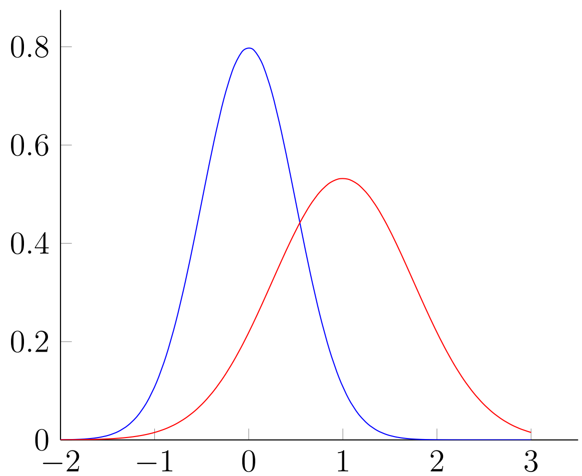
Gaussian / Normal Distribution

A random variable Z has a standard distribution (std. Gaussian) if Z has density

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



Binomial: $S_n \sim \mathrm{Bin}(n, p)$, where S_n is the number of heads in n independent tosses of a coin, where $P(H) = p$.

$$\begin{aligned} Y &= \frac{S_n - E(S_n)}{\sqrt{\mathrm{Var}(S_n)}} \\ &= \frac{S_n - np}{\sqrt{np(1-p)}} \end{aligned}$$

$$S_n = X_1 + \ldots + x_n$$

where x_i are Bernoulli random variables.

Sums of a large number of independent random variables are distributed with a normal distribution.

We write

$$Z \sim N(0,1)$$

Where N denotes the normal distribution, the first parameter of our distribution is the mean of the distribution, and the second parameter is the standard deviation.

cdf of $Z \sim N(0,1)$

$$\Phi(x) = P(Z \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt$$

We cannot solve this integral in solved form.

The cdf is evaluated numerically for different values of x to produce a normal table.

Ex: $Z \sim N(0,1)$. Find $P(-1 \leq x \leq 1.5)$.

$$\begin{aligned}\int_{-1}^{1.5} \phi(x)\, dx &= \int_{-\infty}^{1.5} \phi(x)\, dx - \int_{\infty}^{-1} \phi(x)\, dx \\ &= \Phi(1.5) - \Phi(-1) \\ &= 0.9322 - 1 + 0.8413 \\ &= 0.7745\end{aligned}$$

Ex: Find the c values such that Z has approximately $\frac{2}{3}$ chance to be in $[-c, c]$.

$$\begin{aligned}\frac{2}{3} &= \Phi(c) - \Phi(-c) \\ &= \Phi(c) - (1 - \Phi(c)) \\ &= 2\Phi(c) - 1 \\ &\to \Phi(c) = \frac{5}{6} = 0.833\end{aligned}$$

The value closest to this is $c = 0.97$.

Defining Parameters for the Normal Distribution

Let μ be real and $\sigma > 0$. A random variable x has the normal distribution with mean μ and standard deviation σ if x has density

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-m)^2}{2\sigma^2}}$$

We write $x \sim N(\mu, \sigma^2)$.

1. $S_n \sim N(np, np(1-p))$ as $n \to \infty$.

Quiz: 1, 2, 3, 4, 6, 7