Assignment 8

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#1 Chebyshev's inequality:

P(|X – mu| >= k*s) <= 1/k^2

Only the case k > 1 is useful. When k <= 1 the right-hand side 1/k^2 >=1 and the inequality is trivial as all probabilities are ≤ 1.

mu = 10 s^2 = 100/3 Let co = k*s => k = co/s

```
mu = 10
s = sqrt(100/3)
co = c(2,5,9,20)
k = co/s
k
```

```
## [1] 0.3464102 0.8660254 1.5588457 3.4641016
```

```
upper_bound = ifelse (k > 1, 1/k^2, 1)
upper_bound
```

```
## [1] 1.00000000 1.00000000 0.41152263 0.08333333
```

10 The minimum of exponential random variables X1, ,,, Xn each of which has mean μ. Show that Y = min(X1, ,,, Xn) distributed exponentially with mean μ/n

Exponential: X ~ Exp(m) = e^(-mx) where m = the decay parameter

mean μ = 1/m => m = 1/μ

11 Company buys 100 lightbulbs each of which has an exponential lifetime 1000 hours. What is expected lifetime for the first of this to burn out?

μ = 1000 hours With n = 100 μ/n = 10 hours

14. X1 and X2 independent exponentially distributed random variables with parameter m. Show that Z = X1 - X2 has density

… Theorem: The distribution of the difference of two independent exponential random variables, with population meansα1andα2respectively, has a Laplace distribution with param-etersα1andα2.

14. $X_3 = -X_2$ has density

$$f_{-X_2}(x) = \begin{cases} e^{\lambda x}, & -\infty < x \leq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Thus $Z = X_1 + X_3$ has density

$$\begin{aligned} f_Z(x) &= \int_0^\infty e^{\lambda(x-2y)}\,dy = \frac{1}{2\lambda}e^{\lambda x}, \qquad x < 0; \\ &= \int_x^\infty e^{\lambda(x-2y)}\,dy = \frac{1}{2\lambda}e^{\lambda x}\left(e^{-2\lambda x}\right) = \frac{1}{2\lambda}e^{-\lambda x}, \quad x \geq 0. \end{aligned}$$

Q14