For my parents
Edward E. Tufte and Virginia James Tufte

To the memory of
John W. Tukey (1915–2000)

Introduction to the Second Edition

This new edition provides high-resolution color reproductions of the many graphics of William Playfair, adds color to other images where appropriate, and includes all the changes and corrections accumulated during the 17 printings of the first edition.

This book began in 1975 when Dean Donald Stokes of Princeton's Woodrow Wilson School asked me to teach statistics to a dozen journalists who were visiting that year to learn some economics. I annotated a collection of readings, with a long section on statistical graphics. The literature here was thin, too often grimly devoted to explaining use of the ruling pen and to promulgating "graphic standards" indifferent to the nature of visual evidence and quantitative reasoning. Soon I wrote up some ideas. Then John Tukey, the phenomenal Princeton statistician, suggested that we give a series of joint seminars. Since the mid-1960s, Tukey had opened up the field, as his brilliant technical contributions made it clear that the study of statistical graphics was intellectually respectable and not just about pie charts and ruling pens.

After moving to Yale University, I finished the manuscript in 1982. A publisher was interested but planned to print only 2,000 copies and to charge a very high price, contrary to my hopes for a wide readership. I also sought to design the book so as to make it *self-exemplifying*—that is, the physical object itself would reflect the intellectual principles advanced in the book. Publishers seemed appalled at the prospect that an author might govern design.

Consequently I investigated self-publishing. This required a first-rate book designer, a lot of money (at least for a young professor), and a large garage. I found Howard Gralla who had designed many museum catalogs with great care and craft. He was willing to work closely with this difficult author who was filled with all sorts of opinions about design and typography. We spent the summer in

An early and most worthy use of a map to chart patterns of disease was the famous dot map of Dr. John Snow, who plotted the location of deaths from cholera in central London for September 1854. Deaths were marked by dots and, in addition, the area's eleven water pumps were located by crosses. Examining the scatter over the surface of the map, Snow observed that cholera occurred almost entirely among those who lived near (and drank from) the Broad Street water pump. He had the handle of the contaminated pump removed, ending the neighborhood epidemic which had taken more than 500 lives.[6] The pump is located at the center of the map, just to the right of the D in BROAD STREET. Of course the link between the pump and the disease might have been revealed by computation and analysis without graphics, with some good luck and hard work. But, here at least, graphical analysis testifies about the data far more efficiently than calculation.

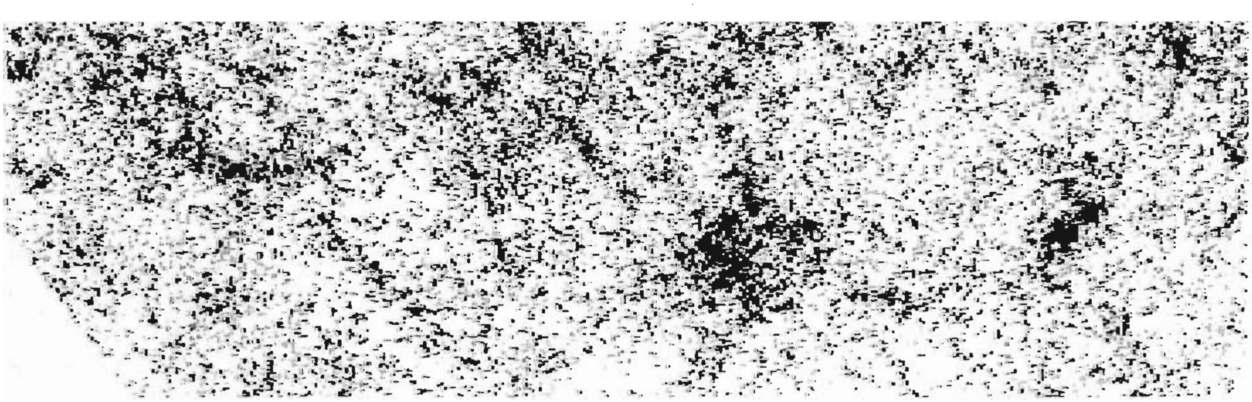
[6] E. W. Gilbert, "Pioneer Maps of Health and Disease in England," *Geographical Journal*, 124 (1958), 172–183. Shown here is a redrawing of John Snow's map. For a reproduction and detailed analysis of the original map, see Edward Tufte, *Visual Explanations: Images and Quantities, Evidence and Narrative* (Cheshire, Connecticut, 1997), Chapter 2. Ideally, see John Snow, *On the Mode of Communication of Cholera* (London, 1855).

Charles Joseph Minard gave quantity as well as direction to the data measures located on the world map in his portrayal of the 1864 exports of French wine:

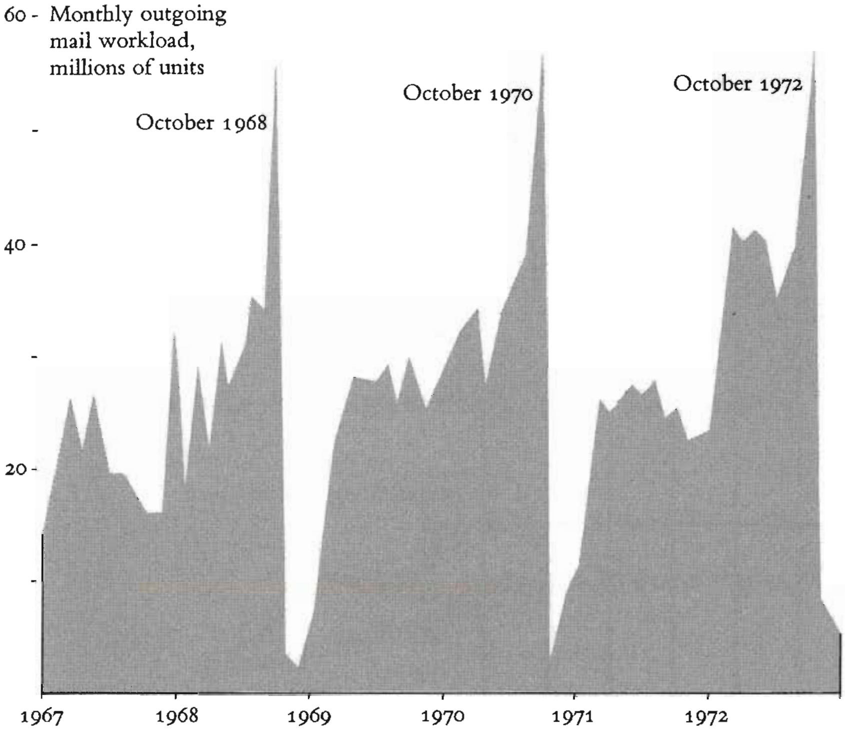
Computerized cartography and modern photographic techniques have increased the density of information some 5,000-fold in the best of current data maps compared to Halley's pioneering effort. This map shows the distribution of 1.3 million galaxies (including some overlaps) in the northern galactic hemisphere. The map divides the sky into 1,024×2,222 rectangles. The number of galaxies counted in each of the 2,275,328 rectangles is represented by ten gray tones; the darker the tone, the greater the number of galaxies counted. The north galactic pole is at the center. The sharp edge on the left results from the earth blocking the view from the observatory. In the area near the perimeter of the map, the view is obscured by the interstellar dust of the galaxy in which we live (the Milky Way) as the line of sight passes through the flattened disk of our galaxy. The curious texture of local clusters of galaxies seen in this truly new view of the universe was not anticipated by students of galaxies, who had, of course, microscopically examined millions of photographs of galaxies before seeing this macroscopic view. Although the clusters are clearly evident (and accounted for by a theory of galactic origins), the seemingly random filaments may be happenstance. The producers of the map note the "strong temptation to conclude that the galaxies are arranged in a remarkable filamentary pattern on scales of approximately 5° to 15°, but we caution that this visual impression may be misleading because the eye tends to pick out linear patterns even in random noise. Indeed, roughly similar patterns are seen on maps constructed from simulated catalogs where no linear structure has been built in. . . ."[7]

[7] Michael Seldner, B. H. Siebers, Edward J. Groth and P. James E. Peebles, "New Reduction of the Lick Catalog of Galaxies," *Astronomical Journal*, 82 (April 1977), 249-314. See Gillian R. Knapp, "Mining the Heavens: The Sloan Digital Sky Survey," *Sky & Telescope* (August 1997), 40-48; Margaret J. Geller and John P. Huchra, "Mapping the Universe," *Sky & Telescope* (August 1991), 134-139.

The most extensive data maps, such as the cancer atlas and the count of the galaxies, place millions of bits of information on a single page before our eyes. No other method for the display of statistical information is so powerful.



The problem with time-series is that the simple passage of time is not a good explanatory variable: descriptive chronology is not causal explanation. There are occasional exceptions, especially when there is a clear mechanism that drives the Y-variable. This time-series does testify about causality: the outgoing mail of the U.S. House of Representatives peaks every two years, just before the election day:



The graphic is worth at least 700 words, the number used in a news report describing how incumbent representatives exploit their free mailing privileges to advance their re-election campaigns:

FRANKED MAIL TIE TO VOTING SHOWN

Testimony Finds the Volume Rises Before Elections

WASHINGTON, June 1 (AP)—New court testimony and documents show that much of the mail Congress sends at taxpayer expense is tied directly to the re-election campaigns of Senate and House members. According to material filed in a lawsuit in Federal Court:

¶Senate Republicans put two direct-mail experts on the public payroll to advise them on how to use their free mailing privileges to get votes.

¶An election manual prepared for Senate Democrats refers to newsletters as a "free forum," and sets up a timetable for sending them as an integral part of a model re-election campaign.

¶Senator John G. Tower, Republican of Texas, mailed more than 800,000 special-interest letters at taxpayer expense as part of his 1972 re-election effort and received campaign volunteer offers and donations in response.

¶Senator Jacob K. Javits, Republican of New York, gave written approval in 1973 for a tax-paid mail program intended to better his image and pay off at the polls. He focused his mail on areas where he needed votes.

¶The volume of "official" Congressional mail rises in election years and peaks just before the general election.

None of this activity necessarily violates any law or regulation, since Congress has wide discretion in the use of tax-paid mail. Congress gave itself the right to send official mail at Government expense at the founding of the republic, and only Congress polices against abuses of the free mailings.

Complaints of political use of the free-mailing privilege, called the franking privilege, are heard every election year. Recently, however, the volume and cost of franked mail has multiplied. A new Federal law will limit what out-of-office challengers can spend to unseat incumbents.

In 1972, Congress passed a law prohibiting mass franked mailings within 28 days before an election. The sponsor of that legislation, Representative Morris K. Udall, Democrat of Arizona, said in an interview that further changes were needed to curtail political abuse of the frank.

Mr. Udall urged a 60-day pre-election cutoff for mass mailings and said he favored closing a loophole that recently allowed defeated Representative Frank M. Clark, Democrat of Pennsylvania, to send a franked newsletter to his old constituents after he had left office. Mr. Clark is seeking to regain his old post.

Practice Documented

Seldom has the political use of franked mail been so well documented as in recent testimony and documents filed in a Federal Court by Common Cause, the lobby group, which is suing for an end to tax-financed mass mailings by Congress.

For example, Joyce P. Baker, a political mail specialist, said in a 1973 job proposal that she wanted to set up direct-mail programs for Republican Senators using franked mail.

"The purpose of such a program is to help an incumbent Senator get re-elected," she said.

She was put on the Senate payroll at $18,810 a year in 1973 and 1974 and testified that during that time she aided Republican Senators Robert J. Dole of Kansas, Peter H. Dominick of Colorado, Charles McC. Mathias Jr. of Maryland

Another political mail specialist, Lee W. MacGregor, wrote a proposal for the use of franked mail by his chief, Senator Javits, in 1973.

"The over-all objective of the franked mail program can be to get the recipient of the mail to identify positively with a particular stand you have taken or a bill you have introduced; the kind of identification that can be translated into a vote at the polls on election day" Mr. MacGregor said.

Mr. Javits was out of the country and could not be reached. His administrative assistant, Donald Kellerman, defended the use of franked mail.

"It is a standard device to let voters, not voters but citizens, know what the Senator is doing here in Washington, he said.

Senator Tower's use of franked mail in his 1972 campaign was documented by memorandums.

Tom Loeffler, a high-ranking campaign aide, wrote in a memorandum dated Oct. 27, 1972, that during the campaign Senator Tower had sent "31 special interest letters totaling approximately 803,333 franked mailings."

Mr. Tower was not available for comment. His administrative assistant, Elwin Skiles, said the Senator's use of franked mail in 1972 was within the law, and he defended the free-mailing privileges.

Postal Service figures show that in the 12 months before November, 1973, Congress sent 222.9 million franked pieces of mail. But in the next 12 months, covering the election season of 1974, Congress sent 350.6 million, a jump of 57 per cent about what's happening," Mr. Skiles said.

2 *Graphical Integrity*

For many people the first word that comes to mind when they think about statistical charts is "lie." No doubt some graphics do distort the underlying data, making it hard for the viewer to learn the truth. But data graphics are no different from words in this regard, for any means of communication can be used to deceive. There is no reason to believe that graphics are especially vulnerable to exploitation by liars; in fact, most of us have pretty good graphical lie detectors that help us see right through frauds.

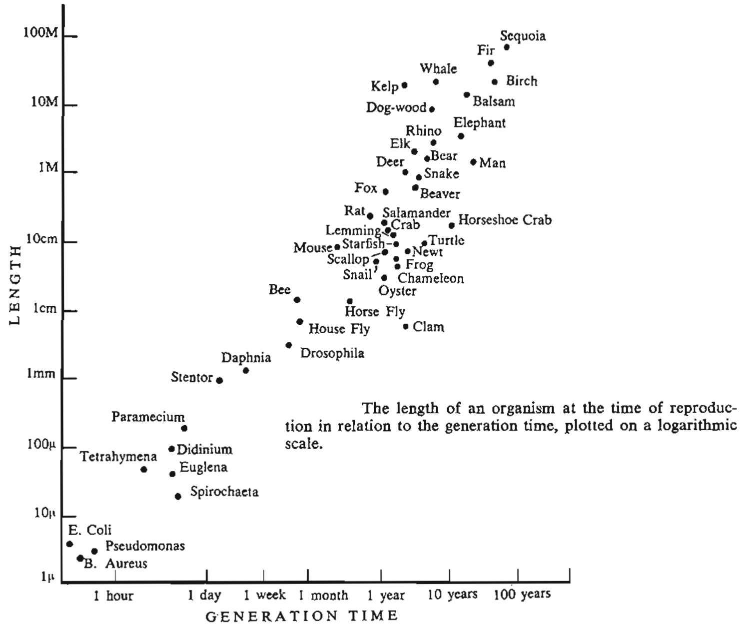
Much of twentieth-century thinking about statistical graphics has been preoccupied with the question of how some amateurish chart might fool a naive viewer. Other important issues, such as the use of graphics for serious data analysis, were largely ignored. At the core of the preoccupation with deceptive graphics was the assumption that data graphics were mainly devices for showing the obvious to the ignorant. It is hard to imagine any doctrine more likely to stifle intellectual progress in a field. The assumption led down two fruitless paths in the graphically barren years from 1930 to 1970: First, that graphics had to be "alive," "communicatively dynamic," overdecorated and exaggerated (otherwise all the dullards in the audience would fall asleep in the face of those boring statistics). Second, that the main task of graphical analysis was to detect and denounce deception (the dullards could not protect themselves).

Then, in the late 1960s, John Tukey made statistical graphics respectable, putting an end to the view that graphics were only for decorating a few numbers. For here was a world-class data analyst spinning off half a dozen new designs and, more importantly, using them effectively to explore complex data.[1] Not a word about deception; no tortured attempts to construct more "graphical standards" in a hopeless effort to end all distortions. Instead, graphics were used as instruments for reasoning about quantitative information. With this good example, graphical work has come to flourish.

Of course false graphics are still with us. Deception must always be confronted and demolished, even if lie detection is no longer at the forefront of research. Graphical excellence begins with telling the truth about the data.

[1] John W. Tukey and Martin B. Wilk, "Data Analysis and Statistics: Techniques and Approaches," in Edward R. Tufte, ed., *The Quantitative Analysis of Social Problems* (Reading, Mass., 1970), 370–390; and John W. Tukey, "Some Graphic and Semigraphic Displays," in T. A. Bancroft, ed., *Statistical Papers in Honor of George W. Snedecor* (Ames, Iowa, 1972), 293–316.

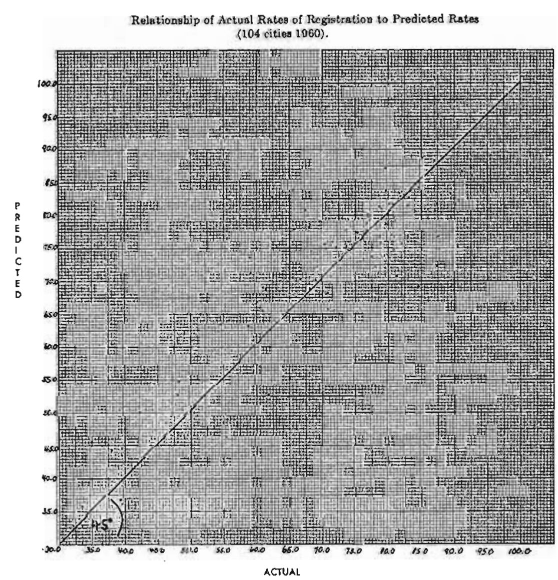
Most of the ink in this graphic is data-ink (the dots and labels on the diagonal), with perhaps 10–20 percent non-data-ink (the grid ticks and the frame):



The length of an organism at the time of reproduction in relation to the generation time, plotted on a logarithmic scale.

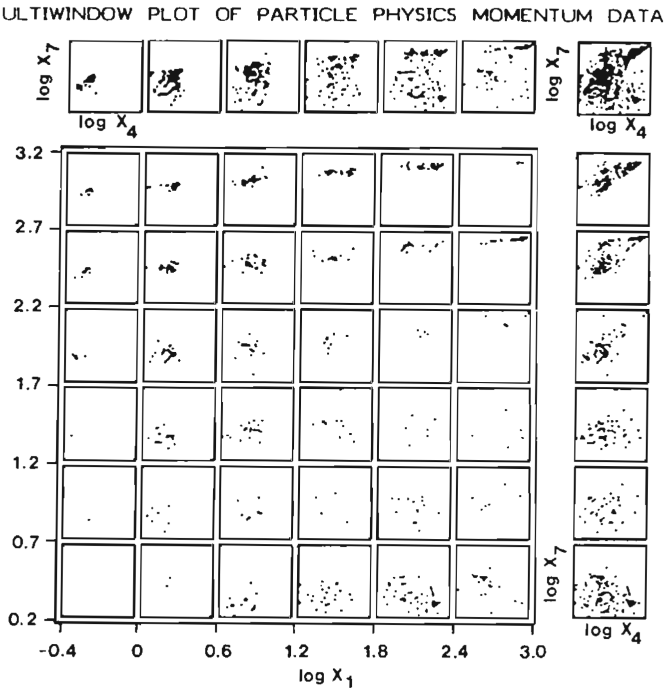
John Tyler Bonner, *Size and Cycle: An Essay on the Structure of Biology* (Princeton, 1965), p. 17.

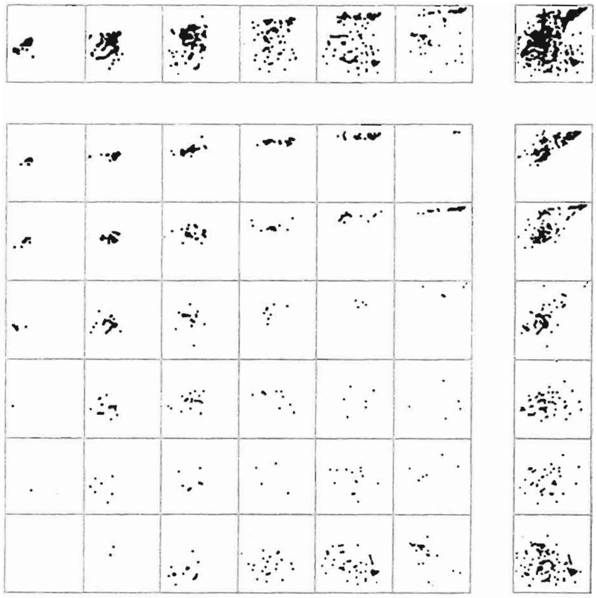
In this display with nearly all its ink devoted to matters other than data, the grid sea overwhelms the numbers (the faint points scattered about the diagonal):



The space occupied by the doubled grid lines consumes 18 percent of the area of this otherwise most ingenious design, a "multiwindow plot." Optical white dots appear at the intersections of the grid lines. (The plot shows the following: The large square contains X_4, X_7 scatterplots for the indicated levels of X_1 and X_3. The marginal plots on the right are conditioned on X_3 and the plots at the top on X_1. The upper right corner shows the unconditional X_4, X_7 scatter.) Redrawing eliminates the vibration:

Paul A. Tukey and John W. Tukey, "Data-Driven View Selection; Agglomeration and Sharpening," in Vic Barnett, ed., *Interpreting Multivariate Data* (Chichester, England, 1981), 231–232.



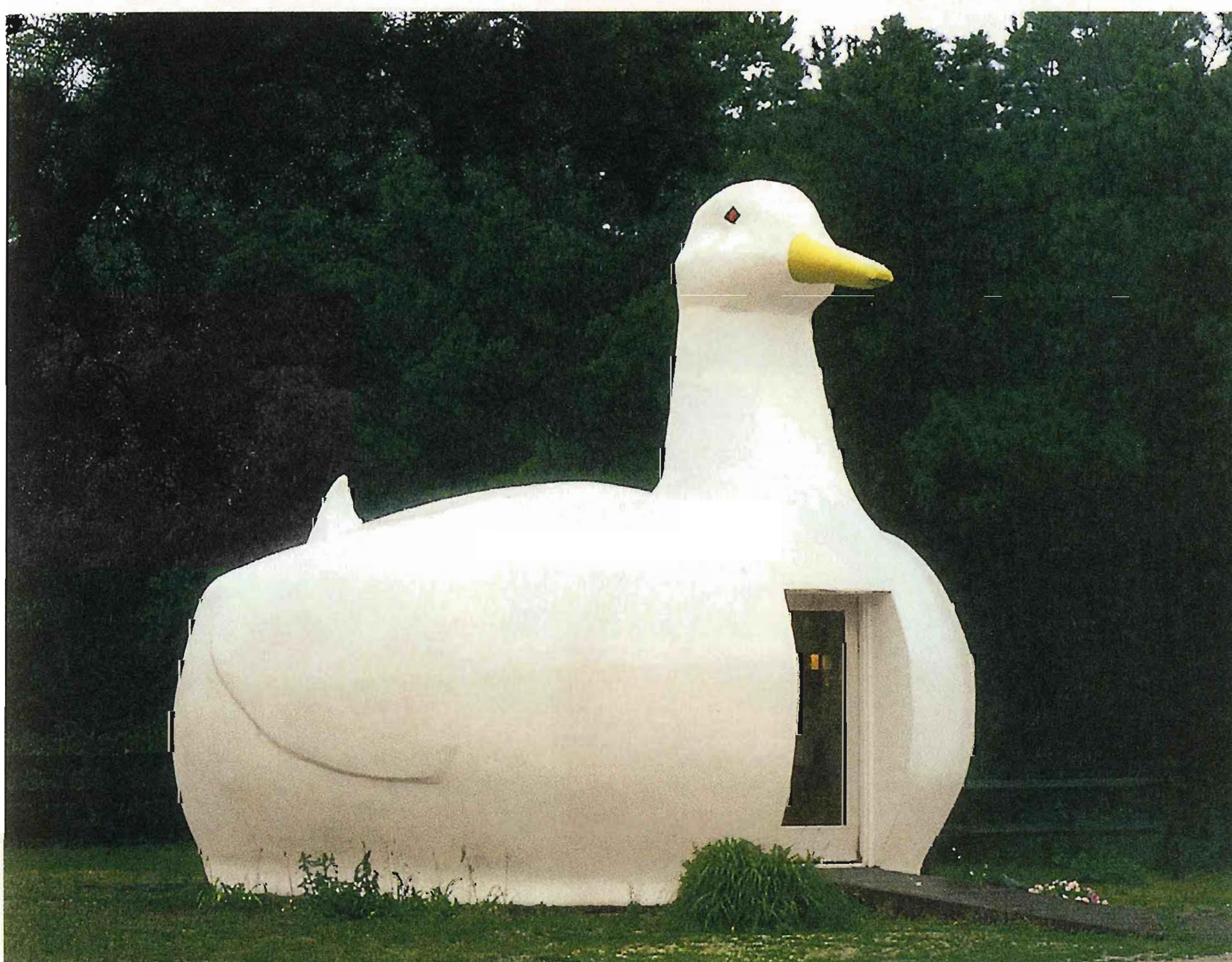


Brown, and Steven Izenour write about the ducks of modern architecture—and their thoughts are relevant to the design of data graphics as well:

> When Modern architects righteously abandoned ornament on buildings, they unconsciously designed buildings that *were* ornament. In promoting Space and Articulation over symbolism and ornament, they distorted the whole building into a duck. They substituted for the innocent and inexpensive practice of applied decoration on a conventional shed the rather cynical and expensive distortion of program and structure to promote a duck. . . . It is now time to reevaluate the once-horrifying statement of John Ruskin that architecture is the decoration of construction, but we should append the warning of Pugin: It is all right to decorate construction but never construct decoration.[2]

[2] Robert Venturi, Denise Scott Brown, and Steven Izenour, *Learning from Las Vegas* (Cambridge, revised edition, 1977), p. 163. The initial statement of the duck concept is found on pp. 87–103.

Big Duck, Flanders, New York; photograph by Edward Tufte, July 2000.

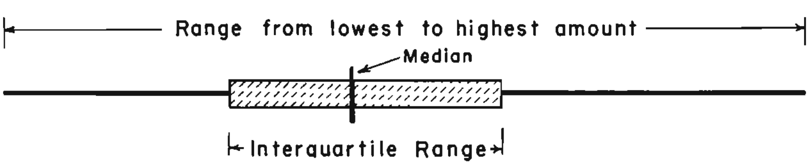


6 *Data-Ink Maximization and Graphical Design*

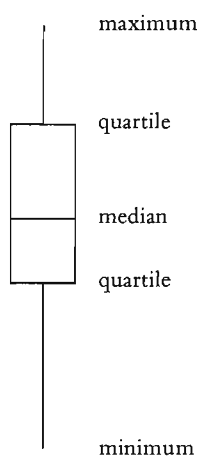
So far the principles of maximizing data-ink and erasing have helped to generate a series of choices in the process of graphical revision. This is an important result, but can the ideas reach beyond the details and particularities of editing? Is it possible to do what a theory of graphics is supposed to do, that is, to derive new graphical forms? In this chapter the principles are applied to many graphical designs, basic and advanced, including box plots, bar charts, histograms, and scatterplots. New designs result.

Redesign of the Box Plot

Mary Eleanor Spear's "range bar"

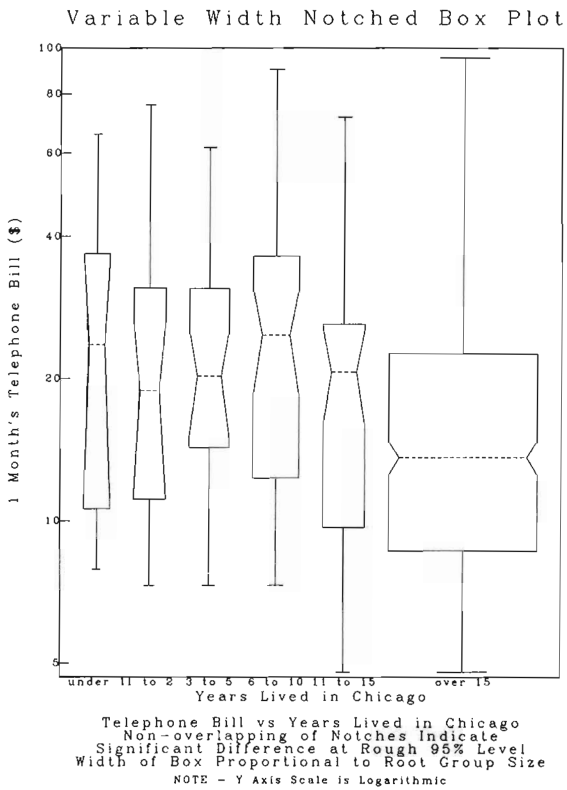


and John Tukey's "box plot"

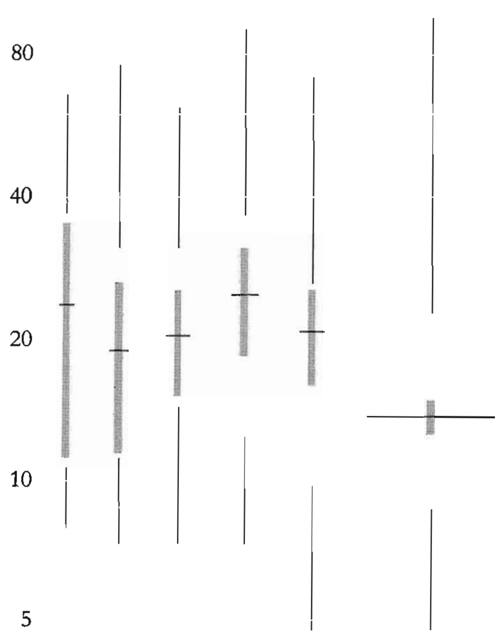


Mary Eleanor Spear, *Charting Statistics* (New York, 1952), p. 166; and John W. Tukey, *Exploratory Data Analysis* (Reading, Massachusetts, 1977).

Erasing and data-ink maximizing have induced changes in the plain old bar chart. The techniques—no frame, no vertical axis, no ticks, and the white grid—apply to other designs:



Robert McGill, John W. Tukey, and Wayne A. Larsen, "Variations of Box Plots," *American Statistician*, 32 (1978), 12–16.



Building data measures out of the data increases the quantitative detail and dimensionality of a graphic. The stem-and-leaf plot constructs the distribution of a variable with numbers themselves:

0 | 9 = 900 feet

Stem-and-leaf displays: heights of 218 volcanoes, unit 100 feet.

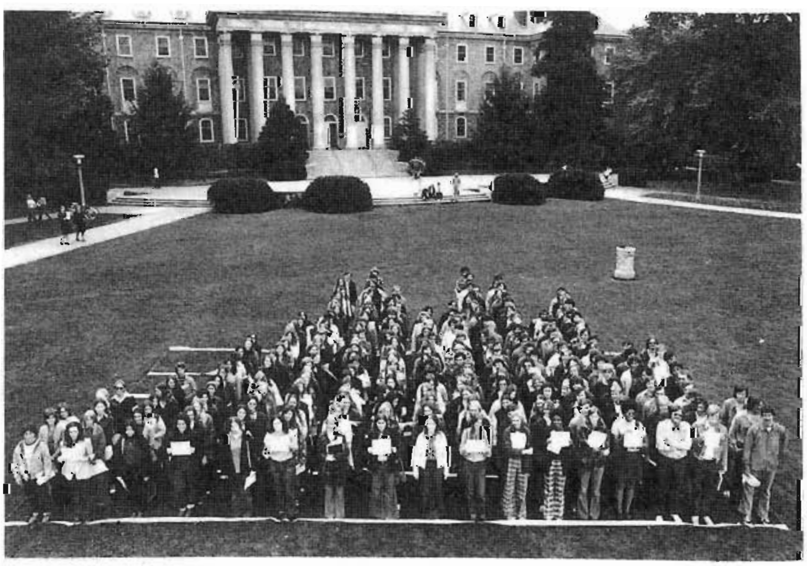
19 | 3 = 19,300 feet

0	98766562
1	97719630
2	69987766544422211009850
3	876655412099551426
4	99988443319294333361107
5	97666666554422210097731
6	898665441077761065
7	98855431100652108073
8	653322122937
9	377655421000493
10	0984433165212
11	4963201631
12	45421164
13	47830
14	00
15	676
16	52
17	92
18	5
19	39730

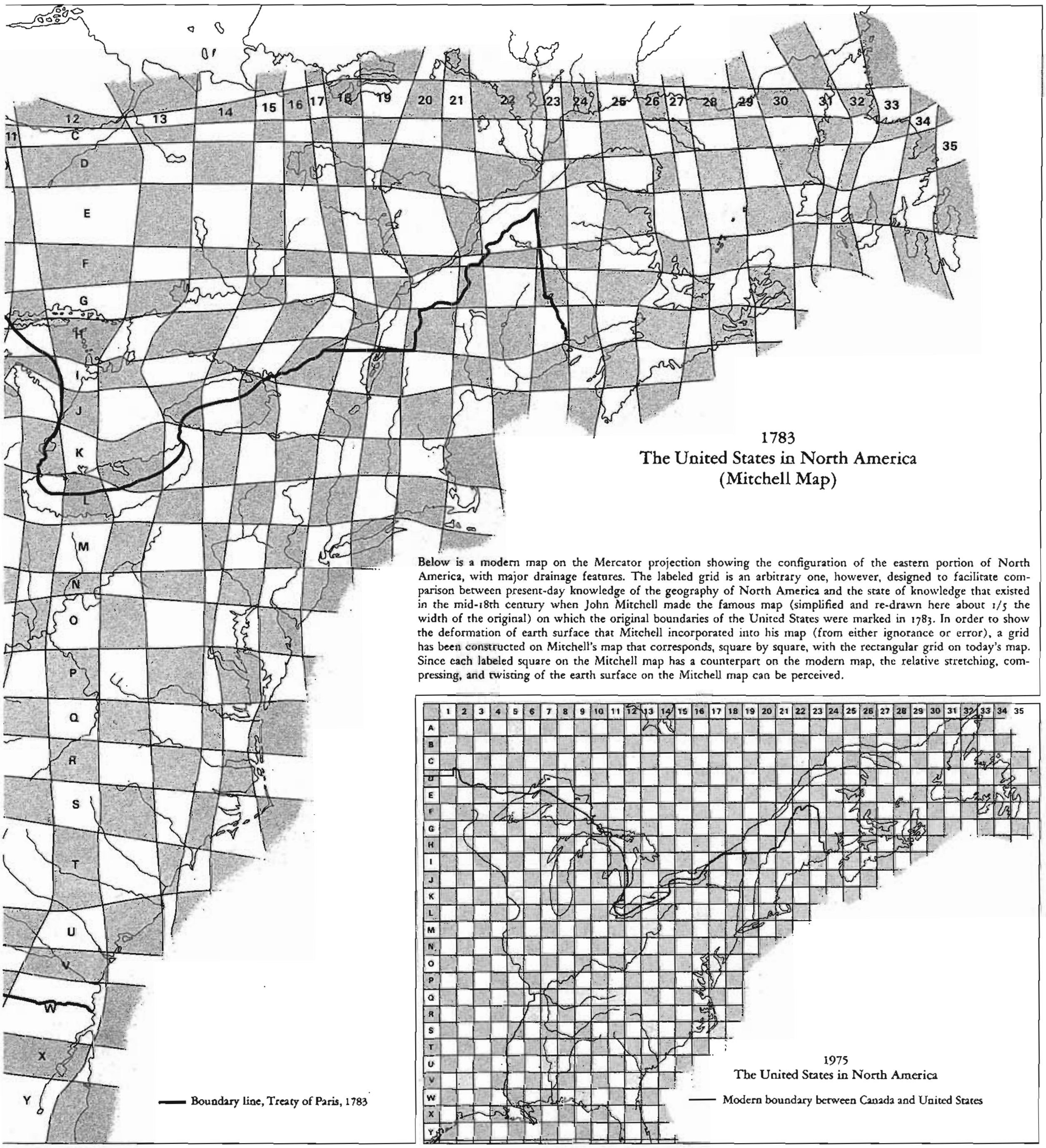
The idea of making every graphical element effective was behind the design of the stem-and-leaf plot. In presenting his invention, John Tukey wrote: "If we are going to make a mark, it may as well be a meaningful one. The simplest—and most useful—meaningful mark is a digit."[2]

Here, too, the data form the data measure. Note the bimodal distribution in the histogram of college students arranged by height.

[2] "Some Graphic and Semigraphic Displays," in T. A. Bancroft, ed., *Statistical Papers in Honor of George W. Snedecor* (Ames, Iowa, 1972), p. 296.



Brian L. Joiner, "Living Histograms," *International Statistical Review*, 43 (1975), 339–340. But, for further developments, see Mark F. Schilling, Ann E. Watkins, and William Watkins, "Is Human Height Bimodal?" *The American Statistician*, 56 (August 2002), 223–229.



Here the grid is the element of interest, rather than the map.

Lester J. Cappon, Barbara Bartz Petchenik, and John Hamilton Long, *Atlas of Early American History* (Princeton, 1976), p. 58.

Maps routinely present even finer detail. A cartographer writes that "the resolving power of the eye enables it to differentiate to 0.1 mm where provoked to do so. Clearly, therefore, conciseness is of the essence and high resolution graphics are a common denominator of cartography."[2] Distinctions at 0.1 mm mean 254 per inch.

[2] D. P. Bickmore, "The Relevance of Cartography," in John C. Davis and Michael J. McCullagh, eds., *Display and Analysis of Spatial Data* (London, 1975), p. 331.

How many statistical graphics take advantage of the ability of the eye to detect large amounts of information in small spaces? And how much information should graphics show? Let us begin by considering an empirical measure of graphical performance, the data density.

Data Density in Graphical Practice

The numbers that go into a graphic can be organized into a data matrix of observations by variables. Taking into account the size of the graphic in relation to the amount of data displayed yields the *data density*:

$$\text{data density of a graphic} = \frac{\text{number of entries in data matrix}}{\text{area of data graphic}}$$

Data matrices and data densities vary enormously in practice. At one extreme, this overwrought display (originally printed in five colors) presents a data matrix of four entries, the names and the numbers for the two bars on the right. The left bar is merely the total of the other two. The graph covers 26.5 square inches (171 square centimeters), resulting in a data density of .15 numbers per square inch (.02 numbers per square centimeter), which is thin indeed.

based on about 50 numbers shown at the rate of 10 per square inch. Among the world's newspapers, the *Wall Street Journal*, *The Times* (London), and *Asahi* publish data-rich graphics, with data densities equal to those of the *Journal of the American Statistical Association*. Most of the American papers and magazines, along with *Pravda*, publish less data per graphic than the major papers of other industrialized countries.

Very few statistical graphics achieve the information display rates found in maps. Highly detailed maps portray 100,000 to 150,000 bits per square inch. For example, the average U.S. Geological Survey topographic quadrangle (measuring 17 by 23 inches) is estimated to contain over 100 million bits of information, or about 250,000 per square inch (40,000 per square centimeter).[3] Perhaps some day statistical graphics will perform as successfully as maps in carrying information.

[3] Morris M. Thompson, *Maps for America* (Washington, D.C., 1979), p. 187.

High-Information Graphics

Data graphics should often be based on large rather than small data matrices and have a high rather than low data density. More information is better than less information, especially when the marginal costs of handling and interpreting additional information are low, as they are for most graphics. The simple things belong in tables or in the text; graphics can give a sense of large and complex data sets that cannot be managed in any other way. If the graphic becomes overcrowded (although several thousand numbers represented may be just fine), a variety of data-reduction techniques—averaging, clustering, smoothing—can thin the numbers out before plotting.[4] Summary graphics can emerge from high-information displays, but there is nowhere to go if we begin with a low-information design.

[4] Paul A. Tukey and John W. Tukey, "Summarization: Smoothing; Supplemented Views," in Vic Barnett, ed., *Interpreting Multivariate Data* (Chichester, England, 1982), ch. 12; and William S. Cleveland, "Robust Locally Weighted Regression and Smoothing Scatterplots," *Journal of the American Statistical Association*, 74 (1979), 829–836.

Data-rich designs give a context and credibility to statistical evidence. Low-information designs are suspect: what is left out, what is hidden, why are we shown so little? High-density graphics help us to compare parts of the data by displaying much information within the view of the eye: we look at one page at a time and the more on the page, the more effective and comparative our eye can be.[5] The principle, then, is:

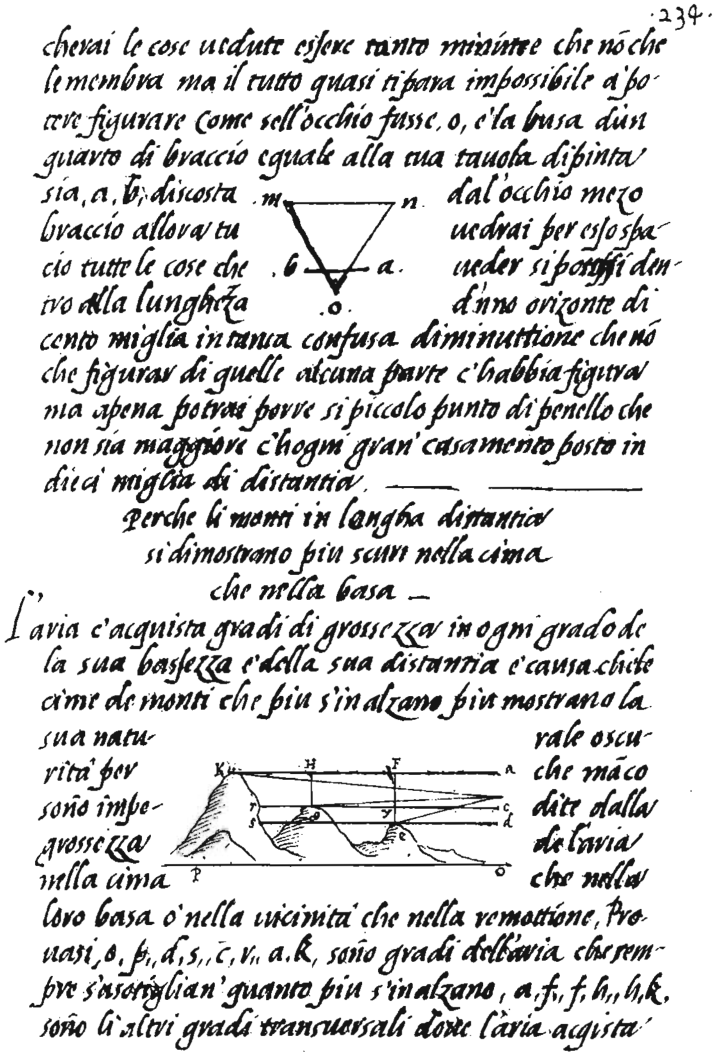
> Maximize data density and the size of the data matrix, within reason.

High-information graphics must be designed with special care. As the volume of data increases, data measures must shrink (smaller dots for scatters, thinner lines for busy time-series). The clutter of

[5] It is suggested in the analysis of x-ray films to "search a reduced image so that the whole display can be perceived on at least one occasion without large eye movement." Edward Llewellyn Thomas, "Advice to the Searcher or What Do We Tell Them?" in Richard A. Monty and John W. Senders, eds., *Eye Movements and Psychological Processes* (Hillsdale, N.J., 1976), p. 349.

Page after page of Leonardo's manuscripts have a gentle but thorough integration of text and figure, a quality rarely seen in modern work:

Leonardo da Vinci, *Treatise on Painting* [*Codex Urbinas Latinus 1270*], vol. 2, facsimile (Princeton, 1956), p. 234, paragraph 827.



Finally, a caveat: the use of words and pictures together requires a special sensitivity to the purpose of the design—in particular, whether the graphic is primarily for communication and illustration of a settled finding or, in contrast, for the exploration of a data set. Words on and around graphics are highly effective—sometimes all too effective—in telling viewers how to allocate their attention to the various parts of the data display.[5] Thus, for graphics in exploratory data analysis, words should tell the viewer *how* to read the design (if it is a technically complex arrangement) and not *what* to read in terms of content.

[5] Experiments in visual perception indicate that word instructions substantially determine eye movements in viewing pictures. See John D. Gould, "Looking at Pictures," in Richard A. Monty and John W. Senders, eds., *Eye Movements and Psychological Processes* (Hillsdale, N.J., 1976), 323–343.

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Cheshire, Connecticut