Example: $S \sim \text{Bin}(40, \frac{1}{2})$.

Check $np(1-p)$

$$np(1-p) = 40 \cdot \frac{1}{2} \cdot \frac{1}{2} = 10$$

Check np^2

$$np^2 = 40 \cdot \frac{1}{4} = 10$$

Our bounds indicate that we should use the normal approximation.

Poisson distributions are good for modeling rare events.

Assume a random variable x counts occurances of events that are infrequent and not strongly dependent.

- Earthquakes
- Arrivals of customers in a line

The distributions of x can be approximated by a Poisson(λ) varable where $\lambda = E(x)$.

i.e $P(x = k)$ is close to

$$\frac{e^{-\lambda}\lambda^k}{k!}$$

Example: A factory experiences 3 accidents a month, on average. What is the probability that there are 2 accidents in January.

1. Assume that the number of accidents in a month are small.
2. Assume that accidents occur independently.

Then we may conclude that the number of accidents in a month can be modeled by Poisson(3).

$$\begin{aligned} P(2 \text{ accidents in a month}) &= \frac{e^{-3} \cdot 3^2}{2!} \\ &= 0.24 \end{aligned}$$

Example: A proofreader notices that a randomly chosen page in a manuscript has no typos with chance 0.9. Estimate the probability that a randomly chosen page has 2 typos.

1. Is a typo infrequent? Yes.
2. Are typos independent? We assume they are **somewhat** independent.

With these assumptions, we let x equal a binomial random variable. Suppose we approximate x by a Poisson(λ).

To find the mean of this Poisson, we should find the mean of x. We have

$$\begin{aligned} P(x=0) &= 0.9 \\ 0.9 &= \frac{e^{-\lambda}\lambda^0}{0!} \\ -\log(0.9) &= \lambda \\ &= .10536 \end{aligned}$$

Then $x \sim \text{Poisson}(.10536)$

$$\begin{aligned} P(x=2) &= \frac{e^{-\lambda}\lambda^2}{2!} \\ &= \frac{0.9 \cdot (.10536)^2}{2!} \\ &= .005 \end{aligned}$$