Factor Market Failures and the Adoption of Irrigation in Rwanda*

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This draft is preliminary and incomplete.

Abstract

We examine constraints to adoption of new technologies in the context of hillside irrigation schemes in Rwanda. We leverage a plot-level spatial regression discontinuity design to produce 3 key results. First, irrigation enables dry season horticultural production, which boosts on farm cash profits by 70%. Second, adoption is constrained: access to irrigation causes farmers to substitute labor and inputs away from their other plots. Eliminating this substitution would increase adoption by at least 21%. Third, this substitution is largest for smaller households and wealthier households. This result can only be explained by labor market failures in a standard agricultural household model.

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1 Introduction

Limited adoption of productive technologies is a prominent explanation of low agricultural productivity in sub-Saharan Africa (World Bank, 2007; Jack, 2013). Productive technologies do exist, but are underutilized (Jack, 2013) due to inefficiencies in the markets faced by farmer households (Udry, 1997). A recent literature has provided robust evidence that these market failures limit technology adoption for at least some farmers, most commonly through experimental manipulation of markets for risk, credit, and information.[1]

Evidence is thinner on the role of constraints to adoption that are difficult to manipulate, such as those generated by failures in factor markets for land and labor. Land and labor markets are characterized by substantial frictions in developing countries (Fafchamps, 1993; Udry, 1997; LaFave & Thomas, 2016), even where these markets are particularly active (Kaur, 2014; Breza et al., 2018). Economic theory suggests land and labor market failures reduce agricultural productivity by generating inefficient allocations of labor and land across farms(Fei & Ranis, 1961; Benjamin, 1992). More recent empirical work has found that these inefficiencies are quantitatively important (Udry, 1997; Adamopoulos & Restuccia, 2014; Adamopoulos et al., 2017; Foster & Rosenzweig, 2017; Adamopoulos & Restuccia, 2018). As many agricultural technologies are land or labor intensive, this work suggests land and labor market failures could generate inefficient adoption of productive technologies.

In this paper, we demonstrate that incomplete land and labor markets also contribute to the productivity gap by limiting technology adoption.[2] We do so in the context of a poten-

[1] Experiments that explore the role of reducing risk in technology adoption include Cai et al. (2015); Cole et al. (2013); Emerick et al. (2016); Karlan et al. (2014). Experiments that examine increasing access to credit on technology adoption include Giné & Yang (2009); Duflo et al. (2011); Carter et al. (2013); Beaman et al. (2014); Karlan et al. (2014); Crépon et al. (2015); Tarozzi et al. (2015). Experiments that examine impacts of improving the information environment on technology adoption include BenYishay & Mobarak (2014); Beaman et al. (2018); Conley & Udry (2010); Cole & Fernando (2016); Kondylis et al. (2017); Jones & Kondylis (2018).

[2] A related question is explored in papers which evaluate the effects of land titling and other formalized property rights on farm investment (Besley, 1995; Goldstein & Udry, 2008; Deininger & Feder, 2009; Besley & Ghatak, 2010; Ali et al., 2014; Goldstein et al., 2018). In our context, farmers have been assigned formal titles to our plots and so we identify the influence of factor market frictions on technology adoption in the presence of formalized rights. Our emphasis on the role of labor market frictions is also distinct.

tially transformative technology: irrigation. Irrigation increases agricultural productivity in several ways: it adds additional agricultural seasons, enables cultivation of water-intensive crops, and reduces production uncertainty. However, irrigation is also costly: it requires large construction and maintenance costs, and is associated with increased usage of complementary inputs, such as labor, fertilizer, and improved seeds. Market failures, including in factor markets, therefore have the potential to cause inefficient levels of irrigation adoption as they induce a wedge between shadow prices and market prices of these inputs.

We proceed in 3 steps. First, we establish that irrigation is a productive technology, but adoption is partial. Second, we demonstrate that this partial adoption is inefficient. Third, we demonstrate that labor market failures generate constraints to adoption of irrigation.

We begin by estimating the returns to irrigation in Rwanda. We identify these returns using a plot-level spatial discontinuity design in newly constructed hillside irrigation schemes. We sample plots within 50 meters of gravity fed canals, which originate from a distant water source and must maintain a consistent gradient along the hillside. We survey 969 cultivators on 1,753 plots for 4 years.[3] We then compare plots just inside the command area, which have access to water for irrigation, to plots just outside the command area, which do not. Treatment on the treated estimates reveal that irrigation enables the transition to dry season cultivation of horticulture. While we find no effects on rainy season yields, labor, or inputs, dry season estimates correspond to 70% growth in annual cash profits. To our knowledge, this is the first study to use a natural experiment to estimate the returns to irrigation in sub-Saharan Africa; our estimate is almost identical to an estimate from Duflo & Pande (2007) in India.[4] Despite the large effects we estimate, adoption is low: only 30% of plots are irrigated 4 years after canals became operational. At this level of adoption, the sustainability of

[3]These numbers are only for the sample of households whose sampled plot is within 50 meters of the associated discontinuity; in full we survey 1,695 cultivators on 3,332 plots.

[4]Existing work that estimates the returns to irrigation using natural experiments is predominantly from groundwater irrigation in South Asia, leveraging variation in slope characteristics of river basins (Duflo & Pande, 2007), aquifer characteristics (Sekhri, 2014), or well-failures (Jacoby, 2017) for identification. Estimates of the return to irrigation in Africa include Dillon (2011), who estimates the returns to irrigation using propensity score matching in Mali.

hillside irrigation systems is in doubt: even the large gains in cash profits to adopters are unable to generate enough surplus to pay for routine maintenance costs.

We investigate the effect of irrigation on inputs to shed light on what might determine farmers decisions to adopt irrigation. In this context, the dominant input associated with irrigation is households' own labor. The shadow wage that prices household labor is notoriously difficult to value, but if this labor were valued at the market wage, estimated effects on household labor would be 6 times as large as estimated effects on expenditures on hired labor and other inputs, and estimated effects on profits would fall from 70% to less than 20%. Valuing household labor at the market wage may not be appropriate: rural market wages are likely to be inefficiently high in developing countries(Kaur, 2014; Breza et al., 2018), and labor market failures in rural areas may generate heterogeneity in the shadow wage (Singh et al., 1986; Benjamin, 1992; LaFave & Thomas, 2016). Heterogeneity in the shadow wage would then cause inefficient adoption of irrigation across households.[5] Alternatively, these results could also be consistent with unconstrained profit maximization if farmers have heterogeneous returns to or costs of adopting irrigation (Suri, 2011) and optimize at market wages.

We derive a test for inefficient adoption of irrigation caused by market failures. To produce this test, we build on the framework from Benjamin (1992) and model households' production decisions, incorporating uncertainty, plot-level heterogeneity, and failures in insurance, credit, and labor markets. Consistent with our reduced form results, we model access to irrigation as a labor- and input-complementing increase in plot-level productivity. Our test is as follows. With complete markets, farmers maximize profits on each plot and access to irrigation on one plot does not affect production decisions on other plots. In contrast, when there are failures in land and other markets, access to irrigation on one plot causes substitution of labor and inputs away from other plots.[6] This test is joint for the null

[5]This heterogeneity could only exist if there were frictions in at least one other market in addition to labor markets.

[6]The mechanism is straightforward: access to irrigation on one plot increases input use on that plot. That increase does not affect input demand on the farmers' other plots; however, if the farmer faces binding

of frictionless land markets: if land markets are frictionless, then markets should reallocate land to farmers who can cultivate most profitably.

We implement our test for inefficient adoption caused by market failures, exploiting the plot-level discontinuity in access to irrigation. We test whether farmers who have a plot just inside the command area reduce their input use on their other plots compared to farmers who have a plot just outside the command area. We find large substitution effects, strongly rejecting complete markets: for farmers with a plot in the command area, an additional irrigated plot caused by access to irrigation is associated with a 68 percentage point decrease in the probability of irrigating the second plot. We find similarly large effects for adoption of horticulture, household labor, and inputs. These results confirm a simple descriptive analysis, which shows that few households are able to irrigate more than one command area plot. Applying these results, a simple back-of-the-envelope calculation implies that, absent this substitution, adoption of irrigation would be at least 21% higher. Moreover, the presence of this substitution implies current adoption of irrigation is inefficient: different households make different adoption decisions on technologically identical plots because of their access to irrigation on their other plots.

The previous test shows that inefficient adoption of irrigation is caused by failures of land markets, and at least one other market; however, it does not establish which other market fails. We produce two tests that suggest that labor market constraints, as opposed to financial constraints, bind in our context.

First, we extend the model and propose a test for whether labor market frictions contribute to inefficiently low adoption in this context. To produce this test, we consider the effects of household size and wealth on input substitution across plots, in the presence of insurance, credit, and labor market failures. We demonstrate that, while many patterns of differential substitution are possible, only labor market failures can explain irrigation access on one plot leading to greater input substitution across plots for richer households, and de-

constraints in input, risk, or labor markets, that increase in input use must be associated with a decrease in input use on other plots.

creased input substitution across plots for larger households. We then estimate differential substitution with respect to household size and wealth to test for labor market failures. We find exactly this pattern: households with two additional members substitute 62-86% less than average size households, while one standard deviation wealthier households substitute 40-80% more than average wealth households. As these patterns of differential substitution can only be explained by labor market failures, and not credit or insurance market failures, these results imply that labor market failures cause substitution and contribute to inefficient adoption of irrigation.

We then complement this result with experimental evidence. We conduct three randomized controlled trials with the farmers who have access to irrigation. Two of these trials focus on characteristics peculiar to irrigation systems: usage fees and failures of operations and maintenance; we find neither plausibly affects farmers' adoption decisions in our context. The third experiment more directly targets the alternative explanations of financial and informational constraints to adoption. In the third experiment, we distribute minikits which contain all necessary inputs for horticulture cultivation to randomly selected farmers. Previous work has shown providing free minikits targets credit, risk, and information constraints: it reduces costs of growing horticulture under irrigation, basis risk, and costs of experimentation, respectively (Emerick et al., 2016; Jones et al., 2018). We find no effects of receiving minikits on adoption of horticulture in our context, in contrast to existing work. A closer analysis indicates that the farmers who take up the minikits are the same farmers who would have been likely to cultivate horticulture absent the intervention. Combining this evidence with the model-based test above, we conclude that financial and informational constraints are unlikely to be a primary explanation for low and inefficient adoption of irrigation.

This paper demonstrates that land and labor market failures cause inefficient adoption of hillside irrigation in Rwanda. This result builds on a deep literature on separation failures, which empirically demonstrates that factor market failures affect the allocation of land and labor across households (Singh et al., 1986; Benjamin, 1992; LaFave & Thomas, 2016; Dillon et al., 2019). The existing literature does so by testing whether households with different characteristics use different levels of inputs; however, this type of test stops short of showing that these allocations are inefficient (Udry, 1997). In particular, it can only conclude that one market has failed, which by Walras' Law is insufficient to imply inefficiencies. We innovate by demonstrating differential adoption of irrigation on technologically identical plots. In doing so, we also contribute to a literature leveraging production function estimates to document misallocation of labor and inputs by inferring their marginal products from their allocations across plots or households (Jacoby, 1993; Skoufias, 1994; Udry, 1996; Restuccia & Santaeulalia-Llopis, 2017). Although demonstrating heterogeneity in the marginal product of labor is sufficient to show that labor market failures generate inefficiencies, the methods employed by this literature are typically not robust to the presence of unobserved heterogeneity across plots or measurement error (Gollin & Udry, 2019). Our test for inefficient technology adoption caused by labor market failures therefore complements this literature, by both imposing less structure and leveraging our plot-level discontinuity in access to irrigation as an exogenous labor- and input-complementing productivity shock.

This paper is organized as follows. Section 2 describes the context we study and our sources of data. Section 3 presents our estimates of the impacts of irrigation in Rwanda. Section 4 presents our model of adoption of irrigation in the presence of market failures. We implement tests of constraints to adoption and labor market failures suggested by the model in Section 5, and experimental tests in Section 6. Section 7 concludes.

2 Data and context

2.1 Irrigation in Rwanda

We study 3 hillside irrigation schemes, located in Karongi and Nyanza districts of Rwanda, that were constructed by the government in 2014; a timeline of construction and our surveys is presented in Figure 1. Rainfed irrigation in and around these sites is seasonal, with three potential seasons per year. During the main rainy season ("Rainy 1"; September - January), rainfall is sufficient for production in most years. In the second rainy season ("Rainy 2"; February - May), rainfall is sufficient in an average year but insufficient in dry years. In the dry season ("Dry"; June - August), rainfall is insufficient for agricultural production for seasonal crops. Absent irrigation, agricultural production in these sites consists of a mix of staples (primarily maize and beans) which are cultivated seasonally and primarily consumed by the cultivator, as well as perennial bananas which are sold commercially.[7] Absent irrigation, therefore, most farmers adopt either a rotation of staples, fallowing land in the dry season, or cultivate bananas.

Irrigation in these schemes is expected to increase yields by reducing risk in the second rainy season and enabling cultivation in the short dry season. As the dry season is relatively short, cultivating the primary staple crops is not possible, even with irrigation, for households that cultivate during the two rainy seasons. Instead, cultivating shorter cycle horticulture during the dry season becomes a possibility with the availability of irrigation. Horticulture production (most commonly eggplant, cabbage, carrots, tomatoes, and onions) can be sold at local markets where it is both consumed locally and traded for consumption in Kigali.[8] As horticultural production is relatively uncommon during the dry season in Rwanda due to limited availability of irrigation, finding buyers for these crops is relatively easy during this time. Absent irrigation, horticulture is familiar but uncommon around these areas; at baseline 3.2% of plots outside of the command area are planted with at least some horticulture, primarily during the rainy seasons.

In this context, the three schemes we study were constructed by the government from 2009 - 2014, with water beginning to flow to some parts of the schemes in 2014 Dry and becoming fully operational by 2015 Rainy 1 (August 2014 - January 2015). The schemes in our study share some common features; a picture from one of the schemes is presented

[7]Staple rotations also include smaller amounts of sorghum and tubers, while there is also some cultivation of the perennial cassava, along with other minor crops. In our data, maize, beans, or bananas are the main crop for 85% of observations excluding horticulture.

[8]Kigali is less than a 3 hour drive from these markets, facilitating trade.

in Figure 2. In each site, land was terraced in preparation for the irrigation works (as hillside irrigation would be infeasible on non-terraced land). Construction and rehabilitation of terraces in these sites began in 2009 - 2010. The schemes are all gravity fed, and use surface water as the source.[9] From these water sources, a main canal (visible in Figure 2) was constructed along a contour of the hillside; engineering specifications required the canal to be sufficiently steep so as to allow water to flow, but sufficiently gradual to control the speed of the flow, preventing manipulation of the path of the canal. Underground pipes run down the terraces from the canal every 200 meters. Farmers draw water from valves on these pipes located on every third terrace, from which flexible hoses and dug furrows enable irrigation on all plots below the canal. The "command area" for these schemes, the land that receives access to irrigation, is the plots which are below the canal and located within 100 meters of one of these valves.

In all sites, sufficient water is available to enable irrigation year-round. To the extent that there is heterogeneity in plot-level water pressure, the plots nearest to the canal face the lowest pressure.[10] The primary cost to farmers of irrigating a plot in this context is their labor associated with the actual irrigation, including maintaining the dug furrows and using the hoses to apply water from the valves to their plots. At the time of the study, there are no fees associated with the use of irrigation water[11]

[9]In two sites, a river provides the water source, while in the third site, a dammed lake is the source.

[10]The lower pressure on these plots is attributable to the design of the pipes, which fill up with water before valves are opened; forces of gravity and the lower volume of water in the pipes above the highest valves generates somewhat weaker pressure than at the lower valves (though pressure is still sufficient for effective irrigation). This difference in pressure could become more serious if lower valves were opened at the same time as higher valves; in practice, schedules of water usage are agreed upon to prevent this from happening.

[11]The government does have an objective of developing the financial self-sufficiency of the schemes. To do so, land taxes are intended to be applied to the plots in the command area, which (as land taxes) should not influence cultivation decisions. These taxes are intended to be small in magnitude compared to potential farmer yields as they are meant to fund only ongoing operations and maintenance costs rather than full cost recovery; the highest fees across the sites were 77,000 RwF/ha/year, while our dry season treatment on the treated estimates presented in Section 3 are 361,000 - 379,000 RwF/ha. The first attempts to collect these taxes were made in 2017 Rainy 1. The survey team engaged in an experiment to test whether these taxes were a barrier to use of the irrigation system by randomizing subsidies across farmers at up to 100%; we failed to find any evidence that the taxes changed farming practices (results available from authors). This is perhaps unsurprising as, based on the original schedule, tax compliance was very low, with less than 20% of taxes collected from farmers who did not receive full subsidies from the research team.

We exploit a spatial discontinuity in irrigation coverage to estimate the impacts of irrigation. Because the main canals must conform to prescribed slopes relative to a distant and originally inaccessible water source, the geologic accident of altitude relative to this source determines which plots will and will not receive access to irrigation water. Hence, before construction, plots just above the canal should be similar to plots just below the canal, and importantly, should be managed by similar farmers. Following construction, however, the plots just below the canal fall inside the command area and have access to irrigation, while the terraces just above the canal fall outside the command area and do not have access to irrigation.

2.2 Data

2.2.1 Aerial sampling

To take advantage of the spatial discontinuity in access generated by the command area boundary, we randomly sampled plots in close proximity to this discontinuity. We ensure a high density of plots just inside and just outside the boundary by using point-based sampling to over-sample plots cultivated just inside and just outside the command area. In practice, we constructed this aerial sample of plots by dropping a uniform grid of points across the site at 2-meter resolution, and then randomly sampling points within the grid. After each point was sampled, we excluded any points within 10m of that that point (to keep from selecting multiple points too close together). In two of the three sites, there is a viable boundary of cultivable land both just inside and just outside the command area. In these sites, to guarantee a high density of observations near the canal, we over sampled points that were within 50m of command area boundary, both inside the command area and outside the command area.[12]

Enumerators were then given GPS devices with the locations of the points, and sent

[12] In both sites, we additionally sampled some points further from the canal inside the command area (at a lower rate). We use these points along with data from the third site primarily to examine experimental treatments described below.

to each point, with a key informant (often the village leader). For each point, they were asked to identify if the point was on cultivable land (this was to discard forest, swamps, thick bushes, bodies of water, or other terrain which would make cultivation impossible). When a point fell on cultivable land, they recorded the name of the cultivator of the plot, their contact information, as well as a sufficiently detailed description of the plot. In the rest of this paper, we refer to all plots thus identified as *sample plots*. Our main household sample was built from this aerial sampling procedure: the data from this listing was used to construct a roster of all the unique names of cultivators, eliminating duplicate names. Finally, for each household, one of the points that fell on their plots was randomly selected to be that household's sample plot.

2.2.2 Survey

Our baseline survey survey was implemented in May 2015 and includes detailed agricultural production data (season-by-season) for seasons 2014 Dry through 2015 Rainy 2, that is, spanning the year from June 2014 - May 2015; the dates of this survey and follow up surveys, along with the agricultural seasons they cover, are presented in Figure 1. As mentioned above, this is not a "true" baseline as some farmers had already gained access to irrigation in 2014 Dry. However, relatively small parts of the site had access to irrigation at this point; in Section 3.2.1 we highlight that 2014 Dry adoption of irrigation is less than 25% of adoption in subsequent dry seasons, and in Section 3.1.1 we show balance across the command area boundary in household and plot characteristics. Production and input data are collected plot-by-plot; in the baseline we conducted this production data for up to four plots, although subsequent surveys maintain a panel of two plots. Each of these plots was also mapped using GPS devices during the baseline; we use this data to construct the area of plots and their locations. The two plots on which panel data is collected represent the primary data for analysis; they include the sample plot (described above) and the farmer's next most important plot (defined at baseline; we refer to this as the "most important plot"). We also collected data on household characteristics, labor force behavior, and a short consumption and food security module. In analysis, we will focus on the sample plots to learn about the effects of the irrigation itself, and the most important plot to learn about how the presence of the irrigation impacts household productive decisions more broadly.

Three follow up household surveys were conducted in May - June 2017, November - December 2017, and November 2018 - February 2019. In each survey, we asked for up to a year of recall data on agricultural production; based on the timing of our surveys we therefore have production for all agricultural seasons from June 2014 through August 2018, with the exception of 2015 Dry (June - August 2015) and 2016 Rainy 1 (September 2015 - February 2016).

The sample for the follow up surveys consists of all the baseline respondents. To build a panel of households and plots, we interviewed households from the baseline and recorded information on all their baseline plots. Whenever a household's sample plot or most important plot was sold or rented out to another household, or a household stopped renting in that plot if it was not the owner, we ran a "tracking survey". Specifically, we tracked and interviewed the new household responsible for cultivation decisions on that plot to record information about cultivation and production, along with household characteristics when the new household was not already in our baseline sample.

2.3 Stylized facts

To motivate our analysis of the impacts of hillside irrigation, we first introduce some stylized facts about irrigation in this context. Table 1 presents summary statistics for agricultural production from our four years of data, pooled across seasons.

Stylized Fact 1. *Irrigation in Rwanda is primarily used to cultivate horticulture in the dry season.*

Farmers in our data rarely irrigate their plots in the wet season, and almost never use irrigation when cultivating staples or bananas (only 2% of plots cultivated with staples or bananas use irrigation in our data). In contrast, 92% of farmers who cultivate horticulture in the dry season use irrigation. This stylized fact makes agronomic sense as the rainfall in rainy seasons in this part of Rwanda is usually sufficient for either staple or horticultural production (and in wet years may be harmfully excessive for horticulture). Additionally, as staples do not have a sufficiently short cycle to permit cultivation during the relatively short dry season (while horticulture does), it is not feasible to use irrigation to cultivate staples during the dry season.

Stylized Fact 2. *Horticultural production is more input intensive than staple cultivation, which in turn is (much) more input intensive than banana cultivation.*

The mean horticultural plot uses about 460 days/ha of household labor, 60 days/ha of hired labor, and 46,000 RwF/ha of inputs, regardless of the season in which it is planted.[13] This contrasts to staple plots (300 days/ha of household labor, 40 days/ha of hired labor, 20,000 - 40,000 RwF/ha of inputs), and bananas (100 days/ha of household labor, 10 days/ha of hired labor, 3,000 RwF/ha of inputs).

Stylized Fact 3. *Horticultural production produces much higher cash profits than other forms of agriculture.*

Horticultural production produces much higher cash profits (defined as yields net of expenditures on inputs and hired labor) than other forms of agricultural production in and around these sites. Plots planted to horticulture yield about 500,000 RwF/ha in cash profits, in both rainy and dry seasons. This contrasts with about 250,000 RwF/ha of cash profits producing either staples or bananas.

Stylized Fact 4. *Household labor is the primary input to production of any crop, and the economic profitability of horticulture depends critically on the shadow wage.*

A large existing literature examines separation failures in labor markets faced by agricultural households (e.g., Benjamin (1992); LaFave & Thomas (2016); Singh et al. (1986)). If

[13] For reference, in the study period, the exchange rate was approximately 800 RwF = 1 USD

households are constrained in the quantity of labor they are able to sell on the labor market, they may work within the household at a marginal product of labor well below the market wage. Here, we see that if we value household labor allocated to horticulture at market wages, then cultivating horticulture appears less profitable than cultivating bananas (though both appear more profitable than cultivating staples).[14] As a result, ultimately the economic profitability of horticulture relative to bananas will depend critically on the constraints on household labor supply decisions.

3 Impacts of irrigation

3.1 Empirical strategy

We start our analysis through a simple OLS framework, and we restrict this and subsequent analysis to sample plots within 50 meters of the discontinuity. If these nearby plots are sufficiently similar so that irrigation access can be taken as random within this sample, we can simply regress

$$y_{1ist} = \beta_0 + \beta_1 \mathrm{CA}_{1is} + \alpha_{st} + \epsilon_{1ist} \tag{1}$$

Where y_{kist} is outcome y for plot k of household i located in site s in season t, CA_{kis} is an indicator for that plot being in the command area, and α_{st} are site-by-season fixed effects meant to control for any differences or trend differences across sites (including market access or prices). We use $k = 1$ to indicate the household's sample plot, as opposed to the household's most important plot.

Next, we consider two primary potential sources of omitted variable bias. First, plots that are positioned relatively higher on the hillside may have different agronomic characteristics, and accordingly farmers may differentially sort into these plots. As plots inside the command area are lower on the hillside (below the canal) and plots outside the command area are higher

[14] Both horticulture and bananas are also primarily commercial crops, unlike staples. Farmers may place higher value on staples if consumer prices are higher than producer prices (Key et al., 2000), or if there is price risk in production and consumption, both of which may contribute to cultivation decisions as well.

on the hillside (above the canal), the command area indicator will be correlated with position on the hillside and β_1 may be biased. Second, as the construction of the canal slices through plots on the hillside, this may differentially change the area of plots that are positioned higher or lower on the hillside. For example, roads are more often located higher on the hillside, leaving less room for plots to extend above the canal relative to below the canal. As we anticipate this will cause plots to be relatively larger just inside the command area, and plots exhibit strong evidence of diminishing returns to scale in this context, this effect will likely bias β_1 downwards.

We account for these two potential sources of omitted variable bias by including controls. First, to account for position on the hillside, we control for distance of the plot from the command area boundary, and distance of the plot from the command area boundary interacted with the command area indicator.[15] This is a standard regression discontinuity specification, and as such compares sample plots that are just inside the command area to sample plots that are just outside the command area. Second, to account for differences in area of plots, we control for the log area of sample plots. Specifically, we estimate

$$y_{1ist} = \beta_0 + \beta_1 \text{CA}_{1is} + \beta_2 \text{Dist}_{1is} + \beta_3 \text{Dist}_{1is} * \text{CA}_{1is} + \alpha_{st} + X'_{1is}\gamma + \epsilon_{1ist} \tag{2}$$

where Dist_{kis} is the distance of that plot from the command area boundary (positive for plots within the command area, negative for plots outside the command area), and X_{1is} is a vector of controls which only includes log area.

Next, we consider additional concerns related to selection into our sample caused by access to irrigation. In particular, plots inside the command area are more productive as they have access to irrigation. If more productive plots are more likely to select into our sample, this can introduce bias. This may arise for two reasons. First, during the construction of the hillside irrigation schemes, forest was deliberately preserved or planted just outside of the command area in order to protect the new investment from erosion. As these forested plots

[15] We calculate distance using the distance of the plot boundary to the command area boundary.

are not agricultural, they are not included in our sampling strategy.[16] Second, marginal plots which, absent irrigation, would have been too unproductive to cultivate, and would thus have been left permanently fallow, may now be sufficiently productive to be worth cultivating with access to irrigation. While our sampling strategy selected both cultivated and uncultivated plots, it did not select plots which had been left overgrown with thick bushes, as it would have been difficult to identify the household responsible for those plots. In practice, the latter is likely uncommon, as typical household landholdings are small in the hillside irrigation schemes we study (around 0.3 ha), and agricultural land is highly valued – median rental prices in our data are 150,000 RwF/ha, approximately 25% of annual yields.

We account for this potential source of bias using spatial fixed effects (SFE; see Goldstein & Udry (2008); Conley & Udry (2010); Magruder (2012, 2013)), which use comparisons between nearby units to adjust for spatially correlated unobservables, such as unobserved heterogeneity in productivity caused by soil characteristics. For example, if some areas of low productivity are left forested outside of the command area, but not inside, then plots inside the command area will be systematically (unobservably) less productive than plots outside the command area. However, because SFE estimators only compare neighboring plots, the low productivity plots inside the command area that are near forested low productivity areas will not have nearby comparison plots outside the command area, and therefore will not contribute to the estimation of the effect of the command area.[17]

In practice, we define a set $\mathcal{N}_{kist}$ to be the group of five closest plots to plot k observed in season t, including the plot itself. Then, for any variable z_{kist}, define $\overline{z}_{kist}$ =

[16] Typically, forests were planted or preserved in areas of low productivity, where the slope of the hillside was relatively high and erosion was relatively common. Therefore, this amounts to selection out of our sample of low productivity plots outside the command area, which would bias β_1 downwards.

[17] Formally, SFE estimators leverage the identification assumption $\lim_{||k-k'||\to 0} E[\epsilon_{kist}|X_{kist}] = E[\epsilon_{k'i'st}|X_{k'i'st}]$, where $||k-k'||$ represents the distance between plot k and plot k'. Similar approaches to identification leveraging a spatial discontinuity include pairwise matching across the boundary (e.g., Dube et al. (2010)) and boundary designs with segment fixed effects (e.g., Dell (2010)).

$(1/|\mathcal{N}_{kist}|)\sum_{k'\in\mathcal{N}_{kist}} z_{k'i'st}$. The SFE specification then estimates

$$y_{1ist} - \overline{y}_{1ist} = \beta_1(\mathrm{CA}_{1is} - \overline{\mathrm{CA}}_{1is}) + (V_{1is} - \overline{V}_{1is})'\gamma + (\epsilon_{1ist} - \overline{\epsilon}_{1ist}) \tag{3}$$

where V_{kis} includes all controls from Equation 2, except the subsumed site-by-season fixed effects.

Our sampling strategy yields the following plot proximity: restricting to the sample plots in our main sample for regression discontinuity analysis, 49% of plots have 3 plots (self inclusive) within 50 meters, and 87% have 3 plots within 100m; 60% of plots have all 5 plots (self inclusive) within 100m, while 83% have all 5 plots within 150m. As reference, Conley & Udry (2010) use 500m as the bandwidth for their estimator, while Goldstein & Udry (2008) use 250m as the bandwidth; we therefore anticipate that underlying land characteristics are likely to be quite similar between each plot and its comparison plots.

3.1.1 Balance

We now use specifications (1), (2), and (3) to examine whether the plots in our sample and the households who cultivate them are comparable at baseline. For each of these specifications, we show balance both with key controls omitted (Columns 3, 5, and 6), and our preferred specifications which we use in our analysis with key controls included (Columns 4, 7, and 8).

First, and crucially, Table 2 indicates that our sample plots are balanced in terms of ownership and rentals, and that the vast majority of sample plot owners on both sides of the canal owned the land over 5 years, or prior to the start of the irrigation construction. There is, however, some imbalance on plot size; as discussed in Section 3.1, log area (measured in hectares) is larger inside the command area than outside the command area. This imbalance is weaker in the SFE specification than in the RDD specification, such that the omnibus test fails to reject the null of balance for the SFE specification (although we reject for the RDD specification). However, we note that this imbalance would bias us against finding the effects we see in Section 3.2 on horticulture, input use, labor use, and yields, as all of these variables are larger in smaller plots in both the command area and outside the command area. We therefore present results estimated using Equation (1), which does not control for log area, and using Equations (2) and (3), which do control for log area.

Following the ownership results, Table 3 examines the characteristics of households whose sample plots are just inside or just outside the command area. First, note that Column 1, which does not restrict to the discontinuity sample, performs poorly here; we find significant imbalance on half of our variables, and the omnibus test rejects the null of balance. However, we fail to reject balance for all specifications which restrict to the discontinuity sample; households with sample plots just inside the command area appear similar to households with sample plots just outside the command area. In some specifications, there are significant differences in whether the household head has completed primary schooling or not, though we note that 1 out of 10 variables significant at the 10% level is what one would expect due to chance.

Lastly, in Section 5.1.1, we consider the characteristics of households' most important plots; we show that these appear similarly balanced.

3.2 Estimating the effects of irrigation

3.2.1 Adoption Dynamics

Figure 3 presents the share of plots irrigated by season for sample plots just inside the command area and sample plots outside the command area. First, as the irrigation sites were already partially online in our baseline, we already observe some increased adoption of irrigation in the command area in 2014 Dry: sample plots in the command area are approximately 5pp more likely to be irrigated than sample plots outside the command area. Second, starting with 2015, adoption of irrigation does not appear to trend, but exhibits meaningful seasonality. Differences remain around 3pp-6pp in the rainy seasons, and 18pp-25pp in the dry seasons.

Given the limited changes in adoption dynamics after 2014 and the stark differences in adoption across dry and rainy seasons, for the remainder of our analysis we estimate (1), (2), and (3) pooling across our three years of follow up surveys, splitting our results across dry and rainy seasons.

3.2.2 Impacts of irrigation

We now present our results on the impact of access to irrigation on crop choices, on input use, and on production. First, we present graphical evidence of the regression discontinuity in Figure 4; for parsimony, we do so only for the dry seasons (2016 Dry, 2017 Dry, and 2018 Dry).[18] In each of the regression discontinuity figures, distance to the canal in meters is represented on the x-axis, with a positive sign indicating that the plot is on the command area side of the boundary. Second, we present regression evidence in Tables 4, 5, and 6. In the discussion below, we focus on results from the tables, but we note that these results are consistent with estimates based on Figure 4.

First, in line with results from Section 3.2.1, command area plots are 15-19pp more likely to be irrigated during the dry season than plots outside the command area, and almost all of this increase is explained by the transition to cultivation of high value horticulture during this dry season. In contrast, adoption of irrigation during the rainy season is much lower, with increases of just 4-6pp. This transition to dry season horticulture substitutes for cultivation of perennial bananas, a less productive but less input intensive commercial crop; we estimate a decrease of 13-15pp in the command area, and as a consequence we observe no impacts on cultivation in the dry season.[19]

Second, we find large increases in dry season input use, which are dominated by increases in household labor. These results are consistent with the transition from perennial

[18] Rainy season differences are always smaller and generally not visually noteworthy; we focus most of our discussion on the dry season results.

[19] As bananas are perennials, plots cultivated with bananas typically have harvests in each season. In contrast, the rotations of staples and horticulture (or simply horticulture) that replace bananas may only involve two plantings and harvests, and we therefore see a modest decrease in cultivation during the rainy seasons of 5-10pp on a baseline of 81%.

bananas, which require little inputs and labor, into horticulture, which is highly input and labor intensive. To interpret these results, we conduct a treatment on the treated analysis under the assumption that the command area increases input use only through it's effect on irrigation. Doing so, we find that adoption of irrigation increases household labor use, input expenditures, and hired labor expenditures by 350-430 person-days/ha, 25,000-35,000 RwF/ha, and 15,000-25,000 RwF/ha, respectively; these numbers are similar to differences in input intensity of dry season horticulture and bananas reported in Table 1. The impacts on household labor are particularly large – valued at a typical wage of 800 RwF/person-day, this labor would be priced at 280,000-345,000 RwF/ha, an order of magnitude larger than the effects on input expenditures or hired labor expenditures. Additionally, as reference, applying this labor to 0.3 ha (median household landholdings) of command area land would require roughly 4 person-months of labor during the 3 month dry season. In contrast, we find no effects on input use during the rainy seasons.

Third, consistent with our estimates of impacts on input use, we find large increases in dry season agricultural production. Treatment on the treated analysis suggests adoption of irrigation increases yields by 300,000-400,000 RwF/ha, 50-70% of annual agricultural production. As horticulture is primarily commercial: each 1 RwF/ha increase in yields is associated with a 0.79-0.86 RwF/ha increase in sales. Once again, these results on outputs are consistent with differences between bananas and horticulture production reported in Table 1. Additionally, these impacts on yields are much larger than our estimates of impacts on input and hired labor expenditures; our results suggest irrigation increases yields net of expenditures by 240,000-360,000 RwF/ha, a 46-70% increase in annual yields net of expenditures. However, we should not interpret this as impacts on profits, as it implicitly places no value on the large increases in household labor. If we instead value household labor at 800 RwF/person-day, the median wage we observe, these impacts vanish completely. Therefore, the profitability of the transition to dry season horticulture enabled by irrigation depends crucially on the shadow wage at which household labor is valued.

Taken together, these results together suggest that irrigation leads to a large change in production practices for a minority of farmers. Those farmers cultivate horticulture in the dry season and a mix of horticulture, staples, and fallowing in the rainy seasons, they have substantially higher earnings in the dry season but similar earnings in the other seasons, and they invest more in inputs and much more in household labor in the dry seasons. Our estimates suggest that irrigation has the potential to be transformative in Africa, in light of the 46-70% increases in yields net of expenditures that we document from just three months of cultivation. At the same time, these results also suggest that the shadow wage, and therefore labor market frictions, are likely to be important for the decision to cultivate horticulture. Building on this result, we next adapt the model from Benjamin (1992) to develop tests for the role of market failures in adoption of irrigation.

4 Testing for binding constraint

4.1 Model

Farmers have 2 plots, indexed by k: $k = 1$ indicates the sample plot, while $k = 2$ indicates the most important plot. On each plot k, they have access to a simple production technology $\sigma A_k F_k(M_k, L_k)$ where A_k is plot productivity, M_k is the inputs applied to plot k and L_k is the labor applied to plot k. The common production shock σ is a random variable such that $\sigma \sim \Psi(\sigma), E[\sigma] = 1$. While this specification assumes a single production function on each plot, we can think of $F_k(M_k, L_k)$ as the envelope of production functions from cultivating different fractions of bananas and horticulture on the dry season; thus we will think of cultivating bananas as optimizing at a low input intensity. Utilizing subscripts to indicate partial derivatives and subsuming arguments we assume $F_{kM} > 0, F_{kL} > 0, F_{kML} > 0, F_{kMM} < 0, F_{kLL} < 0$.[20] Farmers have a budget of $\bar{M}$ which, if not utilized for inputs, can

[20]Among these, $F_{kML} > 0$ is the most controversial. Existing evidence on F_{kML} in developing country agriculture is mixed (see Heisey & Norton (2007) for discussion). In our context, we expect $F_{kML} > 0$ primarily because $F_k(\cdot, \cdot)$ encompasses the transition from bananas to horticulture, which should be associated

be invested in a risk-free asset which appreciates at rate r. In this context, farmers maximize expected utility over consumption and leisure l, considering their budget constraint and a labor constraint $\overline{L}$ which is allocated to labor on each plot, leisure, and up to $\overline{L^O}$ units of off farm labor L^O. Finally, we model irrigation access as an increase in A_1. As we consider the role of each different constraint, we develop the necessary assumptions to imply the results above: that this increase in A_1 generates an increase in demand for inputs and labor on plot A_1.

Farmers maximize expected utility

$$\max_{M_1,M_2,L_1,L_2,l,L^O} E[u(c,l)]$$

subject to the constraints enumerated above

$$\begin{aligned} \sigma A_1 F(M_1, L_1) + \sigma A_2 F(M_2, L_2) + wL^O + r(\overline{M} - M_1 - M_2) = c \\ M_1 + M_2 \leq \overline{M} \\ L_1 + L_2 + l + L^O = \overline{L} \\ L^O \leq \overline{L^O} \end{aligned}$$

In this framework, there are three crucial constraints farmers may face that cause deviations from expected profit maximization: access to insurance may be limited, reducing input use to avoid basis risk; credit or access constraints may limit input use; and farmers' off farm labor allocations may be constrained from above, resulting in overutilization of labor on the household farm. In analyzing model predictions we discuss the cases in which each of these constraints do or do not bind.

After substituting in the constraints which bind with equality, we derive the following

with increased input demands according to Stylized Fact 2.

first order conditions[21]

$$
\begin{aligned}
&(M_k) \quad \left(1+\tfrac{\mathrm{cov}(\sigma,u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kM} &&= (1+\lambda_M) r && (4)\\
&(L_k) \quad \left(1+\tfrac{\mathrm{cov}(\sigma,u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kL} &&= (1-\lambda_L) w && (5)\\
&(\ell) \quad \tfrac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} &&= (1-\lambda_L) w && (6)
\end{aligned}
$$

Intuitively, the first order conditions for inputs and labor include three parts. First, each contains the marginal product of the factor, $A_k F_{kM}$ and $A_k F_{kL}$ respectively, on the left hand side, and the market price of the factor, r and w respectively, on the right hand side. The second piece, $1 + \frac{\mathrm{cov}(\sigma,u_c)}{\mathbf{E}[u_c]}$, is the ratio of the marginal utility from agricultural production to the marginal utility from certain consumption. This ratio scales down the marginal product of the factor. It is less than 1 because agricultural production is uncertain, and higher in periods in which marginal utility is lower, so $\mathrm{cov}(\sigma, u_1) < 0$. With perfect insurance, $\mathrm{cov}(\sigma, u_1) = 0$, and this piece disappears. Without it, however, farmers will underinvest in both inputs and labor relative to the perfect insurance optimum. Third, there are the Lagrange multipliers associated with the input constraint λ_M and with the labor constraint λ_L, which scale the associated factor prices up and down, respectively.

When these constraints do not bind, and with perfect insurance, we have the familiar result that marginal products equal marginal prices. However, if any of these constraints bind, then separation fails: farmer characteristics which are related to λ_L, λ_M, or $\mathrm{cov}(\sigma, u_1)$ will be correlated with inefficient input allocation on all plots (inefficiently low in the case of inputs and inefficiently high in the case of labor).

4.2 A test for separation failures

In this context, we consider a new test of separation: the effect of a change in access to irrigation on the sample plot on production decisions on the most important plot. Much of

[21] The derivation is in Appendix B.

the literature that tests for separation, building on Benjamin (1992), has focused on tests built around the assumption that household characteristics should not affect the household's optimal production decisions under perfect markets. We instead leverage the assumption that access to irrigation on the sample plot (the "sample plot shock") should not affect the optimal production decisions on the household's most important plot.

Following our model, we show how these market failures in insurance, labor, or input markets generate a separation failure between production decisions on the sample plot and production decisions on the most important plot. First, we derive the classic separation result from Singh et al. (1986) in our framework when there are no market imperfections.

Proposition 1. *If no constraint binds, separation holds and input and labor use on the most important plot does not respond to the sample plot shock.*

Showing this result is straightforward: with perfect markets for inputs, labor, and insurance, $\frac{\mathrm{cov}(\sigma, u_c)}{\mathbf{E}[u_c]} = 0$, $\lambda_L = 0$, and $\lambda_M = 0$, respectively. The first order conditions then simplify to

$$\begin{aligned} (M_k) \quad & A_k F_{kM} && = r \\ (L_k) \quad & A_k F_{kL} && = w \\ (\ell) \quad & \frac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} && = w \end{aligned}$$

The household's labor and input allocations on plot 2 depend only on plot 2 productivity A_2, the price of inputs r, and the wage w, and not on access to irrigation on plot 1 (A_1).

In contrast to the case with perfect markets, in the presence of market failures, the sample plot shock can affect the households allocations on its most important plot. Roughly speaking, the sample plot shock increases the household's agricultural production, and increases its labor and input demands on the sample plot. When markets fail, this reduces the value the household places on agricultural production, and increases its opportunity costs of labor and inputs, and the household reduces its labor and input allocations on its most important plot. The following propositions require additional assumptions on the shape of the utility function or on the distribution of σ; we flag those in the text below each proposition.

Proposition 2. *If input, labor, or insurance constraints bind, then input and labor use are reduced on the most important plot in response to the sample plot shock.*[22]

The logic case-by-case is as follows. First, if input constraints bind, then the increase in inputs on the sample plot caused by access to irrigation must be associated with a reduction in inputs on the most important plot. As inputs and labor are complements, this causes labor allocations on the most important plot to fall as well. Second, if labor constraints bind, then the increase in labor on the sample plot caused by access to irrigation must be associated with a reduction in the sum of leisure and labor on the most important plot. Under standard restrictions on the household's on farm labor supply, this must be associated with a reduction in labor on the most important plot.[23] As inputs and labor are complements, this causes input allocations on the most important plot to fall as well. Third, absent insurance, then the increase in agricultural production caused by access to insurance reduces the marginal utility from agricultural production relative to the marginal utility from consumption.[24] In turn, this causes labor and input allocations to the most important plot to fall.

This result produces a test of separation. Rejecting separation with this test implies that irrigation adoption levels are inefficient and that land market failures contribute to this inefficiency. At the same time, this test does not allow us to test for which other constraints interact with land market frictions to generate separation failures. This is because the presence of any set of constraints that generate separation failures yields the same prediction: the sample plot shock should cause input and labor allocations on the most important plot to fall. In particular, the intuition that observing changes in input allocations, labor

[22]See proof in Appendix B.

[23]Specifically, we assume that leisure demand is increasing in consumption; this assumption is not necessary but is sufficient.

[24]This does not generically hold; however, restrictions on the distribution of σ are sufficient to imply that marginal utility from agricultural production relative to the marginal utility from consumption is falling in agricultural production. Details are in Appendix B.

allocations, or cropping decisions on the most important plot might suggest the presence of input constraints, labor constraints, or insurance constraints, respectively, fails, because inputs, labor, and horticulture are all complements in the production function.

4.3 Separating constraints

To shed some light on which other constraints generate separation failures, we leverage the fact that our model offers predictions about how households with different characteristics should *differentially* respond to the sample plot shock. Roughly speaking, depending on which constraint binds, changes in different household characteristics may slacken or tighten the binding constraint. We focus on two important household characteristics in our model: we use household size to shift $\overline{L}$, the household's total available labor, and wealth to shift $\overline{M}$, the household's exogenous income available for input expenditures. We present these predictions below.

Proposition 3. *If input constraints or insurance constraints bind, then the input and labor allocations on the most important plot of larger households (wealthier households) should be less (less) responsive to the sample plot shock.*[25]

Under insurance constraints, both wealth and household size enter the model symmetrically by increasing consumption; therefore, in all cases, wealthier and larger households will respond similarly to the sample plot shock. When risk aversion is decreasing sufficiently quickly in consumption, then the allocations of wealthier and larger households will be closer to those maximizing expected profits, and therefore allocations on the most important plot will be less responsive to the sample plot shock.

Under input constraints, wealthier households are less likely to see the constraint bind. As the allocations on the most important plot of unconstrained households do not respond to the sample plot shock, wealthier households should be less responsive. Now, note that

[25] See proof in Appendix B.

in this model, farmers cannot use labor income to purchase additional inputs. In a more general model with borrowing, they may be able to; in that case, both wealthier households and larger households are less likely to see the constraint bind, and therefore will both be less responsive to the sample plot shock on their most important plots.[26]

Proposition 4. *If labor constraints bind, then the relative responsiveness of input and labor allocations on the most important plot of larger households (wealthier households) to the sample plot shock cannot be signed without further assumptions. If larger households and poorer households have more elastic on farm labor supply schedules, and if on farm labor supply exhibits sufficient curvature, then the input and labor allocations on the most important plot of larger households (wealthier households) should be less (more) responsive to the sample plot shock.*[27]

When labor constraints bind, the household responds to the sample plot shock by allocating additional labor to the sample plot, but they may withdraw that labor from either the most important plot or from leisure. Whether wealthier or larger households withdraw relatively more labor from the most important plot depends on the higher order derivatives of the utility and production functions; in general, these differential responses can not be signed.[28] Additionally, one key difference from the insurance case and input case is that household size and wealth no longer enter the model symmetrically. In one sense, household size and wealth instead enter the model as opposing forces: wealthier households allocate less labor to their plots, as they value leisure relatively more than consumption, while larger households allocate more labor to their plots.

We focus on one particular case that builds on this intuition, presented in Figure 5. When on farm labor supply exhibits sufficient curvature, then changes in responsiveness to

[26]If *all* households are input constrained, then the effect of the sample plot shock on input allocations on the most important plot depends on characteristics of the production function. Note that in this case, larger households will still exhibit a response in the same direction as wealthier households as both effects enter only through the wealth channel.

[27]See proof in Appendix B.

[28]Of course, the potential for ambiguous responses is heightened further if other forms of labor constraints, for example on hiring labor, are also considered.

the sample plot shock of allocations on the most important plot are dominated by changes in the elasticity of on farm labor supply; suppose this to be the case, and further suppose that the elasticity of on farm labor supply is decreasing in the shadow wage. As we can think of household size as shifting out on farm labor supply (by increasing $\overline{L}$), and wealth as shifting in on farm labor supply (by increasing the marginal utility of leisure relative to the marginal utility of consumption), then larger households are located on a more elastic portion of their on farm labor supply schedule, while wealthier households are located on a less elastic portion of their on farm labor supply schedule.[29] As a result, larger households will be less responsive to the sample plot shock, as they will primarily draw labor on the sample plot from leisure, while wealthier households will be more responsive to the sample plot shock, as they will primarily draw labor on the sample plot from the most important plot.

These predictions of the model, summarised in Table 7, generate a test that allows us to reject the absence of labor constraints. In particular, note that while insurance constraints or input constraints can rationalize the allocations of wealthier households to their most important plot as less responsive to the sample plot shock, only the presence of labor constraints can rationalize them as *more* responsive to the sample plot shock. Additionally, note that the model would struggle to rationalize larger households as more responsive to the sample plot shock, although it is possible to do so in the presence of labor constraints. In sum, we would interpret observing larger households as (weakly) less responsive and richer households as less responsive to the sample plot shock as most consistent with the presence of either input or insurance constraints, observing larger households as less responsive and richer households as more responsive as evidence for the presence of labor constraints, and observing larger households as more responsive as inconsistent with our model.

[29]This relationship between household size, wealth, and on farm labor supply elasticity has been posited as far back as Lewis (1954), and is discussed in depth in Sen (1966).

5 Separation failures and adoption of irrigation

5.1 Empirical strategy

Our first specification to test for separation failures mirrors Equation (1), which we use to estimate the impacts of irrigation. We still make use of the discontinuity across the command area boundary, but outcomes are now on the household's most important plot (plot 2) instead of the sample plot (plot 1).

$$y_{2ist} = \beta_0 + \beta_1 \mathrm{CA}_{1is} + \alpha_{st} + \epsilon_{2ist} \tag{7}$$

We report β_1, the effect of the sample plot shock on outcomes on the most important plot. In other specifications, we also consider heterogeneity with respect to the location of the most important plot, and include $\mathrm{CA}_{1is} * \mathrm{CA}_{2is}$ to test for this. In these specifications, we also report this difference in differences coefficient. For both this coefficient and β_1, in line with the model predictions in Table 7, we interpret negative coefficients on labor, inputs, irrigation use, and horticulture, as evidence of separation failures.

As in Section 3, we include specifications with progressively more controls. Specifically, we also estimate

$$y_{2ist} = \beta_0 + \beta_1 \mathrm{CA}_{1is} + \beta_2 \mathrm{Dist}_{1is} + \beta_3 \mathrm{CA}_{1is} * \mathrm{Dist}_{1is} + \beta_4 \mathrm{CA}_{2is} + X'_{1is}\gamma_1 + X'_{2is}\gamma_2 + \alpha_{st} + \epsilon_{2ist} \tag{8}$$

$$y_{2ist} - \overline{y}_{2ist} = \beta_1(\mathrm{CA}_{1is} - \overline{\mathrm{CA}}_{1is}) + (V_{1is} - \overline{V}_{1is})'\gamma_1 + (V_{2is} - \overline{V}_{2is})'\gamma_2 + (\epsilon_{2ist} - \overline{\epsilon}_{2ist}) \tag{9}$$

Equation 8 includes controls CA_{2is}, an indicator for whether the most important plot is in the command area, and X_{1is} and X_{2is}, the log area of the sample plot and the most important plot, respectively. Equation 9 uses spatial fixed effects, as described in Section 3.1.[30]

[30]Note that all differencing in this specification is done using the location of sample plots; in other words, most important plots whose associated sample plots are near each other are compared, as opposed to most important plots which are near each other.

Our benchmark specification to test for which constraints drive the separation failures is similar, but also includes the interaction of households characteristics with the sample plot shock. For parsimony, we only present the specification of this interaction for a specification similar to Equation 8; all tables present results with interactions included in Equation 7 and Equation 9 similarly.

$$y_{2ist} = \beta_0 + \beta_1 \mathrm{CA}_{1is} + W_i'\beta_2 + \mathrm{CA}_{1is} * W_i'\beta_3 + \beta_4 \mathrm{Dist}_{1is} + \beta_5 \mathrm{CA}_{1is} * \mathrm{Dist}_{1is} + \beta_6 \mathrm{CA}_{2is} + X_{1is}'\gamma_1 + X_{2is}'\gamma_2 + \alpha_{st} + \epsilon_{2ist} \tag{10}$$

where W_i is a vector of household characteristics, which includes household size and an asset index in our primary specifications. We focus on β_3: the heterogeneity, with respect to household characteristics, of the impacts of the sample plot shock on outcomes on the most important plot. The signs on β_3 give our main test of which market failures cause separation failures; Table 7 presents which signs map to which market failures.

5.1.1 Balance

We now use specifications (7), (8), and (9) to examine whether the most important plots in our sample are comparable for households whose sample plot is just inside or just outside the command area. As in Section 3.1.1, for each of these specifications, we show balance both with key controls omitted (Columns 3, 5, and 6), and our preferred specifications which we use in our analysis with key controls included (Columns 4, 7, and 8). Balance tests for most important plots are reported in Table 8. First, note that specifications that do not restrict to the discontinuity sample perform particularly poorly here. Most notably, most important plots are more likely to be located in the command area when sample plots are also located in the command area, as households' plots tend to be located near each other. In contrast, specifications which restrict to the discontinuity sample and include site fixed effects correct for this imbalance. Otherwise, we have a p-value of less than 0.1 for one variable in Columns 3 and 4 (an indicator for owning the plot) and in Columns 5 and 6 (an indicator for terracing); in all cases, the omnibus test fails to reject the null of balance.

5.2 Results

5.2.1 A test for separation failures

First, the graphical intuition behind the test for separation failures is captured in Figures 6. In this figure, irrigation use on the sample plot and the most important plot is plotted against the distance of sample plot to the command area boundary. Focusing on the Irrigation panel, and as presented in Figure 4, irrigation use on the sample plot is 17pp higher for sample plots just inside the command area compared to sample plots just outside the command area. However, we now see that on most important plots, irrigation use is 5pp *lower* when the sample plot is just inside the command area relative to when the sample plot is just outside the command area. This result represents a separation failure; as discussed in Section 4.2, the technology on the sample plot does not directly affect optimal allocations on the most important plot.

Note that this result is distinct from many other tests of separation failures, as it implies that in our context, the separation failure generates inefficiencies: we observe technologically identical most important plots, distinct only through the managing household and the technology of their sample plot, receiving different allocations of inputs. This contrasts with tests that consider differences in on farm labor allocations or land cultivated across households of different sizes, either statically or dynamically, or leveraging between or within household variation (e.g., Benjamin (1992); LaFave & Thomas (2016); Dillon & Barrett (2017); Dillon et al. (2019)); in particular, their tests provide evidence that at least one market has failed, which is known to be insufficient to show inefficiency. Alternatively, another literature has used production function estimates to infer marginal products of labor, land, and inputs from their allocations (Jacoby, 1993; Skoufias, 1994; Restuccia & Santaeulalia-Llopis, 2017); although heterogeneity in these marginal products is sufficient for the existence of market failures, these tests are typically not robust to the presence of unobserved heterogeneity across plots or to measurement error (Gollin & Udry, 2019).

We present results on separation failures from our benchmark specification in Tables 9, 10, and 11. For interpretation, a coefficient for sample plots is presented in Columns 1, and the mean outcome on the most important plot for sample plots just outside the command area is presented in Column 2. Columns 3 through 5 present our benchmark estimates of the effect of the sample plot shock on outcomes on the most important plot.

We discuss some key findings. First, irrigation use falls by 4.0-4.7 pp on most important plots; this magnitude represents 39-45% of average irrigation use, and 26% of the command area effect on irrigation use.[31] In addition to being consistent with Figure 6, and with the presence of separation failures, the magnitude of this estimate is important, as it represents a within households negative spillover of the command area; we discuss how this affects our interpretation of our main reduced form estimates in Section 3 in the following paragraphs. Second, we observe similar decreases for horticulture (3.5-4.9 pp), household labor (41.5-43.2 person-days/ha), and inputs (5,600-6,400 RwF/ha). However, although they are less robust, we observe increases in bananas (3.8-8.9 pp); as these are a less labor and input intensive crop, this is consistent with our interpretation of the production function as the envelope of production functions across crop choices.

Next, we expect the results above to be driven primarily by most important plots located in the command area for most outcomes, as there is limited irrigation, and therefore input use or horticulture during the dry season, on plots that cannot be irrigated. Consistent with this, in Columns 6 through 8, we find our results on irrigation, horticulture, and inputs are all driven by plots located in the command area. When the most important plot is located in the command area, the 15-18 pp increase in irrigation use on sample plots in the command area coincides with a 10-12 pp decrease in irrigation use on the most important plot; these

[31] Although the p-value on this result is .120-.129, this specification loses power by considering irrigation use on most important plots outside the command area, which are almost never irrigated. As discussed in the next paragraph, specifications which include the interaction of the sample plot command area indicator with a most important plot command area indicator are more precise for irrigation use as an outcome.

relative magnitudes suggest that separation failures cause few households to be able to use irrigation on more than one plot in the command area.

As discussed in Section 3, the direct effects of the command area appear driven by enabling the transition to dry season horticultural cultivation and substitution away from lower value banana cultivation. However, the model in Section 4 is agnostic about whether decreases in labor and input allocations on the most important plot are driven by extensive margin responses (i.e., decreases in horticulture) or intensive margin responses (i.e., decreases in labor and input allocations conditional on crop choice). To test this, in Tables 12 and 13, we present results of the sample plot shock on labor and input use on sample plots and most important plots, controlling for cultivation and crop choice.[32] Table 12 confirms that the effects we document in Section 3 are driven by the shift to dry season horticulture, as effects on sample plots all but disappear controlling for crop choice. However, Table 13 suggests that much of the effect of the sample plot shock on labor and input use on most important plots is driven by intensive margin responses, as coefficients on household labor and inputs fall by only 7%-33%. Combined with our results on irrigation use and horticulture, this suggests that both intensive and extensive margin responses on most important plots are important in response to the sample plot shock.

These results on separation failures imply the existence of a within household negative spillover, as they show that having one additional plot in the command area causes a household to substitute away from their other plots, reducing their use of irrigation, labor, and input allocations on those plots. In principle, this implies that our reduced form effects on the impacts of irrigation may be biased as sample plots both inside and outside the command area may see their input use, crop choice, and productivity influenced indirectly through these spillovers. This bias would be most concerning if we witnessed a differential

[32] As crop fixed effects are a "bad control" (Angrist & Pischke, 2008), which introduces selection bias, we interpret these results as suggestive. However, we anticipate that selection conditional on crop choice should bias us towards finding no intensive margin effect on most important plots, as the particularly constrained households switching out of horticulture in response to the sample plot shock are likely to be the households who used less labor and inputs.

decrease in input use outside of the command area on most important plots. In practice, our estimates confirm that for most dependent variables, this bias manifests as relatively reduced input use and horticultural cultivation within the command area. Thus, we anticipate that the estimated impacts of irrigation on inputs and horticulture are lower than they would be absent within-household spillovers.

To quantify the degree to which separation failures affect our reduced form estimates of impacts of the command area, we ask what would happen to adoption of irrigation if all households with two or more plots in the command area only had one plot in the command area. To do so, we conduct a simple exercise where we increase adoption of irrigation, on all command area plots held by households with multiple command area plots, by our point estimate for the effect of the sample plot shock on irrigation use on most important plots in the command area. This exercise suggests that adoption would be 21-24% higher with perfect insurance and if inputs to production flowed frictionlessly between househoulds. We interpret this estimate to be conservative for two reasons. First, we treat households with 3 or more command area plots the same as households with 2 command area plots; we do so because our research design has little to say about the impacts of two sample plot shocks as opposed to one sample plot shock on allocations to other plots. Second, this simple exercise abstracts from potential decreases in production driven by reduced labor and input allocations conditional on adopting irrigation; our specification with crop fixed effects provides suggestive evidence that accounting for these responses would decrease an estimate of counterfactual productivity without separation failures.

5.2.2 Separating constraints

We now provide evidence on the source of the separation failure by estimating heterogeneous impacts, with respect to household size and wealth, of the sample plot shock on outcomes on the most important plot. Recall that for this analysis, the key predictions of the model were 1) if only insurance or input constraints bind, wealthier households and larger households should be less responsive, and 2) if only labor constraints bind, differential responsiveness of wealthier and larger households is ambiguous, but under reasonable assumptions wealthier households should be more responsive and larger households should be less responsive. Note that this test does not allow us to reject a null that a particular constraint exists; any pattern of differential responses is consistent with all constraints binding. However, if we observe that wealthier households are more responsive, we can reject the null of no labor constraints. Additionally, we would interpret observing wealthier households to be more responsive and larger households to be less responsive as the strongest evidence of the presence of labor constraints from this test.

We present the results of this test in Tables 14 and 15. First, larger households are less responsive to the sample plot shock across every outcome. A household with 2 additional members, approximately one standard deviation of household size, is less responsive to the sample plot shock on its most important plot by 70-80% for irrigation use, 62-86% for horticulture, 54-58% for household labor, and 18-36% for inputs, with all but the input coefficient statistically significant and robust across specifications.[33] In contrast, wealthier households are more responsive to the sample shock across these same outcomes. A household with a one standard deviation higher asset index is more responsive to the sample plot shock on its most important plot by 40-80% for irrigation use, 40-63% for horticulture, 35-58% for household labor, and 42-100% for input use; however, these results are less robust, as statistical significance drops for all outcomes except inputs in boundary discontinuity specifications. In effect, these results suggest that our estimates of separation failures are driven by the behavior of small, rich households, while large, poor households do not change their allocations on their most important plot in response to the sample plot shock. As discussed in Section 4.3, these results are very difficult to reconcile with a model that does not feature labor market failures.

In sum, these results provide strong evidence for the existence of labor market failures

[33]These percentages, and the remainder of percentages in this paragraph, are expressed relative to the average estimated sample plot shock.

that generate separation failures, which in turn cause inefficient adoption of irrigation.

6 Experimental evidence

6.1 Experimental design

We now complement these results using the spatial discontinuity in access to irrigation with experimental evidence. We conducted three randomized controlled trials in these hillside irrigation schemes. First, we manipulated operations and maintanence (O&M) in the hillside irrigation schemes, by randomly assigning water user groups to different approaches to monitoring. Qualitative work raised concerns that the water user groups as established would not be sufficient to enforce water usage schedules and that routine maintenance tasks would not be performed adequately, as has been documented by Ostrom (1990). Second, we subsidized water usage fees the government had planned to collect from farmers, which were as high as 77,000 RwF/ha/year. For reference, this is roughly 20% of our dry season treatment on the treated estimates, and roughly 50% of median land rental prices. At this size, fees had the potential to influence farmers production decisions, although small relative to potential yield gains from irrigation use. Third, we provided agricultural minikits, which included 0.02 ha of seeds, chemical fertilizer, and insecticide, which could be used for horticulture cultivation. In other contexts, minikits of similar size relative to median landholdings have been shown to increase adoption of new crop varieties or varieties with low levels of adoption (Emerick et al., 2016; Jones et al., 2018). Although horticulture is not unfamiliar in these areas, at baseline 3.2% of plots outside the command area were planted with at least some horticulture, and primarily during the rainy seasons.

Assignment to experimental arms for O&M, minikits, and subsidies were as follows. First, for the O&M intervention, 251 water user groups across three irrigation sites were randomized, stratified across the 33 Zones these irrigation sites are divided into, into three arms.[34] Second, for the minikit intervention, water user groups were randomly assigned to 20%, 60% or 100% saturation, with rerandomization for balance on Zone and O&M treatment status. Following this assignment, individuals on the lists of water user group members provided to us by the sites were randomly assigned to receive minikits with probabilities equal to that water user group's saturation. Minikits were offered to assigned individuals prior to 2017 Rainy A and 2017 Dry. Third, for the subsidy intervention, our implementing partner was concerned with the perception of an assignment rule that might be perceived as hidden, so public lotteries for subsidies were conducted at the Zone level.[35]

6.2 Empirical strategy and results

6.2.1 Minikits

We conduct an intent-to-treat analysis and estimate the impacts of random assignment to receive a minikit. As randomization was stratified on Zone and O&M treatment status, we control for Zone fixed effects and O&M treatment status. To test for spillovers, we include the saturation of minikits, randomly assigned at the water user group level, as a control. Additionally, after matching names from the lists of water user group members to our baseline survey, we found that 32% of households either had multiple household members on the lists of water user group members or had a single household member listed multiple times; as these households are more likely to be assigned to receive a minikit and may differ from other households, we control for dummies for the number of times the household appeared

[34] 40% were assigned to a status quo arm where the irrigator/operators employed by the site were responsible for enforcing water usage schedules and reporting O&M problems to the local Water User Association. 30% were assigned to an arm where the water user group elected a monitor who was tasked with these responsibilities, trained in implementing them, and given worksheets to fill and return to the Water User Association reporting challenges with enforcement of the water usage schedule and any O&M concerns. In an additional 30%, the elected monitor was required to have a plot near the top of the water user group, where the flow of water is most negatively impacted when too many farmers try to irrigate at once. Monitors were trained just before the 2016 Dry season, with refresher trainings during 2016 Dry and 2017 Rainy A.

[35] At these public lotteries, 40% of farmers received no subsidy, 20% received a 50% subsidy for one season, 20% received a 100% subsidy for one season, and 20% received a 100% subsidy for two seasons. The lotteries took place at the start of the 2017 Rainy A, and subsidies were for 2017 Rainy A and 2017 Rainy B; at the time the Water User Associations did not plan to collect fees during the Dry season.

on these lists. Lastly, in some specifications, we include 2016 Dry horticulture adoption as a control to test for selection into using the minikit. We estimate the impact of minikits using the specification

$$y_{1ist} = \beta_0 + \beta_1 \text{Assigned minikit}_i + \beta_2 \text{Minikit saturation}_i + X'_{1is}\gamma + \epsilon_{1ist} \tag{11}$$

Assigned minikit$_i$ is a dummy for whether household i was randomly assigned to receive a minikit, Minikit saturation$_i$ is the probability of receiving a minikit for households in the water user group of household i's sample plot, and X_{1is} includes Zone fixed effects, O&M treatment status, and in some specifications 2016 Dry horticulture adoption. As minikit saturation is assigned at the water user groups level, robust standard errors are clustered at the water user group level.

For our primary outcomes y, we focus on whether households used a minikit (in 2017 Rainy A or in 2017 Dry), adoption of horticulture, and input expenditures. Impacts on minikit use are our first stage, impacts on adoption of horticulture are our measure of learning from the minikits, and impacts on input expenditures provide evidence on whether minikits crowd in or substitute for other inputs. For precision, we restrict to command area plots, and for plot level outcomes we focus on 2017 Dry and 2018 Dry; these are the plots and seasons in which we expect households to adopt horticulture in response to being assigned a minikit.

We present the results of this analysis in Table 16. First, we find a strong first stage; households assigned to receive a minikit are 40-43pp more likely to use a minikit than households not assigned to receive a minikit. Almost all non-compliance is driven by households who were assigned to receieve a minikit but did not pick it up – 4.8% of households not assigned to received a minikit used one, while 43.8% of households assigned to receive a minikit used one. Second, we find no effects of minikits on horticulture use, and we have sufficient precision to reject estimates from other contexts of the effect of minikits on technology adoption (Emerick et al., 2016; Jones et al., 2018). Third, consistent with this null effect on horticulture use, we find no effects of minikit saturation, although these estimates are less precise than those of the impacts of assignment to receive a minikit. Fourth, we fail to find statistically significant effects on input expenditures, although the magnitude of the point estimates is consistent with full crowd out.[36] Fifth, we find strong positive selection into using a minikit: farmers who grew horticulture in 2016 Dry, who are 33.1pp more likely to grow horticulture in 2017 and 2018 Dry, are 12.5pp more likely to use a minikit.

We interpret these results to suggest that information and financial constraints are not dominant constraints to adoption of irrigation. First, minikits should alleviate financial constraints by reducing the costs of growing horticulture under irrigation and by reducing basis risk. As such, we should expect farmers who face financial constraints to use minikits. Instead, most farmers assigned to receive a minikit do not pick it up and use it, and the farmers who do pick it up typically would have grown horticulture even if not assigned to receive a minikit. Second, minikits should alleviate informational constraints, as they enable farmers to experiment with horticulture at lower cost. Therefore, in the presence of informational constraints, we should expect minikits to increase adoption of horticulture and, in turn, irrigation. Instead, we find that minikits do not increase adoption of horticulture. That information is not a binding constraint is also consistent with the moderate, and stable over time, levels of adoption of irrigation that we observe, in contrast to an S-curve of adoption which would be consistent with learning.

6.2.2 Other experiments

We find no effects of empowering monitors and fee subsidies on agricultural decisions in our context; we offer some qualitative evidence and simple descriptives from our data that explain these null effects.[37]

[36] On a typical 0.1 ha plot, the point estimate implies a 300-800 RwF decrease in input expenditures, while typical input expenditures per hectare on horticulture, scaled by the 0.02 ha the minikits were designed to cover, are 900 RwF.

[37] Results are available upon request.

First, we find no impact of empowering monitors. This is because O&M was highly effective in these irrigation schemes, and empowering monitors therefore had limited scope for changing O&M practices. Farmers reported 14% as many days without enough water during the dry seasons as they reported days using irrigation. Any event where conflict among water user group members caused insufficient water at some point during the dry season was reported for 3% of irrigated plots.[38] This success was far from guaranteed in the early years of the schemes; site engineers have suggested that the combination of lower adoption of irrigation than the schemes are designed for and high compliance with water usage schedules among farmers have been the cause of this. Moreover, during the 2018 Dry season we found evidence that control water user groups adopted the intervention, as some members of control water user groups adopted the roles that were assigned to monitors.

Second, we find no impact of fee subsidies. The reason for this is clear – although we have a strong and large first stage on fees owed by farmers in administrative data, the impacts of subsidies on feed paid by farmers were 10% of the size of the impacts on fees owed, both in administrative data and self reports. Moreover, the fees were implemented as land taxes and not charged based on irrigation use so as not to discourage adoption. In sum, at the low levels of enforcement observed during the 2017 Rainy seasons, they should not have affected farmers' production decisions, consistent with the results we find.

7 Conclusions

This paper provides evidence that irrigation has the potential to be a transformative technology in sub-Saharan Africa. Using data from very proximate plots which receive differential access to irrigation, we document that the construction of an irrigation system leads to a 70% increase in cash profits. These profits are generated by a switch in cropping patterns from perennial bananas towards a rotation of dry-season horticulture and rainy-season staples,

[38] This magnitude is small; as reference, Sekhri (2014) finds the share of farmers reporting disputes over ground water in India increases by 29pp when water tables become sufficiently low.

which itself is associated with an increase in input intensity. In our context, the primary increase in input demands is for household labor, which is utilized intensively on horticulture and minimally on banana cultivation.

These results suggest that irrigation may have similar potential in Africa to the transformative role it played in South Asia, where other studies have documented similar impacts of irrigation on farmer revenues and yields. In some ways, this is surprising: other evidence on the use of inputs in Africa and the returns to those inputs often finds lower usage and technological returns in the African context. These two facts together suggest that expanding irrigation access in Africa may be a necessary contributor to shrinking the yield gap.

At the same time, even with access to a new, highly productive technology offered freely by the government we observe a minority of farmers adopting this technology four years after introduction. Given the returns identified above, we take this as evidence that the existence of a productive technology is not itself sufficient to generate majority adoption in all agricultural contexts. We further document that frictions in labor markets contribute to low utilization of irrigation systems by examining farmers' input utilization on other plots in response to irrigation investments. This result provides novel evidence that separation failures in agricultural household production leads to inefficient under-adoption of a new and highly productive technology in Rwanda. It also poses a clear priority for future research: we need more evidence on both the role of factor markets in technology adoption, and the identification of particular institutions which contribute to or which can smooth those market failures. In some cases, these market failures may pose a competing constraint which coexists with other, more conventional constraints to production: if frictions in factor markets similarly constrain adoption of new technologies in other environments, then incomplete factor markets may generate limits to the effectiveness of financial and information interventions in improving agricultural productivity.

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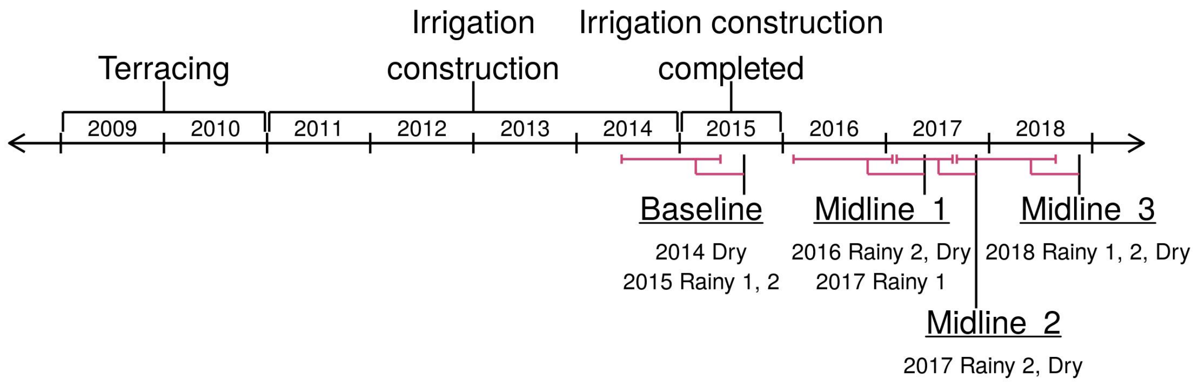
Figure 1: Timeline

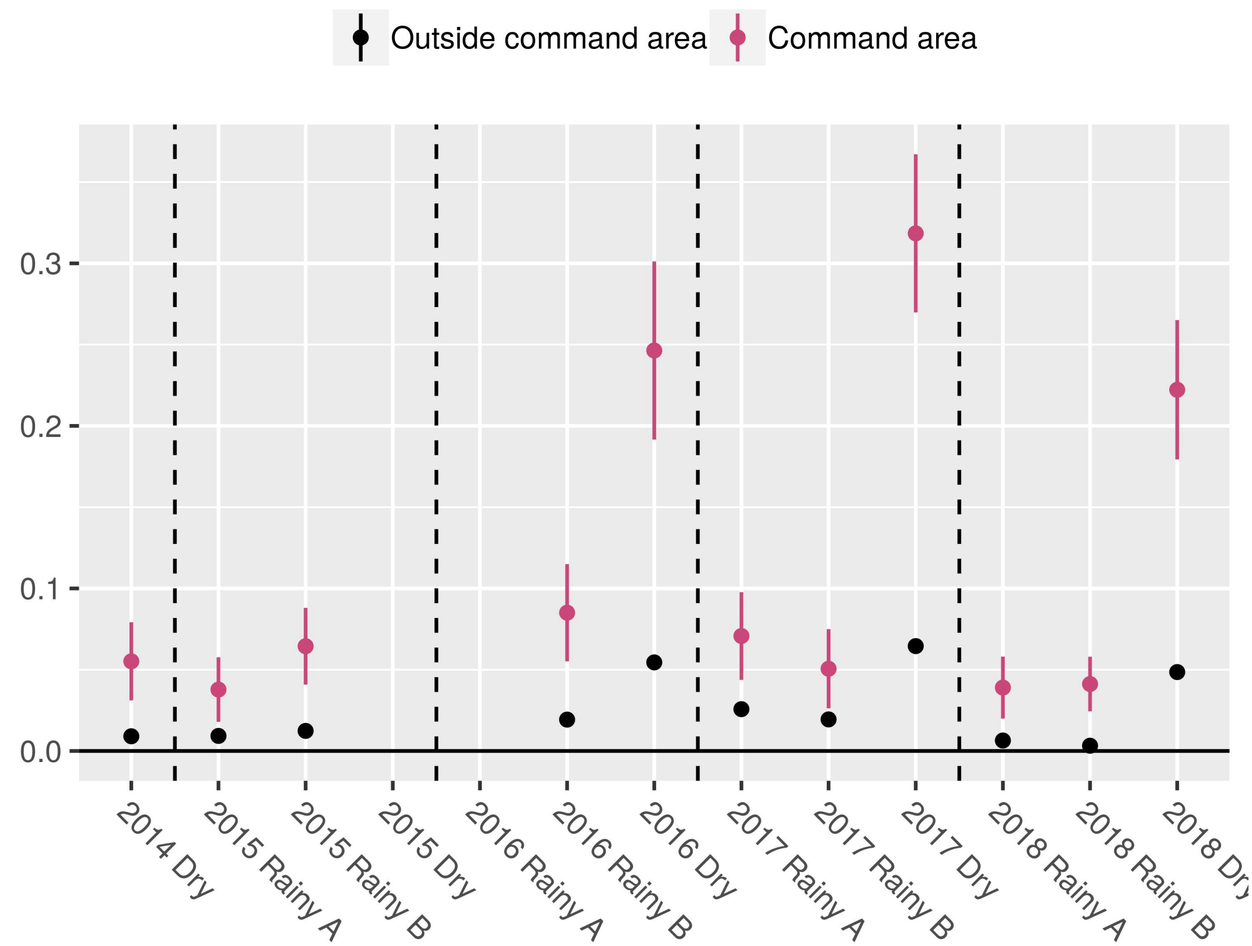


Notes: A timeline of events on the 3 hillside irrigation schemes we study is presented in this figure. Black lines are used to indicate when (or the period during which) events took place, while pink lines are used to indicate survey recall periods.

Figure 2: Hillside irrigation scheme

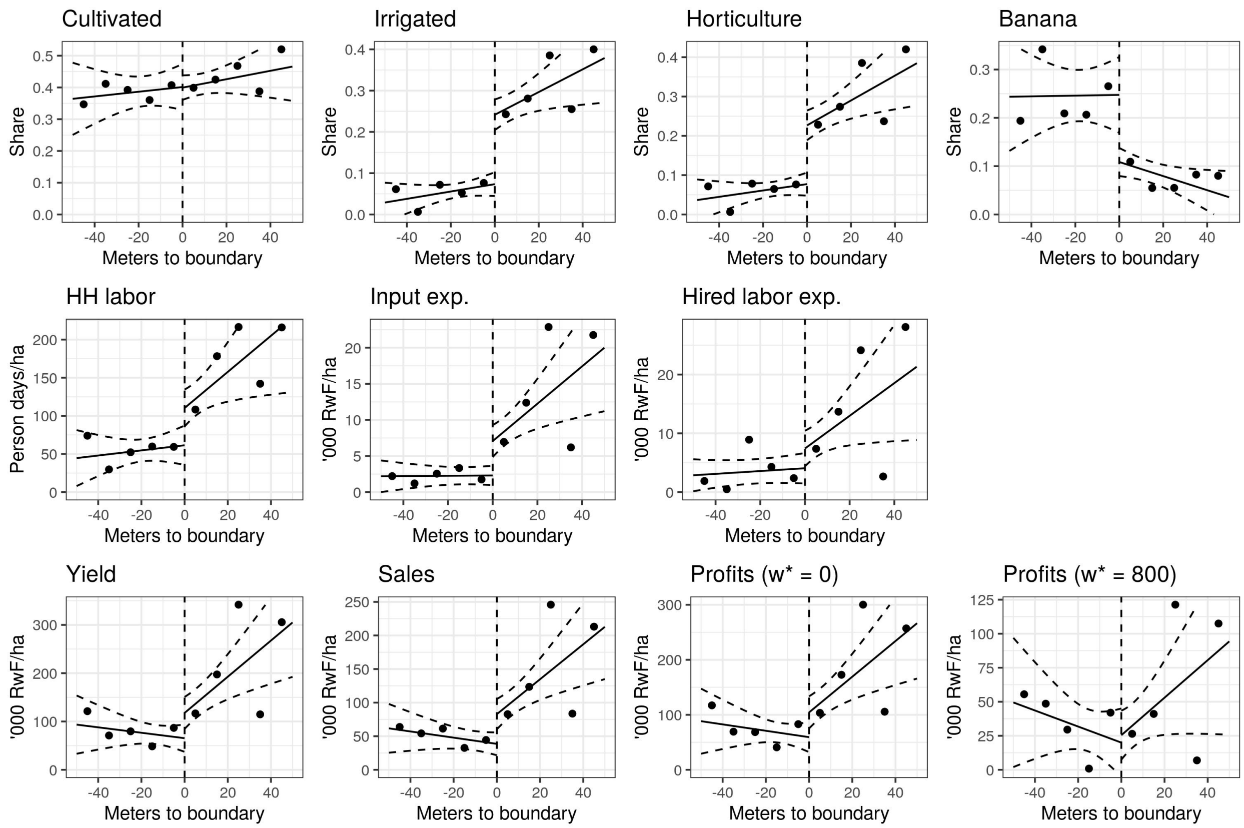
Notes: A photograph of Karongi 12, one of the hillside irrigation schemes in this study, is presented in this figure.

Figure 3: Adoption dynamics



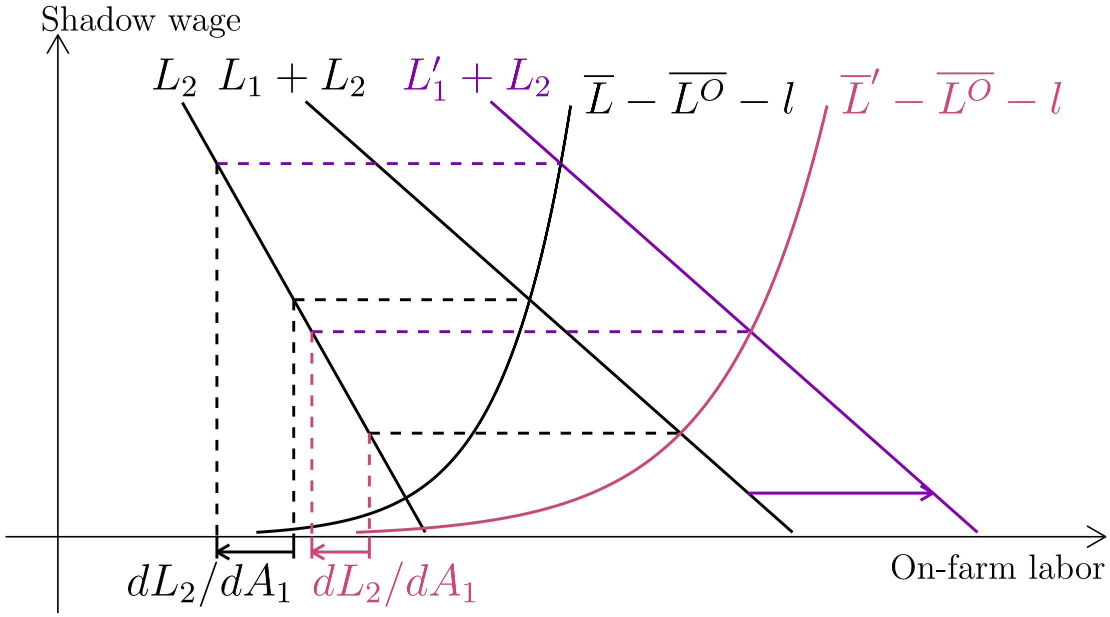
Notes: Average adoption of irrigation by season on sample plots in the main discontinuity sample, inside and outside the command area, is presented in this figure. Averages outside the command area are in black, while averages inside the command area and 95% confidence intervals for the difference are in pink. Robust standard errors are clustered at the nearest water user group level.

Figure 4: RDD



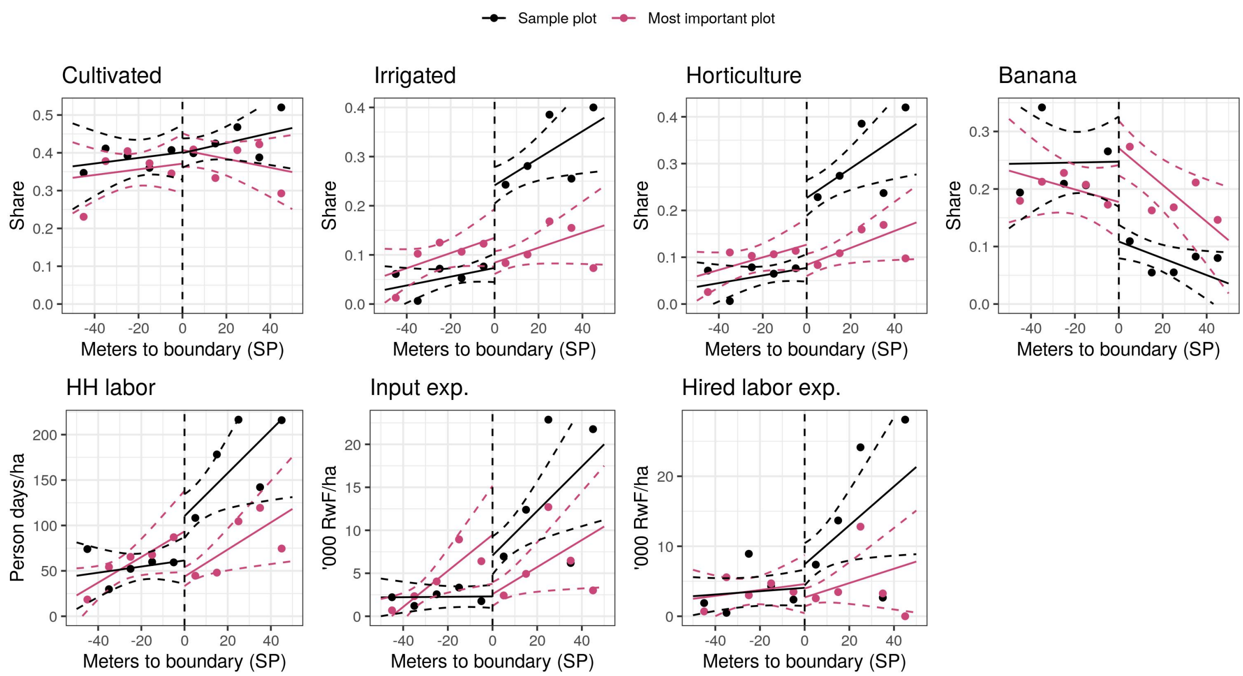
Notes: Visual regression discontinuity analysis on sample plots in the main discontinuity sample during the dry season is presented in this figure. Distance to the boundary is reported in meters, with positive distance corresponding to sample plots inside the command area. Points are binned average outcomes. Predicted outcomes from regressions of outcomes on distance to the command area boundary, a command area dummy, and their interaction are presented with 95% confidence intervals on the prediction. Robust standard errors are clustered at the nearest water user group level.

Figure 5: Differential responses under labor constraints



Notes: Households' labor allocations under a binding off farm labor constraint are presented in this figure. L_k and l are the household's labor allocation on plot k and choice of leisure, respectively, as a function of the shadow wage, with the argument suppressed. $L_1 + L_2$ is total household on farm labor demand; if the household's sample plot ($k = 1$) is in the command area ("sample plot shock"), on farm labor demand shifts out to $L_1' + L_2$. $\overline{L} - \overline{L^O} - l$ is household on farm labor supply; for large households, on farm labor supply is shifted out to $\overline{L}' - \overline{L^O} - l$. The shadow wage is determined by the intersection of on farm labor demand and on farm labor supply, and labor allocations on the most important plot are L_2 evaluated at this shadow wage. In this figure, larger households are on a more elastic portion of their on farm labor supply schedule; as a result, the sample plot shock causes a smaller increase in the shadow wage, and in turn a smaller decrease in labor allocations on the most important plot (smaller in magnitude dL_2/dA_1).

Figure 6: RDD: Most important plot



Notes: Visual regression discontinuity analysis on sample plots and associated most important plots during the dry season, for sample plots in the main discontinuity sample, is presented in this figure. Distance to the boundary is reported in meters, with positive distance corresponding to sample plots inside the command area. Points are binned average outcomes. Predicted outcomes from regressions of outcomes on distance to the command area boundary, a command area dummy, and their interaction are presented with 95% confidence intervals on the prediction. Robust standard errors are clustered at the nearest water user group level.

Table 1: Summary statistics on agricultural production

	Staples				Horticulture		
	Staples	Maize	Beans	Bananas	All	Rainy	Dry
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Yield	299	315	283	270	555	572	542
Hired labor (days)	36	36	36	9	57	63	52
HH labor (days)	263	246	257	97	412	414	411
Inputs	17	27	15	3	42	45	39
Profits							
Shadow wage = 0 RwF/day	255	260	240	260	470	479	463
Shadow wage = 800 RwF/day	45	64	35	182	140	148	134
Sales share	0.19	0.30	0.14	0.46	0.61	0.60	0.63
Irrigated	0.02	0.02	0.02	0.02	0.64	0.25	0.92
Rainy	0.99	1.00	1.00	0.50	0.42	1.00	0.00
log area	-2.42	-2.25	-2.45	-2.09	-2.69	-2.82	-2.59
Share of obs.	0.65	0.13	0.42	0.19	0.11	0.05	0.06

Notes: Sample averages of outcomes by crop per agricultural season are presented in this table. Yield, inputs, and profits are reported in units of '000 RwF/ha, labor variables are reported in units of person-days/ha, and log area is in units of log hectares. All other variables are shares or indicators. For reference, the median wage in our data is 800 RwF/person-day.

Table 2: Balance: Sample plots

	Full sample	RD sample						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log area	0.045	-2.515	0.219	0.285	0.425	0.200		
	(0.077)	(1.179)	(0.087)	(0.087)	(0.121)	(0.128)		
	[0.554]	969	[0.012]	[0.001]	[0.000]	[0.118]		
Own plot	-0.012	0.894	0.003	-0.001	0.004	-0.004	-0.001	-0.006
	(0.020)	(0.309)	(0.023)	(0.024)	(0.032)	(0.038)	(0.032)	(0.038)
	[0.535]	969	[0.897]	[0.966]	[0.907]	[0.921]	[0.972]	[0.877]
Owned plot >5 years	0.045	0.880	0.070	0.072	0.019	0.012	0.007	0.010
	(0.019)	(0.326)	(0.026)	(0.025)	(0.037)	(0.035)	(0.036)	(0.034)
	[0.020]	686	[0.006]	[0.004]	[0.613]	[0.723]	[0.834]	[0.767]
Rented out to farmer	0.027	0.032	0.018	0.019	-0.003	0.009	-0.009	0.007
	(0.012)	(0.177)	(0.014)	(0.014)	(0.023)	(0.027)	(0.023)	(0.027)
	[0.022]	969	[0.197]	[0.182]	[0.884]	[0.726]	[0.699]	[0.796]
Omnibus F-stat [p]	2.6		3.4	4.9	3.2	0.6	0.1	0.1
	[0.038]		[0.010]	[0.001]	[0.013]	[0.639]	[0.979]	[0.984]
Site FE				X	X	X	X	X
Distance to boundary					X	X	X	X
log area							X	X
Spatial FE						X		X

Notes: Balance for sample plot characteristics is presented in this table. Column 2 presents, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 1 and 3 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Controls are listed below. The final row of each column presents the Omnibus F-stat for the null of balance on all outcomes, with the p-value for the associated test in brackets. Column 1 uses the full sample, while Columns 2 through 8 use the discontinuity sample. Column 4 uses the specification in Equation (1), Column 7 uses the specification in Equation (2), and Column 8 uses the specification in Equation (3).

Table 3: Balance: Households

	Full sample	RD sample						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
HHH female	0.041	0.221	0.057	0.055	0.045	0.044	0.043	0.041
	(0.025)	(0.416)	(0.029)	(0.029)	(0.046)	(0.050)	(0.046)	(0.050)
	[0.094]	969	[0.054]	[0.063]	[0.326]	[0.378]	[0.345]	[0.412]
HHH age	0.5	47.5	1.5	1.5	2.1	0.7	1.4	0.3
	(0.8)	(14.5)	(0.9)	(0.9)	(1.4)	(1.8)	(1.4)	(1.9)
	[0.497]	967	[0.096]	[0.087]	[0.127]	[0.694]	[0.298]	[0.863]
HHH completed primary	0.069	0.287	0.044	0.052	0.128	0.102	0.119	0.099
	(0.025)	(0.453)	(0.031)	(0.032)	(0.047)	(0.062)	(0.047)	(0.062)
	[0.005]	966	[0.159]	[0.106]	[0.006]	[0.097]	[0.012]	[0.111]
HHH worked off farm	0.023	0.410	-0.023	-0.033	-0.039	-0.019	-0.024	-0.011
	(0.027)	(0.493)	(0.035)	(0.035)	(0.051)	(0.064)	(0.050)	(0.064)
	[0.392]	969	[0.516]	[0.350]	[0.441]	[0.763]	[0.631]	[0.868]
# of plots	0.61	5.19	0.37	0.16	0.20	0.35	0.36	0.40
	(0.18)	(3.38)	(0.22)	(0.21)	(0.36)	(0.46)	(0.36)	(0.46)
	[0.001]	969	[0.099]	[0.442]	[0.582]	[0.448]	[0.319]	[0.382]
# of HH members	0.17	4.89	0.04	0.02	-0.00	-0.03	-0.01	-0.03
	(0.11)	(2.16)	(0.15)	(0.15)	(0.21)	(0.25)	(0.22)	(0.25)
	[0.104]	969	[0.799]	[0.916]	[0.985]	[0.917]	[0.971]	[0.908]
# who worked off farm	0.10	0.77	0.04	0.02	0.01	0.03	0.01	0.04
	(0.05)	(0.85)	(0.06)	(0.06)	(0.08)	(0.10)	(0.08)	(0.10)
	[0.039]	969	[0.523]	[0.771]	[0.909]	[0.799]	[0.906]	[0.722]
Housing expenditures	-2.3	49.2	3.5	3.3	-5.6	-16.7	-6.5	-18.6
	(6.9)	(127.4)	(9.0)	(9.0)	(14.9)	(19.0)	(14.7)	(19.1)
	[0.739]	962	[0.700]	[0.717]	[0.707]	[0.380]	[0.658]	[0.328]
Asset index	0.11	-0.12	0.06	0.07	0.15	0.06	0.13	0.04
	(0.05)	(0.99)	(0.07)	(0.07)	(0.12)	(0.13)	(0.12)	(0.13)
	[0.034]	967	[0.372]	[0.303]	[0.215]	[0.647]	[0.291]	[0.738]
Omnibus F-stat [p]	3.6		1.6	1.6	1.8	0.8	1.5	0.9
	[0.000]		[0.122]	[0.118]	[0.080]	[0.571]	[0.158]	[0.507]
Site FE				X	X	X	X	X
Distance to boundary					X	X	X	X
log area							X	X
Spatial FE						X		X

Notes: Balance for household characteristics is presented in this table. Column 2 presents, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 1 and 3 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Controls are listed below. The final row of each column presents the Omnibus F-stat for the null of balance on all outcomes, with the p-value for the associated test in brackets. Column 1 uses the full sample, while Columns 2 through 8 use the discontinuity sample. Column 4 uses the specification in Equation (1), Column 7 uses the specification in Equation (2), and Column 8 uses the specification in Equation (3).

Table 4: Sample plots

	Dry season				Rainy seasons			
	Dep. var.	Coef. (SE) [p]			Dep. var.	Coef. (SE) [p]		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cultivated	0.387	0.026	-0.009	0.007	0.814	-0.050	-0.102	-0.067
	(0.487)	(0.031)	(0.041)	(0.044)	(0.389)	(0.023)	(0.031)	(0.036)
	2,442	[0.397]	[0.822]	[0.879]	4,080	[0.030]	[0.001]	[0.059]
Irrigated	0.056	0.194	0.152	0.174	0.015	0.045	0.035	0.062
	(0.230)	(0.019)	(0.023)	(0.029)	(0.121)	(0.007)	(0.009)	(0.013)
	2,442	[0.000]	[0.000]	[0.000]	4,080	[0.000]	[0.000]	[0.000]
Horticulture	0.061	0.173	0.128	0.157	0.072	0.043	0.015	0.052
	(0.240)	(0.019)	(0.023)	(0.029)	(0.258)	(0.011)	(0.018)	(0.025)
	2,441	[0.000]	[0.000]	[0.000]	4,079	[0.000]	[0.396]	[0.039]
Banana	0.246	-0.132	-0.136	-0.149	0.273	-0.145	-0.158	-0.168
	(0.431)	(0.024)	(0.037)	(0.034)	(0.446)	(0.025)	(0.039)	(0.034)
	2,441	[0.000]	[0.000]	[0.000]	4,079	[0.000]	[0.000]	[0.000]
Distance to boundary			X	X			X	X
log area			X	X			X	X
Site-by-season FE		X	X	X		X	X	X
Spatial FE				X				X

Notes: Regression analysis is presented in this table. Columns 1 through 4 restrict to observations during the dry season, while columns 5 through 8 restrict to observations during the rainy season. Columns 1 and 5 present, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 2 through 4 and 6 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Columns 2 and 6 use the specification in Equation (1). Columns 3 and 7 use the regression discontinuity specification in Equation (2). Columns 4 and 8 use the spatial fixed effects specification in Equation (3).

Table 5: Sample plots

	Dry season				Rainy seasons			
	Dep. var.	Coef. (SE) [p]			Dep. var.	Coef. (SE) [p]		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
HH labor/ha	54.8	67.3	65.4	70.8	217.0	-7.7	-2.3	-15.0
	(188.7)	(14.2)	(16.7)	(19.9)	(309.3)	(17.8)	(24.5)	(25.4)
	2,438	[0.000]	[0.000]	[0.000]	4,072	[0.668]	[0.924]	[0.554]
Input exp./ha	2.3	6.3	5.2	4.4	15.2	2.1	-0.6	0.5
	(16.0)	(1.2)	(1.4)	(1.7)	(39.1)	(1.9)	(2.8)	(3.0)
	2,442	[0.000]	[0.000]	[0.011]	4,080	[0.267]	[0.820]	[0.864]
Hired labor exp./ha	3.6	5.1	3.2	2.7	14.9	6.5	2.8	1.2
	(25.2)	(1.8)	(2.1)	(2.5)	(45.3)	(2.4)	(3.2)	(3.7)
	2,442	[0.006]	[0.126]	[0.285]	4,080	[0.006]	[0.385]	[0.751]
Distance to boundary			X	X			X	X
log area			X	X			X	X
Site-by-season FE		X	X	X		X	X	X
Spatial FE				X				X

Notes: Regression analysis is presented in this table. Columns 1 through 4 restrict to observations during the dry season, while columns 5 through 8 restrict to observations during the rainy season. Columns 1 and 5 present, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 2 through 4 and 6 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Columns 2 and 6 use the specification in Equation (1). Columns 3 and 7 use the regression discontinuity specification in Equation (2). Columns 4 and 8 use the spatial fixed effects specification in Equation (3).

Table 6: Sample plots

	Dry season				Rainy seasons			
	Dep. var.	Coef. (SE) [p]			Dep. var.	Coef. (SE) [p]		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Yield	76.6	57.4	62.0	52.2	256.5	-43.1	-27.8	-28.7
	(259.4)	(19.9)	(22.8)	(28.3)	(442.4)	(21.7)	(31.3)	(32.9)
	2,317	[0.004]	[0.007]	[0.065]	3,942	[0.047]	[0.375]	[0.383]
Sales/ha	47.7	47.5	48.7	45.1	78.0	-2.2	-11.6	7.9
	(174.5)	(13.1)	(14.4)	(19.1)	(218.1)	(10.7)	(18.1)	(21.4)
	2,442	[0.000]	[0.001]	[0.018]	4,080	[0.835]	[0.522]	[0.712]
Profits/ha								
Shadow wage = 0	70.9	47.5	54.5	46.5	226.7	-50.3	-28.8	-29.5
	(245.8)	(17.9)	(20.8)	(25.8)	(419.1)	(20.2)	(29.2)	(29.9)
	2,317	[0.008]	[0.009]	[0.071]	3,942	[0.013]	[0.324]	[0.324]
Shadow wage = 800	31.7	-0.1	5.1	-1.8	54.3	-44.0	-25.1	-15.9
	(213.3)	(11.9)	(16.8)	(20.7)	(349.1)	(16.2)	(25.6)	(31.0)
	2,315	[0.991]	[0.761]	[0.929]	3,935	[0.007]	[0.328]	[0.607]
Distance to boundary			X	X			X	X
log area			X	X			X	X
Site-by-season FE		X	X	X		X	X	X
Spatial FE				X				X

Notes: Regression analysis is presented in this table. Columns 1 through 4 restrict to observations during the dry season, while columns 5 through 8 restrict to observations during the rainy season. Columns 1 and 5 present, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 2 through 4 and 6 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Columns 2 and 6 use the specification in Equation (1). Columns 3 and 7 use the regression discontinuity specification in Equation (2). Columns 4 and 8 use the spatial fixed effects specification in Equation (3).

Table 7: Model predictions

	$\frac{dL_2}{dA_1}$	$\frac{d}{d\overline{L}}\frac{dL_2}{dA_1}$	$\frac{d}{d\overline{M}}\frac{dL_2}{dA_1}$
No constraints	0	0	0
Constraints			
Insurance	$-$	$+$	$+$
Inputs	$-$	0/+	$+$
Labor	$-$	$+^*$	$-^*$

Notes: Predicted signs from the model for key comparative statics of interest are presented in this table. Predictions in the no constraints case correspond to Proposition 1. Predictions on $\frac{dL_2}{dA_1}$ correspond to Proposition 2. Predictions on $\frac{d}{d\overline{L}}\frac{dL_2}{dA_1}$ and $\frac{d}{d\overline{M}}\frac{dL_2}{dA_1}$ when insurance or input constraints bind correspond to Proposition 3, and when labor constraints bind correspond to Proposition 4. * is used to indicate predictions that hold when additional assumptions are made.

Table 8: Balance: Most important plot

	Full sample	RD sample						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log area	-0.108	-2.381	0.043	0.089	0.094	0.074		
	(0.068)	(1.041)	(0.083)	(0.082)	(0.128)	(0.136)		
	[0.114]	784	[0.603]	[0.275]	[0.460]	[0.588]		
Own plot	0.025	0.875	0.048	0.043	0.040	0.033	0.039	0.029
	(0.019)	(0.331)	(0.023)	(0.023)	(0.033)	(0.039)	(0.032)	(0.037)
	[0.174]	784	[0.037]	[0.064]	[0.226]	[0.392]	[0.232]	[0.436]
Owned plot >5 years	0.005	0.960	-0.004	-0.003	0.012	0.033	0.011	0.030
	(0.014)	(0.197)	(0.016)	(0.016)	(0.024)	(0.024)	(0.023)	(0.025)
	[0.728]	585	[0.811]	[0.853]	[0.617]	[0.175]	[0.617]	[0.233]
Rented out to farmer	0.013	0.033	-0.006	-0.006	-0.026	-0.040	-0.029	-0.041
	(0.010)	(0.179)	(0.013)	(0.013)	(0.022)	(0.025)	(0.023)	(0.026)
	[0.224]	784	[0.664]	[0.645]	[0.249]	[0.114]	[0.222]	[0.116]
Command area	0.187	0.399	0.074	0.045	-0.053	-0.079		
	(0.032)	(0.491)	(0.039)	(0.037)	(0.058)	(0.059)		
	[0.000]	784	[0.059]	[0.219]	[0.360]	[0.183]		
Terraced	0.017	0.626	-0.030	-0.043	-0.099	-0.091	-0.076	-0.058
	(0.028)	(0.485)	(0.035)	(0.035)	(0.053)	(0.055)	(0.051)	(0.052)
	[0.539]	784	[0.403]	[0.225]	[0.063]	[0.099]	[0.134]	[0.260]
Rented out to investor	0.035	0.081	0.017	0.008	-0.042	-0.016	-0.036	-0.004
	(0.018)	(0.273)	(0.025)	(0.023)	(0.040)	(0.034)	(0.036)	(0.031)
	[0.054]	784	[0.486]	[0.735]	[0.292]	[0.638]	[0.324]	[0.895]
Omnibus F-stat [p]	5.6		1.8	1.6	1.3	1.5	1.4	1.2
	[0.000]		[0.093]	[0.132]	[0.278]	[0.153]	[0.209]	[0.292]
Site FE				X	X	X	X	X
Distance to boundary					X	X	X	X
log area							X	X
MIP log area							X	X
MIP CA							X	X
Spatial FE						X		X

Notes: Balance for most important plot characteristics is presented in this table. Column 2 presents, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 1 and 3 through 8 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Controls are listed below. The final row of each column presents the Omnibus F-stat for the null of balance on all outcomes, with the p-value for the associated test in brackets. Column 1 uses the full sample, while Columns 2 through 8 use the discontinuity sample.

Table 9: Most important plot

	Sample plot	MIP						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cultivated								
CA	0.026	0.356	0.056	0.040	-0.010	0.096	0.082	0.046
	(0.031)	(0.479)	(0.024)	(0.041)	(0.047)	(0.030)	(0.044)	(0.046)
	[0.397]	2,123	[0.021]	[0.333]	[0.826]	[0.002]	[0.063]	[0.321]
CA * MIP CA						-0.105	-0.102	-0.128
						(0.055)	(0.054)	(0.054)
						[0.056]	[0.057]	[0.019]
Joint F-stat [p]						4.9	2.5	2.9
						(0.008)	(0.085)	(0.058)
Irrigated								
CA	0.194	0.104	-0.017	-0.040	-0.042	0.017	0.003	0.004
	(0.019)	(0.306)	(0.016)	(0.026)	(0.031)	(0.008)	(0.020)	(0.026)
	[0.000]	2,123	[0.295]	[0.120]	[0.177]	[0.026]	[0.872]	[0.885]
CA * MIP CA						-0.104	-0.103	-0.105
						(0.036)	(0.036)	(0.044)
						[0.005]	[0.004]	[0.018]
Joint F-stat [p]						5.2	4.3	2.8
						(0.006)	(0.016)	(0.061)
Site-by-season FE	X		X	X	X	X	X	X
Distance to boundary				X	X		X	X
log area				X	X		X	X
Spatial FE					X			X
MIP log area				X	X		X	X
MIP CA				X	X	X	X	X

Notes: Regression analysis is presented in this table. Column 1 uses outcomes on the sample plot (and replicates analysis in Table 4), while Columns 3 through 8 use outcomes on the associated most important plot. All columns restrict to observations during the dry season. Column 2 presents, for the most important plot associated with sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. For Columns 1 and 3 through 8, Rows "CA" present coefficients on a command area indicator for the sample plot, while Rows "CA * MIP in CA" present coefficients on the interaction of a command area indicator for the sample plot with a command area indicator for the most important plot; robust standard errors clustered at the nearest water user group level are in parentheses, and p-values are in brackets.

Table 10: Most important plot

	Sample plot	MIP						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horticulture								
CA	0.173	0.100	-0.013	-0.035	-0.039	0.012	-0.000	-0.010
	(0.019)	(0.300)	(0.016)	(0.024)	(0.028)	(0.008)	(0.018)	(0.023)
	[0.000]	2,123	[0.406]	[0.147]	[0.160]	[0.141]	[0.982]	[0.672]
CA * MIP CA						-0.082	-0.082	-0.067
						(0.037)	(0.036)	(0.046)
						[0.028]	[0.024]	[0.141]
Joint F-stat [p]						2.8	2.6	1.4
						(0.063)	(0.079)	(0.258)
Banana								
CA	-0.132	0.199	0.070	0.089	0.057	0.082	0.094	0.076
	(0.024)	(0.399)	(0.024)	(0.034)	(0.037)	(0.034)	(0.042)	(0.043)
	[0.000]	2,123	[0.003]	[0.008]	[0.121]	[0.015]	[0.027]	[0.077]
CA * MIP CA						-0.017	-0.013	-0.044
						(0.043)	(0.042)	(0.044)
						[0.701]	[0.764]	[0.318]
Joint F-stat [p]						5.9	3.7	1.6
						(0.003)	(0.027)	(0.210)
Site-by-season FE	X		X	X	X	X	X	X
Distance to boundary				X	X		X	X
log area				X	X		X	X
Spatial FE					X			X
MIP log area				X	X		X	X
MIP CA				X	X	X	X	X

Notes: Regression analysis is presented in this table. Column 1 uses outcomes on the sample plot (and replicates analysis in Table 4), while Columns 3 through 8 use outcomes on the associated most important plot. All columns restrict to observations during the dry season. Column 2 presents, for the most important plot associated with sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. For Columns 1 and 3 through 8, Rows "CA" present coefficients on a command area indicator for the sample plot, while Rows "CA * MIP in CA" present coefficients on the interaction of a command area indicator for the sample plot with a command area indicator for the most important plot; robust standard errors clustered at the nearest water user group level are in parentheses, and p-values are in brackets.

Table 11: Most important plot

	Sample plot	MIP						
	Coef. (SE) [p]	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
HH labor/ha								
CA	67.3	65.5	-13.7	-39.7	-39.7	3.6	-17.1	-15.9
	(14.2)	(211.1)	(12.0)	(20.4)	(23.7)	(6.2)	(14.7)	(18.7)
	[0.000]	2,120	[0.256]	[0.052]	[0.094]	[0.567]	[0.245]	[0.394]
CA * MIP CA						-49.8	-53.7	-53.8
						(27.8)	(24.3)	(32.5)
						[0.073]	[0.027]	[0.098]
Joint F-stat [p]						1.7	2.6	1.7
						(0.194)	(0.078)	(0.177)
Input exp./ha								
CA	6.3	5.3	-2.1	-6.2	-6.7	0.2	-3.5	-3.8
	(1.2)	(27.6)	(1.5)	(2.8)	(2.7)	(0.7)	(1.9)	(2.1)
	[0.000]	2,123	[0.154]	[0.024]	[0.013]	[0.819]	[0.062]	[0.072]
CA * MIP CA						-6.4	-6.5	-6.4
						(3.5)	(3.2)	(3.7)
						[0.070]	[0.042]	[0.083]
Joint F-stat [p]						1.6	2.7	3.3
						(0.196)	(0.068)	(0.039)
Hired labor exp./ha								
CA	5.1	3.8	-0.6	-1.7	-0.3	0.8	0.0	1.6
	(1.8)	(24.5)	(1.3)	(2.2)	(2.2)	(1.2)	(2.1)	(2.6)
	[0.006]	2,123	[0.630]	[0.419]	[0.894]	[0.469]	[0.986]	[0.549]
CA * MIP CA						-4.0	-4.2	-4.2
						(2.7)	(2.6)	(3.2)
						[0.148]	[0.105]	[0.188]
Joint F-stat [p]						1.1	1.4	0.9
						(0.342)	(0.250)	(0.418)
Site-by-season FE	X		X	X	X	X	X	X
Distance to boundary				X	X		X	X
log area				X	X		X	X
Spatial FE					X			X
MIP log area				X	X		X	X
MIP CA				X	X	X	X	X

Notes: Regression analysis is presented in this table. Column 1 uses outcomes on the sample plot (and replicates analysis in Table 5), while Columns 3 through 8 use outcomes on the associated most important plot. All columns restrict to observations during the dry season. Column 2 presents, for the most important plot associated with sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. For Columns 1 and 3 through 8, Rows "CA" present coefficients on a command area indicator for the sample plot, while Rows "CA * MIP in CA" present coefficients on the interaction of a command area indicator for the sample plot with a command area indicator for the most important plot; robust standard errors clustered at the nearest water user group level are in parentheses, and p-values are in brackets.

Table 12: Sample plot, intensive margin effects

	Sample plot						
	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
HH labor/ha	54.8	67.3	65.4	70.8	12.2	28.4	24.4
	(188.7)	(14.2)	(16.7)	(19.9)	(10.1)	(13.2)	(13.0)
	2,438	[0.000]	[0.000]	[0.000]	[0.226]	[0.031]	[0.060]
Input exp./ha	2.3	6.3	5.2	4.4	0.5	1.0	-0.8
	(16.0)	(1.2)	(1.4)	(1.7)	(0.9)	(1.2)	(1.4)
	2,442	[0.000]	[0.000]	[0.011]	[0.587]	[0.402]	[0.544]
Hired labor exp./ha	3.6	5.1	3.2	2.7	1.4	0.7	-0.4
	(25.2)	(1.8)	(2.1)	(2.5)	(1.6)	(2.0)	(2.5)
	2,442	[0.006]	[0.126]	[0.285]	[0.353]	[0.750]	[0.885]
Site-by-season FE		X	X	X	X	X	X
Distance to boundary			X	X		X	X
log area			X	X		X	X
Spatial FE				X			X
Crop					X	X	X

Notes: Regression analysis is presented in this table. All columns restrict to observations during the dry season. Column 1 presents, for sample plots in the main discontinuity sample that are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 2 through 7 present regression coefficients on a command area indicator, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Columns 2 and 5 use the specification in Equation (1). Columns 3 and 6 use the regression discontinuity specification in Equation (2). Columns 4 and 7 use the spatial fixed effects specification in Equation (3). Columns 5, 6, and 7 also include controls for cultivation, horticulture, and bananas.

Table 13: Most important plot, intensive margin effects

	MIP						
	Dep. var.	Coef. (SE) [p]					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
HH labor/ha	65.5	-13.7	-39.7	-39.7	-12.7	-32.3	-26.7
	(211.1)	(12.0)	(20.4)	(23.7)	(9.2)	(14.2)	(16.7)
	2,120	[0.256]	[0.052]	[0.094]	[0.165]	[0.023]	[0.109]
Input exp./ha	5.3	-2.1	-6.2	-6.7	-1.6	-4.8	-4.9
	(27.6)	(1.5)	(2.8)	(2.7)	(1.2)	(2.1)	(2.1)
	2,123	[0.154]	[0.024]	[0.013]	[0.188]	[0.023]	[0.019]
Hired labor exp./ha	3.8	-0.6	-1.7	-0.3	-0.7	-1.3	0.5
	(24.5)	(1.3)	(2.2)	(2.2)	(1.2)	(1.9)	(2.1)
	2,123	[0.630]	[0.419]	[0.894]	[0.582]	[0.507]	[0.802]
Site-by-season FE		X	X	X	X	X	X
Distance to boundary			X	X		X	X
log area			X	X		X	X
MIP log area			X	X		X	X
MIP CA			X	X		X	X
Spatial FE				X			X
Crop					X	X	X

Notes: Regression analysis is presented in this table. All columns restrict to observations during the dry season. Column 1 presents, for most important plots whose associated sample plots in the main discontinuity sample are outside the command area, the mean of the dependent variable, the standard deviation of the dependent variable in parentheses, and the total number of observations. Columns 2 through 7 present regression coefficients on a command area indicator for the associated sample plot, with robust standard errors clustered at the nearest water user group level in parentheses, and p-values in brackets. Columns 2 and 5 use the specification in Equation (1). Columns 3 and 6 use the regression discontinuity specification in Equation (2). Columns 4 and 7 use the spatial fixed effects specification in Equation (3). Columns 5, 6, and 7 also include controls for cultivation, horticulture, and bananas.

Table 14: Heterogeneity with respect to household size and wealth

	MIP Coef. (SE) [p] (1)	(2)	(3)		MIP Coef. (SE) [p] (1)	(2)	(3)
Cultivated				Horticulture			
CA	-0.065	-0.092	-0.238	CA	-0.085	-0.109	-0.133
	(0.075)	(0.088)	(0.101)		(0.039)	(0.045)	(0.047)
	[0.388]	[0.295]	[0.018]		[0.029]	[0.016]	[0.005]
CA * # of HH members	0.024	0.026	0.045	CA * # of HH members	0.014	0.015	0.019
	(0.014)	(0.013)	(0.015)		(0.008)	(0.007)	(0.007)
	[0.081]	[0.055]	[0.002]		[0.067]	[0.036]	[0.008]
CA * Asset index	-0.009	-0.016	-0.043	CA * Asset index	-0.018	-0.014	-0.026
	(0.028)	(0.027)	(0.032)		(0.019)	(0.016)	(0.019)
	[0.738]	[0.549]	[0.183]		[0.356]	[0.409]	[0.170]
Joint F-stat [p]	3.3	2.0	3.9	Joint F-stat [p]	1.7	1.9	2.8
	(0.022)	(0.109)	(0.009)		(0.178)	(0.123)	(0.039)
Irrigated				Banana			
CA	-0.086	-0.109	-0.140	CA	0.027	0.036	-0.089
	(0.041)	(0.048)	(0.050)		(0.066)	(0.067)	(0.085)
	[0.037]	[0.022]	[0.005]		[0.679]	[0.586]	[0.297]
CA * # of HH members	0.014	0.014	0.020	CA * # of HH members	0.008	0.010	0.028
	(0.008)	(0.007)	(0.008)		(0.012)	(0.011)	(0.015)
	[0.097]	[0.054]	[0.011]		[0.476]	[0.379]	[0.059]
CA * Asset index	-0.020	-0.015	-0.030	CA * Asset index	0.005	-0.008	-0.018
	(0.019)	(0.016)	(0.018)		(0.024)	(0.023)	(0.027)
	[0.295]	[0.346]	[0.098]		[0.845]	[0.733]	[0.504]
Joint F-stat [p]	1.6	1.7	2.8	Joint F-stat [p]	3.3	2.8	2.1
	(0.181)	(0.159)	(0.039)		(0.023)	(0.043)	(0.094)
# of HH members	X	X	X	# of HH members	X	X	X
Asset index	X	X	X	Asset index	X	X	X
Site-by-season FE	X	X	X	Site-by-season FE	X	X	X
Distance to boundary		X	X	Distance to boundary		X	X
log area		X	X	log area		X	X
MIP log area		X	X	MIP log area		X	X
MIP CA		X	X	MIP CA		X	X
Spatial FE			X	Spatial FE			X

Notes: Regression analysis is presented in this table. All columns use outcomes on most important plots. Rows "CA" present coefficients on a command area indicator for the sample plot, while Rows "CA * W" present coefficients on the interaction of a command area indicator for the sample plot with a household characteristic W; robust standard errors clustered at the nearest water user group level are in parentheses, and p-values are in brackets. The Row "Joint F-stat [p]" presents F-statistics for the null that all 3 coefficients are 0, with the p-value for the associated test in brackets.

Table 15: Heterogeneity with respect to household size and wealth

	MIP		
	Coef. (SE) [p]		
	(1)	(2)	(3)
HH labor/ha			
CA	-82.8	-94.1	-106.3
	(32.6)	(37.1)	(40.3)
	[0.011]	[0.011]	[0.008]
CA * # of HH members	13.7	10.9	13.5
	(5.5)	(4.6)	(4.5)
	[0.012]	[0.018]	[0.003]
CA * Asset index	-23.8	-13.8	-22.7
	(12.9)	(10.5)	(12.5)
	[0.065]	[0.188]	[0.070]
Joint F-stat [p]	2.2	2.2	3.1
	(0.085)	(0.091)	(0.027)
Input exp./ha			
CA	-5.8	-9.2	-10.2
	(3.4)	(4.3)	(4.2)
	[0.091]	[0.035]	[0.017]
CA * # of HH members	0.7	0.6	0.7
	(0.5)	(0.5)	(0.5)
	[0.180]	[0.261]	[0.191]
CA * Asset index	-3.3	-2.7	-4.0
	(1.8)	(1.6)	(1.6)
	[0.061]	[0.094]	[0.013]
Joint F-stat [p]	1.4	2.0	2.8
	(0.250)	(0.115)	(0.036)
# of HH members	X	X	X
Asset index	X	X	X
Site-by-season FE	X	X	X
Distance to boundary		X	X
log area		X	X
MIP log area		X	X
MIP CA		X	X
Spatial FE			X

	MIP		
	Coef. (SE) [p]		
	(1)	(2)	(3)
Hired labor exp./ha			
CA	-4.5	-5.1	-1.4
	(3.0)	(3.7)	(3.8)
	[0.131]	[0.170]	[0.717]
CA * # of HH members	0.7	0.7	0.2
	(0.5)	(0.5)	(0.5)
	[0.138]	[0.184]	[0.708]
CA * Asset index	-0.3	0.1	0.1
	(1.5)	(1.4)	(1.4)
	[0.862]	[0.955]	[0.960]
Joint F-stat [p]	0.8	0.7	0.1
	(0.473)	(0.532)	(0.975)
# of HH members	X	X	X
Asset index	X	X	X
Site-by-season FE	X	X	X
Distance to boundary		X	X
log area		X	X
MIP log area		X	X
MIP CA		X	X
Spatial FE			X

Notes: Regression analysis is presented in this table. All columns use outcomes on most important plots. Rows "CA" present coefficients on a command area indicator for the sample plot, while Rows "CA * W" present coefficients on the interaction of a command area indicator for the sample plot with a household characteristic W; robust standard errors clustered at the nearest water user group level are in parentheses, and p-values are in brackets. The Row "Joint F-stat [p]" presents F-statistics for the null that all 3 coefficients are 0, with the p-value for the associated test in brackets.

Table 16: Minikits

	Minikit takeup		Horticulture		Input exp./ha	
	(1)	(2)	(3)	(4)	(5)	(6)
Assigned minikit	0.398	0.435	-0.001	0.021	-3.1	-1.4
	(0.038)	(0.042)	(0.041)	(0.039)	(3.1)	(3.2)
	[0.000]	[0.000]	[0.978]	[0.586]	[0.327]	[0.669]
Minikit saturation	-0.047	-0.057	-0.072	-0.082	3.6	5.3
	(0.056)	(0.057)	(0.058)	(0.054)	(6.4)	(5.3)
	[0.394]	[0.312]	[0.218]	[0.126]	[0.579]	[0.310]
Horticulture (2016 Dry)		0.125		0.331		11.3
		(0.035)		(0.038)		(3.2)
		[0.000]		[0.000]		[0.000]
# of lotteries entered	X	X	X	X	X	X
O&M treatment	X	X	X	X	X	X
Zone FE	X	X	X	X	X	X
# of observations	910	762	798	707	799	708
# of clusters	187	170	181	166	181	166

Notes: Regression analysis is presented in this table. All columns use outcomes on sample plots. Each row presents coefficients, with robust standard errors clustered at the water user group level in parentheses, and p-values in brackets. "Assigned minikit" is an indicator for whether the household was assigned to receive a minikit, "Minikit saturation" is the probability of minikit assignment that was assigned to the water user group of the household's sample plot, and "Horticulture (2016 Dry)" is an indicator that the household planted horticulture on their sample plot in 2016 Dry.

A Sampling

Work in progress: Sampling appendix.

B Model appendix

Derivation of first order conditions. Substitute for L^O using the household labor constraint, $L_1 + L_2 + \ell + L^O = \overline{L}$, and substitute for c in the household's maximization problem. This leaves two constraints, $M_1 + M_2 \leq \overline{M}$, and $\overline{L} - L_1 - L_2 - \ell \leq \overline{L^O}$; call the multipliers on these constraints $\widetilde{\lambda_M}$ and $\widetilde{\lambda_L}$, respectively. Taking first order conditions yields

$$
\begin{aligned}
(M_k) \quad & \mathbf{E}[u_c\sigma]A_k F_{kM} - \mathbf{E}[u_c]r && = \widetilde{\lambda_M} \\
(L_k) \quad & \mathbf{E}[u_c\sigma]A_k F_{kL} - \mathbf{E}[u_c]w && = -\widetilde{\lambda_L} \\
(\ell) \quad & \mathbf{E}[u_\ell] - \mathbf{E}[u_c]w && = -\widetilde{\lambda_L}
\end{aligned}
$$

To ease interpretation, normalize $\lambda_M \equiv \widetilde{\lambda_M}/r\mathbf{E}[u_c]$ and $\lambda_L \equiv \widetilde{\lambda_L}/w\mathbf{E}[u_c]$, and substitute $\text{cov}(\sigma, u_c) = \mathbf{E}[u_c\sigma] - \mathbf{E}[u_c]\mathbf{E}[\sigma] = \mathbf{E}[u_c\sigma] - \mathbf{E}[u_c]$. This yields

$$
\begin{aligned}
(M_k) \quad & \left(1 + \tfrac{\text{cov}(\sigma,u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kM} && = (1 + \lambda_M)r \\
(L_k) \quad & \left(1 + \tfrac{\text{cov}(\sigma,u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kL} && = (1 - \lambda_L)w \\
(\ell) \quad & \tfrac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} && = (1 - \lambda_L)w
\end{aligned}
$$

No constraints. When no constraints bind, as discussed the first order conditions simplify to

$$
\begin{aligned}
(M_k) \quad & A_k F_{kM} && = r \\
(L_k) \quad & A_k F_{kL} && = w \\
(\ell) \quad & \tfrac{u_\ell}{u_c} && = w
\end{aligned}
$$

Note that the first order conditions for M_2 and L_2 are functions only of (M_2, L_2), and exogenous (A_2, r, w). Therefore, $\frac{dM_2}{dA_1} = \frac{dL_2}{dA_1} = 0$.

Insurance market failure. Consider the case when insurance markets fail. To abstract fully from labor supply, we temporarily remove leisure from the model. To further simplify, we drop other inputs from the production function; when the production function is homogeneous in labor and other inputs, this is without loss of generality. Let $\gamma(c) = \frac{\mathbf{E}[u_c(c)]}{\mathbf{E}[\sigma u_c(c)]}$; $\gamma > 1$ is the ratio of the marginal utility from consumption to the marginal utility from agricultural production. This yields the first order conditions

$$\begin{aligned}
(L_1) \quad & A_1 F_{1L}(L_1) - \gamma(\sigma A_1 F_1(L_1) + \sigma A_2 F_2(L_2) + w(\overline{L} - L_1 - L_2) + r\overline{M})w &= 0 \\
(L_2) \quad & A_2 F_{2L}(L_2) - \gamma(\sigma A_1 F_1(L_1) + \sigma A_2 F_2(L_2) + w(\overline{L} - L_1 - L_2) + r\overline{M})w &= 0
\end{aligned}$$

The central intuition for this case can be captured from just the first order conditions: $\overline{L}$ and $\overline{M}$ enter symmetrically into the model, so larger households should respond similarly to richer households. If absolute risk aversion decreases sufficiently quickly (e.g. with CRRA preferences), then for sufficiently high levels of consumption $\mathbf{E}[\sigma u_c] = \mathbf{E}[\sigma]\mathbf{E}[u_c] = \mathbf{E}[u_c] \Rightarrow \gamma = 1$. Therefore, sufficiently wealthy or sufficiently large households should not respond to the sample plot shock. Below, we will maintain the assumption that preferences exhibit decreasing absolute risk aversion.

Some additional comments for the the derivations. The substitution that $A_k F_{kL} = \gamma w$ will be used frequently to simplify. Additionally, we use γ_c to represent the derivative of γ with respect to consumption (so $\gamma_c = \frac{partial}{partial c}\gamma(c)$), and γ_F to represent the derivative of γ with respect to agricultural production (so $\frac{\partial}{\partial F}\gamma(\sigma F)$). We define γ_{Fc} and γ_{FF} analagously. Additionally, the substitution $\frac{1}{A_k F_{kLL}} = \frac{\partial L_k}{\partial w^*}$, the partial derivative of labor demand on plot k with respect to the uncertainty adjusted wage γw, is useful for interpretation. It is assumed that $\frac{\partial L_k}{\partial w^*} < 0$, so labor demand is downward sloping. Finally, two multipliers frequently emerge in the math. The first, $M_N = 1 - \gamma w \frac{\partial L_1}{\partial w^*} \frac{\gamma w}{A_1 F_1}$, is a production multiplier: it captures when A_1 increases, how much does more does F_1 increase due to increased labor allocations to plot 1, holding fixed risk adjusted wages. The second, $M_D = 1 - \gamma_F w \gamma w \frac{\partial(L_1+L_2)}{\partial w^*}$, is a risk multiplier: its inverse captures when agricultural production is increased, how much less does agricultural production increase due to reduced labor allocations caused by risk aversion. Note that $M_N > 1$, and we will present conditions below sufficient for $\gamma_F > 0$, which implies $M_D > 1$.

Stack the left hand sides of the first order conditions into the vector FOC_I. Define the Jacobian $J_I \equiv D_{(L_1,L_2)}\text{FOC}_I$. Applying the implicit function theorem yields $D_{(A_1)}(L_1,L_2)' = -J_I^{-1}D_{(A_1)}\text{FOC}_I$ and $D_{(\overline{L})}(L_1,L_2)' = -J_I^{-1}D_{(\overline{L})}\text{FOC}_I$. Some algebra yields

$$
\begin{aligned}
J_I &= \begin{pmatrix} A_1F_{1LL} - \gamma_F w\gamma w & -\gamma_F w\gamma w \\ -\gamma_F w\gamma w & A_2F_{2LL} - \gamma_F w\gamma w \end{pmatrix} \\
J_I^{-1} &= \frac{\frac{\partial L_1}{\partial w^*}\frac{\partial L_2}{\partial w^*}}{M_D} \begin{pmatrix} A_2F_{2LL} - \gamma_F w\gamma w & \gamma_F w\gamma w \\ \gamma_F w\gamma w & A_1F_{1LL} - \gamma_F w\gamma w \end{pmatrix} \\
D_{(A_1)}\text{FOC}_I &= \left(\frac{\gamma w}{A_1} - \gamma_F w\frac{A_1F_1}{A_1}, -\gamma_F w\frac{A_1F_1}{A_1}\right)' \\
D_{(\overline{L})}\text{FOC}_I &= \left(-\gamma_c w^2, -\gamma_c w^2\right)' \\
D_{(A_1)}(L_1,L_2)' &= -\frac{\frac{\partial L_1}{\partial w^*}\frac{\partial L_2}{\partial w^*}}{M_D} \begin{pmatrix} \frac{\gamma w}{A_1\frac{\partial L_2}{\partial w^*}} - \left(\frac{\gamma_F w(\gamma w)^2}{A_1} - \frac{\gamma_F w A_1F_1}{A_1\frac{\partial L_2}{\partial w^*}}\right) \\ \frac{\gamma_F w(\gamma w)^2}{A_1} - \frac{\gamma_F w A_1F_1}{A_1\frac{\partial L_1}{\partial w^*}} \end{pmatrix} \\
D_{(\overline{L})}(L_1,L_2)' &= \frac{\gamma_c w^2}{M_D} \begin{pmatrix} \frac{\partial L_1}{\partial w^*} \\ \frac{\partial L_2}{\partial w^*} \end{pmatrix}
\end{aligned}
$$

Some additional simplification will be useful for $\frac{dL_2}{dA_1}$; note that

$$
\frac{dL_2}{dA_1} = \left(\frac{\gamma_F w A_1F_1}{A_1}\right)\frac{\partial L_2}{\partial w^*}\frac{M_N}{M_D}
$$

Under conditions assumed, a sufficient condition for $\frac{dL_2}{dA_1} < 0$ is that $\gamma_F > 0$: that is, an exogenous increase in agricultural production would increase the household's marginal utility of consumption relative to marginal utility from agricultural production. That this should hold seems intuitive, but it need not hold in general: increases in agricultural production, by increasing consumption, may move households to a less risk averse portion of their utility function, and in turn increase marginal utility from agricultural production relative to marginal utility of consumption. Some assumptions on the distribution of σ rule this out: we follow Karlan et al. (2014) and, for some $k > 1$, assume $\sigma = k$ with probability $\frac{1}{k}$ ("the good state") and $\sigma = 0$ with probability $\frac{k-1}{k}$ ("the bad state"); i.e., there is a crop failure with probability $\frac{k-1}{k}$.

Under this assumption, some additional simplification and signing is possible. Define $\overline{R} = -\frac{\mathbf{E}[u_c \frac{u_{cc}}{u_c}]}{\mathbf{E}[u_c]}$ to be the household's average risk aversion, $R_k = -\mathbf{E}[\frac{u_{cc}}{u_c}|\sigma = k]$ to be the household's risk aversion in the good state, and $R_{kc} = -\mathbf{E}[\frac{u_{ccc}}{u_c}|\sigma = k] + R_k^2$ to be the derivative of the household's risk aversion in the good state. Note that by assumption, $R_{kc} < 0$ and $R_k < \overline{R}$. From this, it follows that

$$\begin{aligned}
\gamma_c &= \frac{\mathbf{E}[u_{cc}]}{\mathbf{E}[\sigma u_c]} - \frac{\mathbf{E}[\sigma u_{cc}]\mathbf{E}[u_c]}{\mathbf{E}[\sigma u_c]^2} = \gamma(R_k - \overline{R}) < 0 \\
\gamma_F &= \frac{\mathbf{E}[\sigma u_{cc}]}{\mathbf{E}[\sigma u_c]} - \frac{\mathbf{E}[\sigma^2 u_{cc}]\mathbf{E}[u_c]}{\mathbf{E}[\sigma u_c]^2} = (k-1)\frac{\mathbf{E}[u_c|\sigma = 0]}{\mathbf{E}[u_c|\sigma = k]} R_k = (k\gamma - 1)R_k > 0 \\
\gamma_{Fc} &= \left((k\gamma - 1)\frac{R_{kc}}{R_k} + k\gamma(R_k - \overline{R}) \right) < 0 \\
\gamma_{FF} &= (k^2\gamma - 1)R_{kc} + k(k\gamma - 1)R_k^2 = k\gamma_F R_k \left(-\frac{R_{kc}}{R_k} + R_k \right) > 0
\end{aligned}$$

Since, $\gamma_F > 0$, it follows that $\frac{dL_2}{dA_1} < 0$, so households substitute labor away from their other plots in response to the sample plot shock.

These results are almost sufficient to sign our cross partial of interest. Note that

$$\frac{\frac{1}{w}\frac{d^2 L_2}{d\overline{L} dA_1}}{\frac{dL_2}{dA_1}} = \frac{\frac{1}{w}\frac{d\gamma_F}{d\overline{L}}}{\gamma_F} + \frac{\frac{1}{w}\frac{dF_1}{d\overline{L}}}{F_1} + \frac{\frac{1}{w}\frac{d(\partial L_2/\partial w^*)}{d\overline{L}}}{(\partial L_2/\partial w^*)} + \frac{\frac{1}{w}\frac{dM_N}{d\overline{L}}}{M_N} - \frac{\frac{1}{w}\frac{dM_D}{d\overline{L}}}{M_D}$$

The following substitutions are now useful

$$
\begin{aligned}
\frac{\frac{1}{w}\frac{d\gamma_F}{d\overline{L}}}{\gamma_F} &= \frac{\gamma_{Fc}}{\gamma_F} + \left(\frac{1}{w}\frac{dF}{d\overline{L}}\right)\frac{\gamma_{FF}}{\gamma_F} \\
\frac{1}{w}\frac{dF}{d\overline{L}} &\equiv \frac{1}{w}\frac{d(A_1F_1 + A_2F_2)}{d\overline{L}} = \frac{\gamma w}{w}\left(\frac{dL_1}{d\overline{L}} + \frac{dL_2}{d\overline{L}}\right) = \frac{M_D - 1}{M_D}\left(-\frac{\gamma_c}{\gamma_F}\right) \\
\frac{\gamma_{Fc}}{\gamma_F} &= \frac{R_{kc}}{R_k} + \frac{k\gamma}{k\gamma - 1}\frac{\gamma_c}{\gamma} \\
\frac{\frac{1}{w}\frac{dF_1}{d\overline{L}}}{F_1} &= \frac{M_N - 1}{M_D}\left(-\frac{\gamma_c}{\gamma}\right) \\
\frac{\frac{1}{w}\frac{d(\partial L_k/\partial w^*)}{d\overline{L}}}{(\partial L_k/\partial w^*)} &= \frac{\gamma_c w\frac{\partial L_k}{\partial w^*}}{M_D}\left(-\frac{F_{kLLL}}{F_{kLL}}\right) \\
\frac{\frac{1}{w}\frac{dM_N}{d\overline{L}}}{M_N} &= \frac{M_N - 1}{M_N}\left(2\frac{\gamma_c}{\gamma} + \frac{\frac{1}{w}\frac{d(\partial L_1/\partial w^*)}{d\overline{L}}}{(\partial L_1/\partial w^*)}\right) \\
\frac{\frac{1}{w}\frac{dM_D}{d\overline{L}}}{M_D} &= \frac{M_D - 1}{M_D}\left(\frac{\gamma_{Fc}}{\gamma_F} + \frac{\gamma_c}{\gamma} + \frac{\frac{1}{w}\frac{d(\partial(L_1+L_2)/\partial w^*)}{d\overline{L}}}{(\partial(L_1 + L_2)/\partial w^*)}\right)
\end{aligned}
$$

Making these substitutions, and rearranging terms, yields

$$
\begin{aligned}
\frac{\frac{1}{w}\frac{d^2L_2}{d\overline{L}dA_1}}{\frac{dL_2}{dA_1}} = {} & \frac{1}{M_D}\frac{R_{kc}}{R_k} + \frac{M_D - 1}{M_D}\left(\left(-\frac{\gamma_c}{\gamma_F}\right)\frac{\gamma_{FF}}{\gamma_F} + \frac{k\gamma}{k\gamma - 1}\frac{\gamma_c}{\gamma}\right) + \\
& \left(\frac{k\gamma}{k\gamma - 1}\frac{1}{M_D} + 2\frac{M_N - 1}{M_N} - \left(1 + \frac{k\gamma}{k\gamma - 1}\right)\frac{M_D - 1}{M_D} - \frac{M_N - 1}{M_N M_D}\right)\frac{\gamma_c}{\gamma} + \\
& \frac{M_N - 1}{M_N}\frac{\frac{1}{w}\frac{d(\partial L_1/\partial w^*)}{d\overline{L}}}{(\partial L_1/\partial w^*)} + \frac{\frac{1}{w}\frac{d(\partial L_2/\partial w^*)}{d\overline{L}}}{(\partial L_2/\partial w^*)} - \frac{M_D - 1}{M_D}\frac{\frac{1}{w}\frac{d(\partial(L_1+L_2)/\partial w^*)}{d\overline{L}}}{(\partial(L_1 + L_2)/\partial w^*)}
\end{aligned}
$$

To sign this derivative, I now make the assumption that $F_{kLLL} \approx 0$. Essentially, this is making as assumption about the relative magnitudes of channels through which the effect of increasing $\overline{L}$, through reduced risk aversion, affects the responsiveness of the household to the sample plot shock. In particular, it assumes that changes in the labor demand elasticity caused by increased labor allocations do not dominate the direct effects of reduced risk aversion. This simplifies this expression to

$$\frac{\frac{1}{w}\frac{d^2L_2}{d\overline{L}dA_1}}{\frac{dL_2}{dA_1}} = \frac{1}{M_D}\frac{R_{kc}}{R_k} + \frac{M_D-1}{M_D}\left(\left(-\frac{\gamma_c}{\gamma_F}\right)\frac{\gamma_{FF}}{\gamma_F} + \frac{k\gamma}{k\gamma-1}\frac{\gamma_c}{\gamma}\right) +$$
$$\left(\frac{k\gamma}{k\gamma-1}\frac{1}{M_D} + 2\frac{M_N-1}{M_N} - \left(1+\frac{k\gamma}{k\gamma-1}\right)\frac{M_D-1}{M_D} - \frac{M_N-1}{M_N M_D}\right)\frac{\gamma_c}{\gamma}$$

We now sign each term individually. First, $\frac{1}{M_D}\frac{R_{kc}}{R_k} < 0$, as we assumed decreasing absolute risk aversion. For the second term, note that $\left(-\frac{\gamma_c}{\gamma_F}\right)\frac{\gamma_{FF}}{\gamma_F} + \frac{k\gamma}{k\gamma-1}\frac{\gamma_c}{\gamma} = \left(-\frac{\gamma_c}{\gamma_F}\right)\left(\frac{\gamma_{FF}}{\gamma_F} - \frac{k\gamma}{k\gamma-1}\frac{\gamma_F}{\gamma}\right)$. Therefore,

$$\text{sign}\left[\frac{M_D-1}{M_D}\left(\left(-\frac{\gamma_c}{\gamma_F}\right)\frac{\gamma_{FF}}{\gamma_F} + \frac{k\gamma}{k\gamma-1}\frac{\gamma_c}{\gamma}\right)\right] = \text{sign}\left[\frac{\gamma_{FF}}{\gamma_F} - \frac{k\gamma}{k\gamma-1}\frac{\gamma_F}{\gamma}\right]$$

Substituting yields

$$\frac{\gamma_{FF}}{\gamma_F} - \frac{k\gamma}{k\gamma-1}\frac{\gamma_F}{\gamma} = \frac{k^2-1}{k\gamma^2}\frac{R_{kc}}{R_k} + kR_k - kR_k = \frac{k^2-1}{k\gamma^2}\frac{R_{kc}}{R_k} < 0$$

Therefore, $\frac{M_D-1}{M_D}\left(\left(-\frac{\gamma_c}{\gamma_F}\right)\frac{\gamma_{FF}}{\gamma_F} + \frac{k\gamma}{k\gamma-1}\frac{\gamma_c}{\gamma}\right) < 0$.

For the third term, note that

$$\frac{k\gamma}{k\gamma-1}\frac{1}{M_D} + 2\frac{M_N-1}{M_N} - \left(1+\frac{k\gamma}{k\gamma-1}\right)\frac{M_D-1}{M_D} - \frac{M_N-1}{M_N M_D} = \frac{2(M_N-M_D) + \frac{1}{k\gamma-1}M_N(2-M_D)+1}{M_N M_D}$$

This third term is positive when M_D is sufficiently small. Explained alternatively, M_D sufficiently small means that an exogenous increase in agricultural production cannot cause households to decrease their labor allocations by too much; i.e., households cannot be too risk averse.[39] When this holds, then $\left(\frac{k\gamma}{k\gamma-1}\frac{1}{M_D} + 2\frac{M_N-1}{M_N} - \left(1+\frac{k\gamma}{k\gamma-1}\right)\frac{M_D-1}{M_D} - \frac{M_N-1}{M_N M_D}\right)\frac{\gamma_c}{\gamma} < 0$.

Since each term is negative, we have that $\frac{\frac{1}{w}\frac{d^2L_2}{d\overline{L}dA_1}}{\frac{dL_2}{dA_1}}$. As $\frac{dL_2}{dA_1} < 0$, we have $\frac{d^2L_2}{d\overline{L}dA_1} > 0$. Since $w\overline{L}$ and $r\overline{M}$ enter the household's problem symmetrically, then $\frac{d^2L_2}{d\overline{M}dA_1} > 0$.

[39]Note that this condition is sufficient, but not necessary, to sign this term and also to sign $\frac{\frac{1}{w}\frac{d^2L_2}{d\overline{L}dA_1}}{\frac{dL_2}{dA_1}}$.

Input constraint. When only the input constraint binds, the first order conditions simplify to

$$\begin{aligned}
(M_k) \quad & A_k F_{kM} && = (1+\lambda_M) r \\
(L_k) \quad & A_k F_{kL} && = w \\
(\ell) \quad & \tfrac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} && = w
\end{aligned}$$

Note that the choice of leisure does not enter into the first order conditions for M_k or L_k. Substituting $M_2 = \overline{M} - M_1$ yields the following system of equations

$$\begin{aligned}
A_1 F_{1M}(M_1, L_1) - (1+\lambda_M) r = 0 \\
A_1 F_{1L}(M_1, L_1) - w = 0 \\
A_2 F_{2M}(\overline{M} - M_1, L_2) - (1+\lambda_M) r = 0 \\
A_2 F_{2L}(\overline{M} - M_1, L_2) - w = 0
\end{aligned}$$

Stack the left hand sides into the vector FOC_M. Define the Jacobian $J_M \equiv D_{(M_1, L_1, \lambda_M, L_2)} \text{FOC}_M$. Applying the implicit function theorem yields $D_{(A_1)}(M_1, L_1, \lambda_M, L_2)' = -J_M^{-1} D_{(A_1)} \text{FOC}_M$. Some algebra yields

$$J_M = \begin{pmatrix} A_1 F_{1MM} & A_1 F_{1ML} & -r & 0 \\ A_1 F_{1ML} & A_1 F_{1LL} & 0 & 0 \\ -A_2 F_{2MM} & 0 & -r & A_2 F_{2ML} \\ -A_2 F_{2ML} & 0 & 0 & A_2 F_{2LL} \end{pmatrix}$$

$$\begin{aligned}
D_{(A_1)} \text{FOC}_M &= (F_{1M}, F_{1L}, 0, 0)' \\
\frac{dM_2}{dA_1} &= k_M A_2 F_{2LL} A_1 (F_{1L} F_{1ML} - F_{1M} F_{1LL}) \\
\frac{dL_2}{dA_1} &= -k_M A_2 F_{2ML} A_1 (F_{1L} F_{1ML} - F_{1M} F_{1LL})
\end{aligned}$$

where k_M is positive.[40] As $F_{2LL} < 0$, $\text{sign}\left(\frac{dM_2}{dA_1}\right) = -\text{sign}\left(F_{1L}F_{1ML} - F_{1M}F_{1LL}\right)$. This is negative whenever productivity growth on plot 1 would cause optimal input allocations, holding fixed the shadow price of inputs, to increase on plot 1. Similarly, $\text{sign}\left(\frac{dL_2}{dA_1}\right) = \text{sign}(F_{2LM})\text{sign}\left(\frac{dM_2}{dA_1}\right)$. The labor response and input response on the second plot have the same sign whenver labor and inputs are complements on the second plot.

Labor constraint. When only the labor constraint binds, the first order conditions simplify to

$$\begin{aligned}
(M_k) \quad & A_k F_{kM} && = r \\
(L_k) \quad & A_k F_{kL} && = (1 - \lambda_L) w \\
(\ell) \quad & \frac{u_\ell}{u_c} && = (1 - \lambda_L) w
\end{aligned}$$

Substituting $\ell = \overline{L} - L^O - L_1 - L_2$ and $L^O = \overline{L^O}$, and some rearranging yields

$$\begin{aligned}
A_1 F_{1M}(M_1, L_1) - r = 0 \\
A_1 F_{1L}(M_1, L_1) - (1 + \lambda_L) w = 0 \\
A_2 F_{2M}(M_2, L_2) - r = 0 \\
A_2 F_{2L}(M_2, L_2) - (1 + \lambda_L) w = 0 \\
u_\ell \left(\sum_{k \in \{1,2\}} A_k F_k(M_k, L_k) + r(\overline{M} - M_1 - M_2) + w\overline{L^O}, \overline{L} - L^O - L_1 - L_2 \right) - \\
(1 + \lambda_L) w u_c \left(\sum_{k \in \{1,2\}} A_k F_k(M_k, L_k) + r(\overline{M} - M_1 - M_2) + w\overline{L^O}, \overline{L} - L^O - L_1 - L_2 \right) = 0
\end{aligned}$$

Stack the left hand sides into the vector FOC_L.

Additionally, it will be convenient to define the following derivatives of on farm labor demand on plot k, LD_k, with respect to the shadow wage w^* and productivity A_k, on farm

[40] $k_M = -\frac{1}{(A_1 F_{1LL}) A_2^2 (F_{2MM} F_{2LL} - F_{2ML}^2) + (A_2 F_{2LL}) A_1^2 (F_{1MM} F_{1LL} - F_{1ML}^2)}$. We make standard assumptions required for unconstrained optimization; second order conditions for unconstrained optimization imply k_M is positive.

input demand on plot k, MD_k, with respect to productivity A_k, and on farm labor supply, LS, with respect to the shadow wage w^* and consumption (through shifts to wealth) c. Let

$$\begin{aligned}
\mathrm{LD}_{kw^*} &= \frac{A_k F_{kMM}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\mathrm{LD}_{kA_k} &= \frac{A_k F_{kM}F_{kML} - A_k F_{kL}F_{kMM}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\mathrm{MD}_{kA_k} &= \frac{A_k F_{kL}F_{kML} - A_k F_{kM}F_{kLL}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\mathrm{LS}_{w^*} &= -\frac{u_c}{u_{\ell\ell} - (1+\lambda_L)wu_{c\ell}} \\
\mathrm{LS}_c &= -\frac{u_{c\ell} - (1+\lambda_L)wu_{cc}}{u_{\ell\ell} - (1+\lambda_L)wu_{c\ell}}
\end{aligned}$$

We make standard assumptions required for unconstrained optimization; these imply LD_{kw^*} is negative (labor demand decreasing in shadow wage), and LS_{w^*} is positive (labor supply increasing in shadow wage). We further assume LD_{kA_k} and MD_{kA_k} are positive (labor demand and input demand are increasing in productivity); an additional sufficient assumption for this is that F is homogeneous. We further assume LS_c is negative (labor supply is decreasing in wealth); an additional sufficient assumption for this is that u is additively separable in c and ℓ.

Next, define the Jacobian $J_L \equiv D_{(M_1,L_1,M_2,L_2,\lambda_L)}\mathrm{FOC}_L$. Some algebra yields

$$J_L = \begin{pmatrix} A_1F_{1MM} & A_1F_{1ML} & 0 & 0 & 0 \\ A_1F_{1ML} & A_1F_{1LL} & 0 & 0 & -w \\ 0 & 0 & A_2F_{2MM} & A_2F_{2ML} & 0 \\ 0 & 0 & A_2F_{2ML} & A_2F_{2LL} & -w \\ \frac{d\mathrm{FOC}_{L,\ell}}{dM_1} & \frac{d\mathrm{FOC}_{L,\ell}}{dL_1} & \frac{d\mathrm{FOC}_{L,\ell}}{dM_2} & \frac{d\mathrm{FOC}_{L,\ell}}{dL_2} & -wu_c \end{pmatrix}$$

$$\frac{d\mathrm{FOC}_{L,\ell}}{dM_1} = A_1F_{1M}(u_{c\ell} - (1+\lambda_L)wu_{cc})$$

$$\frac{d\mathrm{FOC}_{L,\ell}}{dL_1} = A_1F_{1L}(u_{c\ell} - (1+\lambda_L)wu_{cc}) - (u_{\ell\ell} - (1+\lambda_L)wu_{c\ell})$$

$$\frac{d\mathrm{FOC}_{L,\ell}}{dM_2} = A_2F_{2M}(u_{c\ell} - (1+\lambda_L)wu_{cc})$$

$$\frac{d\mathrm{FOC}_{L,\ell}}{dL_2} = A_2F_{2L}(u_{c\ell} - (1+\lambda_L)wu_{cc}) - (u_{\ell\ell} - (1+\lambda_L)wu_{c\ell})$$

Applying the implicit function theorem yields $D_{(A_1)}(M_1, L_1, M_2, L_2, \lambda_L)' = -J_L^{-1}D_{(A_1)}\mathrm{FOC}_L$. Some further algebra, and substitution, yields

$$D_{(A_1)}\mathrm{FOC}_L = (F_{1M}, F_{1L}, 0, 0, (u_{c\ell} - (1+\lambda_L)wu_{cc})F_1)'$$

$$\frac{dL_2}{dA_1} = \mathrm{LD}_{2w^*}\frac{\mathrm{LD}_{1A_1} - \mathrm{LS}_c(F_{1M}\mathrm{MD}_{1A_1} + F_{1L}\mathrm{LD}_{1A_1} + F_1)}{\mathrm{LS}_{w^*} - (\mathrm{LD}_{1w^*} + \mathrm{LD}_{2w^*}) - \mathrm{LS}_c(\mathrm{LD}_{1A_1} + \mathrm{LD}_{2A_2})}$$

$$\frac{dL_2}{d\overline{L}} = \mathrm{LD}_{2w^*}\frac{1}{\mathrm{LS}_{w^*} - (\mathrm{LD}_{1w^*} + \mathrm{LD}_{2w^*}) - \mathrm{LS}_c(\mathrm{LD}_{1A_1} + \mathrm{LD}_{2A_2})}$$

$$\frac{dL_2}{d\overline{M}} = \mathrm{LD}_{2w^*}\frac{r\mathrm{LS}_c}{\mathrm{LS}_{w^*} - (\mathrm{LD}_{1w^*} + \mathrm{LD}_{2w^*}) - \mathrm{LS}_c(\mathrm{LD}_{1A_1} + \mathrm{LD}_{2A_2})}$$

$\frac{dL_2}{dA_1} < 0$; for interpretation, note that this expression is the derivative of labor demand on plot 2 with respect to the shadow wage, times the effect of the shock to A_1 on the shadow wage. The numerator of the latter is the effect the shock on negative residual labor supply through direct effects (LD_{1A_1}) and wealth effects, including through adjustments of labor and inputs ($-\mathrm{LS}_c(F_{1M}\mathrm{MD}_{1A_1} + F_{1L}\mathrm{LD}_{1A_1} + F_1)$). The denominator of the latter is the derivative of residual labor supply with respect to the shadow wage, adjusted for wealth effects $(\mathrm{LS}_{w^*} - (\mathrm{LD}_{1w^*} + \mathrm{LD}_{2w^*}) - \mathrm{LS}_c(\mathrm{LD}_{1A_1} + \mathrm{LD}_{2A_2}))$.

The signs of $\frac{d^2 L_2}{d\bar{L} dA_1}$ and $\frac{d^2 L_2}{d\bar{M} dA_1}$ are ambiguous. However, unlike the cases of input market failures or insurance market failures, here these second derivatives may have opposite signs. To see one example of this, consider a case where on farm labor and input demands are approximately linear in the shadow wage and productivity, and on farm labor supply is approximately linear in consumption, but exhibits meaningful curvature with respect to the shadow wage. In this case, $\mathrm{sign}(\frac{d^2 L_2}{d\bar{L} dA_1}) = \mathrm{sign}\left(\frac{d}{d\bar{L}}\mathrm{LS}_{w^*}\right)$ and $\mathrm{sign}(\frac{d^2 L_2}{d\bar{L} dA_1}) = \mathrm{sign}\left(\frac{d}{d\bar{M}}\mathrm{LS}_{w^*}\right)$. To focus on one case, larger households are less responsive to the A_1 shock ($\frac{d^2 L_2}{d\bar{L} dA_1} > 0$) if and only if they are on a more elastic portion of their labor supply curve ($\frac{d}{d\bar{L}}\mathrm{LS}_{w^*} > 0$). That larger households, with more labor available for agriculture, or poorer households, who likely have fewer productive opportunities outside agriculture, would be on a more elastic portion of their labor supply curve is consistent with proposed models of household labor supply dating back to Lewis (1954). This motivates the prediction we focus on: that larger households should be less responsive to the A_1 shock, and richer households should be more responsive to the A_1 shock.