Section 1: Introduction, Solow Model[1]

EEP 152

August 31, 2016

- Introduction (10 min)
- Solow model (math) (20 min)
- Solow model (motivating the course) (20 min)

1 Introduction

See section syllabus.

2 Solow model (math)

2.1 Setup

As we saw the first day of class, per capita income is strongly predictive of welfare, so understanding why some countries have higher incomes and some countries have lower incomes will take us a long way to understand variation in welfare. The Solow model presents one explanation for the source of this variation - differences in the "capital stock" (think industrialization, buildings, machines, . . .).

We assume each country's income is a function of two things - its effective population (number of workers, adjusted for human capital), and its capital stock. We further assume

- Income is constant returns to scale in effective population and capital stock (if we double effective population and capital stock, income doubles)

- Marginal product of capital and effective workers is positive (adding another unit of capital increases income, adding another unit of effective workers increases income)

- Decreasing marginal product of capital (the $(i + 1)$th unit of capital you add increases production by less than the ith unit of capital, all else equal)

[1]These section notes are based off notes created by Zhimin Li.

One convenient function that satisfies these assumptions is

$$\underbrace{Y_t}_{\substack{\text{income,}\\ \text{period } t}} = (\underbrace{K_t}_{\substack{\text{capital stock,}\\ \text{period } t}})^{\alpha}(\underbrace{E_t}_{\substack{\text{labor augmenting}\\ \text{technology, period } t}}\underbrace{P_t}_{\substack{\text{population,}\\ \text{period } t}})^{1-\alpha}$$

E_tP_t is the units of effective workers - doubling E_t (how productive workers are) or doubling P_t (number of workers) has the same effect on income Y_t. We next assume

- A share s of income goes to investment, which increases the capital stock next period (K_{t+1})
- A share δ of the capital stock is lost each period

Together, these imply

$$\underbrace{K_{t+1}}_{\substack{\text{capital stock,}\\ \text{period } t+1}} = \underbrace{(1-\delta)K_t}_{\substack{\text{depreciated period } t\\ \text{capital stock}}} + \underbrace{sY_t}_{\text{investment}}$$

We can simplify both of these expressions by dividing through by the number of effective workers, E_tP_t, to understand the evolution of capital per effective worker (which we can call "capitalization"), which in this model determines the evolution of income per effective worker. This yields

$$\text{Income: } \widehat{y_t} = \left(\widehat{k_t}\right)^{\alpha}$$

$$\text{Law of motion: } (1+\pi)(1+n)\widehat{k_{t+1}} = (1-\delta)\widehat{k_t} + s\widehat{y_t}$$

where π is the growth rate of technology $\frac{E_{t+1}-E_t}{E_t}$, and n is the growth rate of population $\frac{P_{t+1}-P_t}{P_t}$. In the income equation, we can see income per effective worker only depends on capitalization (capital per effective worker), so understanding the evolution of capitalization is sufficient to understand the evolution of income per effective worker. So the law of motion for capital is where we'll focus our analysis.

2.2 Dynamics and a steady state

How does capital evolve? We can rewrite the law of motion as

$$\widehat{k_{t+1}} = \frac{1-\delta}{(1+\pi)(1+n)}\widehat{k_t} + \frac{s}{(1+\pi)(1+n)}\widehat{y_t}$$

Lets plot this evolution for three cases : a country beginning with a very low, low, and high level of initial capitalization. We assume all three countries have the same values of δ, π, n, and s. We'll put $\widehat{k_{t+1}}$ on the vertical axis and $\widehat{k_t}$ on the horizontal axis. We'll also plot the 45 degree line, so once we calculate $\widehat{k_{t+1}}$, we can use that to go back to the horizontal axis and calculate $\widehat{k_{t+2}}$.

You should note two main things:

- All three countries converge to the same steady state, where $\widehat{k_{t+1}} = \widehat{k_t}$, with identical capitalization $\widehat{k^*}$
- Countries further from the steady state converge faster

Finally, given δ, n, π, s, and $Y_t(K_t, E_tL_t)$ (income as a function of capital and effective workers), we'll often use the model to answer two questions.

- How does the capital stock K_t evolve?
  - Plug in parameters and starting values of the capital stock K_t and effective worker stock E_tL_t into the law of motion.
- What is the steady state level of capitalization $\widehat{k^*}$?
  - Solve $\widehat{k_{t+1}} = \widehat{k_t}$. To do this, substitute the law of motion for $\widehat{k_{t+1}}$, which gives $\widehat{k_{t+1}}$ as a function of $\widehat{k_t}$, then solve for $\widehat{k_t}$.

3 Solow model (motivating the course)

We've seen that the Solow model predicts that countries with the same values of the growth parameters (n, δ, π, s) should experience convergence in income per effective worker. As mentioned in class, there are persistent gaps in income per capita across countries, and although variation in n and s may exist, it does not explain the magnitude of variation in income per capita across countries. Technology and depreciation should also be identical across countries.[2] Together, this suggests something is wrong with our model, so lets consider few candidate possibilities:

- Investment in the "wrong capital"
- Poverty traps/violations of decreasing marginal product of capital

[2]For now, let's think about education, which could in our model drive big differences in E_t across countries, as human capital that countries can invest it. The puzzle remains similar - investment in human capital should have higher returns in countries with low initial human capital stocks, once again generating convergence.

3.1 "Wrong capital"

As we mentioned in class, the process by which sY_t becomes K_{t+1} is "magic". Recall that one reason why decreasing marginal product of capital is an attractive assumption is that it will be the case if we have a menu of investments we can make, and we first make the investments with the highest returns. What if there are high return investments out there, but they aren't being made?

Why might firms/governments/... in poor countries not make the highest return investments?

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3.2 Poverty traps

Lets consider a case in which there are "locally increasing" returns to capital - this means that at low levels of capitalization, the marginal product of capital is increasing in capitalization, but for high levels of capital, the marginal product of capital is decreasing in capitalization.

What types of poverty traps might exist? How could we test for the existence of those poverty traps?

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