EECS203A Exam #1 April 28, 2022

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This is an 80 minute, closed book exam. You are allowed one 8.5x11 inch sheet with notes on both sides. Calculators are not allowed. Show all of your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

TOTAL:

Question 1 (8 points) Consider a 512×512 digital image $f(x, y)$ where all pixels have the gray level 100 except for 500 pixels which are randomly selected and have the noise gray level of 255.

4

a) Is a 3×3 averaging filter or a 3×3 median filter better for removing noise in $f(x, y)$? Explain.

Median filter. The median filter will completely remove the noise if fewer than 5 noise pixels occur in a 3x3 window. The averaging filter will not completely remove the noise.

2 for Median filter, 2 for explanation

4

b) Is the filter that you selected in part a) guaranteed to remove all of the effects of noise in $f(x, y)$? Explain.

No. The median filter will not completely remove the noise if 5 or more noise pixels occur in a 3x3 window.

2 for No, 2 for explanation

Question 2 (8 points) Consider a digital image $f(x, y)$ with the pixel values

$$f(0,0) = 4 \quad f(0,1) = 8 \quad f(1,0) = 7 \quad f(1,1) = 4$$

6

a) Find a continuous bilinear function $b(x, y)$ such that $b(x, y) = f(x, y)$ at these four points.

$b(x,y) = c_1 x + c_2 y + c_3 xy + c_4$

at $(0,0)$ $\quad c_4 = 4$

at $(0,1)$ $\quad c_2 + c_4 = 8 \longrightarrow c_2 = 4$

at $(1,0)$ $\quad c_1 + c_4 = 7 \longrightarrow c_1 = 3$

at $(1,1)$ $\quad c_1 + c_2 + c_3 + c_4 = 4 \longrightarrow c_3 = -7$

$b(x,y) = 3x + 4y - 7xy + 4$

2

b) Find $b(0.4, 0.7)$.

$b(0.4, 0.7) = 3(0.4) + 4(0.7) - 7(0.4)(0.7) + 4$

$= 6.04$

Question 3 (12 points) Consider three spatial filters with the frequency responses

$$H_1(u,v) = \frac{1}{1 + K\,(u^2+v^2)} \qquad H_2(u,v) = \frac{1}{1 + K^2\,(u^2+v^2)^2} \qquad H_3(u,v) = \frac{K\,(u^2+v^2)}{1 + K\,(u^2+v^2)}$$

where K is a positive constant.

3 a) Give the name for each of these three filters.

$H_1(u,v)$ Butterworth lowpass filter, order $n=1$, cutoff $D_0 = \frac{1}{\sqrt{K}}$

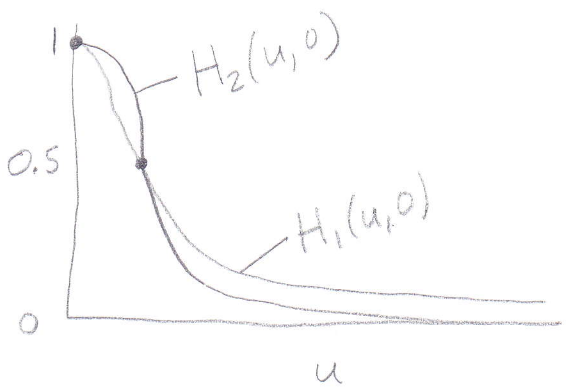
$H_2(u,v)$ Butterworth lowpass filter, order $n=2$, cutoff $D_0 = 1/\sqrt{K}$

$H_3(u,v) = 1 - H_1(u,v)$ Butterworth highpass filter, order $n=1$, cutoff $D_0 = 1/\sqrt{K}$

3 b) How will an input image that is filtered by $H_1(u,v)$ be changed?

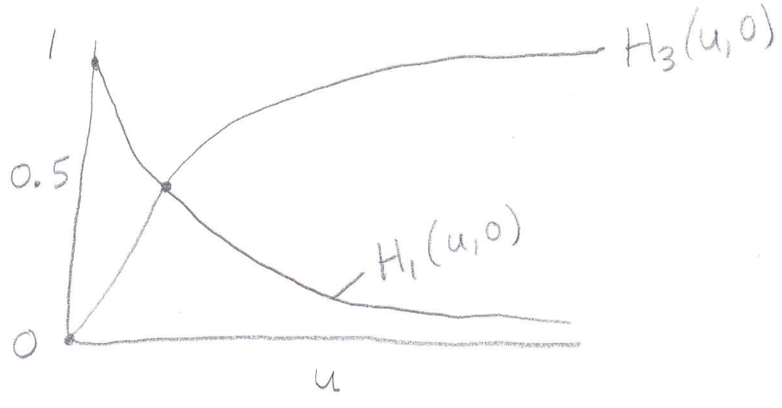
Image will be lowpass filtered or smoothed.

3 c) Plot $H_1(u,0)$ and $H_2(u,0)$ on the same plot as a function of u.



$H_2(u,0)$ has sharper cutoff

3 d) Plot $H_1(u,0)$ and $H_3(u,0)$ on the same plot as a function of u.



$H_1(u,0) + H_3(u,0) = 1$

Question 4 (12 points) Let H be an operator that maps an input image $I(x, y)$ to an output image $O(x, y)$ according to

$$O(x, y) = xyI(x, y)$$

2 a) Is H a linear operator?

Yes.

4 b) Prove your answer to part a).

$$H[af(x,y) + bg(x,y)] = xy(af(x,y) + bg(x,y))$$
$$= axyf(x,y) + bxyg(x,y)$$
$$= aH[f(x,y)] + bH[g(x,y)]$$

2 c) Can H be represented by

$$O(x, y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s, t)I(x + s, y + t)$$

using nine constants $w(-1, -1), w(-1, 0), w(-1, 1), w(0, -1), w(0, 0), w(0, 1), w(1, -1), w(1, 0), w(1, 1)$?

No.

4 d) If you answered Yes to part c) then find the nine $w(s, t)$ constants. If you answered No to part c) then explain why not.

The $w(s,t)$ would need to depend on x,y so they would not be constants.

Question 5 (14 points) Consider a spatial domain operator that transforms an input image $f(x, y)$ to an output image $g(x, y)$ according to

$$g(x,y) = f(x+1,y-1) + 2f(x+1,y) + f(x+1,y+1) - f(x-1,y-1) - 2f(x-1,y) - f(x-1,y+1)$$

Assume the textbook coordinate system where x increases going down and y increases going right. Ignore boundary effects.

2 a) What is this operator called?

Sobel $\frac{\partial f}{\partial x}$ operator

2 b) Find a mask that implements this operator.

$$\begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$$

2 c) For what locations (x, y) in an image would we expect the absolute value of the output image $|g(x, y)|$ to be the smallest?

Locations where the image is not changing in the x direction.

2 d) For what locations (x, y) in an image would we expect the absolute value of the output image $|g(x, y)|$ to be the largest?

Locations where the image is changing rapidly in the x direction.

3 e) Given the input image $f(x, y)$ below

0	0	0	0
0	0	50	10
0	0	50	10
0	0	50	10

Circle the pixel (x, y) in the interior (not boundary) of the image where $|g(x, y)|$ will be the largest.

3 f) Compute $g(x, y)$ for the (x, y) location that you selected in part e).

$2(50) + 10 - 0 = 110$

Question 6 (12 points) Suppose that we apply the mask below to an input image $f(x, y)$ to generate an output image $g(x, y)$. Ignore boundary effects.

-1	-1	-1
-1	9	-1
-1	-1	-1

$$g(x,y) = 9f(x,y) - \begin{bmatrix} f(x-1,y-1) + f(x-1,y) + f(x-1,y+1) \\ + f(x,y-1) + f(x,y+1) \\ + f(x+1,y-1) + f(x+1,y) + f(x+1,y+1) \end{bmatrix}$$

3 a) For a typical input image $f(x, y)$, describe the appearance of $g(x, y)$ compared to $f(x, y)$.

$g(x,y)$ will be a sharpened version of $f(x,y)$.

Suppose that the input image is defined by $f(x, y) = F + n(x, y)$ where F is a constant and $n(x, y)$ is a zero-mean noise source with variance σ^2 that is independent from pixel to pixel.

3 b) What is the expected value of $g(x, y)$ at pixel (x, y)?

$E[g(x,y)] = 9F - 8F = F$

3 c) What is the variance of $g(x, y)$ at pixel (x, y)?

$VAR[g(x,y)] = VAR[9n(x,y) - n(x-1,y-1) - \dots - n(x+1,y+1)]$

$= 81\sigma^2 + \sigma^2 + \dots + \sigma^2 = 81\sigma^2 + 8\sigma^2 = 89\sigma^2$

3 d) Does this filter improve the appearance of the image $f(x, y) = F + n(x, y)$? Explain.

No. The mean of $g(x,y)$ is the same as the mean of $f(x,y)$ but the variance of the noise is higher

$89\sigma^2 > \sigma^2$

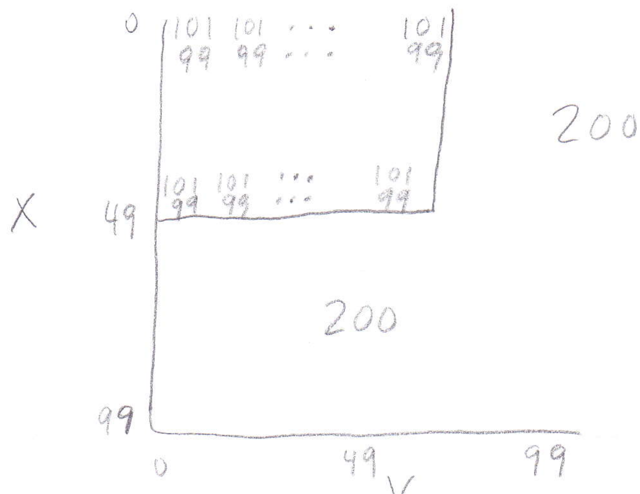
Question 7 (10 points) Let $f(x,y)$ for $x = 0, 1, \ldots, 99, \quad y = 0, 1, \ldots, 99$ be the 100×100 digital image defined by

$$f(x,y) = \begin{cases} 100 + (-1)^x & \text{if } \ 0 \le x \le 49 \text{ and } 0 \le y \le 49 \\ 200 & \text{otherwise} \end{cases}$$

and let $F(u,v)$ for $u = 0, 1, \ldots, 99, \quad v = 0, 1, \ldots, 99$ be the 100×100 DFT of $f(x,y)$. Let

$$F'(u,v) = \begin{cases} 100 & \text{if } \ u = 0 \text{ and } v = 0 \\ F(u,v) & \text{otherwise} \end{cases}$$

2 a) Draw the image $f(x,y)$. Assume the textbook coordinate system where x increases going down and y increases going right.



8 b) Find the 100×100 image $f'(x,y)$ that is the inverse DFT of $F'(u,v)$.

mean of $f(x,y)$ is $\frac{1}{4}(100) + \frac{3}{4}(200) = 175$

mean of $f'(x,y)$ is 100

$$f'(x,y) = \begin{cases} 25 + (-1)^x & \text{if } 0 \le x \le 49 \text{ and } 0 \le y \le 49 \\ 125 & \text{otherwise} \end{cases}$$

Question 8 (14 points) Consider the 8×8 digital image $f(x, y)$ below with 8 gray levels from 0 to 7.

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	1	2	2	3	4
1	1	1	2	2	2	3	4
1	1	1	2	2	2	3	4
1	1	1	2	2	2	3	4
2	2	2	2	2	5	5	5
6	6	6	6	7	7	5	5

10 a) Use the method described in class to determine the gray level transformation $M(r_k)$ for $r_k = 0, 1, 2, \ldots, 7$ that corresponds to histogram equalization. The output image will also have 8 gray levels from 0 to 7.

3

r_K	P_r	$T(r_K)$
0	8	8
1	21	29
2	16	45
3	4	49
4	4	53
5	5	58
6	4	62
7	2	64

Input

3

Z_K	P_Z	$G(Z_K)$
0	8	8
1	8	16
2	8	24
3	8	32
4	8	40
5	8	48
6	8	56
7	8	64

Desired

4

r_K	$M(r_K)$
0	0
1	3
2	5
3	5
4	6
5	6
6	7
7	7

Map each r_K to Z_K so that $T(r_K)$ is as close as possible to $G(Z_K)$

4 b) Find the 8×8 digital image that results after applying histogram equalization to $f(x, y)$.

0	0	0	0	0	0	0	0
3	3	3	3	3	3	3	3
3	3	3	3	5	5	5	6
3	3	3	5	5	5	5	6
3	3	3	5	5	5	5	6
3	3	3	5	5	5	5	6
5	5	5	5	5	6	6	6
7	7	7	7	7	7	6	6

10

Question 9 (10 points) Define the masks for two 3×3 spatial filters $h_1(x, y)$ and $h_2(x, y)$ as shown below

$$h1 = \begin{matrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 3 & 2 & 1 \end{matrix} \qquad h2 = \begin{matrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{matrix}$$

Suppose that the output image $g_1(x, y)$ is obtained by applying $h_1(x, y)$ to an input image $f(x, y)$. Another output image $g_2(x, y)$ is obtained by applying $h_2(x, y)$ to $g_1(x, y)$. Find a single mask that when applied to an input image $f(x, y)$ is equivalent to the double filtering operation that is used to generate $g_2(x, y)$ from $f(x, y)$.

$$\underset{f(x,y)}{\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}} \xrightarrow{h1} \underset{g_1(x,y)}{\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}} \xrightarrow{h2} \underset{g_2(x,y)}{\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -2 & 2 & 3 \\ 0 & -4 & 0 & 4 & 0 \\ -3 & -2 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}}$$

Mask will give $g_2(x,y)$ when applied to $f(x,y)$

$$\text{Mask} \quad \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & -2 & -3 \\ 0 & 4 & 0 & -4 & 0 \\ 3 & 2 & -2 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$