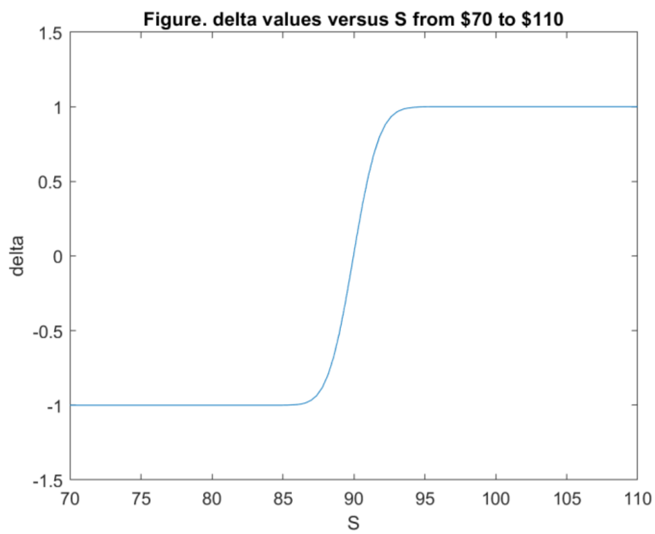
Question 3

```
(a)
randn('state',100);
%
T = 1.00;            %expiry time
sigma = 0.25;        %voliatility
mu = .018;           %P measure drift
S_init = 90;        %initial value
N_sim = 10000;       % number of simulations
N=250;               % number of timesteps
delt = T/N;          % timestep
r = 0.03;            % risk-free interest rate
K = S_init;

drift = r*delt;      %under risk neutrail
sigma_sqrt_delt = sigma*sqrt(delt);

u = exp(sigma*sqrt(delt));
d = 1/u;
S = linspace(70,110,100);     %the stock price at T = N-1


delta_S = 0.01;      %initialized the delt_S value
%delta_S(1,:) = S(1,:).*(drift + sigma_sqrt_delt*randn(1,100));

%get simulated V0 price
%V = zeros(1,100);             % straddle option payoff initialization

%V(1,:) = max(delta_S(1,:) + delta_S - K,0) +  max(K- delta_S - S(1,:),0);
%V0_tao = zeros(1,100);
%V0_tao(1,:) = exp(-r*delt*(N-1)).*V(1,:);

%get V0 price

V0 = zeros(1,100);
[call0,put0] = blsprice(S(1,:), K, r, delt, .25);
V0 = call0 + put0;

[call,put] = blsprice(S(1,:)+delta_S, K, r, delt, .25);
V = call +put;
delta = zeros(1,100);
delta = (V(1,:) - V0(1,:))./delta_S;
%ratio = zeros(1,100);
%ratio(1,:) = delta(1,:)./S(1,:);
plot(S,delta);
title('Figure. delta values versus S from $70 to $110')
xlabel('S');
ylabel('delta');
```

The plot is shown below,



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Question 3 (b)

```
S_init = 90;            % initial stock price
K = 90;                 % strike price
N_sim = 10000;                      % number of simulation
T = 1.00;                           % time to expiry
sigma = 0.25;                       % volatility
r = 0.03;                           % risk free rate
mu = 0.18;                          % expected stock return
N = 250;                            % timesteps
delt = T / N;                       % time interval


%S_old = S_init * ones(N_sim, 1);     % a vector to store simulated stock price
%S_new = zeros(N_sim, 1);             % a vector to store simulated stock price
%S_sim = S_old;                       % a matrix to store all sample paths

% calculating option price using MATLAB function blsprice
[C0,P0] = blsprice(S_init, K, r, T, sigma);
V0 = C0 + P0;
% MC to generate the S prices by BS SDE formula
S_sim = zeros(N_sim,N+1);
S_sim(:,1) = S_init;
for i = 1:N
    S_sim(:,i+1) = S_sim(:,i).*exp((mu-0.5*sigma^2)*delt +
sigma*sqrt(delt)*randn(N_sim,1));
end
VN = max(S_sim(:, N+1) - K, 0) + max(K - S_sim(:, N+1),0);    %calculate the payoff

Delta_sim = zeros(N_sim,N);
for i = 1:N
    Delta_sim(:,i) = blsdelta(S_sim(:,i),K,r,delt*(N-i+1),sigma);
end

 %for no hedging

B0 = V0;                                %get initial bond value

pie_T_0 = -VN + B0 .* exp(r*T);           %get time T pie value
PL_0 = (pie_T_0) .* exp(-r*T) ./ V0;
```

```matlab
figure('Name','No Hedging');
hist(PL_0,50);


% hedging position rebalanced at n = 0
B0 = V0 - Delta_sim(:,1) .* S_sim(:,1); % computing B0
 % option value at expiry time
pie_T_1 = -VN + Delta_sim(:,1) .* S_sim(:,N+1) + B0 .* exp(r*T);
 % value of portfolio at expiry T
PL_1 = pie_T_1 .* exp(-r*T) ./ V0; % relative hedging error
% plot histogram
figure('Name',' Hedging at n=0');
hist(PL_1,50);




% daily hedging
B1 = B0; % initial hedging position

% computing first hedging
for n = 2 : 1 : N
    B1 = exp(r * delt) .* B1 + S_sim(:, n) .* (Delta_sim(:, n-1) - Delta_sim(:, n));
     % computing hegding positon of bonds
end
pie_T_2 = -VN + Delta_sim(:, N) .* S_sim(:, N+1) + B1*exp(delt*r);
PL_2 = pie_T_2 .* exp(-r*T) ./ V0;

figure('Name','Daily Hedging ');
hist(PL_2,50);



% hedging position rebalanced weekly
Bw = B0;

for n = 6 : 5 : N
    Bw = exp(r * delt * 5) .* Bw + S_sim(:, n) .* (Delta_sim(:, n-5)-Delta_sim(:, n));
     % computing hegding positon of bonds
end
% option value at expiry time
pie_T_3 = -VN + Delta_sim(:, n) .* S_sim(:, N+1) + Bw*exp(r*(N-n)*delt);
PL_3 = pie_T_3 .* exp(-r*T) ./ V0;
% plot histogram

figure('Name','Weekly Hedging ');
hist(PL_3,50);


% hedging position rebalanced monthly
Bm = B0;
for n = 21 : 20 : N
    Bm = exp(r * delt * 20) .* Bm + S_sim(:, n) .* (Delta_sim(:, n-20) - Delta_sim(:,
n) );
     % computing hegding positon of bonds
end
% option value at expiry time
pie_T_4 = -VN + Delta_sim(:, n) .* S_sim(:, N+1) + Bm * exp(r * delt * (N-n));
PL_4 = pie_T_4 .* exp(-r*T) ./ V0;
% plot histogram

figure('Name','Monthly Hedging ');
hist(PL_4,50);
```

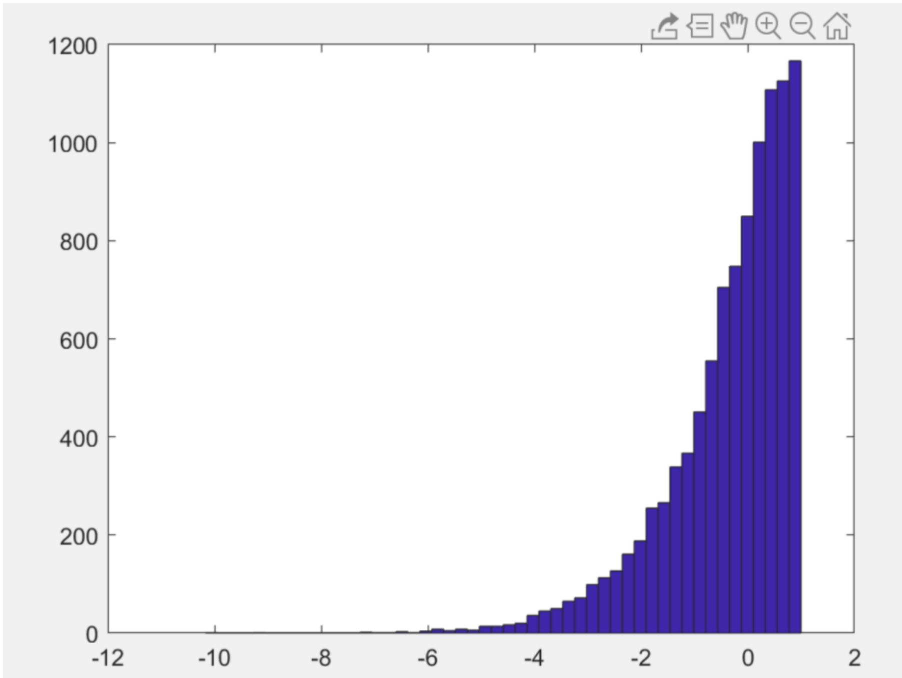

Figure. No hedging

As we can see from the figure above, the Profit and Loss are randomly scattered from about -7 to 1. And there are more portfolio has a positive profit. Later compared to the other hedging strategies, it is easily seen that the profit or loss by other hedging strategy is more closer to 0.

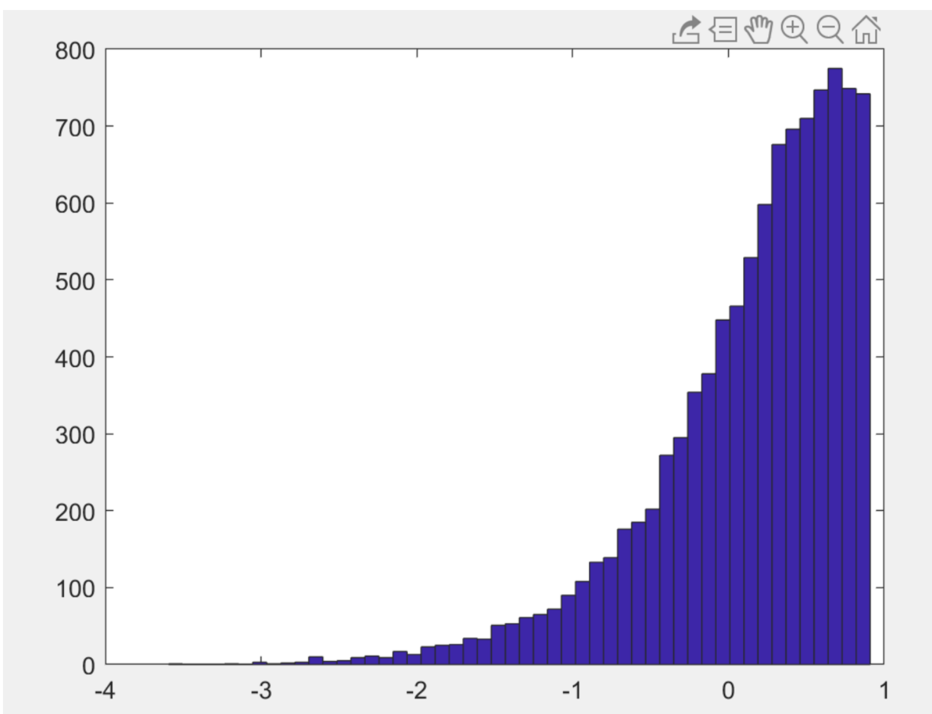

Figure. Hedging at time t = 0

Although there is only one time hedging, we do see improved from the figure above, there is not more portfolio has a more value, more and more portfolio are scattered from -1 to 1. There are very few portfolio has a value less than -3.

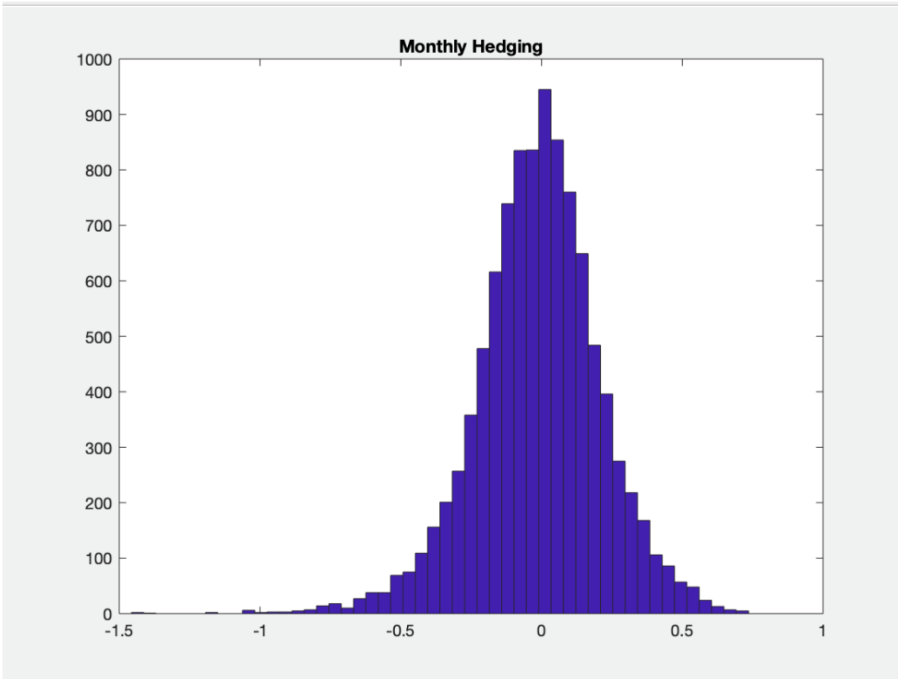

Figure. Monthly Hedging

The above figure is for monthly hedging strategy, as we can see from the figure above, very few portfolio has a value less than -2. And more and more portfolio has a value from -0.5 to 0.5.

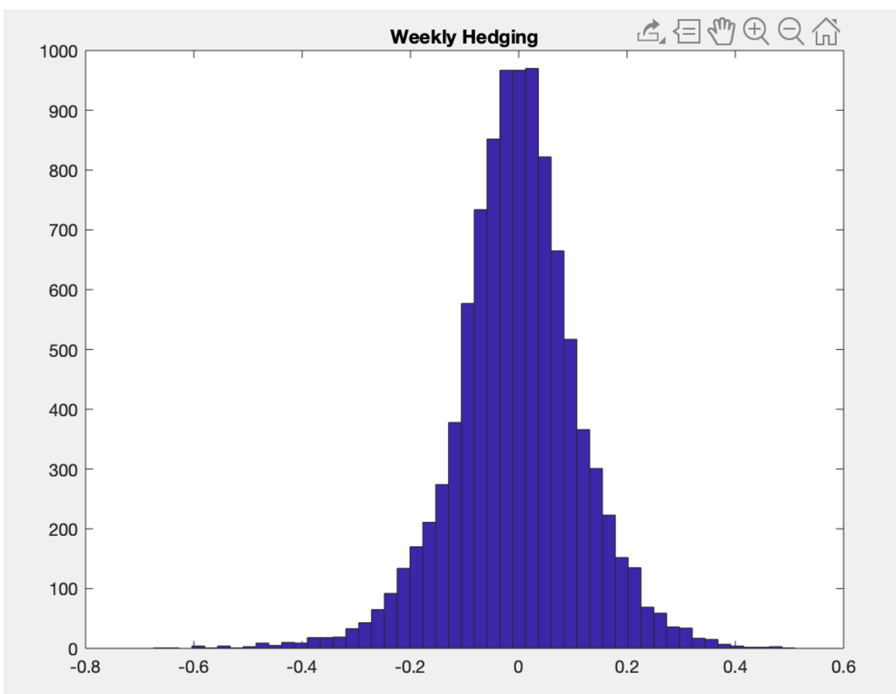

Figure. Weekly Hedging

We see an improvement compared to the monthly hedging, most of portfolio are from -0.5 to 0.5. and very less portfolio have a value less than -1.

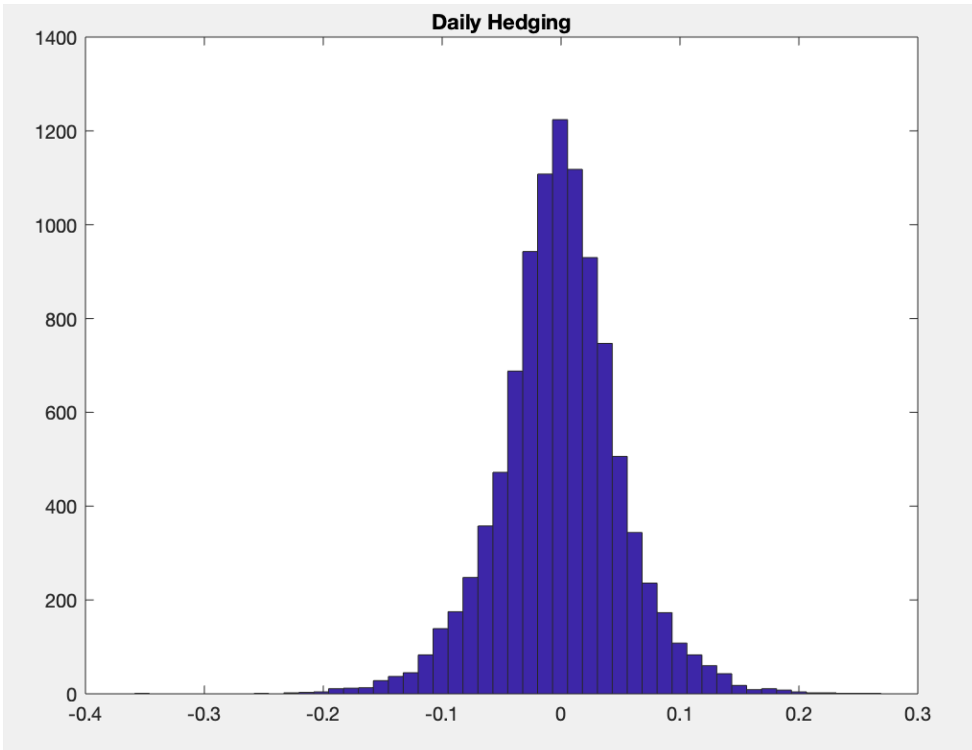

Figure. Daily Hedging

We see an improvement compared to the weekly hedging, most of portfolio are at 0. and very less portfolio have a value less than -1.

(c)

```
function [Var,CVaR]=dVaRCVaR(PL, beta)
    %% this function is used to calculate the VaR and CVaR value
    %% It reqiure an input of Profit and Loss and beta
    sorted_PL = sort(PL);        % sort P&L in increasing order
    index = floor((1-beta)*length(PL));     % find the index of M simulated P&L for
beta (s.t. index/M <= 1-beta)
    Var = sorted_PL(index);      % compute VaR
    CVaR = mean(sorted_PL(1:index));    %compte CVaR
end
```

(d)

```
[var0, cvar0] = dVaRCVaR(PL_0, 0.98)
mean(PL_0)
std(PL_0)

[var1, cvar1] = dVaRCVaR(PL_1, 0.98)
mean(PL_1)
std(PL_1)
```

```
[var2, cvar2] = dVaRCVaR(PL_2, 0.98)
mean(PL_2)
std(PL_2)

[var3, cvar3] = dVaRCVaR(PL_3, 0.98)
mean(PL_3)
std(PL_3)

[var4, cvar4] = dVaRCVaR(PL_4, 0.98)
mean(PL_4)
std(PL_4)
```

	No hedge	Hedge once	Monthly hedge	Weekly hedge	Daily hedge
mean	-0.3635	0.14	0.2329	0.239	0.2397
sd	1.179	0.627	0.4154	0.4036	0.3985
VaR(98%)	-3.6506	-1.603	-1.1237	-1.0917	-1.0827
CVaR(98%)	-4.5215	-2.0312	-1.4234	-1.3812	-1.3738

Table : The mean, sd, VaR and CVaR for no hedging, heding once, monthly hedging, weekly hedging and daily hedging

As we can see from the table above, we do see an improvement no matter VaR and CVaR (we can see the loss value gradually approaches to 0). And as for the mean and sd, it should congerve to 0, and we do see from no hedging to daily hedging, the mean value and sd value converges to 0.

(e)

In American option, we have to detect the time when point is the optimal exercise time(whether the intrinsic value is greater than continuation value) because American option can be exercised anytime when intrinsic value exceeds continuation value.

The value of American option on the each nodes of binomial tree changes, since American option is more valuable than European option, and thus leading to the changes of delta hedging position, and then the accumulated bond value alters as delta position shifts.