Question 4

(a) From the question, we know that the delta = 1 when $S_t > K$, which means we will buy one share of the stock, and delta = 0 when $S_t < K$, which means we have no more share of stocks in our hand.

$$\delta_t = \begin{cases} 0, S_t < K \\ 1, S_t > K \end{cases}$$

(b) From part a, we know that the writer will borrow money from bank to buy one share of stock if $S_t > K$, and this happens immediately after $S_t > K$. Therefore, the money we used to buy the stock is K. And since the writer shorts the call option at time t=0, which means we collect C_0 money at the beginning. Therefore, from the equation of the portfolio, we have

$$\pi_0 = -C_0 + B_0 + K$$

Since the initial portfolio has value 0 ($\pi_0 = 0$). Therefore, we get $B_0 = C_0 - K$. Thu the value of the portfolio

$$\delta_t S_t + B_t = S_t + C_0 - K$$

When $S_t < K$, the writer sells the stock at K and pay back to the bank. So the writer has no stock and the only asset left is $B_t = C_0 - K + K = C_0$.

Therefore, at any time $0 < t < T$, $\delta_t S_t + B_t = C_0$.

Therefore, we can conclude that the portfolio value $\delta_t S_t + B_t$is either $S_t + C_0 - K$ or C_0 at time $0 < t < T$.

(c) When $S_T > K$, the option holder will exercise the option, thus the $-C_T = -(S_T - K) = K - S_T$. Also, from previous part, we see that when $B_0 = C_0 - K$ when $S_t > K$. Therefore, at $S_T > K$, the hedging error is

$$-C_T + \delta_T S_T + B_T = K - S_T + S_T + C_0 - K = C_0.$$

The hedging error is $\frac{C_0}{C_0} = 1$

When $S_T < K$, the option holder will not exercise the option, and the writer will sell the stock to pay back to the bank, from part (b) as well, we can know that

$$-C_T + \delta_T S_T + B_T = 0 + 0 + C_0 = C_0.$$

The hedging error is $\frac{C_0}{C_0} = 1$ as well.

(d)

```
S_init = 99;     % initial stock price
K = 100;         % strike price
T = 1.00;        % time to expiry
sigma = 0.25;    % volatility
r = 0.03;        % risk free rate
mu = 0.18;       % expected stock return
N = [100,200,400,800];  % rebalancing times
delt = T ./ N;          % time interval
```

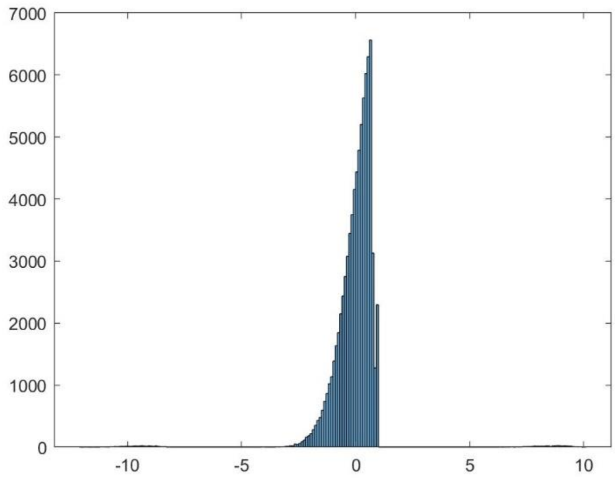
```
% calculating option price using MATLAB function blsprice
C0 = blsprice(S_init, K, r, T, sigma);

% B0 = C0 at t = 0
B0 = C0;
% simulating sample path
pie = zeros(80000,4);   % pie value initialization
for j = 1:4
    S_sim = zeros(80000,N(j)+1);
    B_sim = zeros(80000,N(j)+1);
    S_sim(:,1) = S_init;
    B_sim(:,1) = B0;
    for i = 1:N(j)
        % calculating  relative hedging error value
        % if currently holding the stock & next period S < K, then sell the stock
        % if currently not holding the stock & next period S > K, then buy stock
        S_sim(:,i+1) = S_sim(:,i).*exp((mu-0.5*sigma^2)*delt(j) +
sigma*sqrt(delt(j))*randn(80000,1));
        B_sim(:,i+1) = B_sim(:,i).*exp(r*delt(j)) - ...
            (S_sim(:,i+1) > K & S_sim(:,i) <=K).*S_sim(:,i+1) +(S_sim(:,i+1) < K &
S_sim(:,i) >= K).*S_sim(:,i+1);
    end
    pie(:,j) = exp(-r*T).*(-max(S_sim(:,N(j)+1)-K,0) + (S_sim(:,N(j)+1) >
K) .*S_sim(:,N(j)+1) +B_sim(:,i))./C0;
end

%mean, standard deviation, VaR and CVaR initialization
mea = zeros(4,1);
sd = zeros(4,1);
VaR = zeros(4,1);
CVaR = zeros(4,1);

for j = 1:4
    mea(j) = mean(pie(:,j));
    sd(j) = std(pie(:,j));
    [VaR(j),CVaR(j)] = dVaRCVaR(pie(:,j),0.95); % call the function that we write
in question 3

end
%generate a table
Table = table(mea,sd,VaR,CVaR);
Table.Properties.VariableNames = {'mean','sd','VaR','CVaR'};
Table.Properties.RowNames = {'N=100','N=200','N=400','N=800'};

histogram(pie(:,4));
```

Thus from above code, we have generated an table to summarize all mean,sd, VaR and CVaR value. From the table, we can see that it's not really always 0.

	mean	sd	VaR	CVaR
N=100	0.0037766	1.6275	-1.4123	-3.7599
N=200	-0.014008	1.4237	-1.3864	-3.2417
N=400	0.00048623	1.2634	-1.3413	-2.8158
N=800	-0.0075547	1.1238	-1.3352	-2.5042

Table 2: mean, sd, VaR and CVaR with timsteps from 100 to 800

No matter how many times the position is rebalanced, we can see that the standard deviation still exists around 1, and the mean value is not always 0 even though we increase the number or rebalancing time. However, we do see the VaR and CVaR value approaches to 0, which means with the increase of the number of rebalancing time, the portfolio is closer and closer to perfect hedging.



As we can see from the figure above, there seems exist a left tail but does not exist a right tail. Although the peak is at the 0. It has a fatter tail at the left side, which seems reasonable to have the data CVaR = -2.5042.