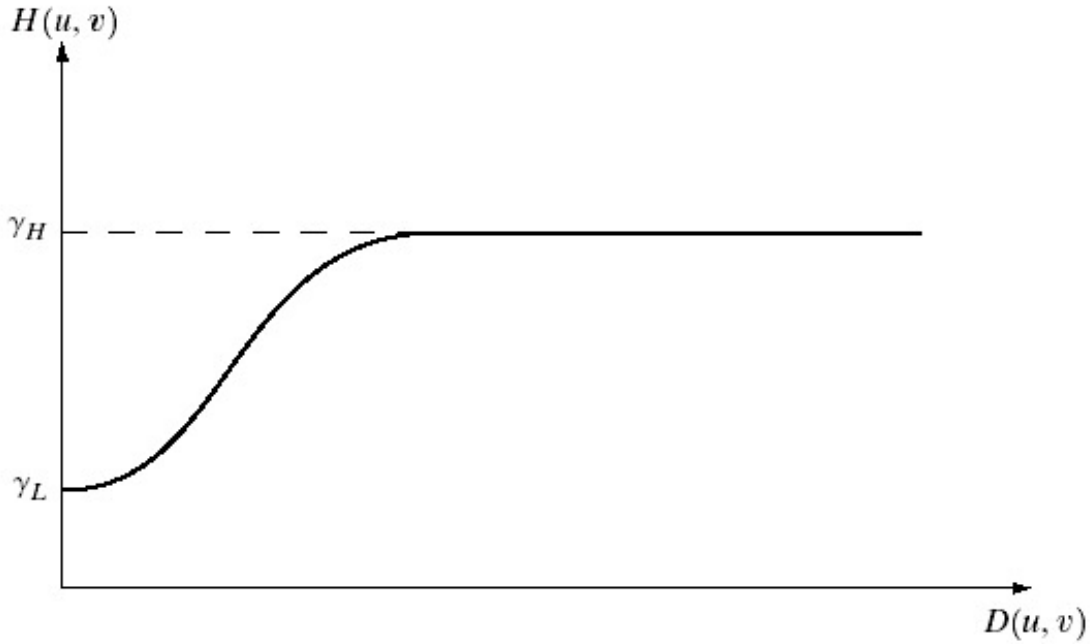

FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

Possible implementation is similar to GHPF

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\,(D^2(u,v)/D_0^2)}\right] + \gamma_L$$

c controls the sharpness of the transition from γ_L to γ_H

a b

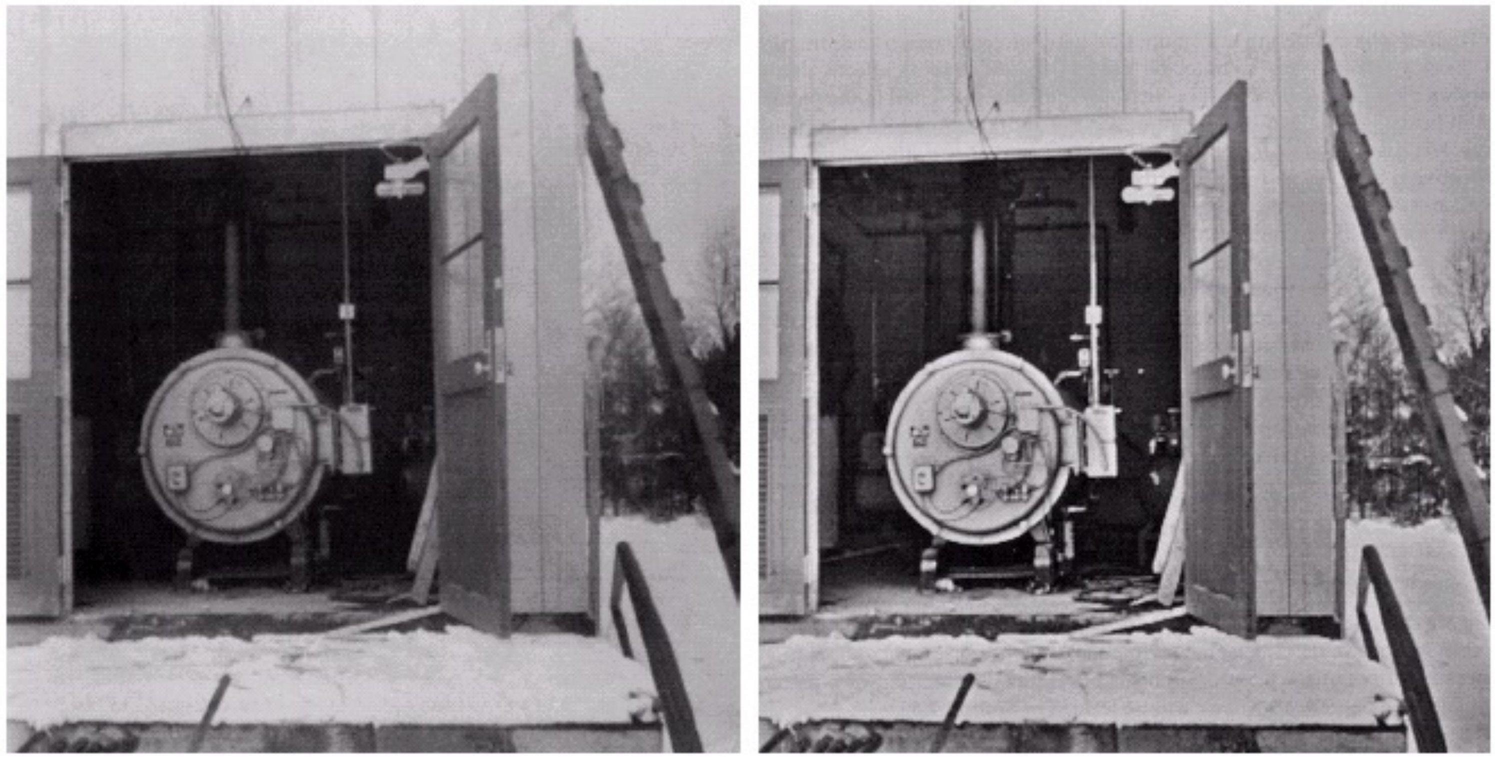

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

Finding the Frequency Response of a Filter

$$f(x - x_0, y - y_0) \xleftrightarrow{\text{DFT}} F(u,v)\, e^{-j2\pi(ux_0/M + vy_0/N)}$$

Ex $g(x,y) = f(x, y+1) + f(x+1, y)$

$$G(u,v) = F(u,v)\, e^{-j2\pi(v(-1)/N)} + F(u,v)\, e^{-j2\pi(u(-1)/M)}$$
$$= F(u,v)\left[e^{j2\pi v/N} + e^{j2\pi u/M}\right]$$

$$G(u,v) = F(u,v)\, H(u,v)$$

Input image $F(u,v)$, Output image $G(u,v)$, Filter response $H(u,v)$

$$H(u,v) = \frac{G(u,v)}{F(u,v)} = e^{j2\pi v/N} + e^{j2\pi u/M}$$