Section 7.3: Systems of Linear Algebraic Equations

- Linear Independence
- Eigenvalues
- Eigenvectors

Suppose we had

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

We may write this as

$$Ax = b$$

where we consider $b = 0$ a homogeneous system, and $b \neq 0$ a nonhomogeneous system.

- If the coefficient matrix A is nonsingular, then $\det(A) \neq 0$, and A^{-1} exists. Thus, $x = A^{-1}b$.
- If A is singular, then A^{-1} does not exist, and

  $$Ax = b$$

  has no solutions unless b satisfies the following conditions.

$$(b, y) = 0 \ \forall \ y \ \text{ satisfying } \ A^*y = 0$$

Ex: Linear Algebra Review

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ -1 & 1 & -2 & -5 \\ 2 & -1 & -1 & 4 \end{array}\right]$$

$R_2 + R_1$, $R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 3 & -7 & -10 \end{array}\right]$$

$R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

$$x = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Ex: Discuss solutions of the system

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ -1 & 1 & -2 & b_2 \\ 2 & -1 & 3 & b_3 \end{array}\right]$$

$$\vdots$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & 1 & -1 & -b_1 - b_2 \\ 0 & 0 & 0 & b_1 + 3b_2 + b_3 \end{array}\right]$$

$$x = \begin{bmatrix} -b_1 - 2b_2 - t \\ -b_1 - b_2 + t \\ t \end{bmatrix}$$

Linear Independence and Dependence

$x_1, x_2, \ldots, x_n$ are linearly dependent if there exists $C = c_1, c_2, \ldots, c_n$, C is not the zero vector, such that

$$c_1 x_1 + c_2 x_2 + \ldots + c_n x_n = 0$$

If the only vector C which may satisfy the above equation is the zero vector, then $x_1, \ldots, x_n$ are linearly independent.

The set of vectors $x_1, \ldots, x_n$ is linearly independent if and only if the matrix composed of x_i is nonsingular.