DATA 605 - Homework #10

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Chapter 11 - Markov Chains

Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars.A guard agrees to make a series of bets with him. If Smith bets A dollars,he wins A dollars with probability .4 and loses A dollars with probability .6.Find the probability that he wins 8 dollars before losing all of his money if

a. he bets 1 dollar each time (timid strategy).
b. he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy).
c. Which strategy gives Smith the better chance of getting out of jail?

Timid Strategy

If we use timid play, then this means Smith's fortune is a Markov chain (Xn, n = 0, 1, . . .) which is representing the evolution of Smith's money

```
w <- 0.4
x <- 1-w
y <- 1
z <- 8
Prob_timid <- round((1 - (x/w)^y) / ( 1 - (x/w)^z),4)

print(Prob_timid)
```

```
## [1] 0.0203
```

We see about little over a 2% chance Smith will win these bets, with this strategy.

If we use simulation to calculate this one

```
a = 0.4
b = 0.6
c = b/a

for (i in seq(1, 7 , 1)){
  print ((1-c^i)/(1-c^8))
}
```

```
## [1] 0.02030135
## [1] 0.05075337
## [1] 0.0964314
## [1] 0.1649485
## [1] 0.267724
## [1] 0.4218874
## [1] 0.6531324
```

Bold Strategy

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 0 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 4 & 0 & 8 \end{array} \\ \begin{pmatrix} 0 & .4 & 0 & .6 & 0 \\ 0 & 0 & .4 & .6 & 0 \\ 0 & 0 & 0 & .6 & .4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array},$$

Smiths fortune is a Markov chain with

$$\mathbf{B} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 8 \end{array} \\ \begin{pmatrix} .936 & .064 \\ .84 & .16 \\ .6 & .4 \end{pmatrix} \end{array}$$

transition matrix P Absorption matrix

```
dbinom(3,3,0.4)
```

```
## [1] 0.064
```

```
m <- matrix(c(1,0,0,0,0,0.6,0,0.4,0,0,0.6,0,0,0.4,0,0.6,0,0,0,0.4,0,0,0,0,1), ncol=5,nrow=5, byr
ow = TRUE)
m
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  1.0    0  0.0  0.0  0.0
## [2,]  0.6    0  0.4  0.0  0.0
## [3,]  0.6    0  0.0  0.4  0.0
## [4,]  0.6    0  0.0  0.0  0.4
## [5,]  0.0    0  0.0  0.0  1.0
```

```
f <- matrix(c(0,1,0,0,0), ncol=5,nrow = 1,byrow = TRUE)
f
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    0    0    0
```

```
u <- f%*%m
u
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  0.6    0  0.4    0    0
```

```
c <- u%*%m
c
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0.84    0    0 0.16    0
```

```
k <- c%*%m
k
```

```
##       [,1] [,2] [,3] [,4]  [,5]
## [1,] 0.936    0    0    0 0.064
```

```
b <- k%*%m
b
```

```
##       [,1] [,2] [,3] [,4]  [,5]
## [1,] 0.936    0    0    0 0.064
```

Since the Smith bets A dollars and will lose or gain A dollars, and Smith bets his entire money each time until $8. Smith must win each time or lose. If Smith wins sequence is: 1,2,4,8. He starts with 1 dollar and must win 3 bets in a row at p=0.4 p=0.4 Smith probability of winning $8 with bold strategy is 0.064

Conclusion

Bold Strategy would be the best. It gives Smith a better chance to get out of jail.