JMazon_Assignment14

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IS 605 FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS - ASSIGNMENT 14 - TAYLOR SERIES

For each function, only consider its valid ranges as indicated in the notes when you arecomputing the Taylor Series expansion. Please submit your assignment as a R-Markdowndocument.

library(pracma)

1)

$f(x) = \frac{1}{(1-x)}$

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$f(x) = \frac{1}{(1-x)}$

If centered at c=0,

$f'(0) = \frac{1}{(1-x)^2} = 1$
$f''(0) = \frac{2}{(1-x)^3} = 2$
$f'''(0) = \frac{6}{(1-x)^4} = 6$
$f''''(0) = \frac{24}{(1-x)^5} = 24$

So,

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \frac{1}{(1-0)} + \frac{\frac{1}{(1-0)^2}}{1!} x^1 + \frac{\frac{2}{(1-0)^3}}{2!} x^2 + \frac{\frac{6}{(1-0)^4}}{3!} x^3 + \frac{\frac{24}{(1-0)^5}}{4!} x^4 + \ldots$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = 1 + x^1 + x^2 + x^3 + x^4 + \ldots$

$\sum_{n=0}^{\infty} x^n$

$$f(x) = 1 + x + x^2 + x^3 + x^4 + O(x^5)$$

```
library(pracma)

f <- function(x) 1/(1-x)
p <- taylor(f, 0, 4)
p
```

```
## [1] 1.000029 1.000003 1.000000 1.000000 1.000000
```

2)

$f(x) = e^x$

$$f(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + O(x^5)$$

$f(x) = e^x$

If centered at c=0,

$f'(0) = e^x = 1$
$f''(0) = e^x = 1$
$f'''(0) = e^x = 1$
$f''''(0) = e^x = 1$

So,

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = e^0 + \frac{e^1}{1!} x^1 + \frac{e^2}{2!} x^2 + \frac{e^3}{3!} x^3 + \frac{e^4}{4!} x^4 + \ldots$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = 1 + \frac{1}{1!} x^1 + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \frac{1}{4!} x^4 + \ldots$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

```
library(pracma)

f <- function(x) exp(x)
p <- taylor(f, 0, 4)
p
```

```
## [1] 0.04166657 0.16666673 0.50000000 1.00000000 1.00000000
```

3)

$f(x) = ln(1+x)$

$$f(x) = ln(1+x) = \sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^5)$$

$f(x) = ln(1+x)$

If centered at c=0,

$\frac{d}{dx}[ln(x+1] = \frac{1}{x+1}\frac{d}{dx}[x+1]$
$\frac{d}{dx}[ln(x+1] = \frac{\frac{d}{dx}[x]+\frac{d}{dx}[1]}{x+1}$
$\frac{d}{dx}[ln(x+1] = \frac{1+0}{x+1}$
$\frac{d}{dx}[ln(x+1] = \frac{1}{x+1}$

$f'(0) = \frac{1}{x+1} = 1$
$f''(0) = -\frac{1}{(1+x)^2} = -1$
$f'''(0) = \frac{2}{(1+x)^3} = 2$
$f''''(0) = -\frac{6}{(1+x)^4} = -6$

So,

$\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = ln(1+0) + \frac{\frac{1}{0+1}}{1!}x^1 + \frac{-\frac{1}{(1+0)^2}}{2!}x^2 + \frac{\frac{2}{(1+0)^3}}{3!}x^3 + \frac{-\frac{6}{(1+0)^4}}{4!}x^4 + \dots$

$\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = 0 + \frac{1}{1!}x^1 + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 + \dots$

$\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$

$\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = \sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{n}x^n$

```
library(pracma)

f <- function(x) log(1+x)
p <- taylor(f, 0, 4)
p
```

```
## [1] -0.2500044  0.3333339 -0.5000000  1.0000000  0.0000000
```