

$$\text{let } f(x,y) = \frac{3x^2 - y^2}{5x^2 + 2y^2} \quad (\text{similar to example 1, §14.2})$$

- ① What is the limit of  $f(x,y)$  as  $(x,y) \rightarrow (0,0)$  ...
- (a) along the  $x$ -axis?
  - (b) along the  $y$ -axis?
  - (c) along the line  $y=x$ ?
  - (d) along the line  $y=mx$ ?
- ② Does  $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$  exist?

Solution

① a)  $f(x,0) = \frac{3x^2}{5x^2} = \frac{3}{5}$

b)  $f(0,y) = \frac{-y^2}{2y^2} = -\frac{1}{2}$

c)  $f(x,x) = \frac{2x^2}{7x^2} = \frac{2}{7}$

d)  $f(x,mx) = \frac{3x^2 - m^2x^2}{5x^2 + 2m^2x^2} = \frac{3-m^2}{5+2m^2}$

② no, limit doesn't exist, as different paths do not agree.