- 1. There are 5 closed boxes on a table. Three of the boxes have good prizes inside and the other two don't. You open boxes one at a time until you find a prize. Let X be the number of boxes you open.
  - (a). Find the pmf of X.

**Ans:** X may take values  $\in [1,3]$ . Let f(X) be the pmf of X.

$$f(X) = \begin{cases} \frac{3}{5} & x = 1\\ \frac{2}{5} \cdot \frac{3}{4} & x = 2\\ \frac{2}{5} \cdot \frac{1}{4} \cdot 1 & x = 3\\ 0 & x > 3 \end{cases}$$

(b). Find E(X).

Ans:

$$E(X) = \sum_{k} kP(X = k) \tag{1}$$

Alternate symbol of  $E[X] = \mu$ 

Thus

$$E(X) = 1 \cdot \frac{2}{5} + 2\left(\frac{2}{5} \cdot \frac{3}{4}\right) + 3 \cdot \left(\frac{2}{5} \cdot \frac{1}{4}\right)$$
  
= 1.3

(c). Find Var(X).

Ans:

$$Var(X) = \sum_{k} (k - \mu)^2 P(X = k)$$
(2)

$$Var(X) = (1 - 1.3)^{2} \cdot \frac{2}{5} + (2 - 1.3)^{2} \left(\frac{2}{5} \cdot \frac{3}{4}\right) + (3 - 1.3)^{2} \left(\frac{2}{5} \cdot \frac{1}{4}\right)$$
$$= .4720$$

(d). Suppose the good prize inside each of the three boxes is \$100, but each empty box you open costs you \$100. What is your expected gain or loss in the game? (Hint: Express the gain or loss as a function of X.)

Ans:

$$f(X) = \begin{cases} 100 & x = 1\\ 0 & x = 2\\ -100 & x = 3 \end{cases}$$

$$E[X] = 100 \cdot \frac{3}{5} + 0 - 100 \cdot \left(\frac{2}{5} \cdot \frac{1}{4}\right)$$
  
= 50

2. Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{1}{4} & 1 < x < 2\\ c & 3 < x < 5\\ 0 & \text{otherwise} \end{cases}$$

(a). Find the value of c.

**Ans:** By the properties of a density function,

$$\int_{-\infty}^{\infty} f(x) \, dx = 1$$

which is only possible given c is equal to

$$c(5-3) = 1 - \frac{1}{4}(2-1)$$
$$2c = .75$$
$$c = \frac{3}{8}$$

(b). Find P(1.5 < X < 4)

Ans:

$$P(a < X < b) = \int_{b}^{a} f(x) dx$$

$$P(1.5 < X < 4) = \int_{1.5}^{4} f(x) dx$$

$$= \int_{1.5}^{2} f(x) dx + \int_{2}^{3} f(x) dx + \int_{3}^{4} f(x) dx$$

$$= \frac{1}{2} (2 - 1.5) + 0 + \frac{3}{8} (4 - 3)$$

$$= .50$$

(c). Find the cdf of X.

Ans:

$$F(s) = P(X \le s) = \int_{-\infty}^{s} f(x) dx$$

$$= \begin{cases} 0 & s < 1 \\ \frac{s-1}{4} & 1 < s < 2 \\ \frac{3}{8}(s-3) + .25 & 3 < s < 5 \\ 1 & x > 5 \end{cases}$$

- 3. Choose a point (X, Y) uniformly inside the triangle with vertices (0,0), (30,0) and (30,20).
  - (a). Find the cdf of X.
  - (b). Using (a), find the density function of X.