

Let X and Y be independent geometric random variables with the same success probability p . Let $V = \min(X, Y)$ and $W = 0$ if $X < Y$, 1 if $X = Y$, and 2 if $X > Y$.

The joint probability mass function (pmf) of V and W is given by $P_{V,W}(k_1, k_2) = P(V = k_1, W = k_2)$, where k_1 and k_2 are the possible values of V and W , respectively.

To calculate the joint pmf, we need to consider the different cases for the value of W . If $W = 0$, then $X < Y$ and $V = X$. The probability of this event is given by $P(W = 0) = P(X < Y) = P(X = 1, Y = 2) + P(X = 1, Y = 3) + \dots + P(X = 1, Y = X - 1) + P(X = 2, Y = 3) + \dots + P(X = 2, Y = X - 1) + \dots + P(X = X - 1, Y = X) = p^2 * (1 + 2 + \dots + (X - 1)) = p^2 * (X - 1) * X/2$.

If $W = 1$, then $X = Y$ and $V = X = Y$. The probability of this event is given by $P(W = 1) = P(X = Y) = P(X = 1, Y = 1) + P(X = 2, Y = 2) + \dots + P(X = X, Y = X) = p^2 * (1 + 2 + \dots + X) = p^2 X(X + 1)/2$.

If $W = 2$, then $X > Y$ and $V = Y$. The probability of this event is given by $P(W = 2) = P(X > Y) = P(X = 2, Y = 1) + P(X = 3, Y = 1) + \dots + P(X = X, Y = 1) + P(X = 3, Y = 2) + \dots + P(X = X, Y = 2) + \dots + P(X = X, Y = X - 1) = p^2 * (1 + 2 + \dots + (X - 1)) = p^2 * (X - 1) * X/2$.

Therefore, the joint pmf of V and W is given by:

$$P_{V,W}(k_1, k_2) = \begin{cases} p^2 * (k_1 - 1)k_1/2 & \text{if } k_2 = 0 \\ p^2 k_1 * (k_1 + 1)/2 & \text{if } k_2 = 1 \\ p^2 * (k_1 - 1) * k_1/2 & \text{if } k_2 = 2 \end{cases}$$

The marginal distribution of V is given by $p_v(k_1) = P(V = k_1) = P_{V,W}(k_1, 0) + P_{V,W}(k_1, 1) + P_{V,W}(k_1, 2) = p^2 * ((k_1 - 1)k_1 + k_1(k_1 + 1) + (k_1 - 1) * k_1)$