Linear Systems of Differential Equations: A system of differential equations is called linear if each of $F_1, F_2, \ldots, F_n$ is a linear function of $x_1, x_2, \ldots, x_n$. Otherwise, it is called non-linear.

We are only considering linearity of the variable x.

$$x_1' = P_{11}(t)x_1 + p_{12}x_2(t) + \ldots + p_{1n}(t)x_n + g_1(t)$$
$$x_2' = P_{21}(t)x_1 + p_{22}x_2(t) + \ldots + p_{2n}(t)x_n + g_1(t)$$
$$\vdots$$
$$x_n' = P_{n1}(t)x_1 + p_{n2}x_2(t) + \ldots + p_{nn}(t)x_n + g_1(t)$$

$$\begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} p_{11}(t) & p_{12}(t) & \ldots & p_{1n}(t) \\ p_{21}(t) & p_{22}(t) & \ldots & p_{2n}(t) \\ \vdots & & \ddots & \vdots \\ p_{n1}(t) & p_{n2}(t) & \ldots & p_{nn}(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Theorem 7.1.2: If the functions p_{11}, p_{12}, $\ldots$, p_{nn}, g_1, g_2, $\ldots$, g_n are continuous on an open interval I, $\alpha < t < \beta$, then there exists an unique solution $x_1 = \phi_1(t), \ldots, x_n = \phi_n(t)$ of the system that satisfies the initial condition problem where t_0 is any point in I and $x_1^{[0]}, \ldots, x_n^{[0]}$ are any prescribed numbers.

Then the solution exists throughout the interval I.

Linear Algebra Review

AB

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 0 & -1 & -1 \\ 7 & 0 & -1 \end{bmatrix}$$

BA

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -3 & 0 \\ 1 & -4 & 2 \\ 4 & -5 & 4 \end{bmatrix}$$

$AB \neq BA$