

Section 14: Confidence regions

ARE 210

December 5, 2017

1. Prove the result on the duality of $(1 - \alpha)$ 100% confidence regions and level- α tests.

Let δ_v be a level- α test of the null $H_v : \nu(P) = v$. Define $C_v = \{X : \delta_v(X) = 1\}$ and $A_v = \mathcal{X} \setminus C_v$. Let $S(X) = \{v \in \mathcal{N} : X \in A_v\}$. Then $P_P[\nu(P) \in S(X)] = P_P[X \in A_{\nu(P)}] \geq 1 - \alpha$.

Note that when ν is invertible, we can reverse this logic.