3.(1)(a)
From Eqn(8)

$$[V_\tau]_i^{n+1} = \left[\frac{\sigma^2 S^2}{2} V_{SS}\right]_i^{n+1} + [(r - \lambda(J-1)S V_S]_i^{n+1} - [rV]_i^{n+1}$$
$$+ \lambda[V(JS_i, \tau^n)]_i^n - \lambda[V]_i^n$$

Eqn(7)

$$V_i^{n+1}[1 + \Delta\tau(r + \alpha_i + \beta_i)] = V_i^n + \Delta\tau\alpha_i V_{i-1}^{n+1} + \Delta\tau\beta_i V_{i+1}^{n+1}$$

divide $\Delta\tau$ both side

$$\frac{V_i^{n+1}}{\Delta\tau} + V_i^{n+1}(r + \alpha_i + \beta_i) = \frac{V_i^n}{\Delta\tau} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1}$$

$$\frac{V_i^{n+1} - V_i^n}{\Delta\tau} = -(r + \alpha_i + \beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1}$$

$$[V_\tau]_i^{n+1} = -(r + \alpha_i + \beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1}$$

By Eqn(6), we also know that

$$[V_\tau]_i^{n+1} = \left[\frac{\sigma^2 S^2}{2} V_{SS}\right]_i^{n+1} + \left[[r - \lambda(J-1)]S V_S\right]_i^{n+1} - [rV]_i^{n+1}$$

Thus, in Eqn(8),

$$[V_\tau]_i^{n+1} = (-r + \alpha_i + \beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1}$$
$$+ \lambda[V(JS_i, \tau^n)]_i^n - \lambda V_i^n$$

$$\frac{V_i^{n+1} - V_i^n}{\Delta\tau} = -(r + \alpha_i + \beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1} +$$
$$\lambda[V(JS_i, \tau^n)]_i^n - \lambda V_i^n$$

$$[1 + \Delta\tau(r + \alpha_i + \beta_i)]V_i^{n+1} = (1 - \Delta\tau\lambda)V_i^n + \Delta\tau\alpha_i V_{i-1}^{n+1}$$
$$+ \Delta\tau\beta_i V_{i+1}^{n+1} + \Delta\tau\lambda[V(JS_i, \tau^n)]_i^n$$

To make it stable, $1 - \Delta\tau\lambda \geq 0$

$$\Delta\tau \leq \frac{1}{\lambda}$$

3. (1) (b)

$$\frac{V_i^{n+1}-V_i^n}{\Delta\tau} = -(r+\alpha_i+\beta_i)V_i^{n+1}+\alpha_i V_{i-1}^{n+1}+\beta_i V_{i+1}^{n+1}$$
$$+\lambda[V(JS_i,\tau^n]_i^n-\lambda V_i^n$$

$$=-(r+\alpha_i+\beta_i)V_i^{n+1}+\alpha_i V_{i-1}^{n+1}+\beta_i V_{i+1}^{n+1}+$$
$$\lambda(w_i V(S_k,\tau^n)+(1-w_i)V(S_{k+1},\tau^n))-$$
$$\lambda V_i^n.$$

$$\Rightarrow V_i^{n+1}(1+\Delta\tau(r+\alpha_i+\beta_i)) = V_i^n+\Delta\tau\alpha_i V_{i-1}^{n+1}+\Delta\tau\beta_i V_{i+1}^{n+1}+$$
$$\Delta\tau\lambda(w_i V_k^n+(1-w_i)V_{k+1}^n)-\lambda\Delta\tau V_i^n.$$

noting that r, α_i, β_i are all non-negative,

$$|V_i^{n+1}|[1+\Delta\tau(r+\alpha_i+\beta_i)] \le (1-\lambda\Delta\tau)|V_i^n|+\Delta\tau\alpha_i|V_{i-1}^{n+1}|$$
$$+\Delta\tau\beta_i|V_{i+1}^{n+1}|+\Delta\tau\lambda w_i|V_k^n|+(1-w_i)\Delta\tau\lambda|V_{k+1}^n|$$

Let $\|V^n\|=\max_i|V_i^n|$

$$|V_i^{n+1}|[1+\Delta\tau(r+\alpha_i+\beta_i)] \le (1-\lambda\Delta\tau)\|V^n\|+\Delta\tau(\alpha_i+\beta_i)\|V^{n+1}\|$$
$$+\Delta\tau\lambda w_i\|V^n\|+(1-w_i)\Delta\tau\lambda\|V^n\|$$

Let $|V_{i*}^{n+1}|=\max_i|V_i^{n+1}|$

$$\|V^{n+1}\|[1+\Delta\tau(r+\cancel{\alpha_i+\beta_i})] \le (1-\lambda\Delta\tau)\|V^n\|+\cancel{\Delta\tau(\alpha_i+\beta_i)\|V^{n+1}\|}$$
$$+\Delta\tau\lambda w_i\|V^n\|+(1-w_i)\Delta\tau\lambda\|V^n\|$$

We have assumed $1-\lambda\Delta\tau\ge 0$.

$$\Rightarrow \|V^{n+1}\|[1+\Delta\tau r] \le (1-\lambda\Delta\tau+\lambda\Delta\tau)\|V^n\|$$
$$\|V^{n+1}\| \le \frac{\|V^n\|}{1+\Delta\tau r} \le \|V^n\|$$

unconditionally converge if $\Delta\tau\le\frac{1}{\lambda}$.

Thus, the stable condition is $\Delta\tau\le\frac{1}{\lambda}$

3.(2)(a)

From Equ (10).

$$[V_\tau]_i^{n+1} = \left[\frac{\sigma^2 S^2}{2} V_{SS}\right]_i^{n+1} + [(r-\lambda(J-1))SV_S]_i^{n+1} - [rV]_i^{n+1}$$
$$+ \lambda[V(JS_i, \tau^n)]_i^n - \lambda[V]_i^{n+1}$$

From Equ (7)

$$V_i^{n+1}[1+\Delta\tau(r+\alpha_i+\beta_i)] = V_i^n + \Delta\tau\alpha_i V_{i-1}^{n+1} + \Delta\tau\beta_i V_{i+1}^{n+1}$$
$$\Rightarrow [V_\tau]_i^{n+1} = -(r+\alpha_i+\beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1}$$

By Equ(6), we also know that

$$[V_\tau]_i^{n+1} = \left[\frac{\sigma^2 S^2}{2} V_{SS}\right]_i^{n+1} + [(r-\lambda(J-1))SV_S]_i^{n+1} - [rV]_i^{n+1}$$

Thus, In Equ(8).

$$[V_\tau]_i^{n+1} = -(r+\alpha_i+\beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1} +$$
$$\lambda[V(JS_i, \tau^n)]_i^n - \lambda[V_i]_i^{n+1}$$
$$\frac{V_i^{n+1} - V_i^n}{\Delta\tau} = -(r+\alpha_i+\beta_i)V_i^{n+1} + \alpha_i V_{i-1}^{n+1} + \beta_i V_{i+1}^{n+1} +$$
$$\lambda[V(JS_i, \tau^n)]_i^n - \lambda[V]_i^{n+1}$$
$$V_i^{n+1}(1+\Delta\tau(r+\alpha_i+\beta_i+\lambda)) = V_i^n + \alpha_i\Delta\tau V_{i-1}^{n+1} + \beta_i\Delta\tau V_{i+1}^{n+1}$$
$$+ \lambda\Delta\tau[V(JS_i, \tau^n)]_i^n$$

3. (a)(b). It follows that α_i, β_i, r are all nonnegative

$$|V_i^{n+1}|\left(1+\Delta\tau(r+\alpha_i+\beta_i+\lambda)\right) \le |V_i^n| + \alpha_i\Delta\tau|V_{i-1}^{n+1}| + \beta_i\Delta\tau|V_{i+1}^{n+1}| + \lambda\Delta\tau|w_i V_k^n + (1-w_i)V_{k+1}^n|$$

Let $\|V^n\| = \max_i |V_i^n|$

$$\Rightarrow |V_i^{n+1}|\left(1+\Delta\tau(r+\alpha_i+\beta_i+\lambda)\right) \le \|V^n\| + \Delta\tau(\alpha_i+\beta_i)\|V^{n+1}\| + \lambda\Delta\tau w_i\|V^n\| + (1-w_i)\lambda\Delta\tau\|V^n\|$$

Then let $|V_{i^*}^{n+1}| = \max_i |V_i^{n+1}|$.

$$\|V^{n+1}\|\left(1+\Delta\tau(r+\cancel{\alpha_i}+\beta_i+\lambda)\right) \le \|V^n\| + \cancel{\Delta\tau(\alpha_i+\beta_i)\|V^{n+1}\|} + \cancel{\lambda\Delta\tau w_i\|V^n\|+}(1-\cancel{w_i})\lambda\Delta\tau\|V^n\|.$$

$$\|V^{n+1}\|\left(1+\Delta\tau(r+\lambda)\right) \le (1+\lambda\Delta\tau)\|V^n\|$$

$$\|V^{n+1}\| \le \frac{1+\lambda\Delta\tau}{1+\Delta\tau(r+\lambda)}\|V^n\|$$

$\frac{1+\lambda\Delta\tau}{1+\Delta\tau(r+\lambda)} \le 1$ for sure since r is non-negative

$\Rightarrow$ It is unconditionally stable.

$$\|V^{n+1}\| \le \frac{1+\lambda\Delta\tau}{1+\Delta\tau(r+\lambda)}\|V^n\| \le \|V^n\|$$